

(19) World Intellectual Property Organization
International Bureau



(43) International Publication Date
3 May 2007 (03.05.2007)

PCT

(10) International Publication Number
WO 2007/051078 A2

(51) International Patent Classification:
G06F 19/00 (2006.01)

[US/US]; 27 ROSEDALE ROAD, Princeton, NJ 08540 (US).

(21) International Application Number:
PCT/US2006/042692

(74) Agent: MELCHER, Jeffrey, S.; MANELLI DENISON & SELTER, PLLC, 2000 M STREET, N.W., 7TH FLOOR, Washington, DC 20036-3307 (US).

(22) International Filing Date: 30 October 2006 (30.10.2006)

(25) Filing Language: English

(26) Publication Language: English

(30) Priority Data:

60/730,882	28 October 2005 (28.10.2005)	US
60/732,154	2 November 2005 (02.11.2005)	US
60/737,744	18 November 2005 (18.11.2005)	US
60/758,528	13 January 2006 (13.01.2006)	US
60/780,518	9 March 2006 (09.03.2006)	US
60/788,694	4 April 2006 (04.04.2006)	US
60/812,590	12 June 2006 (12.06.2006)	US
60/815,253	21 June 2006 (21.06.2006)	US

(81) Designated States (unless otherwise indicated, for every kind of national protection available): AE, AG, AL, AM, AT, AU, AZ, BA, BB, BG, BR, BW, BY, BZ, CA, CH, CN, CO, CR, CU, CZ, DE, DK, DM, DZ, EC, EE, EG, ES, FI, GB, GD, GE, GH, GM, GT, HN, HR, HU, ID, IL, IN, IS, JP, KE, KG, KM, KN, KP, KR, KZ, LA, LC, LK, LR, LS, LT, LU, LV, LY, MA, MD, MG, MK, MN, MW, MX, MY, MZ, NA, NG, NI, NO, NZ, OM, PG, PH, PL, PT, RO, RS, RU, SC, SD, SE, SG, SK, SL, SM, SV, SY, TJ, TM, TN, TR, TT, TZ, UA, UG, US, UZ, VC, VN, ZA, ZM, ZW.

(71) Applicant (for all designated States except US): BLACK-LIGHT POWER, INC. [US/US]; 493 OLD TRENTON ROAD, Cranbury, NJ 08512 (US).

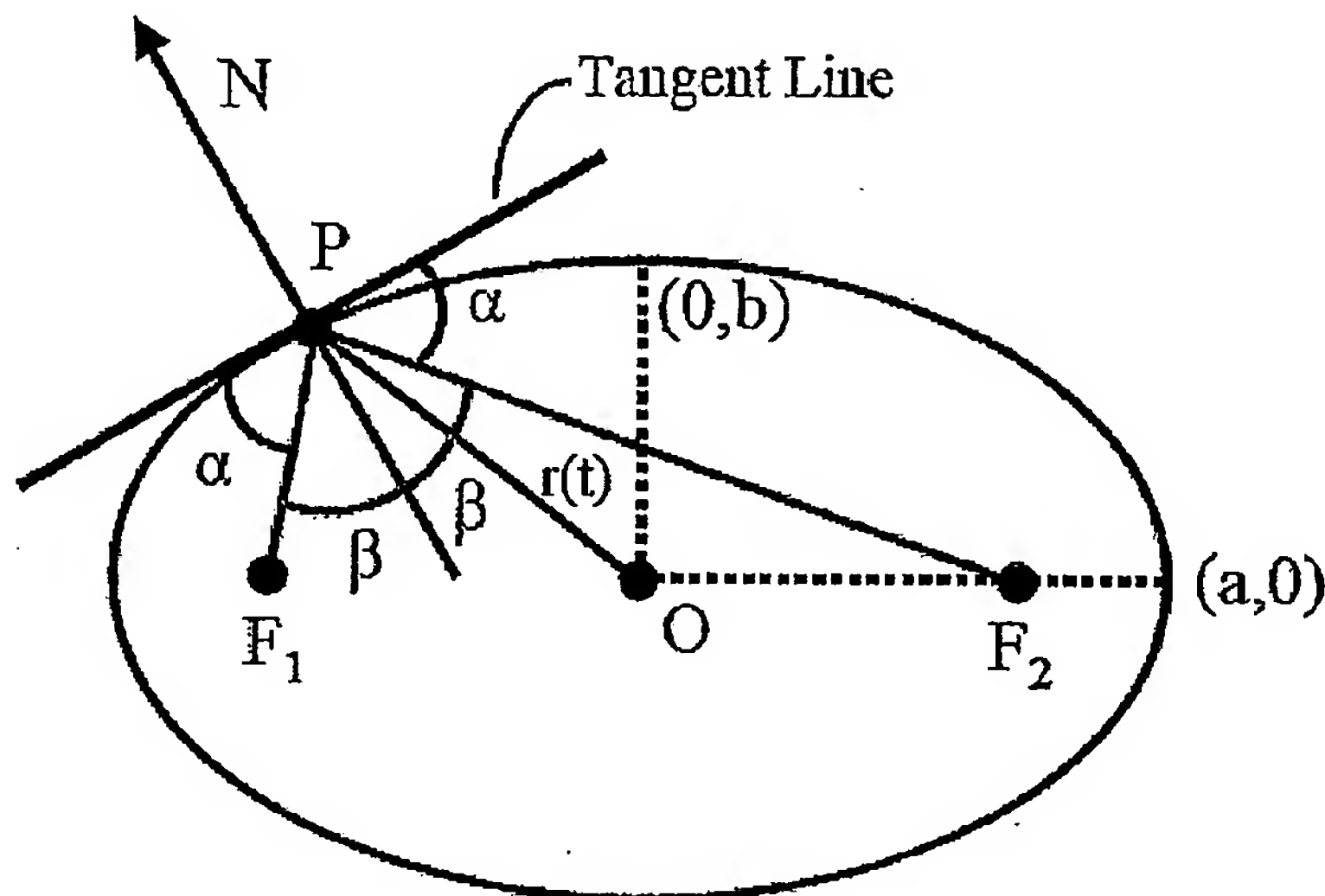
(72) Inventor; and

(75) Inventor/Applicant (for US only): MILLS, Randell, L.

(84) Designated States (unless otherwise indicated, for every kind of regional protection available): ARIPO (BW, GH, GM, KE, LS, MW, MZ, NA, SD, SL, SZ, TZ, UG, ZM, ZW), Eurasian (AM, AZ, BY, KG, KZ, MD, RU, TJ, TM), European (AT, BE, BG, CH, CY, CZ, DE, DK, EE, ES, FI, FR, GB, GR, HU, IE, IS, IT, LT, LU, LV, MC, NL, PL, PT, RO, SE, SI, SK, TR), OAPI (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, ML, MR, NE, SN, TD, TG).

[Continued on next page]

(54) Title: SYSTEM AND METHOD OF COMPUTING AND RENDERING THE NATURE OF POLYATOMIC MOLECULES AND POLYATOMIC MOLECULAR IONS



(57) Abstract: A method and system of physically solving the charge, mass, and current density functions of polyatomic molecules, polyatomic molecular ions, diatomic molecules, molecular radicals, molecular ions, or any portion of these species using Maxwell's equations and computing and rendering the physical nature of the chemical bond using the solutions. The results can be displayed on visual or graphical media. The display can be static or dynamic such that electron motion and specie's vibrational, rotational, and translational motion can be displayed in an embodiment. The displayed information is useful to anticipate reactivity and physical properties. The insight into the nature of the chemical bond of at least one specie can permit the solution and display of those of other species to provide utility to anticipate their reactivity and physical properties.

WO 2007/051078 A2



Published:

— without international search report and to be republished upon receipt of that report

For two-letter codes and other abbreviations, refer to the "Guidance Notes on Codes and Abbreviations" appearing at the beginning of each regular issue of the PCT Gazette.

SYSTEM AND METHOD OF COMPUTING AND RENDERING THE NATURE OF POLYATOMIC MOLECULES AND POLYATOMIC MOLECULAR IONS

This application claims priority to U.S. Application Nos.: 60/730,882, filed October 28,
5 2005; 60/732,154, filed November 2, 2005; 60/737,744, filed November 18, 2005;
60/758,528, filed January 13, 2006; 60/780,518, filed March 9, 2006; 60/788,694, filed April
4, 2006; 60/812,590, filed June 12, 2006; and 60/815,253, June 21, 2006, the complete
disclosures of which are incorporated herein by reference.

10 Field of the Invention:

This invention relates to a system and method of physically solving the charge, mass,
and current density functions of polyatomic molecules, polyatomic molecular ions, diatomic
molecules, molecular radicals, molecular ions, or any portion of these species, and computing
and rendering the nature of these species using the solutions. The results can be displayed on
15 visual or graphical media. The displayed information provides insight into the nature of these
species and is useful to anticipate their reactivity, physical properties, and spectral absorption
and emission, and permits the solution and display of other species.

Rather than using postulated unverifiable theories that treat atomic particles as if they
were not real, physical laws are now applied to atoms and ions. In an attempt to provide
20 some physical insight into atomic problems and starting with the same essential physics as
Bohr of the e^- moving in the Coulombic field of the proton with a true wave equation, as
opposed to the diffusion equation of Schrödinger, a classical approach is explored which
yields a model that is remarkably accurate and provides insight into physics on the atomic
level. The proverbial view deeply seated in the wave-particle duality notion that there is no
25 large-scale physical counterpart to the nature of the electron is shown not to be correct.
Physical laws and intuition may be restored when dealing with the wave equation and
quantum atomic problems.

Specifically, a theory of classical quantum mechanics (CQM) was derived from first
principles as reported previously [reference Nos. 1-8] that successfully applies physical laws
30 to the solution of atomic problems that has its basis in a breakthrough in the understanding of
the stability of the bound electron to radiation. Rather than using the postulated Schrödinger
boundary condition: " $\Psi \rightarrow 0$ as $r \rightarrow \infty$ ", which leads to a purely mathematical model of the
electron, the constraint is based on experimental observation. Using Maxwell's equations, *the*

classical wave equation is solved with the constraint that the bound $n = 1$ -state electron cannot radiate energy. Although it is well known that an accelerated *point* particle radiates, an *extended distribution* modeled as a superposition of accelerating charges does not have to radiate. A simple invariant physical model arises naturally wherein the predicted results are
5 extremely straightforward and internally consistent requiring minimal math, as in the case of the most famous equations of Newton, Maxwell, Einstein, de Broglie, and Planck on which the model is based. No new physics is needed; only the known physical laws based on direct observation are used.

Applicant's previously filed WO2005/067678 discloses a method and system of
10 physically solving the charge, mass, and current density functions of atoms and atomic ions and computing and rendering the nature of these species using the solutions. The complete disclosure of this published PCT application is incorporated herein by reference.

Applicant's previously filed WO2005/116630 discloses a method and system of
15 physically solving the charge, mass, and current density functions of excited states of atoms and atomic ions and computing and rendering the nature of these species using the solutions. The complete disclosure of this published PCT application is incorporated herein by reference.

Applicant's previously filed U.S. Published Patent Application No. 20050209788A1, relates to a method and system of physically solving the charge, mass, and current density
20 functions of hydrogen-type molecules and molecular ions and computing and rendering the nature of the chemical bond using the solutions. The complete disclosure of this published application is incorporated herein by reference.

Background of the Invention

25 The old view that the electron is a zero or one-dimensional point in an all-space probability wave function $\Psi(x)$ is not taken for granted. The theory of classical quantum mechanics (CQM), derived from first principles, must successfully and consistently apply physical laws on all scales [1-8]. Stability to radiation was ignored by all past atomic models. Historically, the point at which QM broke with classical laws can be traced to the issue of
30 nonradiation of the one electron atom. Bohr just postulated orbits stable to radiation with the further postulate that the bound electron of the hydrogen atom does not obey Maxwell's equations—rather it obeys different physics [1-12]. Later physics was replaced by "pure mathematics" based on the notion of the inexplicable wave-particle duality nature of electrons

which lead to the Schrödinger equation wherein the consequences of radiation predicted by Maxwell's equations were ignored. Ironically, Bohr, Schrödinger, and Dirac used the Coulomb potential, and Dirac used the vector potential of Maxwell's equations. But, all ignored electrodynamics and the corresponding radiative consequences. Dirac originally
5 attempted to solve the bound electron physically with stability with respect to radiation according to Maxwell's equations with the further constraints that it was relativistically invariant and gave rise to electron spin [13]. He and many founders of QM such as Sommerfeld, Bohm, and Weinstein wrongly pursued a planetary model, were unsuccessful, and resorted to the current mathematical-probability-wave model that has many problems [9-
10 16]. Consequently, Feynman for example, attempted to use first principles including Maxwell's equations to discover new physics to replace quantum mechanics [17].

Physical laws may indeed be the root of the observations thought to be "purely quantum mechanical", and it was a mistake to make the assumption that Maxwell's electrodynamic equations must be rejected at the atomic level. Thus, in the present approach,
15 the classical wave equation is solved with the constraint that a bound $n = 1$ -state electron cannot radiate energy.

Herein, derivations consider the electrodynamic effects of moving charges as well as the Coulomb potential, and the search is for a solution representative of the electron wherein there is acceleration of charge motion without radiation. The mathematical formulation for
20 zero radiation based on Maxwell's equations follows from a derivation by Haus [18]. The function that describes the motion of the electron must not possess spacetime Fourier components that are synchronous with waves traveling at the speed of light. Similarly, nonradiation is demonstrated based on the electron's electromagnetic fields and the Poynting power vector.

25 It was shown previously [1-8] that CQM gives closed form solutions for the atom including the stability of the $n = 1$ state and the instability of the excited states, the equation of the photon and electron in excited states, and the equation of the free electron and photon, which predict the wave particle duality behavior of particles and light. The current and charge density functions of the electron may be directly physically interpreted. For example,
30 spin angular momentum results from the motion of negatively charged mass moving systematically, and the equation for angular momentum, $\mathbf{r} \times \mathbf{p}$, can be applied directly to the wave function (a current density function) that describes the electron. The magnetic moment of a Bohr magneton, Stern Gerlach experiment, g factor, Lamb shift, resonant line width and

shape, selection rules, correspondence principle, wave particle duality, excited states, reduced mass, rotational energies, and momenta, orbital and spin splitting, spin-orbital coupling, Knight shift, and spin-nuclear coupling, and elastic electron scattering from helium atoms, are derived in closed-form equations based on Maxwell's equations. The calculations agree with
 5 experimental observations.

The Schrödinger equation gives a vague and fluid model of the electron. Schrödinger interpreted $e\Psi^*(x)\Psi(x)$ as the charge-density or the amount of charge between x and $x + dx$ (Ψ^* is the complex conjugate of Ψ). Presumably, then, he pictured the electron to be spread over large regions of space. After Schrödinger's interpretation, Max Born, who
 10 was working with scattering theory, found that this interpretation led to inconsistencies, and he replaced the Schrödinger interpretation with the probability of finding the electron between x and $x + dx$ as

$$\int \Psi(x)\Psi^*(x) dx \quad (1)$$

Born's interpretation is generally accepted. Nonetheless, interpretation of the wave function
 15 is a never-ending source of confusion and conflict. Many scientists have solved this problem by conveniently adopting the Schrödinger interpretation for some problems and the Born interpretation for others. This duality allows the electron to be everywhere at one time—yet have no volume. Alternatively, the electron can be viewed as a discrete particle that moves here and there (from $r = 0$ to $r = \infty$), and $\Psi\Psi^*$ gives the time average of this motion.

20 In contrast to the failure of the Bohr theory and the nonphysical, adjustable-parameter approach of quantum mechanics, multielectron atoms [1, 4] and the nature of the chemical bond [1, 5] are given by exact closed-form solutions containing fundamental constants only. Using the nonradiative wave equation solutions that describe the bound electron having conserved momentum and energy, the radii are determined from the force balance of the
 25 electric, magnetic, and centrifugal forces that corresponds to the minimum of energy of the system. The ionization energies are then given by the electric and magnetic energies at these radii. The spreadsheets to calculate the energies from exact solutions of one through twenty-electron atoms are given in '06 Mills GUT [1] and are available from the internet [19]. For 400 atoms and ions, as well as hundreds of molecules, the agreement between the predicted
 30 and experimental results is remarkable.

The background theory of classical quantum mechanics (CQM) for the physical solutions of atoms and atomic ions is disclosed in R. Mills, *The Grand Unified Theory of*

Classical Quantum Mechanics, January 2000 Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '00 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512; R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, September 2001 Edition, BlackLight Power, Inc., Cranbury, New Jersey, Distributed by
5 Amazon.com (" '01 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512; R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, July 2004 Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '04 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512; R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, January 2005 Edition,
10 BlackLight Power, Inc., Cranbury, New Jersey, (" '05 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512 (posted at www.blacklightpower.com); R. L. Mills, "The Grand Unified Theory of Classical Quantum Mechanics", June 2006 Edition, Cadmus Professional Communications-Science Press Division, Ephrata, PA, ISBN 0963517171, Library of Congress Control Number
15 2005936834, (" '06 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ, 08512 (posted at <http://www.blacklightpower.com/bookdownload.shtml>); in prior PCT applications PCT/US02/35872; PCT/US02/06945; PCT/US02/06955; PCT/US01/09055; PCT/US01/25954; PCT/US00/20820; PCT/US00/20819; PCT/US00/09055; PCT/US99/17171;
20 PCT/US99/17129; PCT/US 98/22822; PCT/US98/14029; PCT/US96/07949; PCT/US94/02219; PCT/US91/08496; PCT/US90/01998; and PCT/US89/05037 and U.S. Patent No. 6,024,935; the entire disclosures of which are all incorporated herein by reference (hereinafter "Mills Prior Publications").

The following list of references, which are also incorporated herein by reference in
25 their entirety, are referred to in the above sections using [brackets]:

1. R. L. Mills, "The Grand Unified Theory of Classical Quantum Mechanics", June 2006 Edition, Cadmus Professional Communications-Science Press Division, Ephrata, PA, ISBN 0963517171, Library of Congress Control Number 2005936834; posted at
30 <http://www.blacklightpower.com/bookdownload.shtml>.
2. R. L. Mills, "Classical Quantum Mechanics", Physics Essays, Vol. 16, No. 4, December, (2003), pp. 433-498; posted with spreadsheets at www.blacklightpower.com/techpapers.shtml.

3. R. Mills, "Physical Solutions of the Nature of the Atom, Photon, and Their Interactions to Form Excited and Predicted Hydrino States", submitted.
4. R. L. Mills, "Exact Classical Quantum Mechanical Solutions for One- Through Twenty-Electron Atoms", in press, posted with spreadsheets at
5 <http://www.blacklightpower.com/techpapers.shtml>.
5. R. L. Mills, "The Nature of the Chemical Bond Revisited and an Alternative Maxwellian Approach", Physics Essays, Vol. 17, (2004), pp. 342-389, posted with spreadsheets at
<http://www.blacklightpower.com/techpapers.shtml>.
6. R. L. Mills, "Maxwell's Equations and QED: Which is Fact and Which is Fiction", in
10 press, posted with spreadsheets at <http://www.blacklightpower.com/techpapers.shtml>.
7. R. L. Mills, "Exact Classical Quantum Mechanical Solution for Atomic Helium Which Predicts Conjugate Parameters from a Unique Solution for the First Time", submitted, posted with spreadsheets at <http://www.blacklightpower.com/theory/theory.shtml>.
8. R. Mills, "The Grand Unified Theory of Classical Quantum Mechanics", Int. J. Hydrogen
15 Energy, Vol. 27, No. 5, (2002), pp. 565-590.
9. R. L. Mills, "The Fallacy of Feynman's Argument on the Stability of the Hydrogen Atom According to Quantum Mechanics", Annales de la Fondation Louis de Broglie, Vol. 30, No. 2, (2005), pp. 129-151, posted at <http://www.blacklightpower.com/techpapers.shtml>.
10. R. Mills, The Nature of Free Electrons in Superfluid Helium—a Test of Quantum
20 Mechanics and a Basis to Review its Foundations and Make a Comparison to Classical Theory, Int. J. Hydrogen Energy, Vol. 26, No. 10, (2001), pp. 1059-1096.
11. R. Mills, "The Hydrogen Atom Revisited", Int. J. of Hydrogen Energy, Vol. 25, Issue 12, December, (2000), pp. 1171-1183.
12. F. Laloë, Do we really understand quantum mechanics? Strange correlations, paradoxes,
25 and theorems, Am. J. Phys. 69 (6), June 2001, 655-701.
13. P. Pearle, Foundations of Physics, "Absence of radiationless motions of relativistically rigid classical electron", Vol. 7, Nos. 11/12, (1977), pp. 931-945.
14. V. F. Weisskopf, Reviews of Modern Physics, Vol. 21, No. 2, (1949), pp. 305-315.
15. H. Wergeland, "The Klein Paradox Revisited", *Old and New Questions in Physics, Cosmology, Philosophy, and Theoretical Biology*, A. van der Merwe, Editor, Plenum
30 Press, New York, (1983), pp. 503-515.
16. A. Einstein, B. Podolsky, N. Rosen, Phys. Rev., Vol. 47, (1935), p. 777.
17. F. Dyson, "Feynman's proof of Maxwell equations", Am. J. Phys., Vol. 58, (1990), pp.

209-211.

18. Haus, H. A., "On the radiation from point charges", American Journal of Physics, 54, (1986), pp. 1126-1129.

19. <http://www.blacklightpower.com/new.shtml>.

5

SUMMARY OF THE INVENTION

The present invention, an exemplary embodiment of which is also referred to as Millsian software, stems from a new fundamental insight into the nature of the atom.

Applicant's new theory of Classical Quantum Mechanics (CQM) reveals the nature of atoms
10 and molecules using classical physical laws for the first time. As discussed above, traditional quantum mechanics can solve neither multi-electron atoms nor molecules exactly. By contrast, CQM produces exact, closed-form solutions containing physical constants only for even the most complex atoms and molecules.

The present invention is the first and only molecular modeling program ever built on
15 the CQM framework. All the major functional groups that make up most organic molecules have been solved exactly in closed-form solutions with CQM. By using these functional groups as building blocks, or independent units, a potentially infinite number of organic molecules can be solved. As a result, the present invention can be used to visualize the exact 3D structure and calculate the heat of formation of almost any organic molecule.

20 For the first time, the significant building-block molecules of chemistry have been successfully solved using classical physical laws in exact closed-form equations having fundamental constants only. The major functional groups have been solved from which molecules of infinite length can be solved almost instantly with a computer program. The predictions are accurate within experimental error for over 375 exemplary molecules.

25 Applicant's CQM is the theory that physical laws (Maxwell's Equations, Newton's Laws, Special and General Relativity) must hold on all scales. The theory is based on an often overlooked result of Maxwell's Equations, that an extended distribution of charge may, under certain conditions, accelerate without radiating. This "condition of no radiation" is invoked to solve the physical structure of subatomic particles, atoms, and molecules.

30 In exact closed-form equations with physical constants only, solutions to thousands of known experimental values arise that were beyond the reach of previous outdated theories. These include the electron spin, g-factor, multi-electron atoms, excited states, polyatomic molecules, wave-particle duality and the nature of the photon, the masses and families of

fundamental particles, and the relationships between fundamental laws of the universe that reveal why the universe is accelerating as it expands. CQM is successful to over 85 orders of magnitude, from the level of quarks to the cosmos. Applicant now has over 65 peer-reviewed journal articles and also books discussing the CQM and supporting experimental evidence.

5 The molecular modeling market was estimated to be a two-billion-dollar per year industry in 2002, with hundreds of millions of government and industry dollars invested in computer algorithms and supercomputer centers. This makes it the largest effort of computational chemistry and physics.

 The present invention's advantages over other models includes: Rendering true
10 molecular structures; Providing precisely all characteristics, spatial and temporal charge distributions and energies of every electron in every bond, and of every bonding atom; Facilitating the identification of biologically active sites in drugs; and Facilitating drug design.

 An objective of the present invention is to solve the charge (mass) and current-density
15 functions of polyatomic molecules, polyatomic molecular ions, diatomic molecules, molecular radicals, molecular ions, or any portion of these species from first principles. In an embodiment, the solution for the polyatomic molecules, polyatomic molecular ions, diatomic molecules, molecular radicals, molecular ions, or any portion of these species is derived from Maxwell's equations invoking the constraint that the bound electron before excitation does
20 not radiate even though it undergoes acceleration.

 Another objective of the present invention is to generate a readout, display, or image of the solutions so that the nature of polyatomic molecules, polyatomic molecular ions, diatomic molecules, molecular radicals, molecular ions, or any portion of these species be better understood and potentially applied to predict reactivity and physical and optical
25 properties.

 Another objective of the present invention is to apply the methods and systems of solving the nature of polyatomic molecules, polyatomic molecular ions, diatomic molecules, molecular radicals, molecular ions, or any portion of these species and their rendering to numerical or graphical form to all atoms and atomic ions.

30 These objectives and other objectives are obtained by a system of computing and rendering the nature of at least one specie selected from a group of diatomic molecules having at least one atom is other than hydrogen, polyatomic molecules, molecular ions, polyatomic molecular ions, or molecular radicals, or any functional group therein, comprising

physical, Maxwellian solutions of charge, mass, and current density functions of said specie, said system comprising processing means for processing physical, Maxwellian equations representing charge, mass, and current density functions of said specie; and an output device in communication with the processing means for displaying said physical, Maxwellian
5 solutions of charge, mass, and current density functions of said specie.

Also provided is a composition of matter comprising a plurality of atoms, the improvement comprising a novel property or use discovered by calculation of at least one of a bond distance between two of the atoms, a bond angle between three of the atoms, and a bond energy between two of the atoms, orbital intercept distances and angles, charge-density
10 functions of atomic, hybridized, and molecular orbitals, the bond distance, bond angle, and bond energy being calculated from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that a bound electron(s) does not radiate under acceleration.

The presented exact physical solutions for known species of the group of polyatomic
15 molecules, polyatomic molecular ions, diatomic molecules, molecular radicals, molecular ions, or any functional group therein, can be applied to other species. These solutions can be used to predict the properties of other species and engineer compositions of matter in a manner which is not possible using past quantum mechanical techniques. The molecular solutions can be used to design synthetic pathways and predict product yields based on
20 equilibrium constants calculated from the heats of formation. Not only can new stable compositions of matter be predicted, but now the structures of combinatorial chemistry reactions can be predicted.

Pharmaceutical applications include the ability to graphically or computationally render the structures of drugs that permit the identification of the biologically active parts of
25 the specie to be identified from the common spatial charge-density functions of a series of active species. Novel drugs can now be designed according to geometrical parameters and bonding interactions with the data of the structure of the active site of the drug.

The system can be used to calculate conformations, folding, and physical properties, and the exact solutions of the charge distributions in any given specie are used to calculate the
30 fields. From the fields, the interactions between groups of the same specie or between groups on different species are calculated wherein the interactions are distance and relative orientation dependent. The fields and interactions can be determined using a finite-element-analysis approach of Maxwell's equations.

Embodiments of the system for performing computing and rendering of the nature of the polyatomic molecules, polyatomic molecular ions, diatomic molecules, molecular radicals, molecular ions, or any portion of these species using the physical solutions may comprise a general purpose computer. Such a general purpose computer may have any
 5 number of basic configurations. For example, such a general purpose computer may comprise a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage device, an input means, such as a keyboard or mouse, a display device, and a printer or other output device. A system implementing the present invention can also comprise a special
 10 purpose computer or other hardware system and all should be included within its scope.

Although not preferred, any of the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

BRIEF DESCRIPTION OF THE DRAWINGS

15

Fig. 1 illustrates an elliptical current element of the prolate spheroidal MO;

Fig. 2 illustrates the ellipsoidal current-density surface obtained by stretching $Y_0^0(\theta, \phi)$ along the semimajor axis;

Fig. 3 illustrates the angular momentum components of the MO and S;

20

Fig. 4 illustrates cross section of an atomic orbital;

Fig. 5 illustrates A. Prolate spheroid MO;

Fig. 6 illustrates the equilateral triangular $H_3^+(1/p)$;

Fig. 7 illustrates the cross section of the OH MO;

25

Fig. 8 illustrates OH MO comprising the superposition of the H_2 -type ellipsoidal MO and the $O2p_y$ AO with a relative charge-density of 0.75 to 1.25;

Fig. 9 illustrates H_2O MO comprising the linear combination of two $O-H$ -bond MOs;

Fig. 10 illustrates the cross section of the NH MO showing the axes, angles, and point of intersection of the H_2 -type ellipsoidal MO with the $N2p_x$ AO;

30

Fig. 11 illustrates NH MO comprising the superposition of the H_2 -type ellipsoidal MO and the $N2p_x$ AO with a relative charge-density of 0.75 to 1.25;

Fig. 12 illustrates NH_2 MO comprising the linear combination of two $N-H$ -bond MOs;

Fig. 13 illustrates NH_3 MO comprising the linear combination of three $N-H$ -bonds;

Fig. 14 illustrates the cross section of the CH MO showing the axes, angles, and point of intersection of the H_2 -type ellipsoidal MO with the $C2sp^3$ HO;

Fig. 15 illustrates CH MO comprising the superposition of the H_2 -type ellipsoidal MO and
5 the $C2sp^3$ HO with a relative charge-density of 0.75 to 1.25;

Fig. 16 illustrates CH_2 MO comprising the linear combination of two $C-H$ -bond MOs;

Fig. 17 illustrates CH_3 MO comprising the linear combination of three $C-H$ -bond MOs;

Fig. 18 illustrates CH_4 MO comprising the linear combination of four $C-H$ -bond MOs
formed by the superposition of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO;

10 Fig. 19 illustrates the cross section of the N_2 MO;

Fig. 20 illustrates N_2 MO comprising the σ MO (H_2 -type MO) with N atoms at the foci;

Fig. 21 illustrates the cross section of the O_2 MO;

Fig. 22 illustrates O_2 MO comprising the σ MO (H_2 -type MO);

Fig. 23 illustrates the cross section of the F_2 MO;

15 Fig. 24 illustrates F_2 MO comprising the σ MO (H_2 -type MO) with F atoms at the foci;

Fig. 25 illustrates the cross section of the Cl_2 MO;

Fig. 26 illustrates Cl_2 MO comprising the superposition of the H_2 -type ellipsoidal MO and
the two $Cl3sp^3$ HOs;

Fig. 27 illustrates the cross section of the CN MO;

20 Fig. 28 illustrates CN MO;

Fig. 29 illustrates the cross section of the CO MO;

Fig. 30 illustrates CO MO;

Fig. 31 illustrates the cross section of the NO MO;

Fig. 32 illustrates NO MO;

25 Fig. 33 illustrates the cross section of the CO_2 MO;

Fig. 34 illustrates CO_2 MO;

Fig. 35 illustrates the cross section of the NO_2 MO;

Fig. 36 illustrates NO_2 MO;

Fig. 37 illustrates the cross section of the $C-C$ -bond MO (σ MO) and one $C-H$ -bond

MO of ethane;

Fig. 38 illustrates the cross section of one $C-H$ -bond MO of ethane showing the axes, angles, and point of intersection of the H_2 -type ellipsoidal MO with the $C_{ethane} 2sp^3$ HO;

- 5 Fig. 39 illustrates CH_3CH_3 MO comprising the linear combination of two sets of three $C-H$ -bond MOs and a $C-C$ -bond MO;

Fig. 40 illustrates the cross section of the $C=C$ -bond MO (σ MO) and one $C-H$ -bond MO of ethylene showing the axes, angles, and point of intersection of each H_2 -type ellipsoidal MO with the corresponding $C_{ethylene} 2sp^3$ HO;

- 10 Fig. 41 illustrates the cross section of one $C-H$ -bond MO of ethylene showing the axes, angles, and point of intersection of the H_2 -type ellipsoidal MO with the $C_{ethylene} 2sp^3$ HO;

Fig. 42 illustrates CH_2CH_2 MO comprising the linear combination of two sets of two $C-H$ -bond MOs and a $C=C$ -bond MO;

- 15 Fig. 43 illustrates the cross section of the $C\equiv C$ -bond MO (σ MO) and one $C-H$ -bond MO of acetylene showing the axes, angles, and point of intersection of each H_2 -type ellipsoidal MO with the corresponding $C_{acetylene} 2sp^3$ HO;

Fig. 44 illustrates $CHCH$ MO comprising the linear combination of two $C-H$ -bond MOs and a $C\equiv C$ -bond MO;

- 20 Fig. 45 illustrates the cross section of one $C=C$ -bond MO (σ MO) and one $C-H$ -bond MO of benzene showing the axes, angles, and point of intersection of each H_2 -type ellipsoidal MO with the corresponding $C_{benzene} 2sp^3$ HO;

Fig. 46 illustrates the cross section of one $C-H$ -bond MO of benzene showing the axes, angles, and point of intersection of the H_2 -type ellipsoidal MO with the $C_{benzene} 2sp^3$ HO;

- 25 Fig. 47 illustrates C_6H_6 MO comprising the linear combination of six sets of $C-H$ -bond MOs bridged by $C=C$ -bond MOs;

Fig. 48 illustrates the cross section of one $C-C$ -bond MO (σ MO) and one $C-H$ -bond MO of C_nH_{2n+2} showing the axes, angles, and point of intersection of each H_2 -type ellipsoidal MO with the corresponding $C_{alkane} 2sp^3$ HO;

30

Fig. 49 illustrates the cross section of one $C-H$ -bond MO of C_nH_{2n+2} showing the axes, angles, and point of intersection of the H_2 -type ellipsoidal MO with the $C_{alkane} 2sp^3$ HO;

Fig. 50 illustrates C_3H_8 MO comprising a linear combination of $C-H$ -bond MOs and
5 $C-C$ -bond MOs of the two methyl groups and one methylene group;

Fig. 51 illustrates C_4H_{10} MO comprising a linear combination of $C-H$ -bond MOs and
 $C-C$ -bond MOs of the two methyl and two methylene groups;

Fig. 52 illustrates C_5H_{12} MO comprising a linear combination of $C-H$ -bond MOs and
 $C-C$ -bond MOs of the two methyl and three methylene groups;

10 Fig. 53 illustrates C_6H_{14} MO comprising a linear combination of $C-H$ -bond MOs and
 $C-C$ -bond MOs of the two methyl and four methylene groups;

Fig. 54 illustrates C_7H_{16} MO comprising a linear combination of $C-H$ -bond MOs and
 $C-C$ -bond MOs of the two methyl and five methylene groups;

Fig. 55 illustrates C_8H_{18} MO comprising a linear combination of $C-H$ -bond MOs and
15 $C-C$ -bond MOs of the two methyl and six methylene groups;

Fig. 56 illustrates C_9H_{20} MO comprising a linear combination of $C-H$ -bond MOs and
 $C-C$ -bond MOs of the two methyl and seven methylene groups. (A) Opaque view
of the charge-density of the $C-C$ -bond and $C-H$ -bond MOs;

Fig. 57 illustrates $C_{10}H_{22}$ MO comprising a linear combination of $C-H$ -bond MOs and
20 $C-C$ -bond MOs of the two methyl and eight methylene groups;

Fig. 58 illustrates $C_{11}H_{24}$ MO comprising a linear combination of $C-H$ -bond MOs and
 $C-C$ -bond MOs of the two methyl and nine methylene groups;

Fig. 59 illustrates $C_{12}H_{26}$ MO comprising a linear combination of $C-H$ -bond MOs and
 $C-C$ -bond MOs of the two methyl and ten methylene groups;

25 Fig. 60 illustrates $C_{18}H_{38}$ MO comprising a linear combination of $C-H$ -bond MOs and
 $C-C$ -bond MOs of the two methyl and sixteen methylene groups;

Fig. 61.A illustrates 1,3 Butadiene;

Fig. 61.B illustrates 1,3 Pentadiene;

Fig. 61.C illustrates 1,4 Pentadiene;

30 Fig. 61.D illustrates 1,3 Cyclopentadiene;

- Fig. 61E illustrates Cyclopentene;
Fig. 62 illustrates Naphthalene;
Fig. 63 illustrates Toluene;
Fig. 64 illustrates Benzoic acid;
5 Fig. 65 illustrates Pyrrole;
Fig. 66 illustrates Furan;
Fig. 67 illustrates Thiophene;
Fig. 68 illustrates Imidazole;
Fig. 69 illustrates Pyridine;
10 Fig. 70 illustrates Pyrimidine;
Fig. 71 illustrates Pyrazine;
Fig. 72 illustrates Quinoline;
Fig. 73 illustrates Isoquinoline;
Fig. 74 illustrates Indole;
15 Fig. 75 illustrates Adenine;
Fig. 76 illustrates a block diagram of an exemplary software program; and
Figs. 77 and 78 illustrate pictures of an exemplary software program.

20

25

30

35

Section I

THE NATURE OF THE CHEMICAL BOND OF HYDROGEN-TYPE MOLECULES AND MOLECULAR IONS

5

With regard to the Hydrino Theory—BlackLight Process section, the possibility of states with $n = 1/p$ is also predicted in the case of hydrogen molecular species wherein $H(1/p)$ reacts a proton or two $H(1/p)$ atoms react to form $H_2^+(1/p)$ and $H_2(1/p)$, respectively. The natural molecular-hydrogen coordinate system based on symmetry is ellipsoidal coordinates.

10 The magnitude of the central field in the derivations of molecular hydrogen species is taken as the general parameter p wherein p may be an integer which may be predictive of new possibilities. Thus, p replaces the effective nuclear charge of quantum mechanics and corresponds to the physical field of a resonant photon superimposed with the field of the proton. The case with $p = 1$ is evaluated and compared with the experimental results for
15 hydrogen species in Table 11.1, and the consequences that $p = \text{integer}$ are considered in the Nuclear Magnetic Resonance Shift section.

Two hydrogen atoms react to form a diatomic molecule, the hydrogen molecule.

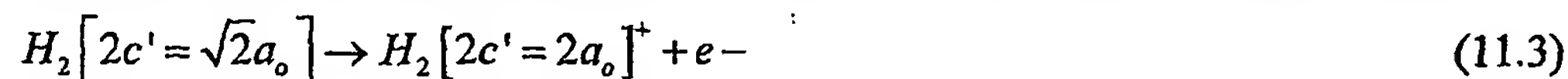


where $2c'$ is the internuclear distance. Also, two hydrino atoms react to form a diatomic
20 molecule, a dihydrino molecule.

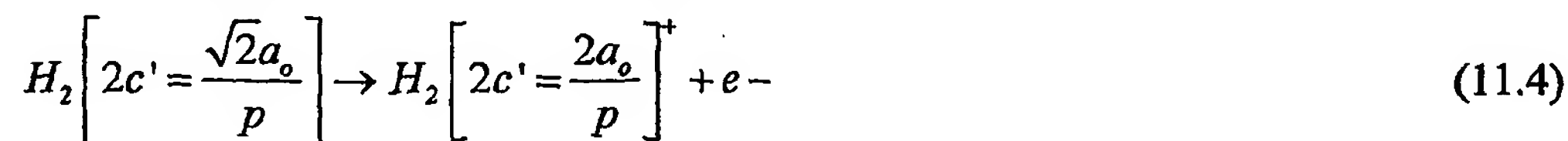


where p is an integer.

Hydrogen molecules form hydrogen molecular ions when they are singly ionized.



25 Also, dihydrino molecules form dihydrino molecular ions when they are singly ionized.



HYDROGEN-TYPE MOLECULAR IONS

Each hydrogen-type molecular ion comprises two protons and an electron where the equation of motion of the electron is determined by the central field which is p times that of a proton at each focus (p is one for the hydrogen molecular ion, and p is an integer greater than one for each $H_2^+(1/p)$, called dihydrino molecular ion). The differential equations of motion in the case of a central field are [1]

$$m(\ddot{r} - r\dot{\theta}^2) = f(r) \quad (11.5)$$

$$m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = 0 \quad (11.6)$$

The second or transverse equation, Eq. (11.6), gives the result that the angular momentum is constant.

$$r^2\dot{\theta} = \text{constant} = L/m \quad (11.7)$$

where L is the angular momentum (\hbar in the case of the electron). The central force equations can be transformed into an orbital equation by the substitution, $u = \frac{1}{r}$. The differential equation of the orbit of a particle moving under a central force is

$$\frac{\partial^2 u}{\partial \theta^2} + u = \frac{-1}{mL^2 u^2} f(u^{-1}) \quad (11.8)$$

Because the angular momentum is constant, motion in only one plane need be considered; thus, the orbital equation is given in polar coordinates. The solution of Eq. (11.8) for an inverse-squared force

$$f(r) = -\frac{k}{r^2} \quad (11.9)$$

is

$$r = r_0 \frac{1+e}{1+e \cos \theta} \quad (11.10)$$

$$e = A \frac{m \frac{L^2}{m^2}}{k} \quad (11.11)$$

$$r_0 = \frac{m \frac{L^2}{m^2}}{k(1+e)} \quad (11.12)$$

where e is the eccentricity of the ellipse and A is a constant. The equation of motion due to a central force can also be expressed in terms of the energies of the orbit. The square of the speed in polar coordinates is

$$v^2 = (\dot{r}^2 + r^2 \dot{\theta}^2) \quad (11.13)$$

- 5 Since a central force is conservative, the total energy, E , is equal to the sum of the kinetic, T , and the potential, V , and is constant. The total energy is

$$\frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) + V(r) = E = \text{constant} \quad (11.14)$$

Substitution of the variable $u = \frac{1}{r}$ and Eq. (11.7) into Eq. (11.14) gives the orbital energy equation.

$$10 \quad \frac{1}{2} m \frac{L^2}{m^2} \left(\frac{\partial^2 u}{\partial \theta^2} + u^2 \right) + V(u^{-1}) = E \quad (11.15)$$

Because the potential energy function $V(r)$ for an inverse-squared force field is

$$V(r) = -\frac{k}{r} = -ku \quad (11.16)$$

the energy equation of the orbit, Eq. (11.15),

$$\frac{1}{2} m \frac{L^2}{m^2} \left(\frac{\partial^2 u}{\partial \theta^2} + u^2 \right) - ku = E \quad (11.17)$$

- 15 which has the solution

$$r = \frac{m \frac{L^2}{m^2} k^{-1}}{1 + \left(1 + 2Em \frac{L^2}{m^2} k^{-2} \right)^{1/2} \cos \theta} \quad (11.18)$$

where the eccentricity, e , is

$$e = \left(1 + 2Em \frac{L^2}{m^2} k^{-2} \right)^{1/2} \quad (11.19)$$

- Eq. (11.19) permits the classification of the orbits according to the total energy, E , as
20 follows:

$E < 0,$	$e < 1$	closed orbits (ellipse or circle)
$E = 0,$	$e = 1$	parabolic orbit
$E > 0,$	$e > 1$	hyperbolic orbit

Since $E = T + V$ and is constant, the closed orbits are those for which $T < |V|$, and the open orbits are those for which $T \geq |V|$. It can be shown that the time average of the kinetic energy, $\langle T \rangle$, for elliptical motion in an inverse-squared field is $1/2$ that of the time average of the magnitude of the potential energy, $\langle |V| \rangle$. $\langle T \rangle = 1/2 \langle |V| \rangle$ [1].

- 5 As demonstrated in the One-Electron Atom section, the electric inverse-squared force is conservative; thus, the angular momentum of the electron, \hbar , and the energy of atomic orbitspheres are constant. In addition, the orbitspheres are nonradiative when the boundary condition is met.

The central force equation, Eq. (11.14), has orbital solutions, which are circular,
10 elliptical, parabolic, or hyperbolic. The former two types of solutions are associated with atomic and molecular orbitals. These solutions are nonradiative. The boundary condition for nonradiation given in the One-Electron Atom section, is the absence of components of the spacetime Fourier transform of the current-density function synchronous with waves traveling at the speed of light. The boundary condition is met when the velocity for the charge density
15 at every coordinate position on the orbitsphere is

$$v_n = \frac{\hbar}{m_e r_n} \quad (11.20)$$

The allowed velocities and angular frequencies are related to r_n by

$$v_n = r_n \omega_n \quad (11.21)$$

$$\omega_n = \frac{\hbar}{m_e r_n^2} \quad (11.22)$$

- 20 As demonstrated in the One-Electron Atom section and by Eq. (11.22), this condition is met for the product function of a radial Dirac delta function and a time harmonic function where the angular frequency, ω , is constant and given by Eq. (11.22).

$$\omega_n = \frac{\hbar}{m_e r_n^2} = \frac{\pi L}{A} \quad (11.23)$$

- where L is the angular momentum and A is the area of the closed orbit. Consider the
25 solution of the central force equation comprising the product of a two-dimensional ellipsoid and a time harmonic function. The spatial part of the product function is the convolution of a radial Dirac delta function with the equation of an ellipsoid. The Fourier transform of the convolution of two functions is the product of the individual Fourier transforms of the functions; thus, the boundary condition is met for an ellipsoidal-time harmonic function when

$$\omega_n = \frac{\pi \hbar}{m_e A} = \frac{\hbar}{m_e ab} \quad (11.24)$$

where the area of an ellipse is

$$A = \pi ab \quad (11.25)$$

where b and $2b$ are the lengths of the semiminor and minor axes, respectively, and a and $2a$ are the lengths of the semimajor and major axes, respectively. The geometry of molecular hydrogen is ellipsoidal with the internuclear axis as the principal axis; thus, the electron orbital is a two-dimensional ellipsoidal-time harmonic function. The mass follows an elliptical path, time harmonically as determined by the central field of the protons at the foci. Rotational symmetry about the internuclear axis further determines that the orbital is a prolate spheroid. In general, ellipsoidal orbits of molecular bonding, hereafter referred to as ellipsoidal molecular orbitals (MOs), have the general equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (11.26)$$

The semiprincipal axes of the ellipsoid are a , b , c .

In ellipsoidal coordinates the Laplacian is

$$(\eta - \zeta)R_\xi \frac{\partial}{\partial \xi} \left(R_\xi \frac{\partial \phi}{\partial \xi} \right) + (\zeta - \xi)R_\eta \frac{\partial}{\partial \eta} \left(R_\eta \frac{\partial \phi}{\partial \eta} \right) + (\xi - \eta)R_\zeta \frac{\partial}{\partial \zeta} \left(R_\zeta \frac{\partial \phi}{\partial \zeta} \right) = 0 \quad (11.27)$$

An ellipsoidal MO is equivalent to a charged perfect conductor (i.e. no dissipation to current flow) whose surface is given by Eq. (11.26). It is a two-dimensional equipotential membrane where each MO is supported by the outward centrifugal force due to the corresponding angular velocity, which conserves its angular momentum of \hbar . It satisfies the boundary conditions for a discontinuity of charge in Maxwell's equations, Eq. (11.48). It carries a total charge $q = -e$, and its potential is a solution of the Laplacian in ellipsoidal coordinates, Eq. (11.27).

Excited states of orbitspheres are discussed in the Excited States of the One-Electron Atom (Quantization) section. In the case of ellipsoidal MOs, excited electronic states are created when photons of discrete frequencies are trapped in the ellipsoidal resonator cavity of the MO. The photon changes the effective charge at the MO surface where the central field is ellipsoidal and arises from the protons and the effective charge of the "trapped photon" at the foci of the MO. Force balance is achieved at a series of ellipsoidal equipotential two-dimensional surfaces confocal with the ground state ellipsoid. The "trapped photons" are solutions of the Laplacian in ellipsoidal coordinates, Eq. (11.27).

As is the case with the orbitsphere, higher and lower energy states are equally valid. The photon standing wave in both cases is a solution of the Laplacian in ellipsoidal coordinates. For an ellipsoidal resonator cavity, the relationship between an allowed circumference, $4aE$, and the photon standing wavelength, λ , is

$$5 \quad 4aE = n\lambda \quad (11.28)$$

where n is an integer and where the elliptic integral E of Eq. (11.28) is given by

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2 \phi} d\phi \quad (11.29)$$

$$k = e = \frac{\sqrt{a^2 - b^2}}{a} \quad (11.30)$$

Applying Eqs. (11.28) and (11.29-11.30), the relationship between an allowed angular
10 frequency given by Eq. (11.24) and the photon standing wave angular frequency, ω , is:

$$\frac{\pi \hbar}{m_e A} = \frac{\hbar}{m_e n a_1 n b_1} = \frac{\hbar}{m_e a_n b_n} = \frac{1}{n^2} \omega_1 = \omega_n \quad (11.31)$$

where $n = 1, 2, 3, 4, \dots$

$$n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

ω_1 is the allowed angular frequency for $n = 1$

15 a_1 and b_1 are the allowed semimajor and semiminor axes for $n = 1$

The potential, ϕ , and distribution of charge, σ , over the conducting surface of an ellipsoidal MO are sought given the conditions: 1.) the potential is equivalent to that of a charged ellipsoidal conductor whose surface is given by Eq. (11.26), 2.) it carries a total charge $q = -e$, and 3.) initially there is no external applied field. To solve this problem, a
20 potential function must be found which satisfies Eq. (11.27), which is regular at infinity, and which is constant over the given ellipsoid. The solution is well known and is given after Stratton [2]. Consider that the Laplacian is solved in ellipsoidal coordinates wherein ξ is the parameter of a family of ellipsoids all confocal with the standard surface $\xi = 0$ whose axes have the specified values a, b, c . The variables ζ and η are the parameters of confocal
25 hyperboloids and as such serve to measure position on any ellipsoid $\xi = \text{constant}$. On the surface $\xi = 0$; therefore, ϕ must be independent of ζ and η . Due to the uniqueness property of solutions of the Laplacian, a function which satisfies Eq. (11.27), behaves

properly at infinity, and depends only on ξ , can be adjusted to represent the potential correctly at any point outside the ellipsoid $\xi = 0$.

Thus, it is assumed that $\phi = \phi(\xi)$. Then, the Laplacian reduces to

$$\frac{\partial}{\partial \xi} \left(R_{\xi} \frac{\partial \phi}{\partial \xi} \right) = 0 \quad R_{\xi} = \sqrt{(\xi + a^2)(\xi + b^2)(\xi + c^2)} \quad (11.32)$$

5 which on integration leads to

$$\phi(\xi) = C_1 \int_{\xi}^{\infty} \frac{\partial \xi}{R_{\xi}} \quad (11.33)$$

where C_1 is an arbitrary constant. The upper limit is selected to ensure the proper behavior at infinity. When ξ becomes very large, R_{ξ} approaches $\xi^{3/2}$ and

$$\phi \sim \frac{2C_1}{\sqrt{\xi}} \quad (\xi \rightarrow \infty) \quad (11.34)$$

10 Furthermore, the equation of an ellipsoid can be written in the form

$$\frac{x^2}{1 + \frac{a^2}{\xi}} + \frac{y^2}{1 + \frac{b^2}{\xi}} + \frac{z^2}{1 + \frac{c^2}{\xi}} = \xi \quad (11.35)$$

If $r^2 = x^2 + y^2 + z^2$ is the distance from the origin to any point on the ellipsoid ξ , it is apparent that as ξ becomes very large $\xi \rightarrow r^2$. Thus, at great distances from the origin, the potential becomes that of a point charge at the origin:

$$15 \quad \phi \sim \frac{2C_1}{r} \quad (11.36)$$

The solution Eq. (11.33) is, therefore, regular at infinity, and the constant C_1 is then determined. It has been shown by Stratton [2] that whatever the distribution, the dominant term of the expansion at remote points is the potential of a point charge at the origin equal to the total charge of the distribution—in this case q . Hence $C_1 = \frac{q}{8\pi\epsilon_0}$, and the potential at

20 any point is

$$\phi(\xi) = \frac{q}{8\pi\epsilon_0} \int_{\xi}^{\infty} \frac{\partial \xi}{R_{\xi}} \quad (11.37)$$

The equipotential surfaces are the ellipsoids $\xi = \text{constant}$. Eq. (11.37) is an elliptic integral and its values have been tabulated [3].

Since the distance along a curvilinear coordinate u^1 is measured not by du^1 but by $h_1 du^1$, the normal derivative in ellipsoidal coordinates is given by

$$\frac{\partial \phi}{\partial n} = \frac{1}{h_1} \frac{\partial \phi}{\partial \xi} = \frac{-q}{4\pi\epsilon_0} \frac{1}{\sqrt{(\xi-\eta)(\xi-\zeta)}} \quad (11.38)$$

where

$$h_1 = \frac{1}{2} \frac{\sqrt{(\xi-\eta)(\xi-\zeta)}}{R_\xi} \quad (11.39)$$

The density of charge, σ , over the surface $\xi = 0$ is

$$\sigma = \epsilon_0 \left(\frac{\partial \phi}{\partial n} \right)_{\xi=0} = \frac{q}{4\pi\sqrt{\eta\zeta}} \quad (11.40)$$

Defining x, y, z in terms of ξ, η, ζ , we put $\xi = 0$, it may be easily verified that

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4} = \frac{\zeta\eta}{a^2 b^2 c^2} \quad (\xi = 0) \quad (11.41)$$

Consequently, the charge density in rectangular coordinates is

$$\sigma = \frac{q}{4\pi abc} \frac{1}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \quad (11.42)$$

(The mass-density function of an MO is equivalent to its charge-density function where m replaces q of Eq. (11.42)). The equation of the plane tangent to the ellipsoid at the point x_0, y_0, z_0 is

$$X \frac{x_0}{a^2} + Y \frac{y_0}{b^2} + Z \frac{z_0}{c^2} = 1 \quad (11.43)$$

where X, Y, Z are running coordinates in the plane. After dividing through by the square root of the sum of the squares of the coefficients of X, Y , and Z , the right member is the distance D from the origin to the tangent plane. That is,

$$D = \frac{1}{\sqrt{\frac{x_0^2}{a^4} + \frac{y_0^2}{b^4} + \frac{z_0^2}{c^4}}} \quad (11.44)$$

so that for an electron MO

$$\sigma = \frac{-e}{4\pi abc} D \quad (11.45)$$

In other words, the surface density at any point on a charged ellipsoidal conductor is proportional to the perpendicular distance from the center of the ellipsoid to the plane tangent

to the ellipsoid at the point. The charge is thus greater on the more sharply rounded ends farther away from the origin.

In the case of hydrogen-type molecules and molecular ions, rotational symmetry about the internuclear axis requires that two of the axes be equal. Thus, the MO is a spheroid, and Eq. (11.37) can be integrated in terms of elementary functions. If $a > b = c$, the spheroid is prolate, and the potential is given by

$$\phi = \frac{1}{8\pi\epsilon_0} \frac{-e}{\sqrt{a^2 - b^2}} \ln \frac{\sqrt{\xi + a^2} + \sqrt{a^2 - b^2}}{\sqrt{\xi + a^2} - \sqrt{a^2 - b^2}} \quad (11.46)$$

SPHEROIDAL FORCE EQUATIONS

10

Electric Force

The spheroidal MO is a two-dimensional surface of constant potential given by Eq. (11.46) for $\xi = 0$. For an isolated electron MO the electric field inside is zero as given by Gauss' Law

$$\int_S \mathbf{E} dA = \int_V \frac{\rho}{\epsilon_0} dV \quad (11.47)$$

15 where the charge density, ρ , inside the MO is zero. Gauss' Law at a two-dimensional surface with continuity of the potential across the surface according to Faraday's law in the electrostatic limit [4-6] is

$$\mathbf{n} \cdot (\mathbf{E}_1 - \mathbf{E}_2) = \frac{\sigma}{\epsilon_0} \quad (11.48)$$

\mathbf{E}_2 is the electric field inside which is zero. The electric field of an ellipsoidal MO with 20 semimajor and semiminor axes a and $b = c$, respectively, is given by substituting σ given by Eq. (11.38-11.42) into Eq. (11.48).

$$\mathbf{E} = \frac{\sigma}{\epsilon_0} \mathbf{i}_\xi = \frac{-e}{4\pi\epsilon_0} \frac{1}{\sqrt{(\xi - \eta)(\xi - \zeta)}} \mathbf{i}_\xi = \frac{-e}{4\pi\epsilon_0 abc} D \mathbf{i}_\xi = \frac{-e}{4\pi\epsilon_0 abc} \frac{1}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \mathbf{i}_\xi \quad (11.49)$$

wherein the ellipsoidal-coordinate parameter $\xi = 0$ at the surface of the MO and D is the 25 distance from the origin to the tangent plane given by Eq. (11.44). The electric field and thus the force and potential energy between the protons and the electron MO can be solved based on three principles: (1) Maxwell's equations require that the electron MO is a equipotential energy surface that is a function of ξ alone; thus, it is a prolate spheroid, (2) stability to

radiation, and conservation first principles require that the angular velocity is constant and given in polar coordinates with respect to the origin by Eq. (11.24), and (3) the equations of motion due to the central force of each proton (Eqs. (11.5-11.19) and Eqs. (11.68-11.70)) also determine that the current is ellipsoidal, and based on symmetry, the current is a prolate
 5 spheroid. Thus, based on Maxwell's equations, conservation principles, and Newton's Laws for the equations of motion, the electron MO constraints and the motion under the force of the protons both give rise to a prolate spheroid. Since the energy of motion is determined from the Coulombic central field (Eqs. (11.5-11.19), the protons give rise to a prolate spheroidal energy surface (a surface of constant energy) that is matched to the equipotential, prolate
 10 spheroidal electron MO.

The force balance equation between the protons and the electron MO is solved to give the position of the foci, then the total energy is determined including the repulsive energy between the two protons at the foci to determine whether the original assumption of an elliptic orbit was valid. If the condition that $E < 0$ is met, then the problem of the stable
 15 elliptic orbit is solved. In any case that this condition is not found to be met, then a stable orbit can not be formed.

The force and energy equations of a point charge(mass) (Eqs. (11.5-11.24)) are reformulated in term of densities for charge, current, mass, momentum, and potential, kinetic, and total energies. Consider an elliptical orbit shown in Figure 1 that applies to a point
 20 charge(mass) as well as a point on a continuous elliptical current loop that comprises a basis element of the continuous current density of the ellipsoidal MO. The tangent plane at any point on the ellipsoid makes equal angles with the foci radii at that point and the sum of the distance to the foci is a constant, $2a$. Thus, the normal is the bisector of the angle between the foci radii at that point as shown in Figure 1.

25 The unit vector normal to the ellipsoidal MO at a point (x, y, z) is

$$\hat{\mathbf{d}} = \frac{\left(\frac{x}{a^2}, \frac{y}{b^2}, \frac{z}{c^2} \right)}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \quad (11.50)$$

$F_1(r(t))$ and $F_2(r(t))$ are defined as the components of the central forces centered on F_1 and F_2 . The components of the central forces that are normal to the ellipsoidal MO in the direction of $\hat{\mathbf{d}}$, the unit vector in the \mathbf{i}_ξ -direction are defined as $F_{1\perp}(r(t))$ and $F_{2\perp}(r(t))$.

The normalized projections or projection factor of the sum of these central forces in the $\hat{\mathbf{d}}$ -direction at the point (x, y, z) is

$$\begin{aligned}
 \frac{F_{1\perp}(r(t)) + F_{2\perp}(r(t))}{|F_1(r(t)) + F_2(r(t))|} &= \frac{(\mathbf{r}_1 + \mathbf{r}_2) \cdot \hat{\mathbf{d}}}{2a} \\
 &= \frac{((x-c, y, z) + (x+c, y, z)) \cdot \left(\frac{x}{a^2} + \frac{y}{b^2} + \frac{z}{c^2}\right)}{2a\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \\
 &= \frac{\left(\frac{x^2 - cx}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right) + \left(\frac{x^2 + cx}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}\right)}{2a\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \\
 &= \frac{1}{a\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}}
 \end{aligned} \tag{11.51}$$

where \mathbf{r}_1 and \mathbf{r}_2 are the radial vectors of the central forces from the corresponding focus to the point (x, y, z) on the ellipsoidal MO.

The polar-coordinate elliptical orbit of a point charge due to its motion in a central inverse-squared-radius field is given by Eqs. (11.10-11.12) as the solution of the polar-coordinate-force equations, Eqs. (11.5-11.19) and (11.68-11.70). The orbit is also completely specified in Cartesian coordinates by the solution of Eqs. (11.5-11.19) and (11.68-11.70) for the semimajor and semiminor axes. Then, the corresponding polar-coordinate elliptical orbit is given as a plane cross section through the foci of the Cartesian-coordinate-system ellipsoid having the same axes given by Eq. (11.26) where $c = b$. Thus, the Columbic central force can be determined in terms of the general Cartesian coordinates from the polar-coordinate central force equations (Eqs. (11.5-11.19)). Consider separately the elliptical solution at each focus given in polar coordinates by Eq. (11.10):

$$r_1 = a(1-e) \frac{1+e}{1+e \cos \theta} = \frac{a(1-e^2)}{1+e \cos \theta} \tag{11.52}$$

$$r_2 = \frac{a(1-e^2)}{1+e \cos(\theta + \pi)} = \frac{a(1-e^2)}{1-e \cos \theta} \tag{11.53}$$

where

$$r_0 = a - c' = a \left(1 - \frac{c'}{a}\right) = a(1-e) \tag{11.54}$$

The magnitude of the sum of the central forces centered on F_1 and F_2 that are normal to the ellipsoidal MO are

$$\begin{aligned}
 |F_1(r_1)| + |F_2(r_2)| &= \frac{k}{r_1^2} + \frac{k}{r_2^2} \\
 &= k \frac{(1+e \cos \theta)^2 + (1-e \cos \theta)^2}{a^2 (1-e^2)^2} \\
 &= k \frac{1+2e \cos \theta + e^2 \cos^2 \theta + 1-2e \cos \theta + e^2 \cos^2 \theta}{a^2 (1-e^2)^2} \\
 &= k \frac{2+2e^2 \cos^2 \theta}{a^2 (1-e^2)^2}
 \end{aligned} \tag{11.55}$$

The vector central forces centered on F_1 and F_2 that are normal to the ellipsoidal MO are then given by the product of the corresponding magnitude and vector projection given by Eqs. (11.55) and (11.51), respectively:

$$\mathbf{F}_{1\perp}(r_1) + \mathbf{F}_{2\perp}(r_2) = k \frac{2+2e^2 \cos^2 \theta}{a^2 (1-e^2)^2} \frac{1}{a \sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \mathbf{i}_\xi \tag{11.56}$$

Eq. (11.56) is based on a single point charge e . For a charge-density distribution that is given as an ellipsoidal equipotential, the θ -dependence must vanish. In addition to the elliptical orbit being completely specified in Cartesian coordinates by the solution of Eqs. (11.5-11.19) and Eqs. (11.68-11.70) for the semimajor and semiminor axes in Eq. (11.26), the polar-coordinate elliptical orbit is also completely specified by the total constant total energy E and the angular momentum which for the electron is the constant \hbar . Considering Eq. (11.56), the corresponding total energy of the electron is conserved and is determined by the integration over the MO to give the average:

$$\mathbf{F}_{1\perp}(r_1) + \mathbf{F}_{2\perp}(r_2) = k \frac{2+e^2}{a^2 (1-e^2)^2} \frac{1}{a \sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \mathbf{i}_\xi \tag{11.57}$$

Eq. (11.57) is transformed from a two-centered-central force to a one-centered-central force to match the form of the potential of the ellipsoidal MO. In this case,

$$\mathbf{r}_1, \mathbf{r}_2 \rightarrow r(t) \mathbf{i}_\xi \tag{11.58}$$

In the case that

$$r_1 = r_2 = a \quad (11.59)$$

then,

$$r(t) = b \quad (11.60)$$

and the one-centered-central force is in the \mathbf{i}_ξ -direction. Thus, Eq. (11.57) transforms as

$$\begin{aligned} \mathbf{F}_{11}(r(t)) + \mathbf{F}_{21}(r(t)) &= k \frac{(1-e^2)^2}{b^2(2+e^2)} \frac{1}{a \sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \\ &= \frac{2+e^2}{(1-e^2)^2} k \frac{1}{ab^2 \sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \mathbf{i}_\xi \end{aligned} \quad (11.61)$$

Eq. (11.61) has the same form as that of the electric field of the ellipsoidal MO given by Eq. (11.49), except for the scaling factor of two-centered coordinates h_{2cc} :

$$h_{2cc} = \frac{2+e^2}{(1-e^2)^2} \quad (11.62)$$

As shown in the case of the derivation of the Laplacian charge-density and electric field, if $r^2 = x^2 + y^2 + z^2$ is the distance from the origin to any point on the ellipsoid ξ , it is apparent that as ξ becomes very large $\xi \rightarrow r^2$. Thus, at great distances from the origin, the potential becomes that of a point charge at the origin as given by Eq. (11.36). The same boundary condition applies to the potential and field of the protons. The limiting case is also given as $e \rightarrow 0$. Then, to transform the scale factor to that of one-centered coordinates for an ellipsoidal MO, the reciprocal of the scaling factor multiplies the Laplacian-MO-electric-field term. The reciprocal of Eq. (11.62) is

$$h_{2cc}^{-1} = \frac{(1-e^2)^2}{2+e^2} \quad (11.63)$$

such that as $e \rightarrow 0$, $h_{2cc}^{-1} \rightarrow \frac{1}{2}$. This transform scale factor corresponds to the interchange of

the points of highest and lowest velocity on the surface and the distribution of the charge-density in the opposite manner as shown *infra*. The charge-density distribution corrects the angular variation in central force over the surface such that a solution of the central force equation of motion and the Laplacian MO are solved simultaneously. It can also be considered as a multipole normalization factor such those of the spherical harmonics and the

spherical geometric factor of atomic electrons that gives the central force as a function of ξ only.

The reciprocal of the h_{2cc} form-factor with the dependence of the charge density on the distance parameter $r(t)$ gives

$$5 \quad \mathbf{F}_{1\perp}(r(t)) + \mathbf{F}_{2\perp}(r(t)) = k \frac{(1-e^2)^2}{b^2(2+e^2)} \frac{1}{a \sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \mathbf{i}_\xi \quad (11.64)$$

From Eq. (11.31), the magnitude of the ellipsoidal field corresponding to a below "ground state" hydrogen-type molecular ion is an integer p . The integer is one in the case of the hydrogen molecular ion and an integer greater than one in the case of each dihydrino molecular ion. The central-electric-force constant, k , from the two protons that includes the
10 central-field contribution due photons of lower-energy states is

$$k = \frac{Ze^2}{4\pi\epsilon_0} = \frac{p2e^2}{4\pi\epsilon_0} \quad (11.65)$$

Substitution of Eq. (11.65) for k in Eq. (11.64) gives the one-center-coordinate electric force \mathbf{F}_{ele} between the protons and the ellipsoidal MO:

$$\mathbf{F}_{ele} = \mathbf{F}_{1\perp}(r(t)) + \mathbf{F}_{2\perp}(r(t)) = \frac{2pe^2}{4\pi\epsilon_0} \frac{\left(1 - \left(\frac{c'}{a}\right)^2\right)^2}{ab^2 \left(2 + \left(\frac{c'}{a}\right)^2\right)} \frac{1}{a \sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \mathbf{i}_\xi \quad (11.66)$$

15 where e is the charge and with the distance from the origin to a nucleus at a focus defined as c' , the eccentricity, e , is

$$e = \frac{c'}{a} \quad (11.67)$$

From the orbital equations in polar coordinates, Eqs. (11.10-11.12), the following relationship can be derived [1]:

$$20 \quad a = \frac{m \frac{L^2}{m^2}}{k(1-e^2)} \quad (11.68)$$

For any ellipse,

$$b = a\sqrt{1-e^2} \quad (11.69)$$

Thus,

$$b = a \sqrt{\frac{\frac{L^2}{m^2} m}{ka}} \text{ (point charge (mass) in polar coordinates)} \quad (11.70)$$

From, the equal energy condition, it can be shown that b for the motion of a point charge (mass) in polar coordinates due to a proton at one focus corresponds to

$$c' = \sqrt{a^2 - b^2} \quad (11.71)$$

5 of the MO in ellipsoidal coordinates, and k_1 of one attracting focus is replaced by $k = 2k_1$ of ellipsoidal coordinates with two attracting foci. In ellipsoidal coordinates, k is given by Eq. (11.65) and L for the electron equals \hbar .

Consider the force balance equation for the point on the ellipse at the intersection of the semiminor axis b with the ellipse. At this point called $(0, b)$, the distances from each
10 focus, r_1 and r_2 , to the ellipse are equal. The relationship for the sum of the distances from the foci to any point on the ellipse is

$$r_1 + r_2 = 2a \quad (11.72)$$

Thus, at point $(0, b)$,

$$r_1 = r_2 = a \quad (11.73)$$

15 Using Eq. (11.5), the magnitude of the force balance in the radial ($r(t)$) direction, from the origin, is given by

$$mr\dot{\theta}^2 = \frac{2pe^2}{4\pi\epsilon_0 a^2} \sin \theta = \frac{2pe^2}{4\pi\epsilon_0 a^2} \frac{b}{a} \quad (11.74)$$

wherein the $m\ddot{r}$ term is zero and θ is the angle from the focus to point $(0, b)$. Using Eqs. (11.24), (11.94), and (11.95), Eq. (11.74) becomes

$$20 \quad mr\omega^2 = mb \frac{\hbar^2}{m^2 a^2 b^2} = \frac{2pe^2}{4\pi\epsilon_0 a^2} \frac{b}{a} \quad (11.75)$$

In order for the prolate spheroidal MO to be an equipotential surface, the mass and charge density must be according to Eq. (11.45). In this case, the mass and charge density along the ellipse is such that the magnitudes of the radial and transverse forces components at point $(0, b)$ are equivalent. Furthermore, according to Eq. (11.5), the central force of each proton
25 at a focus is separable and symmetrical to that at the other focus. Based on symmetry, the transverse forces of the two protons are in opposite directions and the radial components are in the same direction. But, the relationship between the magnitudes must still hold wherein at

point $(0, b)$ the transverse force is equivalent to that due to the sum of the charges at one focus. The sum of the magnitudes of the transverse forces which is equivalent to a force of $2e$ at each focus in turn is

$$|f(r)e_\theta| = \frac{2pe^2}{4\pi\epsilon_0 a^2} \cos\theta = \frac{2pe^2}{4\pi\epsilon_0 a^2} \frac{c'}{a} \quad (11.76)$$

5 Thus, using the mass and charge-density scaling factor, $\frac{c'}{a} = \frac{c'}{b}$, to match the equipotential

condition in Eq. (75) gives

$$b \frac{\hbar^2}{m_e \frac{c'}{b} a^2 b^2} = \frac{2 \frac{c'}{b} p e^2}{4\pi\epsilon_0 a^2} \frac{b}{a} \quad (11.77)$$

$$c'^2 = \frac{\hbar^2 4\pi\epsilon_0 a}{m_e 2pe^2} \quad (11.78)$$

Using Eq. (1.235)

$$10 \quad c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e^2 2pa}} = \sqrt{\frac{aa_0}{2p}} \quad (11.79)$$

Then, the length of the semiminor axis of the prolate spheroidal MO, $b = c$, is

$$b = \sqrt{a^2 - c'^2} \quad (11.80)$$

Correspondingly, c' is given by Eq. (11.71).

Substitution of Eq. (11.79) into Eq. (11.66) gives the electric force:

$$\begin{aligned}
 F_{ele} &= \frac{2pe^2}{4\pi\epsilon_0} \frac{\left(1 - \left(\frac{\sqrt{aa_0}}{\sqrt{2p}}\right)^2\right)^2}{ab^2 \left(2 + \left(\frac{\sqrt{aa_0}}{\sqrt{2p}}\right)^2\right) \sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \mathbf{i}_\xi \\
 &= \frac{2pe^2}{4\pi\epsilon_0} \frac{\left(1 - \frac{a_0}{2ap}\right)^2}{ab^2 \left(2 + \frac{a_0}{2ap}\right) \sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \mathbf{i}_\xi \\
 &= \frac{2pe^2}{4\pi\epsilon_0} \frac{\left(1 - \frac{a_0}{2ap}\right)^2}{ab^2 \left(2 + \frac{a_0}{2ap}\right)} D\mathbf{i}_\xi
 \end{aligned} \tag{11.81}$$

Centrifugal Force

5 The centrifugal force along the radial vector from each proton at each focus of the ellipsoid is given by the $mr\dot{\theta}^2$ term of Eq. (11.5). The tangent plane at any point on the ellipsoid makes equal angles with the foci radii at that point and the sum of the distance to the foci is a constant, $2a$. Thus, the normal is the bisector of the angle between the foci radii at that point as shown in Figure 1. In order to satisfy the equation of motion for an equal energy surface

10 for both foci, the transverse component of the central force of one foci at any point on the elliptic orbit due to the central force of the other (Eq. (11.5)) must cancel on average and vice versa. Thus, the centrifugal force due to the superposition of the central forces in the direction of each foci must be normal to an ellipsoidal surface in the direction perpendicular to the direction of motion. Thus, it is in the ξ -direction. This can be only be achieved by a

15 time rate of change of the momentum density that compensates for the variation of the distances from each focus to each point on an elliptical cross section. Since the angular momentum must be conserved, there can be no net force in the direction transverse to the elliptical path over each orbital path. The total energy must also be conserved; thus, as shown *infra*, the distribution of the mass must also be a solution of Laplace's equation in the

20 parameter ξ only. Thus, the mass-density constraint is the same as the charge-density constraint. As further shown *infra*, the distribution and concomitantly the centrifugal force is

a function of D , the time-dependent distance from the center of the ellipsoid to a tangent plane given by Eq. (11.44) where D and the Cartesian coordinates are the time-dependent parameters.

Each point or coordinate position on the continuous two-dimensional electron MO
5 defines an infinitesimal mass-density element which moves along an orbit comprising an elliptical plane cross section of the spheroidal MO through the foci. The kinetic energy of the electron is conserved. Then, the corresponding radial conservative force balance equation is

$$m(\ddot{r} + C_1 r) = 0 \quad (11.82)$$

The motion is such that eccentric angle, θ , changes at a constant rate at each point. That is
10 $\theta = \omega t$ at time t where the angular velocity ω is a constant. The solution of the homogeneous equation with $C_1 = \omega^2$ is

$$r(t) = ia \cos \omega t + jb \sin \omega t \quad (11.83)$$

where a is the semimajor axis, b is semiminor axis, and the boundary conditions of $r(t) = a$
for $\omega t = 0$ and $r(t) = b$ for $\omega t = \frac{\pi}{2}$ were applied. Eq. (11.83) is the parametric equation of

15 the ellipse of the orbit. The velocity is given by the time derivative of the parametric position vector:

$$v(t) = \dot{r}(t) = -ia\omega \sin \omega t + jb\omega \cos \omega t \quad (11.84)$$

The velocity is $\frac{\pi}{2}$ out of phase with the charge density at $r(t) = a$ ($\omega t = 0$) and $r(t) = b$
($\omega t = \frac{\pi}{2}$) such that the lowest charge density has the highest velocity and the highest charge

20 density has the lowest velocity. In this case, it can be shown that the current is constant along each elliptical path of the MO. Recall that nonradiation results when $\omega = \text{constant}$ given by Eq. (11.24) that corresponds to a constant current, which further maintains the current continuity condition.

Consider Eq. (11.32) for the prolate spheroidal MO. From this equation, the mass and
25 current-densities, the angular momentum, and the potential and kinetic energies are a function of ξ alone, and any dependence on the orthogonal coordinate parameters averages to unity. From Eq. (11.32),

$$R_\xi \frac{\partial \phi}{\partial \xi} = C_1 \quad (11.85)$$

Substitution of Eq. (11.40) into Eq. (11.85) gives

$$\int_{\xi}^{\infty} R_{\xi} h_1 \frac{e}{4\pi\sqrt{\eta\zeta}} \delta(\xi) d\xi = \epsilon_0 C_1 = \frac{e}{8\pi} \quad (11.86)$$

where C_1 is from Eq. (11.36). Substitution of Eq. (11.39) into Eq. (11.86) gives

$$\int_{\xi}^{\infty} R_{\xi} \frac{e}{4\pi\sqrt{\eta\zeta}} \frac{1}{2} \frac{\sqrt{(\xi-\eta)(\xi-\zeta)}}{R_{\xi}} \delta(\xi) d\xi = \frac{e}{8\pi} \quad (11.87)$$

Comparison of Eq. (11.86) with Eq. (11.87) demonstrates that the

$$5 \quad 8\pi \int_{\xi}^{\infty} R_{\xi} \frac{e}{4\pi\sqrt{\eta\zeta}} \frac{1}{2} \frac{\sqrt{(\xi-\eta)(\xi-\zeta)}}{R_{\xi}} \delta(\xi) d\xi = e \quad (11.88)$$

The current density J is given by the product of the constant frequency (Eq. (11.24)) and the charge density (Eq. (11.40)):

$$J = \frac{\hbar}{2\pi m_e ab} \frac{e}{4\pi\sqrt{\eta\zeta}} \quad (11.89)$$

The total constant current is dependent on ξ alone according to Eq. (11.32). Then, applying
10 the result of Eq. (11.88) to Eq. (11.89) gives

$$\mathbf{i} = 8\pi \int_{\xi}^{\infty} R_{\xi} \frac{\hbar}{2\pi m_e ab} \frac{e}{4\pi\sqrt{\eta\zeta}} \frac{1}{2} \frac{\sqrt{(\xi-\eta)(\xi-\zeta)}}{R_{\xi}} \delta(\xi) d\xi \mathbf{e}_{\zeta} \times \mathbf{e}_{\xi} = \frac{e\hbar}{2\pi m_e ab} \mathbf{e}_{\eta} \quad (11.90)$$

the constant current that is nonradiative.

If $\mathbf{a}(t)$ denotes the acceleration vector, then

$$\mathbf{a}(t) = -\omega^2 \mathbf{r}(t) \hat{\mathbf{i}}, \quad (11.91)$$

15 In other words, the acceleration is centrifugal as in the case of circular motion with constant angular speed ω . The dot product of $\mathbf{r}(t)$ with $\hat{\mathbf{d}}$, the unit vector normal to the ellipsoidal MO at a point (x, y, z) given by Eq. (11.50), is

$$\mathbf{r}(t) \cdot \hat{\mathbf{d}} = \frac{(x, y, z) \cdot \left(\frac{x}{a^2}, \frac{y}{b^2}, \frac{z}{c^2} \right)}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} = \frac{\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \right)}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \quad (11.92)$$

Using Eq. (11.26), the normal component projection is

$$20 \quad \mathbf{r}(t) \cdot \hat{\mathbf{d}} = \frac{1}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} = D \quad (11.93)$$

where D , the distance from the origin to the tangent plane, is given by Eq. (11.44).

The centrifugal force, F_{cl} , on mass element m_i [7] given by the second term of Eq. (11.82) is

$$F_{cl} = m_i a = -m_i \omega^2 r(t) \quad (11.94)$$

Substitution of the angular velocity given by Eq. (11.24) and m_e for m into Eq. (11.94) gives the centrifugal force F_c on the electron that is normal to the MO surface according to Eq. (11.93):

$$F_c = \frac{-\hbar^2}{m_e a^2 b^2} \mathbf{r}(t) \cdot \hat{\mathbf{d}}\mathbf{i}_\xi = \frac{-\hbar^2}{m_e a^2 b^2} D \mathbf{i}_\xi \quad (11.95)$$

F_c has an equivalent dependence on D as the electric force based on the charge distribution (Eq. (11.45)). This is expected based on the invariance of $\frac{e}{m_e}$ which results in the same

10 distribution of the mass and charge.

The equipotential charge-density distribution gives rise to the constant current condition. It also gives rise to a constant total kinetic energy condition wherein the angular velocity given by Eq. (11.24) is a constant. Recall from Eq. (11.32), that on the surface $\xi = 0$; ϕ must be independent of ζ and η and depend only on ξ at any point outside the ellipsoid $\xi = 0$. Since the current and total kinetic energy are also constant on the surface $\xi = 0$, the total kinetic energy depends only on ξ . Thus, the centrifugal force on the mass of the electron, m_e , must be in the same direction as the electric field corresponding to ϕ , normal to the electron surface wherein any tangential component in Eq. (11.94) averages to zero over the electron MO by the mass distribution given by Eqs. (11.40) and (11.45) with m_e replacing e .

The cancellation of tangential acceleration over each elliptical path maintains the charge density distribution given by Eq. (11.40) with constant current at each point on each elliptical path of the MO. Since the centrifugal force is given by Eq. (11.94), the multiplication of the mass density by the scaling factor h_i and integration with respect to ξ gives a constant net centrifugal force. Thus, the result matches those of the determination of the constant current (Eq. (11.90)) and angular momentum shown *infra*. (Eq. (11.101)) wherein the charge and mass densities given in Eqs. (11.90-11.91) and (11.100), respectively, were integrated over.

Specifically, consider the normal-directed centrifugal force, F_{cl} , on mass element m_i :

$$\mathbf{F}_{ci} = -m_i \omega^2 D \mathbf{i}_\xi \quad (11.96)$$

The mass density is given by Eq. (11.40) with m_e replacing e . Then, the substitution of the mass density for m_i in Eq. (11.96) and using Eq. (11.24) for ω gives the centrifugal force density \mathbf{F}_{ca} :

$$5 \quad \mathbf{F}_{ca} = \frac{m_e}{4\pi\sqrt{\eta\zeta}} \frac{\hbar^2}{m_e^2 a^2 b^2} D \mathbf{i}_\xi \quad (11.97)$$

Eq. (11.32) determines that the centrifugal force is a function of ξ alone, and any dependence on the transverse coordinate parameters averages to zero. Using the result of Eq. (11.88) gives the net centrifugal force \mathbf{F}_c :

$$\mathbf{F}_c = 8\pi \int_{\xi}^{\infty} \frac{1}{4\pi\sqrt{\eta\zeta}} \frac{\hbar^2}{m_e a^2 b^2} R_\xi \frac{1}{2} \frac{\sqrt{(\xi-\eta)(\xi-\zeta)}}{R_\xi} D \delta(\xi) d\xi \mathbf{i}_\xi = \frac{\hbar^2}{m_e a^2 b^2} D \mathbf{i}_\xi \quad (11.98)$$

10 In the limit as the ellipsoidal coordinates go over into spherical coordinates, Eq. (11.95) reduces to the centrifugal force of the spherical orbitsphere given by Eq. (1.232) with Eq. (1.47). This condition must be and is met as a further boundary condition that parallels that of Eqs. (11.32-11.37). Using the same dependence of the total mass(charge) on the scale factor h_i according to Eqs. (11.32-11.40), the further boundary conditions on the angular
15 momentum and kinetic energy are met.

Specifically, the constant potential and current conditions and the use of Eq. (11.32) in the derivation of Eq. (11.95) also satisfy another condition, the conservation of \hbar of angular momentum of the electron. The angular momentum \mathbf{p}_i at each point i of mass m_i is

$$\begin{aligned} \mathbf{p}_i(t) &= m_i \mathbf{r}(t) \times \mathbf{v}(t) \\ &= m_i (ia \cos \omega t + jb \sin \omega t) \times (-ia \omega \sin \omega t + jb \omega \cos \omega t) \\ &= m_i ab \omega (\cos^2 \omega t + \sin^2 \omega t) \mathbf{i} \times \mathbf{j} \\ &= m_i ab \omega \mathbf{k} \end{aligned} \quad (11.99)$$

20 The mass density is given by Eq. (11.40) with m_e replacing e . Then, substitution of m_i in Eq. (11.99) by the mass density and using Eq. (11.24) for ω gives the angular momentum density $\mathbf{p}(t)$:

$$\mathbf{p}(t) = ab \omega \frac{m_e}{4\pi\sqrt{\eta\zeta}} \mathbf{k} = ab \frac{\hbar}{m_e ab} \frac{m_e}{4\pi\sqrt{\eta\zeta}} \mathbf{k} \quad (11.100)$$

Using the result of Eq. (11.88) gives the total constant angular momentum \mathbf{L} :

$$\mathbf{L} = 8\pi \int_{\xi}^{\infty} \hbar \frac{1}{4\pi\sqrt{\eta\zeta}} R_{\xi} \frac{1}{2} \frac{\sqrt{(\xi-\eta)(\xi-\zeta)}}{R_{\xi}} \delta(\xi) d\xi \mathbf{k} = \hbar \mathbf{k} \quad (11.101)$$

Eq. (11.101) demonstrates conservation of angular momentum that is a function of ξ alone that parallels the case of atomic electrons where \mathbf{L} conservation is a function of the radius r alone as given by Eq. (1.57).

5 Similarly, the kinetic energy $T(t)$ at each point i of mass m_i is

$$\begin{aligned} T(t) &= \frac{1}{2} m_i v(t)^2 \\ &= \frac{1}{2} m_i (-ia\omega \sin \omega t + jb\omega \cos \omega t)^2 \\ &= \frac{1}{2} m_i \omega^2 (a^2 \sin^2 \omega t + b^2 \cos^2 \omega t) \end{aligned} \quad (11.102)$$

In Eqs. (11.96-11.98), m_i was replaced by the mass density and the ξ integral was determined to give the centrifugal force in terms of the mass of the electron. The kinetic energy can also be determined from the ξ integral of the centrifugal force:

$$10 \quad T = h_{2cc} F_c \frac{ab^2}{2D} \int_{\xi}^{\infty} \frac{d\xi}{R_{\xi}} \quad (11.103)$$

The result is given in Eq. (11.119). From Eq. (11.102), the kinetic energy is time (position) dependent, but the total kinetic energy corresponding to the centrifugal force given by Eq. (11.95) satisfies the condition that the time-averaged kinetic energy is 1/2 the time-averaged potential energy for elliptic motion in an inverse-squared central force [1]. (Here, the
15 potential and total kinetic energies are constant and correspond to the time-averaged energies of the general case.) Thus, as shown by Eqs. (11.122) (11.124), (11.262), and (11.264) energy is conserved.

Force Balance of Hydrogen-type Molecular Ions

20 Consider the case of spheroidal coordinates based on the rotational symmetry about the semimajor axis [2]. In the limit, as the focal distance $2c$ and the eccentricity of the series of confocal ellipses approaches zero, spheroidal coordinates go over into spherical coordinates with $\xi \rightarrow r$ and $\eta \rightarrow \cos \theta$. The field of an equipotential two-dimensional charge surface of constant radius $r = R$ is equivalent to that of a point charge of the total charge of the
25 spherical shell at the origin. The force balance between the centrifugal force and the central Coulomb force for spherical symmetry is given by Eq. (1.232).

Similarly, the centrifugal force is the direction of ξ and balances the central Coulombic force between the protons at the foci and the electron MO. In the case of the prolate spheroidal MO, the inhomogeneous equation given by Eq. (11.5) must hold for each fixed position of $r(t)$ since the MO is static in time due to the constant current condition.

- 5 With $r(t)$ fixed, the $m\ddot{r}$ term of Eq. (11.5) is zero, and the force balanced equation is the balance between the centrifugal force and the Coulombic force which are both normal to the surface of the elliptic orbit:

$$mr\dot{\theta}^2 = f(r) \quad (11.104)$$

- Substitution of Eq. (11.81) and Eq. (11.95) into Eq. (11.104) gives the force balance between
10 the centrifugal and electric central forces:

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{2pe^2}{4\pi\epsilon_0} \frac{\left(1 - \frac{a_0}{2ap}\right)^2}{ab^2 \left(2 + \frac{a_0}{2ap}\right)} D \quad (11.105)$$

$$\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2ap} \left(2 + \frac{a_0}{2ap}\right) = \left(1 - \frac{a_0}{2ap}\right)^2 \quad (11.106)$$

$$\frac{a_0}{ap} + \left(\frac{a_0}{2ap}\right)^2 = \left(1 - \frac{a_0}{2ap}\right)^2 \quad (11.107)$$

$$\frac{a_0}{ap} + \left(\frac{a_0}{2ap}\right)^2 = 1 - \frac{a_0}{ap} + \left(\frac{a_0}{2ap}\right)^2 \quad (11.108)$$

15 $a = 2 \frac{a_0}{p} \quad (11.109)$

Substitution of a given by Eq. (11.109) into Eq. (11.79) gives

$$c' = \frac{a_0}{p} \quad (11.110)$$

The internuclear distance from Eq. (11.110) is

$$2c' = \frac{2a_0}{p} \quad (11.111)$$

- 20 Substitution of $a = \frac{2a_0}{p}$ and $c' = \frac{a_0}{p}$ into Eq. (11.80) gives the length of the semiminor axis

of the prolate spheroidal MO, $b = c$:

$$b = \frac{\sqrt{3}}{p} a_o \quad (11.112)$$

Substitution of $a = \frac{2a_o}{p}$ and $c' = \frac{a_o}{p}$ into Eq. (11.67) gives the eccentricity, e :

$$e = \frac{1}{2} \quad (11.113)$$

From Eqs. (11.63-11.65), the result of Eq. (11.113) can be used to obtain the electric force F_{ele} between the protons and the ellipsoidal MO as

$$F_{ele} = ZeEi_\xi = h_{2cc}^{-1} \frac{p2e^2}{4\pi\epsilon_o ab^2} Di_\xi = \frac{pe^2}{8\pi\epsilon_o} Di_\xi \quad (11.114)$$

where the electric field E of the MO is given by Eq. (11.49). Then, the force balance of the hydrogen-type molecular ion is given by

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{pe^2}{8\pi\epsilon_o} D \quad (11.115)$$

which has the parametric solution given by Eq. (11.83) when

$$a = \frac{2a_o}{p} \quad (11.116)$$

The solutions for the prolate spheroidal axes and eccentricity are given by Eqs. (11.109-11.113).

15 ENERGIES OF HYDROGEN-TYPE MOLECULAR IONS

From Eq. (11.31), the magnitude of the ellipsoidal field corresponding to a below "ground state" hydrogen-type molecule is an integer, p . The force balance equation (Eq. (11.115)) applies for each point of the electron MO having non-constant charge (mass)-density and velocity over the equipotential and equal energy surface. The electron potential and kinetic energies are thus determined from an ellipsoidal integral.

The potential energy is doubled due to the transverse electric force. The force normal to the MO is given by the dot product of the sum of the force vectors from each focus with \hat{d} where the angle β is $\beta = \frac{\pi}{2} - \alpha$, and the transverse forces are given by the cross product with \hat{d} . As shown in Figure 1, equivalently, the transverse projection is given with the angle α replacing β where the range of α is the same as β . The two contributions to the

potential energy doubles it. The potential energy, V_e , of the electron MO in the field of magnitude p times that of the two protons at the foci is

$$\begin{aligned}
 V_e &= 2 \frac{-2pe^2}{4\pi\epsilon_0} D \frac{ab^2}{2D} \int_{\xi}^{\infty} \frac{d\xi}{R_{\xi}} \\
 &= \frac{-4pe^2}{8\pi\epsilon_0} \int_{\xi}^{\infty} \frac{d\xi}{(\xi+b)\sqrt{\xi+a}} \\
 &= \frac{-4pe^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \\
 &= \frac{-4pe^2}{8\pi\epsilon_0 c'} \ln \frac{a+c'}{a-c'}
 \end{aligned} \tag{11.117}$$

where

$$5 \quad \sqrt{a^2-b^2} = c' \tag{11.118}$$

$2c'$ is the distance between the foci which is the internuclear distance. The kinetic energy, T , of the electron MO follows from the same type of integral as V_e using Eqs. (7-14) of Stratton [8], Eqs. (11.37-11.46), and integral #147 of Lide [9]. T is given by the corresponding integral of the centrifugal force (LHS of Eq. (11.115)) with the constraint that the current motion allows the equipotential and equal energy condition with a central field due to the protons; thus, it is corrected by the scale factor h_{2cc} given by Eq. (11.62). The h_{2cc} correction can be considered the scaling factor of the moment of inertial such that the kinetic energy is equivalent to the rotational energy for constant angular frequency ω . The kinetic energy, T , of the electron MO is given by

$$15 \quad T = h_{2cc} \frac{-\hbar^2}{m_e a^2 b^2} D \frac{ab^2}{2D} \int_{\xi}^{\infty} \frac{d\xi}{R_{\xi}} = \frac{-4\hbar^2}{2m_e a} \int_{\xi}^{\infty} \frac{d\xi}{(\xi+b)\sqrt{\xi+a}} = \frac{-2\hbar^2}{m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \tag{11.119}$$

The potential energy, V_p , due to proton-proton repulsion in the field of magnitude p times that of the protons at the foci ($\xi = 0$) is

$$V_p = \frac{pe^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \tag{11.120}$$

20 The total energy, E_T , is given by the sum of the energy terms

$$E_T = V_e + V_p + T \tag{11.121}$$

Substitution of a and b given by Eqs. (11.109) and (11.112), respectively, into Eqs. (11.117), (11.119), (11.120), and (11.121) gives

$$V_e = \frac{-4p^2 e^2}{8\pi\epsilon_0 a_0} \ln 3 \quad (11.122)$$

$$V_p = \frac{p^2 e^2}{8\pi\epsilon_0 a_0} \quad (11.123)$$

$$T = \frac{2p^2 e^2}{8\pi\epsilon_0 a_0} \ln 3 \quad (11.124)$$

$$E_T = -13.6 \text{ eV} (4p^2 \ln 3 - p^2 - 2p^2 \ln 3) = -p^2 16.28 \text{ eV} \quad (11.125)$$

- 5 The total energy, which includes the proton-proton-repulsion term is negative which justifies the original treatment of the force balance using the analytical-mechanics equations of an ellipse that considered only the binding force between the protons and the electron and the electron centrifugal force. T is one-half the magnitude of V_e as required for an inverse-squared force [1] wherein V_e is the source of T .

10

VIBRATION OF HYDROGEN-TYPE MOLECULAR IONS

A charge, q , oscillating according to $r_0(t) = d \sin \omega_0 t$ has a Fourier spectrum

$$J(k, \omega) = \frac{q\omega_0 d}{2} J_m(k \cos \theta d) \{ \delta[\omega - (m+1)\omega_0] + \delta[\omega - (m-1)\omega_0] \} \quad (11.126)$$

where J_m 's are Bessel functions of order m . These Fourier components can, and do, acquire

- 15 phase velocities that are equal to the velocity of light [10]. The protons of hydrogen-type molecular ions and molecules oscillate as simple harmonic oscillators; thus, vibrating protons will radiate. Moreover, non-oscillating protons may be excited by one or more photons that are resonant with the oscillatory resonance frequency of the molecule or molecular ion, and oscillating protons may be further excited to higher energy vibrational states by resonant
20 photons. The energy of a photon is quantized according to Planck's equation

$$E = \hbar\omega \quad (11.127)$$

The energy of a vibrational transition corresponds to the energy difference between the initial and final vibrational states. Each state has an electromechanical resonance frequency, and the emitted or absorbed photon is resonant with the difference in frequencies. Thus, as a general
25 principle, quantization of the vibrational spectrum is due to the quantized energies of photons and the electromechanical resonance of the vibrationally excited ion or molecule.

It is shown by Fowles [11] that a perturbation of the orbit determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit. In a circular orbit in spherical coordinates, the transverse equation of motion gives

$$\dot{\theta} = \frac{L/m}{r^2} \quad (11.128)$$

5 where L is the angular momentum. The radial equation of motion is

$$m(\ddot{r} - r\dot{\theta}^2) = f(r) \quad (11.129)$$

Substitution of Eq. (11.128) into Eq. (11.129) gives

$$m\ddot{r} - \frac{m(L/m)^2}{r^3} = f(r) \quad (11.130)$$

For a circular orbit, r is a constant and $\ddot{r} = 0$. Thus, the radial equation of motion is given by

$$10 \quad -\frac{m(L/m)^2}{a^3} = f(a) \quad (11.131)$$

where a is the radius of the circular orbit for central force $f(a)$ at $r = a$. A perturbation of the radial motion may be expressed in terms of a variable x defined by

$$x = r - a \quad (11.132)$$

The differential equation can then be written as

$$15 \quad m\ddot{x} - m(L/m)^2 (x+a)^{-3} = f(x+a) \quad (11.133)$$

Expanding the two terms involving $x+a$ as a power series in x , gives

$$m\ddot{x} - m(L/m)^2 a^{-3} \left(1 - 3\frac{x}{a} + \dots\right) = f(a) + f'(a)x + \dots \quad (11.134)$$

Substitution of Eq. (11.131) into Eq. (11.134) and neglecting terms involving x^2 and higher powers of x gives

$$20 \quad m\ddot{x} + \left[\frac{-3}{a} f(a) - f'(a) \right] x = 0 \quad (11.135)$$

For an inverse-squared central field, the coefficient of x in Eq. (11.135) is positive, and the equation is the same as that of the simple harmonic oscillator. In this case, the particle, if perturbed, oscillates harmonically about the circle $r = a$, and an approximation of the angular frequency of this oscillation is

$$25 \quad \omega = \sqrt{\frac{\left[\frac{-3}{a} f(a) - f'(a) \right]}{m}} = \sqrt{\frac{k}{m}} \quad (11.136)$$

An apsis is a point in an orbit at which the radius vector assumes an extreme value (maximum or minimum). The angle swept out by the radius vector between two consecutive apsides is called the apsidal angle. Thus, the apsidal angle is π for elliptical orbits under the inverse-squared law of force. In the case of a nearly circular orbit, Eq. (11.135) shows that r oscillates about the circle $r = a$, and the period of oscillation is given by

$$\tau_r = 2\pi \sqrt{\frac{m}{-\left[\frac{3}{a}f(a) + f'(a)\right]}} \quad (11.137)$$

The apsidal angle in this case is just the amount by which the polar angle θ increases during the time that r oscillates from a minimum value to the succeeding maximum value which is τ_r . From Eq. (11.128), $\dot{\theta} = \frac{L/m}{r^2}$; therefore, θ remains constant, and Eq. (11.131) gives

$$\dot{\theta} \approx \frac{L/m}{a^2} = \left[-\frac{f(a)}{ma} \right]^{1/2} \quad (11.138)$$

Thus, the apsidal angle is given by

$$\psi = \frac{1}{2}\tau_r\dot{\theta} = \pi \left[3 + a \frac{f'(a)}{f(a)} \right]^{-1/2} \quad (11.139)$$

Thus, the power force of $f(r) = -cr^n$ gives

$$\psi = \pi(3+n)^{-1/2} \quad (11.140)$$

The apsidal angle is independent of the size of the orbit in this case. The orbit is re-entrant, or repetitive, in the case of the inverse-squared law ($n = -2$) for which $\psi = \pi$.

A prolate spheroid MO and the definition of axes are shown in Figures 5A and 5B, respectively. Consider the two nuclei A and B, each at focus of the prolate spheroid MO. From Eqs. (11.115), (11.117), and (11.119), the attractive force between the electron and each nucleus at a focus is

$$f(a) = -\frac{pe^2}{4\pi\epsilon_0 a^2} \quad (11.141)$$

and

$$f'(a) = \frac{2pe^2}{4\pi\epsilon_0 a^3} \quad (11.142)$$

In addition to the attractive force between the electron and the nuclei, there is a repulsive force between the two nuclei that is the source of a corresponding reactive force on

the reentrant electron orbit. Consider an elliptical orbital plane cross section of the MO in the xy-plane with a nucleus A at $(-c', 0)$ and a nucleus B at $(c', 0)$. For B acting as the attractive focus, the reactive repulsive force at the point $(a, 0)$, the positive semimajor axis, depends on the distance from $(a, 0)$ to nucleus A at $(-c', 0)$ (i.e. the distance from the position of the electron MO at the semimajor axis to the opposite nuclear repelling center at the opposite focus). The distance is given by the sum of the semimajor axis, a , and c' , 1/2 the internuclear distance. The contribution from the repulsive force between the two protons is

$$f(a+c') = \frac{pe^2}{8\pi\epsilon_0(a+c')^2} \quad (11.143)$$

and

$$f'(a+c') = -\frac{pe^2}{4\pi\epsilon_0(a+c')^3} \quad (11.144)$$

Thus, from Eqs. (11.136) and (11.141-11.144), the angular frequency of this oscillation is

$$\begin{aligned} \omega &= \sqrt{\frac{\frac{pe^2}{4\pi\epsilon_0 a^3} - \frac{pe^2}{8\pi\epsilon_0 (a+c')^3}}{\mu}} \\ &= \sqrt{\frac{\frac{pe^2}{4\pi\epsilon_0 \left(\frac{2a_H}{p}\right)^3} - \frac{pe^2}{8\pi\epsilon_0 \left(\frac{3a_H}{p}\right)^3}}{\mu}} \\ &= p^2 4.44865 \times 10^{14} \text{ rad/s} \end{aligned} \quad (11.145)$$

where the semimajor axis, a , is $a = \frac{2a_H}{p}$ according to Eq. (11.116) and c' is $c' = \frac{a_H}{p}$ according to Eq. (11.110).

In the case of a hydrogen molecule or molecular ion, the electrons which have a mass of 1/1836 that of the protons move essentially instantaneously, and the charge density is that of a continuous membrane. Thus, a stable electron orbit is maintained with oscillatory motion of the protons. Hydrogen molecules and molecular ions are symmetrical along the semimajor axis; thus, the oscillatory motion of protons is along this axis. Let x be the increase in the semimajor due to the reentrant orbit with a corresponding displacement of the protons along the semimajor axis from the position of the initial foci of the stationary state. The equation of proton motion due to the perturbation of an orbit having a central inverse-squared central force [1] and neglecting terms involving x^2 and higher is given by

$$\mu\ddot{x} + kx = 0 \quad (11.146)$$

which has the solution in terms of the maximum amplitude of oscillation, A , the reduced nuclear mass, μ , the restoring constant or spring constant, k , the resonance angular frequency, ω_0 , and the vibrational energy, E_{vib} , [12]

$$5 \quad A \cos \omega_0 t \quad (11.147)$$

where

$$\omega_0 = \sqrt{\frac{k}{\mu}} \quad (11.148)$$

For a symmetrical displacement x , the potential energy corresponding to the oscillation E_{pvib} is given by

$$10 \quad E_{pvib} = 2 \left(\frac{1}{2} kx^2 \right) = kx^2 \quad (11.149)$$

The total energy of the oscillating molecular ion, $E_{Totalvib}$, is given as the sum of the kinetic and potential energies

$$E_{Totalvib} = \frac{1}{2} \mu \dot{x}^2 + kx^2 \quad (11.150)$$

The velocity is zero when x is the maximum amplitude, A . The total energy of the
15 oscillating molecular ion, $E_{Totalvib}$, is then given as the potential energy with $x = A$

$$E_{Totalvib} = kA^2 \quad (11.151)$$

Thus,

$$A = \sqrt{\frac{E_{Totalvib}}{k}} \quad (11.152)$$

It is shown in the Excited States of the One-Electron Atom (Quantization) section that
20 the change in angular frequency of the electron orbitsphere (Eq. (2.21)) is identical to the angular frequency of the photon necessary for the excitation, ω_{photon} (Eq. (2.19)). The energy of the photon necessary to excite the equivalent transition in an electron orbitsphere is one-half of the excitation energy of the stationary cavity because the change in kinetic energy of the electron orbitsphere supplies one-half of the necessary energy. The change in the angular
25 frequency of the orbitsphere during a transition and the angular frequency of the photon corresponding to the superposition of the free space photon and the photon corresponding to the kinetic energy change of the orbitsphere during a transition are equivalent. The correspondence principle holds. It can be demonstrated that the resonance condition between

these frequencies is to be satisfied in order to have a net change of the energy field [13]. The bound electrons are excited with the oscillating protons. Thus, the mechanical resonance frequency, ω_0 , is only one-half that of the electromechanical frequency which is equal to the frequency of the free space photon, ω , which excites the vibrational mode of the hydrogen molecule or hydrogen molecular ion. The vibrational energy, E_{vib} , corresponding to the photon is given by

$$E_{vib} = \hbar\omega = \hbar\omega_0 = \hbar\sqrt{\frac{k}{\mu}} = 2kA^2 \quad (11.153)$$

where Planck's equation (Eq. (11.127)) was used. The reduced mass is given by

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \quad (11.154)$$

10 Thus,

$$A = \sqrt{\frac{\hbar\omega_0}{2k}} \quad (11.155)$$

Since the protons and electron are not fixed, but vibrate about the center of mass, the maximum amplitude is given by the reduced amplitude, $A_{reduced}$, given by

$$A_{reduced} = \frac{A_1 A_2}{A_1 + A_2} \quad (11.156)$$

15 where A_n is the amplitude n if the origin is fixed. Thus, Eq. (11.155) becomes

$$A_{reduced} = \frac{1}{2} \sqrt{\frac{\hbar\omega_0}{2k}} \quad (11.157)$$

and from Eq. (11.148), $A_{reduced}$ is

$$A_{reduced} = \frac{1}{2} \sqrt{\frac{\hbar\omega_0}{2k}} = \frac{1}{2} \sqrt{\frac{\hbar}{2k}} \left(\frac{k}{\mu} \right)^{1/4} = \frac{\sqrt{\hbar}}{2^{3/2} (k\mu)^{1/4}} \quad (11.158)$$

Then, from Eq. (11.67), A_c , the displacement of c' is the eccentricity e given by Eq. (11.113) times $A_{reduced}$ (Eq. (11.158)):

$$A_c = eA_{reduced} = \frac{A_{reduced}}{2} = \frac{\sqrt{\hbar}}{2^{5/2} (k\mu)^{1/4}} \quad (11.159)$$

Thus, during bond formation, the perturbation of the orbit determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit, and the

corresponding frequency, $\omega(0)$, for a hydrogen-type molecular ion $H_2^+(1/p)$ given by Eqs. (11.136) and (11.145) is

$$\omega(0) = p^2 \sqrt{\frac{k(0)}{\mu}} = p^2 \sqrt{\frac{165.51 \text{ Nm}^{-1}}{\mu}} = p^2 4.449 \times 10^{14} \text{ radians/s} \quad (11.160)$$

where the reduced nuclear mass of hydrogen given by Eq. (11.154) is

$$\mu = 0.5m_p \quad (11.161)$$

and the spring constant, $k(0)$, given by Eqs. (11.136) and (11.145) is

$$k(0) = p^4 165.51 \text{ Nm}^{-1} \quad (11.162)$$

The transition-state vibrational energy, $E_{vib}(0)$, is given by Planck's equation (Eq. (11.127)):

$$E_{vib}(0) = \hbar\omega = \hbar p^2 4.44865 \times 10^{14} \text{ rad/s} = p^2 0.2928 \text{ eV} \quad (11.163)$$

10 The amplitude of the oscillation, $A_{reduced}(0)$, given by Eq. (11.158) and Eqs. (11.161-11.162) is

$$A_{reduced}(0) = \frac{\sqrt{\hbar}}{2^{3/2} (p^4 165.51 \text{ Nm}^{-1} \mu)^{1/4}} = \frac{5.952 \times 10^{-12} \text{ m}}{p} = 0.1125 \frac{a_0}{p} \quad (11.164)$$

Then, from Eq. (11.67), $A_c(0)$, the displacement of c' is the eccentricity e given by Eq. (11.113) times $A_{reduced}(0)$ (Eq. (11.164)):

$$15 \quad A_c(0) = e A_{reduced}(0) = \frac{A_{reduced}(0)}{2} = \frac{\sqrt{\hbar}}{2^{5/2} (k\mu)^{1/4}} = \frac{0.05624 a_0}{p} \quad (11.165)$$

The spring constant and vibrational frequency for the formed molecular ion are then obtained from Eqs. (11.136) and (11.141-11.145), using the increases in the semimajor axis and internuclear distances due to vibration in the transition state. The vibrational energy, $E_{vib}(1)$, for the $H_2^+(1/p)$ $\nu=1 \rightarrow \nu=0$ transition given by adding $A_c(0)$ (Eq. (11.159)) to the
20 distances a and $a+c'$ in Eqs. (11.145) and (11.163) is

$$E_{vib}(1) = p^2 0.270 \text{ eV} \quad (11.166)$$

where ν is the vibrational quantum number.

A harmonic oscillator is a linear system as given by Eq. (11.146). In this case, the predicted resonant vibrational frequencies and energies, spring constants, and amplitudes for
25 $H_2^+(1/p)$ for vibrational transitions to higher energy $\nu_i \rightarrow \nu_f$ are given by $(\nu_f - \nu_i)$ times the corresponding parameters given by Eq. (11.160) and Eqs. (11.162-11.164). However,

excitation of vibration of the molecular ion by external radiation causes the semimajor axis and, consequently, the internuclear distance to increase as a function of the vibrational quantum number ν . Consequently, the vibrational energies of hydrogen-type molecular ions are nonlinear as a function of the vibrational quantum number ν . The lines become more
 5 closely spaced and the change in amplitude, $\Delta A_{\text{reduced}}$, between successive states becomes larger as higher states are excited due to the distortion of the molecular ion in these states. The energy difference of each successive transition of the vibrational spectrum can be obtained by considering nonlinear terms corresponding to anharmonicity.

The harmonic oscillator potential energy function can be expanded about the
 10 internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter (K&P) [14] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods. The energy $\tilde{\nu}_\nu$ of state ν is

$$\tilde{\nu}_\nu = \nu\omega_0 - \nu(\nu-1)\omega_0x_0, \quad \nu = 0, 1, 2, 3... \quad (11.167)$$

15 where

$$\omega_0x_0 = \frac{hc\omega_0^2}{4D_0} \quad (11.168)$$

ω_0 is the frequency of the $\nu = 1 \rightarrow \nu = 0$ transition corresponding to Eq. (11.166), and D_0 is the bond dissociation energy given by Eq. (11.198). From Eqs. (11.166), (11.168), and (11.198),

$$20 \quad \omega_0x_0 = \frac{100hc \left(8.06573 \times 10^3 \frac{\text{cm}^{-1}}{\text{eV}} p^2 0.270 \text{ eV} \right)^2}{4e(p^2 2.535 \text{ eV} + p^3 0.118755 \text{ eV})} \text{cm}^{-1} \quad (11.169)$$

The vibrational energies of successive states are given by Eqs. (11.166-11.167) and (11.169).

Using Eqs. (11.145), (11.158-11.160), (11.162-11.169), and (11.199) the corresponding parameters for deuterium-type molecular ions with

$$\mu = m_p \quad (11.170)$$

25 are

$$\omega(0) = p^2 \sqrt{\frac{k(0)}{\mu}} = p^2 \sqrt{\frac{165.65 \text{ Nm}^{-1}}{\mu}} = p^2 3.147 \times 10^{14} \text{ radians/s} \quad (11.171)$$

$$k(0) = p^4 165.65 \text{ Nm}^{-1} \quad (11.172)$$

$$E_{\text{vib}}(0) = p^2 0.20714 \text{ eV} \quad (11.173)$$

$$A_{reduced}(0) = \frac{\sqrt{\hbar}}{2^{3/2} (p^4 165.65 \text{ Nm}^{-1} \mu)^{1/4}} = \frac{5.004 \times 10^{-12} \text{ m}}{p} = 0.09457 \frac{a_0}{p} \quad (11.174)$$

$$E_{vib}(1) = p^2 0.193 \text{ eV} \quad (11.175)$$

$$\omega_0 x_0 = \frac{100hc \left(8.06573 \times 10^3 \frac{\text{cm}^{-1}}{\text{eV}} p^2 0.193 \text{ eV} \right)^2}{4e(p^2 2.5770 \text{ eV} + p^3 0.118811 \text{ eV})} \text{cm}^{-1} \quad (11.176)$$

The vibrational energies of successive states are given by Eqs. (11.167) and (11.175-11.176).

5

THE DOPPLER ENERGY TERM OF HYDROGEN-TYPE MOLECULAR IONS

As shown in the Vibration of Hydrogen-type Molecular Ions section, the electron orbiting the nuclei at the foci of an ellipse may be perturbed such that a stable reentrant orbit is established that gives rise to a vibrational state corresponding to time harmonic oscillation of the nuclei and electron. The perturbation is caused by a photon that is resonant with the frequency of oscillation of the nuclei wherein the radiation is electric dipole with the corresponding selection rules.

Oscillation may also occur in the transition state. The perturbation arises from the decrease in internuclear distance as the molecular bond forms. Relative to the unperturbed case given in the Force Balance of Hydrogen-type Molecular Ions section, the reentrant orbit may give rise to a decrease in the total energy while providing a transient kinetic energy to the vibrating nuclei. However, as an additional condition for stability, radiation must be considered. Regarding the potential for radiation, the nuclei may be considered point charges.

20 A point charge undergoing periodic motion accelerates and as a consequence radiates according to the Larmor formula (cgs units) [15]:

$$P = \frac{2e^2}{3c^3} |\dot{\mathbf{v}}|^2 \quad (11.177)$$

where e is the charge, $\dot{\mathbf{v}}$ is its acceleration, and c is the speed of light. The radiation has a corresponding force that can be determined based on conservation of energy with radiation.

25 The radiation reaction force, \mathbf{F}_{rad} , given by Jackson [16] is

$$\mathbf{F}_{rad} = \frac{2}{3} \frac{e^2}{c^3} \ddot{\mathbf{v}} \quad (11.178)$$

Then, the Abraham-Lorentz equation of motion is given by [16]

$$m \left(\dot{v} - \frac{2}{3} \frac{e^2}{mc^3} \ddot{v} \right) = F_{\text{ext}} \quad (11.179)$$

where F_{ext} is the external force and m is the mass. The external force for the vibrating system is given by Eq. (11.146).

$$F_{\text{ext}} = kx \quad (11.180)$$

5 where x is the displacement of the protons along the semimajor axis from the position of the initial foci of the stationary state in the absence of vibration with a reentrant orbit of the electron. A nonradiative state must be achieved after the emission due to transient vibration wherein the nonradiative condition given by Eq. (11.24) must be satisfied.

As shown in the Resonant Line Shape and Lamb Shift section, the spectroscopic
10 linewidth arises from the classical rise-time band-width relationship, and the Lamb Shift is due to conservation of energy and linear momentum and arises from the radiation reaction force between the electron and the photon. The radiation reaction force in the case of the vibration of the molecular ion in the transition state corresponds to a Doppler energy, E_D , that is dependent on the motion of the electron and the nuclei. The Doppler energy of the
15 electron is given by Eq. (2.146)

$$\bar{E}_D \cong 2\sqrt{E_K E_R} = E_{\text{mv}} \sqrt{\frac{2\bar{E}_K}{Mc^2}} \quad (11.181)$$

where E_R is the recoil energy which arises from the photon's linear momentum given by Eq. (2.141), E_K is the vibrational kinetic energy of the reentrant orbit in the transition state, and M is the mass of the electron m_e .

20 As given in the Vibration of Hydrogen-Type Molecular Ions section, for inverse-squared central field, the coefficient of x in Eq. (11.135) is positive, and the equation is the same as that of the simple harmonic oscillator. Since the electron of the hydrogen molecular ion is perturbed as the internuclear separation decreases with bond formation, it oscillates harmonically about the semimajor axis given by Eq. (11.116), and an approximation of the
25 angular frequency of this oscillation is

$$\omega = \sqrt{\frac{\left[\frac{-3}{a} f(a) - f'(a) \right]}{m_e}} = \sqrt{\frac{k}{m_e}} \quad (11.182)$$

From Eqs. (11.115), (11.117), and (11.119), the central force terms between the electron MO and the two protons are

$$f(a) = -\frac{2pe^2}{4\pi\epsilon_0 a^2} \quad (11.183)$$

and

$$f'(a) = \frac{4pe^2}{4\pi\epsilon_0 a^3} \quad (11.184)$$

Thus, the angular frequency of this oscillation is

$$\omega = \sqrt{\frac{\frac{2pe^2}{4\pi\epsilon_0 \left(\frac{2a_H}{p}\right)^3}}{m_e}} = p^2 2.06538 \times 10^{16} \text{ rad/s} \quad (11.185)$$

where the semimajor axis, a , is $a = \frac{2a_H}{p}$ according to Eq. (11.116) including the reduced electron mass. The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar p^2 2.06538 \times 10^{16} \text{ rad/s} = p^2 13.594697 \text{ eV} \quad (11.186)$$

In Eq. (11.181), substitution of the total energy of the hydrogen molecular ion, E_T , (Eq. (11.125)) for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (11.186) for \bar{E}_K gives the Doppler energy of the electron for the reentrant orbit.

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -p^2 16.28034 \text{ eV} \sqrt{\frac{2e(p^2 13.594697 \text{ eV})}{m_e c^2}} = -p^3 0.118755 \text{ eV} \quad (11.187)$$

The total energy of the molecular ion is decreased by \bar{E}_D .

In addition to the electron, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency given in the Vibration of Hydrogen-Type Molecular Ions section. On average, the total energy of vibration is equally distributed between kinetic energy and potential energy [17]. Thus, the average kinetic energy of vibration corresponding to the Doppler energy of the electrons, \bar{E}_{Kvib} , is 1/2 of the vibrational energy of the molecular ion given by Eq. (11.166). The decrease in the energy of the hydrogen molecular ion due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electron and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D and \bar{E}_{Kvib} . Using Eq. (11.187) and E_{vib} from Eq. (11.166) gives

51

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar p^2 \sqrt{\frac{k}{\mu}} \quad (11.188)$$

$$\bar{E}_{osc} = -p^3 0.118755 \text{ eV} + \frac{1}{2} p^2 (0.29282 \text{ eV}) \quad (11.189)$$

To the extent that the MO dimensions are the same, the electron reentrant orbital energies \bar{E}_K are the same independent of the isotope of hydrogen, but the vibrational energies are related by Eq. (11.148). Thus, the differences in bond energies are essentially given by 1/2 the differences in vibrational energies. Using Eq. (11.187) with the deuterium reduced electron mass for E_T and \bar{E}_D , and E_{vib} for D_2^+ ($1/p$) given by Eq. (11.173), that corresponds to the deuterium reduced nuclear mass (Eq. (11.170)), the corresponding \bar{E}_{osc} is

$$\bar{E}_{osc} = -p^3 0.118811 \text{ eV} + \frac{1}{2} p^2 (0.20714 \text{ eV}) \quad (11.190)$$

10

TOTAL, IONIZATION, AND BOND ENERGIES OF HYDROGEN AND DEUTERIUM MOLECULAR IONS

The total energy of the hydrogen molecular ion which is equivalent to the negative of the ionization energy is given by the sum of E_T (Eqs. (11.121) and (11.125)) and \bar{E}_{osc} given by Eqs. (11.185-11.188). Thus, the total energy of the hydrogen molecular ion having a central field of $+pe$ at each focus of the prolate spheroid molecular orbital including the Doppler term is

$$E_T = V_e + V_p + T + \bar{E}_{osc} \quad (11.191)$$

$$E_T = -p^2 \left\{ \frac{e^2}{8\pi\epsilon_0 a_H} (4\ln 3 - 1 - 2\ln 3) \left[1 + p \sqrt{\frac{2\hbar \sqrt{\frac{2e^2}{4\pi\epsilon_0 (2a_H)^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} \quad (11.192)$$

$$= -p^2 16.2803 \text{ eV} - p^3 0.118811 \text{ eV} + \frac{1}{2} p^2 \hbar \sqrt{\frac{k}{\mu}}$$

From Eqs. (11.189) and (11.191-11.192), the total energy for hydrogen-type molecular ions is

$$\begin{aligned}
E_T &= -p^2 16.28033 \text{ eV} + \bar{E}_{osc} \\
&= -p^2 16.28033 \text{ eV} - p^3 0.118755 \text{ eV} + \frac{1}{2} p^2 (0.29282 \text{ eV}) \\
&= -p^2 16.13392 \text{ eV} - p^3 0.118755 \text{ eV}
\end{aligned} \tag{11.193}$$

The total energy of the deuterium molecular ion is given by the sum of E_T (Eq. (11.125)) corrected for the reduced electron mass of D and \bar{E}_{osc} given by Eq. (11.190):

$$\begin{aligned}
E_T &= -p^2 16.284 \text{ eV} + \bar{E}_{osc} \\
&= -p^2 16.284 \text{ eV} - p^3 0.118811 \text{ eV} + \frac{1}{2} p^2 (0.20714 \text{ eV}) \\
&= -p^2 16.180 \text{ eV} - p^3 0.118811 \text{ eV}
\end{aligned} \tag{11.194}$$

- 5 The bond dissociation energy, E_D , is the difference between the total energy of the corresponding hydrogen atom or $H(1/p)$ atom [18-19], called hydrino atom having a principal quantum number $1/p$ where p is an integer, and E_T .

$$E_D = E(H(1/p)) - E_T \tag{11.195}$$

where [18]

$$10 \quad E(H(1/p)) = -p^2 13.59844 \text{ eV} \tag{11.196}$$

and [19]

$$E(D(1/p)) = -p^2 13.603 \text{ eV} \tag{11.197}$$

The hydrogen molecular ion bond energy, E_D , is given by Eq. (11.193) with the reduced electron mass and Eqs. (11.195-11.196):

$$\begin{aligned}
E_D &= -p^2 13.59844 - E_T \\
15 \quad &= -p^2 13.59844 - (-p^2 16.13392 \text{ eV} - p^3 0.118755 \text{ eV}) \\
&= p^2 2.535 \text{ eV} + p^3 0.118755 \text{ eV}
\end{aligned} \tag{11.198}$$

The deuterium molecular ion bond energy, E_D , is given by Eq. (11.194) with the reduced electron mass of D and Eqs. (11.195) and (11.197):

$$\begin{aligned}
E_D &= -p^2 13.603 - E_T \\
&= -p^2 13.603 - (-p^2 16.180 \text{ eV} - p^3 0.118811 \text{ eV}) \\
&= p^2 2.5770 \text{ eV} + p^3 0.118811 \text{ eV}
\end{aligned} \tag{11.199}$$

20 HYDROGEN-TYPE MOLECULES

FORCE BALANCE OF HYDROGEN-TYPE MOLECULES

Hydrogen-type molecules comprise two indistinguishable electrons bound by an elliptic field. Each electron experiences a centrifugal force, and the balancing centripetal force (on each electron) is produced by the electric force between the electron and the elliptic electric field and the magnetic force between the two electrons causing the electrons to pair. In addition to nonradiation, the angular frequency given by Eq. (11.24) corresponds to a Lorentzian invariant magnetic moment of a Bohr magneton, μ_B , as given in the Magnetic Moment of an Ellipsoidal MO section. The internal field is uniform along the major axis, and the far field is that of a dipole as shown in the Magnetic Field of an Ellipsoidal MO section. The magnetic force is derived by first determining the interaction of the two electrons due to the field of the outer electron 2 acting on the magnetic moments of electron 1 and vice versa. Insight to the behavior is given by considering the physics of a single bound electron in an externally applied uniform magnetic field as discussed in the Two-Electron Atoms section. The orbitosphere-cvf and the uniform current- (charge-) density function $Y_0^0(\theta, \phi)$ was given in the Orbitosphere Equation of Motion for $\ell = 0$ section and Appendix III. The resultant angular momentum projections of the spherically-symmetric orbitosphere current density, $Y_0^0(\theta, \phi)$, are $L_{xy} = \frac{\hbar}{4}$ and $L_z = \frac{\hbar}{2}$. As shown in the Resonant Precession of the Spin-1/2-Current-Density Function Gives Rise to the Bohr Magnetron section, the electron spin angular momentum gives rise to a trapped photon with \hbar of angular momentum along an S-axis. Then, the spin state of an orbitosphere comprises a photon standing wave that is phase-matched to a spherical harmonic source current, a spherical harmonic dipole $Y_1^m(\theta, \phi) = \sin \theta$ with respect to the S-axis. The dipole spins about the S-axis at the angular velocity given by Eq.(1.55) with \hbar of angular momentum. S rotates about the z-axis at the Larmor frequency at $\theta = \frac{\pi}{3}$ such that it has a static projection of the angular momentum of $S_{\parallel} = \pm \hbar \cos \frac{\pi}{3} = \pm \frac{\hbar}{2} \mathbf{i}_{z_R}$ as given by Eq. (1.85), and from Eq. (1.84), the projection of S onto the transverse plane (xy-plane) is $S_{\perp} = \hbar \sin \frac{\pi}{3} = \pm \sqrt{\frac{3}{4}} \hbar \mathbf{i}_{r_R}$. Then, the vector projection of the radiation-reaction-type magnetic force of the Two Electron Atom section given by Eqs. (7.24)

and (7.31) contain the factor $\sqrt{\frac{3}{4}}\hbar$. This represents the maximum projection of the time-dependent magnetic moment onto an axis of the spherical-central-force system.

The orbitsphere can serve as a basis element to form a molecular orbital (MO). The total magnitude of the angular momentum of \hbar is conserved for each member of the linear combinations of $Y_0^0(\theta, \phi)$'s in the transition from the $Y_0^0(\theta, \phi)$'s to the MO. Since the charge and current densities are equivalent by the ratio of the frequency, the solution of Laplace's equation for the charge density that is an equipotential energy surface also determines the current density. The frequency and the velocity are given by Newton's laws. Specifically, the further constraint from Newton's laws that the orbital surface is a constant total energy surface and the condition of nonradiation provide that the angular velocity of each point on the surface is constant, the current is continuous and constant, and determines the corresponding velocity function. In non-spherical coordinates, the nonuniform charge distribution given by Laplace's equation is compensated by a nonuniform velocity distribution such that the constant current condition is met. Then, the conservation of the angular momentum is provided by symmetrically stretching the current density along an axis perpendicular to the plane defined by the orthogonal components of angular momentum. The angular momentum projection may be determined by first considering the case of the hydrogen molecular ion. Specifically, the angular momentum must give the results of the Stern-Gerlach experiment as shown for atomic electrons and free electrons in the Resonant Precession of the Spin-1/2-Current-Density Function Gives Rise to the Bohr Magneton section and Stern-Gerlach Experiment section, respectively.

The hydrogen-molecular-ion MO, and all MOs in general, have cylindrical symmetry along the bond axis. Thus, for the hydrogen molecular ion, the two orthogonal semiminor axes are equivalent and interchangeable. Then, in general, $Y_0^0(\theta, \phi)$ can serve as a basis element for an MO having equal angular momentum projections along each of the semiminor axes. This defines the plane and the orthogonal axis for stretching the $Y_0^0(\theta, \phi)$ basis element to form the MO. Thus, to conserve angular momentum, $Y_0^0(\theta, \phi)$ is stretched along the semimajor axis as shown in Figure 2. This gives rise to an ellipsoidal surface comprised of the equivalent of elliptical-orbit, plane cross sections in the direction parallel to the semimajor axis with equal angular momentum projections along the orthogonal semiminor axes when the basis element has equal orthogonal angular momentum components.

As shown in the Exact Generation of $Y_0^0(\theta, \phi)$ from the Orbitsphere-cvf section, the orbitsphere is comprised of the uniform function $Y_0^0(\theta, \phi)$ corresponding to STEP ONE having the angular momentum components $L_{xy} = 0$ and $L_z = \frac{\hbar}{4}$ and the uniform function $Y_0^0(\theta, \phi)$ corresponding to STEP TWO having the angular momentum components $L_{xy} = \frac{\hbar}{4}$ and $L_z = \frac{\hbar}{4}$. These components are separable. Then, the basis element $Y_0^0(\theta, \phi)$ for the construction of an MO that conserves the total magnitude of the angular momentum of \hbar (Eq. (1.57)) that matches the MO conditions of equal orthogonal components of angular momentum along each semiminor axis is a single $Y_0^0(\theta, \phi)$ that is generated according to STEP TWO but with twice the angular momentum in each great-circle basis element to give $L_{xy} = \frac{\hbar}{2}$ and $L_z = \frac{\hbar}{2}$.

Now consider the behavior of the hydrogen molecular ion in a magnetic field. As shown in the Resonant Precession of the Spin-1/2-Current-Density Function Gives Rise to the Bohr Magneton section, the photon angular momentum corresponding to the resonant excitation of the Larmor excited state is \hbar , and the angular momentum change corresponding to the spin-flip transition is also \hbar . Furthermore, torque balance for the orbitsphere was determined by considering the energy minimum due to the interaction of the magnetic moments corresponding to the components of angular momentum. In the case of the hydrogen molecular ion, the Larmor-excitation photon carries \hbar of angular momentum that gives rise to a prolate spheroidal dipole current about an S-axis in the same manner as in the case of the spherical dipole of the Larmor excited orbitsphere shown in Figures 1.15 and 1.16 in Chapter 1. The former are given by the prolate angular function, which comprises an associated Legendre function $P_\ell^m(\eta)$ [20], and the latter comprises the spherical harmonic dipole $Y_\ell^m(\theta, \phi) = \sin \theta$. Both are with respect to the S-axis. For hydrogen molecular ion, $\frac{\hbar}{2}$ of intrinsic spin is along each of the semiminor axes of the prolate spheroidal MO. Torque balance is achieved with S along the semimajor axis as shown in Figure 3. Thus, the Larmor excitation is along the semimajor axis. In general, all bonds are cylindrically symmetrical about the internuclear or semimajor axis; thus, the Larmor precession occurs about the bond

axis of an MO wherein the intrinsic angular momentum components rotate about S at the Larmor frequency.

In the coordinate system rotating at the Larmor frequency (denoted by the axes labeled X_R , Y_R , and Z_R in Figure 2), the angular momentum of S of magnitude \hbar is stationary. The Y_R -component of magnitude $\frac{\hbar}{2}$ and the Z_R -component of magnitude $\frac{\hbar}{2}$ rotate about S at the Larmor frequency. The rotation occurs due to a resonant excitation that results in a balance between the magnetic moment of S of μ_B corresponding to its angular momentum of \hbar (Eq. (28) of Box 1.3 and Eq. (2.65)) and those of the orthogonal $\frac{\hbar}{2}$ angular momentum components along Z_R and Y_R of $\frac{\mu_B}{2}$.

Then, the S-axis is the direction of the magnetic moment of each unpaired electron of a molecule or molecular ion. The magnetic moment of S of μ_B corresponding to its \hbar of angular momentum is consistent with the Stern-Gerlach experiment wherein the Larmor excitation can only be parallel or antiparallel to the magnetic field in order to conserve the angular momentum of the electron, the photon corresponding to the Larmor excitation, and the \hbar of angular momentum of the photon that causes a 180° flip of the direction of S. The result is the same as that for the atomic electron and the free electron given in the Resonant Precession of the Spin-1/2-Current-Density Function Gives Rise to the Bohr Magneton section and Stern-Gerlach Experiment section, respectively. The magnetic field is given in the Magnetic Field of an Ellipsoidal Molecular Orbital section.

20

Next, consider the magnetic-pairing force of the hydrogen molecule due to the spin-angular-momentum components. The magnetic moments of electrons 1 and 2 of the hydrogen molecule cancel as they are spin paired to form an energy minimum at the radius (i.e. $r_1 = r_2$). The magnetic force follows the derivation for that between the electrons of two-electron atoms as given in the Two-Electron Atoms section. The latter force was derived by first determining the interaction of the two electrons due to the field of the outer electron 2 acting on the magnetic moments of electron 1 and vice versa. It was also given by the relationship between the angular momentum, energy, and frequency for the transition of electron 2 from the continuum to the ground state. The magnitude of the magnetic force given by Eqs. (7.24) and (7.31) is equivalent to that of the centrifugal force given by Eqs.

30

(7.1-7.2) multiplied by $\frac{1}{Z\hbar}$ times the magnitude of the photon angular momentum vector that precesses at the Larmor frequency given by Eq. (7.4). In the present case of hydrogen-type molecules, the radiation-reaction-type magnetic force arises between the electrons, each having the components shown in Figure 3. With the photon angular momentum projection of \hbar and the total nuclear (non-photon-field) of 2, the magnitude of the magnetic force between the two electrons is 1/2 that of the centrifugal force given by Eq. (11.95).

The hydrogen-type molecule is formed by the binding of an electron 2 to the hydrogen-type molecular ion comprising two protons at the foci of the prolate spheroidal MO of electron 1. The ellipsoids of electron 1 and electron 2 are confocal; thus, the electric fields and the corresponding forces are normal to the each MO of electron 1 and electron 2. The two electrons are bound by the central field of the two protons as in the case of the molecular ion. Since the field of the protons is only ellipsoidal on average, the field of the hydrogen-type molecular ion is not equivalent to an ellipsoid of charge +1 outside of the electron MO. In addition there is a spin pairing force between the two electrons. Due to the force between electron 2 and electron 1 as well as the central force of the protons, the balance between the centrifugal force and the central field of electron 2 of the hydrogen-type molecule formed by electron 2 binding to a hydrogen-type molecular ion also given by Eq. (11.115). The force balance between the centrifugal force and the sum of the Coulombic and magnetic spin-pairing forces to solve for the semimajor axis is

20

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{pe^2}{8\pi\epsilon_0 ab^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D \quad (11.200)$$

$$\frac{2a_0}{pa} - \frac{a_0}{pa} = 1 \quad (11.201)$$

$$a = \frac{a_0}{p} \quad (11.202)$$

Substitution of Eq. (11.202) into Eq. (11.79) is

$$c' = \frac{1}{p\sqrt{2}} a_0 \quad (11.203)$$

The internuclear distance given by multiplying Eq. (11.203) by two is

$$2c' = \frac{a_0 \sqrt{2}}{p} \quad (11.204)$$

Substitution of Eqs. (11.202-11.203) into Eq. (11.80) is

$$b = c = \frac{1}{p\sqrt{2}} a_o \quad (11.205)$$

Substitution of Eqs. (11.202-11.203) into Eq. (11.67) is

$$e = \frac{1}{\sqrt{2}} \quad (11.206)$$

For hydrogen, $r(t) = D$ for $\theta = n\frac{\pi}{2}$, $n = 0, 1, 2, 3, 4$. Thus, there is no dipole moment and the molecule is not predicted to be infrared active. However, it is predicted to be Raman active due to the quadrupole moment. The liquefaction temperature of H_2 is also predicted to be significantly higher than isoelectronic helium.

ENERGIES OF HYDROGEN-TYPE MOLECULES

The energy components defined previously for the molecular ion, Eqs. (11.117), (11.119), (11.120), and (11.121), apply in the case of the corresponding molecule except that all of the field lines of the protons must end on the MO comprising two-paired electrons. With spin pairing of the mirror-image-current electrons, the scaling factors due to the non-ellipsoidal variation of the electric field of the protons is unity as in the case of the sum of squares of spherical harmonics. Thus, the hydrogen-type molecular energies are given by the integral of the forces without correction. Then, each molecular-energy component is given by the integral of corresponding force in Eq. (11.200) where each energy component is the total for the two equivalent electrons with the central-force action at the position of the electron MO where the parameters a and b are given by Eqs. (11.202) and (11.205), respectively.

The potential energy, V_e , of the two-electron MO comprising equivalent electrons in the field of magnitude p times that of the two protons at the foci is

$$\begin{aligned} V_e &= 2 \frac{-pe^2}{4\pi\epsilon_o} D \frac{ab^2}{2D} \int_{\xi}^{\infty} \frac{d\xi}{R_{\xi}} \\ &= \frac{-2pe^2}{8\pi\epsilon_o} \int_{\xi}^{\infty} \frac{d\xi}{(\xi+b)\sqrt{\xi+a}} \\ &= \frac{-2pe^2}{8\pi\epsilon_o \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \end{aligned} \quad (11.207)$$

which is equivalent to $Ze = 2pe$ times the potential of the MO given by Eq. (11.46) after Eq. (11.114). The potential energy, V_p , due to proton-proton repulsion in the field of magnitude p times that of the protons at the foci ($\xi = 0$) is

$$V_p = \frac{p}{8\pi\epsilon_0} \frac{e^2}{\sqrt{a^2 - b^2}} \quad (11.208)$$

The kinetic energy, T , of the two-electron MO of total mass $2m_e$ is

$$\begin{aligned} T &= 2 \frac{-\hbar^2}{2m_e a^2 b^2} D \frac{ab^2}{2D} \int_{\xi}^{\infty} \frac{d\xi}{R_{\xi}} \\ &= \frac{-\hbar^2}{2m_e a} \int_{\xi}^{\infty} \frac{d\xi}{(\xi + b)\sqrt{\xi + a}} \\ &= \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \end{aligned} \quad (11.209)$$

The magnetic energy, V_m , of the two-electron MO of total mass $2m_e$ corresponding to the
5 magnetic force of Eq. (11.200) is

$$\begin{aligned} V_m &= 2 \frac{-\hbar^2}{2(2m_e) a^2 b^2} D \frac{ab^2}{2D} \int_{\xi}^{\infty} \frac{d\xi}{R_{\xi}} \\ &= \frac{-\hbar^2}{4m_e a} \int_{\xi}^{\infty} \frac{d\xi}{(\xi + b)\sqrt{\xi + a}} \\ &= \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \end{aligned} \quad (11.210)$$

The total energy, E_T , is given by the sum of the energy terms (Eqs. (11.207-11.210)):

$$E_T = V_e + T + V_m + V_p \quad (11.211)$$

$$E_T = -13.60 \text{ eV} \left[\left(2p^2 \sqrt{2} - p^2 \sqrt{2} + \frac{p^2 \sqrt{2}}{2} \right) \ln \frac{\sqrt{2} + 1}{\sqrt{2} - 1} - p^2 \sqrt{2} \right] = -p^2 31.63 \quad (11.212)$$

10 where a and b are given by Eqs. (11.202) and (11.205), respectively. The total energy, which includes the proton-proton-repulsion term is negative which justifies the original treatment of the force balance using the analytical mechanics equation of an ellipse that considered only the binding force between the protons and the electrons and the electron centrifugal force. As shown by Eqs. (11.290) and (11.292), T is one-half the magnitude of
15 V_e as required for an inverse-squared force [1] wherein V_e is the source of T .

VIBRATION OF HYDROGEN-TYPE MOLECULES

The vibrational energy levels of hydrogen-type molecules may be solved in the same manner as hydrogen-type molecular ions given in the Vibration of Hydrogen-type Molecular Ions
20 section. The corresponding central force terms of Eq. (11.136) are

$$f(a) = -\frac{pe^2}{8\pi\epsilon_0 a^2} \quad (11.213)$$

and

$$f'(a) = \frac{pe^2}{4\pi\epsilon_0 a^3} \quad (11.214)$$

The distance for the reactive nuclear-repulsive terms is given by the sum of the semimajor axis, a , and c' , 1/2 the internuclear distance. The contribution from the repulsive force between the two protons is

$$f(a+c') = \frac{pe^2}{8\pi\epsilon_0 (a+c')^2} \quad (11.215)$$

and

$$f'(a+c') = -\frac{pe^2}{4\pi\epsilon_0 (a+c')^3} \quad (11.216)$$

10 Thus, from Eqs. (11.136) and (11.213-11.216), the angular frequency of the oscillation is

$$\omega = \sqrt{\frac{\frac{pe^2}{8\pi\epsilon_0 a^3} - \frac{pe^2}{8\pi\epsilon_0 (a+c')^3}}{\mu}} = \sqrt{\frac{\frac{pe^2}{8\pi\epsilon_0 \left(\frac{a_0}{p}\right)^3} - \frac{pe^2}{8\pi\epsilon_0 \left(\left(1+\frac{1}{\sqrt{2}}\right)a_0\right)^3}}{\mu}} = p^2 8.62385 \times 10^{14} \text{ rad/s} \quad (11.217)$$

where the semimajor axis, a , is $a = \frac{a_0}{p}$ according to Eq. (11.202) and c' is $c' = \frac{a_0}{p\sqrt{2}}$

according to Eq. (11.203). Thus, during bond formation, the perturbation of the orbit
15 determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit, and the corresponding frequency, $\omega(0)$, for a hydrogen-type molecule $H_2(1/p)$ given by Eqs. (11.136) and (11.145) is

$$\omega(0) = p^2 \sqrt{\frac{k(0)}{\mu}} = p^2 \sqrt{\frac{621.98 \text{ Nm}^{-1}}{\mu}} = p^2 8.62385 \times 10^{14} \text{ radians/s} \quad (11.218)$$

where the reduced nuclear mass of hydrogen is given by Eq. (11.161) and the spring constant,
20 $k(0)$, given by Eqs. (11.136) and (11.217) is

$$k(0) = p^4 621.98 \text{ Nm}^{-1} \quad (11.219)$$

The transition-state vibrational energy, $E_{\text{vib}}(0)$, is given by Planck's equation (Eq. (11.127)):

$$E_{vib}(0) = \hbar\omega = \hbar p^2 8.62385 \times 10^{14} \text{ rad/s} = p^2 0.56764 \text{ eV} \quad (11.220)$$

The amplitude of oscillation, $A_{reduced}(0)$, given by Eqs. (11.158), (11.161), and (11.219) is

$$A_{reduced}(0) = \frac{\sqrt{\hbar}}{2^{3/2} (p^4 621.98 \text{ Nm}^{-1} \mu)^{1/4}} = \frac{4.275 \times 10^{-12} \text{ m}}{p} = 0.08079 \frac{a_0}{p} \quad (11.221)$$

Then, from Eq. (11.67), $A_c(0)$, the displacement of c' is the eccentricity e given by Eq. 5 (11.206) times $A_{reduced}(0)$ (Eq. (11.221)):

$$A_c(0) = e A_{reduced}(0) = \frac{A_{reduced}(0)}{\sqrt{2}} = \frac{\sqrt{\hbar}}{4(k\mu)^{1/4}} = \frac{0.05713 a_0}{p} \quad (11.222)$$

The spring constant and vibrational frequency for the formed molecule are then obtained from Eqs. (11.136) and (11.213-11.222) using the increases in the semimajor axis and internuclear distances due to vibration in the transition state. The vibrational energy, $E_{vib}(1)$, for the 10 $H_2(1/p)$ $\nu=1 \rightarrow \nu=0$ transition given by adding $A_c(0)$ (Eq. (11.222)) to the distances a and $a+c'$ in Eqs. (11.213-11.220) is

$$E_{vib}(1) = p^2 0.517 \text{ eV} \quad (11.223)$$

where ν is the vibrational quantum number. Using Eq. (11.176) with Eqs. (11.223) and (11.252), the anharmonic perturbation term, $\omega_0 x_0$, of $H_2(1/p)$ is

$$15 \quad \omega_0 x_0 = \frac{100hc \left(8.06573 \times 10^3 \frac{\text{cm}^{-1}}{\text{eV}} p^2 0.517 \text{ eV} \right)^2}{4e(p^2 4.151 \text{ eV} + p^3 0.326469 \text{ eV})} \text{ cm}^{-1} \quad (11.224)$$

where ω_0 is the frequency of the $\nu=1 \rightarrow \nu=0$ transition corresponding to Eq. (11.223) and D_0 is the bond dissociation energy given by Eq. (11.252). The vibrational energies of successive states are given by Eqs. (11.167) and (11.223-11.224).

Using the reduced nuclear mass given by Eq. (11.170), the corresponding parameters 20 for deuterium-type molecules $D_2(1/p)$ (Eqs. (11.213-11.224) and (11.253)) are

$$\omega(0) = p^2 \sqrt{\frac{k(0)}{\mu}} = p^2 \sqrt{\frac{621.98 \text{ Nm}^{-1}}{\mu}} = p^2 6.09798 \times 10^{14} \text{ radians/s} \quad (11.225)$$

$$k(0) = p^4 621.98 \text{ Nm}^{-1} \quad (11.226)$$

$$E_{vib}(0) = p^2 0.4014 \text{ eV} \quad (11.227)$$

$$A_{\text{reduced}}(0) = \frac{\sqrt{\hbar}}{2^{3/2} (p^4 621.98 \text{ Nm}^{-1} \mu)^{1/4}} = \frac{3.595 \times 10^{-12} \text{ m}}{p} = 0.06794 \frac{a_0}{p} \quad (11.228)$$

$$E_{\text{vib}}(1) = p^2 0.371 \text{ eV} \quad (11.229)$$

$$\omega_0 x_0 = \frac{100 \hbar c \left(8.06573 \times 10^3 \frac{\text{cm}^{-1}}{\text{eV}} p^2 0.371 \text{ eV} \right)^2}{4e (p^2 4.229 \text{ eV} + p^3 0.326469 \text{ eV})} \text{ cm}^{-1} \quad (11.230)$$

The vibrational energies of successive states are given by Eqs. (11.167) and (11.229-11.230).

5

THE DOPPLER ENERGY TERM OF HYDROGEN-TYPE MOLECULES

The radiation reaction force in the case of the vibration of the molecule in the transition state also corresponds to the Doppler energy, E_D , given by Eq. (11.181) that is dependent on the motion of the electrons and the nuclei. Here, a nonradiative state must also be achieved after
 10 the emission due to transient vibration wherein the nonradiative condition given by Eq. (11.24) must be satisfied. Typically, a third body is required to form hydrogen-type molecules. For example, the exothermic chemical reaction of $H + H$ to form H_2 does not occur with the emission of a photon. Rather, the reaction requires a collision with a third
 15 body, M , to remove the bond energy— $H + H + M \rightarrow H_2 + M^*$ [21]. The third body distributes the energy from the exothermic reaction, and the end result is the H_2 molecule and an increase in the temperature of the system. Thus, a third body removes the energy corresponding to the additional force term given by Eq. (11.180). From Eqs. (11.200), (11.207) and (11.209), the central force terms between the electron MO and the two protons are

$$20 \quad f(a) = -\frac{pe^2}{4\pi\epsilon_0 a^2} \quad (11.231)$$

and

$$f'(a) = \frac{2pe^2}{4\pi\epsilon_0 a^3} \quad (11.232)$$

Thus, the angular frequency of this oscillation is

$$\omega = \sqrt{\frac{\frac{pe^2}{4\pi\epsilon_0 \left(\frac{a_0}{p}\right)^3}}{m_e}} = p^2 4.13414 \times 10^{16} \text{ rad/s} \quad (11.233)$$

where the semimajor axis, a , is $a = \frac{a_0}{p}$ according to Eq. (11.202). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar p^2 4.13414 \times 10^{16} \text{ rad/s} = p^2 27.2116 \text{ eV} \quad (11.234)$$

In Eq. (11.181), substitution of the total energy of the hydrogen molecule, E_T , (Eq. (11.212)) for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (11.234) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit.

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.635 p^2 \text{ eV} \sqrt{\frac{2e(p^2 27.216 \text{ eV})}{m_e c^2}} = -p^3 0.326469 \text{ eV} \quad (11.235)$$

The total energy of the molecule is decreased by \bar{E}_D .

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency given in the Vibration of Hydrogen-Type Molecules section. On average, the total energy of vibration is equally distributed between kinetic energy and potential energy [17]. Thus, the average kinetic energy of vibration corresponding to the Doppler energy of the electrons, \bar{E}_{Kvib} , is 1/2 of the vibrational energy of the molecule given by Eq. (11.148). The decrease in the energy of the hydrogen molecule due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D and \bar{E}_{Kvib} . Using Eq. (11.235) and E_{vib} from Eq. (11.220) gives

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar p^2 \sqrt{\frac{k}{\mu}} \quad (11.236)$$

$$\bar{E}_{osc} = -p^3 0.326469 \text{ eV} + \frac{1}{2} p^2 (0.56764 \text{ eV}) \quad (11.237)$$

To the extent that the MO dimensions are the same, the electron reentrant orbital energies, \bar{E}_K , are the same independent of the isotope of hydrogen, but the vibrational energies are related by Eq. (11.148). Thus, the differences in bond energies are essentially given by 1/2 the differences in vibrational energies. Using Eq. (11.235) and E_{vib} for D_2 (1/p) given by Eq. (11.227), that corresponds to the deuterium reduced nuclear mass (Eq. (11.170)), the corresponding \bar{E}_{osc} is

$$\bar{E}_{osc} = -p^3 0.326469 \text{ eV} + \frac{1}{2} p^2 (0.401380 \text{ eV}) \quad (11.238)$$

TOTAL, IONIZATION, AND BOND ENERGIES OF HYDROGEN AND DEUTERIUM MOLECULES

The total energy of the hydrogen molecule is given by the sum of E_T (Eqs. (11.211-11.212)) and \bar{E}_{osc} given Eqs. (11.233-11.236). Thus, the total energy of the hydrogen molecule having a central field of $+pe$ at each focus of the prolate spheroid molecular orbital including the Doppler term is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc} \quad (11.239)$$

$$E_T = -p^2 \left\{ \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] \left[1 + p \sqrt{\frac{2\hbar \sqrt{\frac{e^2}{4\pi\epsilon_0 a_0^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} \\ = -p^2 31.635 \text{ eV} - p^3 0.326469 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (11.240)$$

From Eqs. (11.237) and (11.239-11.240), the total energy for hydrogen-type molecules is

$$E_T = -p^2 31.635 \text{ eV} + \bar{E}_{osc} \\ = -p^2 31.635 \text{ eV} - p^3 0.326469 \text{ eV} + \frac{1}{2} p^2 (0.56764 \text{ eV}) \\ = -p^2 31.351 \text{ eV} - p^3 0.326469 \text{ eV} \quad (11.241)$$

The total energy of the deuterium molecule is given by the sum of E_T (Eq. (11.212)) and \bar{E}_{osc} given by Eq. (11.238):

$$E_T = -p^2 31.6354 \text{ eV} + \bar{E}_{osc} \\ = -p^2 31.6354 \text{ eV} - p^3 0.326469 \text{ eV} + \frac{1}{2} p^2 (0.401380 \text{ eV}) \\ = -p^2 31.4345 \text{ eV} - p^3 0.326469 \text{ eV} \quad (11.242)$$

The total energy, which includes the proton-proton-repulsion term is negative which justifies the original treatment of the force balance using the analytical mechanics equation of an ellipse that considered only the binding force between the protons and the electrons, the spin-pairing force, and the electron centrifugal force.

The first ionization energy of the hydrogen molecule, IP_1 ,

$$H_2(1/p) \rightarrow H_2^+(1/p) + e^- \quad (11.243)$$

is given by the difference of Eqs. (11.193) and (11.241):

$$\begin{aligned} IP_1 &= E_T(H_2^+(1/p)) - E_T(H_2(1/p)) \\ &= -p^2 16.13392 \text{ eV} - p^3 0.118755 \text{ eV} - (-p^2 31.351 \text{ eV} - p^3 0.326469 \text{ eV}) \quad (11.244) \\ &= p^2 15.2171 \text{ eV} + p^3 0.207714 \text{ eV} \end{aligned}$$

The second ionization energy, IP_2 , is given by the negative of Eq. (11.193).

$$5 \quad IP_2 = p^2 16.13392 \text{ eV} + p^3 0.118755 \text{ eV} \quad (11.245)$$

The first ionization energy of the deuterium molecule, IP_1 ,

$$D_2(1/p) \rightarrow D_2^+(1/p) + e^- \quad (11.246)$$

is given by the difference of Eqs. (11.194) and (11.242):

$$\begin{aligned} IP_1 &= E_T(D_2^+(1/p)) - E_T(D_2(1/p)) \\ &= -p^2 16.180 \text{ eV} - p^3 0.118811 \text{ eV} - (-p^2 31.4345 \text{ eV} - p^3 0.326469 \text{ eV}) \quad (11.247) \\ &= p^2 15.255 \text{ eV} + p^3 0.2077 \text{ eV} \end{aligned}$$

10 The second ionization energy, IP_2 , is given by the negative of Eq. (11.194).

$$IP_2 = p^2 16.180 \text{ eV} + p^3 0.118811 \text{ eV} \quad (11.248)$$

The bond dissociation energy, E_D , is the difference between the total energy of the corresponding hydrogen atoms and E_T

$$E_D = E(2H(1/p)) - E_T \quad (11.249)$$

15 where [18]

$$E(2H(1/p)) = -p^2 27.20 \text{ eV} \quad (11.250)$$

and [19]

$$E(2D(1/p)) = -p^2 27.206 \text{ eV} \quad (11.251)$$

The hydrogen bond energy, E_D , is given by Eqs. (11.249-11.250) and (11.241):

$$\begin{aligned} E_D &= -p^2 27.20 \text{ eV} - E_T \\ 20 \quad &= -p^2 27.20 \text{ eV} - (-p^2 31.351 \text{ eV} - p^3 0.326469 \text{ eV}) \quad (11.252) \\ &= p^2 4.151 \text{ eV} + p^3 0.326469 \text{ eV} \end{aligned}$$

The deuterium bond energy, E_D , is given by Eqs. (11.249), (11.251), and (11.242):

$$\begin{aligned}
 E_D &= -p^2 27.206 \text{ eV} - E_r \\
 &= -p^2 27.206 \text{ eV} - (-p^2 31.4345 \text{ eV} - p^3 0.326469 \text{ eV}) \\
 &= p^2 4.229 \text{ eV} + p^3 0.326469 \text{ eV}
 \end{aligned}
 \tag{11.253}$$

THE HYDROGEN MOLECULAR ION $\text{H}_2[2c' = 2a_0]^+$

5 FORCE BALANCE OF HYDROGEN MOLECULAR ION

Force balance between the electric and centrifugal forces is given by Eq. (11.115) where $p = 1$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D \tag{11.254}$$

which has the parametric solution given by Eq. (11.83) when

$$10 \quad a = 2a_0 \tag{11.255}$$

The semimajor axis, a , is also given by Eq. (11.116) where $p = 1$. The internuclear distance, $2c'$, which is the distance between the foci is given by Eq. (11.111) where $p = 1$.

$$2c' = 2a_0 \tag{11.256}$$

The experimental internuclear distance is $2a_0$. The semiminor axis is given by Eq. (11.112) where $p = 1$.

$$b = \sqrt{3}a_0 \tag{11.257}$$

The eccentricity, e , is given by Eq. (11.113).

$$e = \frac{1}{2} \tag{11.258}$$

20 ENERGIES OF THE HYDROGEN MOLECULAR ION

The potential energy, V_e , of the electron MO in the field of the protons at the foci ($\xi = 0$) is given by Eq. (11.117) where $p = 1$

$$V_e = \frac{-4e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \tag{11.259}$$

The potential energy, V_p , due to proton-proton repulsion is given by Eq. (11.120) where $p = 1$

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2 - b^2}} \quad (11.260)$$

The kinetic energy, T , of the electron MO is given by Eq. (11.119) where $p=1$

$$T = \frac{2\hbar^2}{m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (11.261)$$

Substitution of a and b given by Eqs. (11.255) and (11.257), respectively, into Eqs. (11.259-
5 11.261) is

$$V_e = \frac{-4e^2}{8\pi\epsilon_0 a_H} \ln 3 = -59.7575 \text{ eV} \quad (11.262)$$

$$V_p = \frac{e^2}{8\pi\epsilon_0 a_H} = 13.5984 \text{ eV} \quad (11.263)$$

$$T = \frac{2e^2}{8\pi\epsilon_0 a_H} \ln 3 = 29.8787 \text{ eV} \quad (11.264)$$

The Doppler term, \bar{E}_{osc} , for hydrogen and deuterium are given by Eqs. (11.189) and (11.190),
10 respectively, where $p=1$

$$\bar{E}_{osc}(H_2^+) = \bar{E}_D + \bar{E}_{Kvib} = -0.118755 \text{ eV} + \frac{1}{2}(0.29282 \text{ eV}) = 0.027655 \quad (11.265)$$

$$\bar{E}_{osc}(D_2^+) = -0.118811 \text{ eV} + \frac{1}{2}(0.20714 \text{ eV}) = -0.01524 \text{ eV} \quad (11.266)$$

The total energy, E_T , for the hydrogen molecular ion given by Eqs. (11.191-11.193) is

$$\begin{aligned} E_T &= - \left\{ \frac{e^2}{8\pi\epsilon_0 a_H} (4 \ln 3 - 1 - 2 \ln 3) \left[1 + \sqrt{\frac{2\hbar^2 \sqrt{\frac{2e^2}{4pe_0(2a_H)^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{m}} \right\} \\ &= -16.2803 \text{ eV} - 0.118811 \text{ eV} + \frac{1}{2}(0.29282 \text{ eV}) \\ &= -16.2527 \text{ eV} \end{aligned} \quad (11.267)$$

15 where in Eqs. (11.262-11.267), the radius of the hydrogen atom a_H (Eq. (1.287)) was used in place of a_0 to account for the corresponding electrodynamic force between the electron and the nuclei as given in the case of the hydrogen atom by Eq. (1.231). The negative of Eq. (11.267) is the ionization energy of H_2^+ and the second ionization energy, IP_2 , of H_2 . From

Eqs. (11.191-11.192) and (11.194), the total energy, E_T , for the deuterium molecular ion (the ionization energy of D_2^+ and the second ionization energy, IP_2 , of D_2) is

$$E_T = -16.284 \text{ eV} - 0.118811 \text{ eV} + \frac{1}{2}(0.20714 \text{ eV}) = -16.299 \text{ eV} \quad (11.268)$$

The bond dissociation energy, E_D , is the difference between the total energy of the 5 corresponding hydrogen atom and E_T . The hydrogen molecular ion bond energy, E_D , including the reduced electron mass given by Eq. (11.198) where $p=1$ is

$$E_D = 2.535 \text{ eV} + 0.118755 \text{ eV} = 2.654 \text{ eV} \quad (11.269)$$

The experimental bond energy of the hydrogen molecular ion [22] is

$$E_D = 2.651 \text{ eV} \quad (11.270)$$

10 From Eq. (11.199) where $p=1$, the deuterium molecular ion bond energy, E_D , including the reduced electron mass of D is

$$E_D = 2.5770 \text{ eV} + 0.118811 \text{ eV} = 2.6958 \text{ eV} \quad (11.271)$$

The experimental bond energy of the deuterium molecular ion [23] is

$$E_D = 2.691 \text{ eV} \quad (11.272)$$

15

VIBRATION OF THE HYDROGEN MOLECULAR ION

It can be shown that a perturbation of the orbit determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit [11]. The resonant vibrational frequency for H_2^+ given by Eq. (11.160) is

$$20 \quad \omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{165.51 \text{ Nm}^{-1}}{\mu}} = 4.449 \times 10^{14} \text{ radians/s} \quad (11.273)$$

wherein $p=1$. The spring constant, $k(0)$, for H_2^+ given by Eq. (11.162) is

$$k(0) = 165.51 \text{ Nm}^{-1} \quad (11.274)$$

The vibrational energy, $E_{vib}(0)$, of H_2^+ during bond formation given by Eq. (11.163) is

$$E_{vib}(0) = 0.29282 \text{ eV} \quad (11.275)$$

25 The amplitude of oscillation given by Eq. (11.164) is

$$A(0) = \frac{\sqrt{\hbar}}{2^{3/2} (165.51 \text{ Nm}^{-1} \mu)^{1/4}} = 5.952 \times 10^{-12} \text{ m} = 0.1125 a_0 \quad (11.276)$$

The vibrational energy for the H_2^+ $\nu=1 \rightarrow \nu=0$ transition given by Eq. (11.166) is

$$E_{vib}(1) = 0.270 \text{ eV} \quad (11.277)$$

The experimental vibrational energy of H_2^+ [14, 19] is

$$E_{vib} = 0.271 \text{ eV} \quad (11.278)$$

5 The anharmonicity term of H_2^+ given by Eq. (11.169) is

$$\omega_0 x_0 = 55.39 \text{ cm}^{-1} \quad (11.279)$$

The experimental anharmonicity term of H_2^+ from NIST [19] is

$$\omega_e x_e = 66.2 \text{ cm}^{-1} \quad (11.280)$$

The vibrational energy for the D_2^+ $\nu=1 \rightarrow \nu=0$ transition given by Eq. (11.175) is

$$10 \quad E_{vib} = 0.193 \text{ eV} \quad (11.281)$$

The vibrational energy of the D_2^+ [19] based on calculations from experimental data is

$$E_{vib} = 0.196 \text{ eV} \quad (11.282)$$

The anharmonicity term of D_2^+ given by Eq. (11.176) is

$$\omega_0 x_0 = 27.86 \text{ cm}^{-1} \quad (11.283)$$

15 The experimental anharmonicity term of D_2^+ for the state $X \sum_g^{+1} s\sigma$ is not given, but the term for state $B \sum_g^{+3} d\sigma$ from NIST [19] is

$$\omega_e x_e = 2.62 \text{ cm}^{-1} \quad (11.284)$$

THE HYDROGEN MOLECULE $H_2 [2c' = \sqrt{2}a_0]$

20

FORCE BALANCE OF THE HYDROGEN MOLECULE

The force balance equation for the hydrogen molecule is given by Eq. (11.200) where $p=1$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D \quad (11.285)$$

which has the parametric solution given by Eq. (11.83) when

$$25 \quad a = a_0 \quad (11.286)$$

The semimajor axis, a , is also given by Eq. (11.202) where $p=1$. The internuclear distance, $2c'$, which is the distance between the foci is given by Eq. (11.204) where $p=1$.

$$2c' = \sqrt{2}a_o \quad (11.287)$$

The experimental internuclear distance is $\sqrt{2}a_o$. The semiminor axis is given by Eq. (11.205) where $p=1$.

$$b = \frac{1}{\sqrt{2}}a_o \quad (11.288)$$

5 The eccentricity, e , is given by Eq. (11.206).

$$e = \frac{1}{\sqrt{2}} \quad (11.289)$$

The finite dimensions of the hydrogen molecule are evident in the plateau of the resistivity versus pressure curve of metallic hydrogen [24].

10 ENERGIES OF THE HYDROGEN MOLECULE

The energies of the hydrogen molecule are given by Eqs. (11.207-11.210) where $p=1$

$$V_e = \frac{-2e^2}{8\pi\epsilon_o\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -67.8358 \text{ eV} \quad (11.290)$$

$$V_p = \frac{e^2}{8\pi\epsilon_o\sqrt{a^2-b^2}} = 19.2415 \text{ eV} \quad (11.291)$$

$$T = \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 33.9179 \text{ eV} \quad (11.292)$$

15 The energy, V_m , of the magnetic force is

$$V_m = \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -16.9589 \text{ eV} \quad (11.293)$$

The Doppler terms, \bar{E}_{osc} , for hydrogen and deuterium molecules are given by Eqs. (11.237) and (11.238), respectively, where $p=1$

$$\bar{E}_{osc}(H_2) = \bar{E}_D + \bar{E}_{Kvib} = -0.326469 \text{ eV} + \frac{1}{2}(0.56764 \text{ eV}) = -0.042649 \text{ eV} \quad (11.294)$$

$$20 \quad \bar{E}_{osc}(D_2) = -0.326469 \text{ eV} + \frac{1}{2}(0.401380 \text{ eV}) = -0.125779 \text{ eV} \quad (11.295)$$

The total energy, E_T , for the hydrogen molecule given by Eqs. (11.239-11.241) is

$$\begin{aligned}
 E_T &= - \left\{ \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{e^2}{4\pi\epsilon_0 a_0^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} \\
 &= -31.635 \text{ eV} - 0.326469 \text{ eV} + \frac{1}{2} (0.56764 \text{ eV}) \\
 &= -31.6776 \text{ eV}
 \end{aligned} \tag{11.29}$$

6)

From Eqs. (11.239-11.240) and (11.242), the total energy, E_T , for the deuterium molecule is

$$E_T = -31.635 \text{ eV} - 0.326469 \text{ eV} + \frac{1}{2} (0.401380 \text{ eV}) = -31.7608 \text{ eV} \tag{11.297}$$

- 5 The first ionization energies of the hydrogen and deuterium molecules, IP_1 , (Eqs. (11.243) and (11.246)) are given by the differences in the total energy of corresponding molecular ions and molecules which are given by Eqs. (11.244) and (11.247), respectively, where $p=1$:

$$IP_1(H_2) = 15.2171 \text{ eV} + 0.207714 \text{ eV} = 15.4248 \text{ eV} \tag{11.298}$$

$$IP_1(D_2) = 15.255 \text{ eV} + 0.2077 \text{ eV} = 15.4627 \text{ eV} \tag{11.299}$$

- 10 The bond dissociation energy, E_D , is the difference between the total energy of two of the corresponding hydrogen atoms and E_T . The hydrogen molecular bond energy, E_D , given by Eq. (11.252) where $p=1$ is

$$E_D = 4.151 \text{ eV} + 0.326469 \text{ eV} = 4.478 \text{ eV} \tag{11.300}$$

The experimental bond energy of the hydrogen molecule [22] is

$$15 \quad E_D = 4.478 \text{ eV} \tag{11.301}$$

The deuterium molecular bond energy, E_D , given by Eq. (11.253) where $p=1$ is

$$E_D = 4.229 \text{ eV} + 0.326469 \text{ eV} = 4.556 \text{ eV} \tag{11.302}$$

The experimental bond energy of the deuterium molecule [22] is

$$E_D = 4.556 \text{ eV} \tag{11.303}$$

20

VIBRATION OF THE HYDROGEN MOLECULE

It can be shown that a perturbation of the orbit determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit [11]. The resonant vibrational frequency for H_2 given by Eq. (11.218) is

72

$$\omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{621.98 \text{ Nm}^{-1}}{\mu}} = 8.62385 \times 10^{14} \text{ radians/s} \quad (11.304)$$

The spring constant, $k(0)$, for H_2 given by Eq. (11.219) is

$$k(0) = 621.98 \text{ Nm}^{-1} \quad (11.305)$$

wherein $p = 1$. The vibrational energy, $E_{vib}(0)$, of H_2 during bond formation given by Eq. 5 (11.220) is

$$E_{vib}(0) = 0.56764 \text{ eV} \quad (11.306)$$

The amplitude of oscillation given by Eq. (11.221) is

$$A(0) = \frac{\sqrt{\hbar}}{2^{3/2} (p^4 621.98 \text{ Nm}^{-1} \mu)^{1/4}} = 4.275 \times 10^{-12} \text{ m} = 0.08079 a_0 \quad (11.307)$$

The vibrational energy for the H_2 $\nu = 1 \rightarrow \nu = 0$ transition given by Eq. (11.223) is

$$10 \quad E_{vib}(1) = 0.517 \text{ eV} \quad (11.308)$$

The experimental vibrational energy of H_2 [25-26] is

$$E_{vib}(1) = 0.5159 \text{ eV} \quad (11.309)$$

The anharmonicity term of H_2 given by Eq. (11.224) is

$$\omega_0 x_0 = 120.4 \text{ cm}^{-1} \quad (11.310)$$

15 The experimental anharmonicity term of H_2 from Huber and Herzberg [23] is

$$\omega_e x_e = 121.33 \text{ cm}^{-1} \quad (11.311)$$

The vibrational energy for the D_2 $\nu = 1 \rightarrow \nu = 0$ transition given by Eq. (11.229) is

$$E_{vib} = 0.371 \text{ eV} \quad (11.312)$$

The experimental vibrational energy of D_2 [14, 19] is

$$20 \quad E_{vib} = 0.371 \text{ eV} \quad (11.313)$$

The anharmonicity term of D_2 given by Eq. (11.230) is

$$\omega_0 x_0 = 60.93 \text{ cm}^{-1} \quad (11.314)$$

The experimental anharmonicity term of D_2 from NIST [19] is

$$\omega_e x_e = 61.82 \text{ cm}^{-1} \quad (11.315)$$

25 The results of the determination of the bond, vibrational, total, and ionization energies, and internuclear distances for hydrogen and deuterium molecules and molecular

ions are given in Table 11.1. The calculated results are based on first principles and given in closed form equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

Despite the predictions of standard quantum mechanics that preclude the imaging of a molecule orbital, the full three-dimensional structure of the outer molecular orbital of N_2 has been recently tomographically reconstructed [27]. The charge-density surface observed is similar to that shown in Figure 5 for H_2 which is direct evidence that electrons are not point-particle probability waves that have no form until they are "collapsed to a point" by measurement. Rather they are physical, two-dimensional equipotential charge density surfaces.

Table 11.1. The calculated and experimental parameters of H_2 , D_2 , H_2^+ and D_2^+ .

Parameter	Calculated	Experimental	Eqs.	Ref. for Exp.
H_2 Bond Energy	4.478 eV	4.478 eV	11.300	22
D_2 Bond Energy	4.556 eV	4.556 eV	11.302	22
H_2^+ Bond Energy	2.654 eV	2.651 eV	11.269	22
D_2^+ Bond Energy	2.696 eV	2.691 eV	11.271	23
H_2 Total Energy	31.677 eV	31.675 eV	11.296	22, 28, 18 ^a
D_2 Total Energy	31.760 eV	31.760 eV	11.297	19, 23 ^b
H_2 Ionization Energy	15.425 eV	15.426 eV	11.298	28
D_2 Ionization Energy	15.463 eV	15.466 eV	11.299	23
H_2^+ Ionization Energy	16.253 eV	16.250 eV	11.267	22, 18 ^c
D_2^+ Ionization Energy	16.299 eV	16.294 eV	11.268	19, 23 ^d
H_2^+ Magnetic Moment	$9.274 \times 10^{-24} \text{ JT}^{-1}$ μ_B	$9.274 \times 10^{-24} \text{ JT}^{-1}$ μ_B	13.1-13.7	29
Absolute H_2 Gas- Phase NMR Shift	-28.0 ppm	-28.0 ppm	11.411	30-31
H_2 Internuclear Distance ^e	0.748 Å $\sqrt{2}a_0$	0.741 Å	11.287	32
D_2 Internuclear Distance ^e	0.748 Å $\sqrt{2}a_0$	0.741 Å	11.287	32
H_2^+ Internuclear Distance ^f	1.058 Å $2a_0$	1.06 Å	11.256	22
D_2^+ Internuclear Distance ^e	1.058 Å $2a_0$	1.0559 Å	11.256	23
H_2 Vibrational Energy	0.517 eV	0.516 eV	11.308	25, 26
D_2 Vibrational Energy	0.371 eV	0.371 eV	11.313	14, 19
H_2 $\omega_e x_e$	120.4 cm^{-1}	121.33 cm^{-1}	11.310	23
D_2 $\omega_e x_e$	60.93 cm^{-1}	61.82 cm^{-1}	11.314	19
H_2^+ Vibrational Energy	0.270 eV	0.271 eV	11.277	14, 19

				75		
D_2^+ Vibrational Energy		0.193 eV	0.196 eV	11.281	19	
H_2 J=1 to J=0		0.0148 eV	0.01509 eV	13.45	22	
Rotational Energy ^e						
D_2 J=1 to J=0		0.00741 eV	0.00755 eV	13.37-13.45	22	
Rotational Energy ^e						
H_2^+ J=1 to J=0		0.00740 eV	0.00739 eV	13.49	22	
Rotational Energy ^f						
D_2^+ J=1 to J=0		0.00370 eV	0.003723 eV	13.37-13.43, 13.49	23	
Rotational Energy ^e						

^a The experimental total energy of the hydrogen molecule is given by adding the first (15.42593 eV) [28] and second (16.2494 eV) ionization energies where the second ionization energy is given by the addition of the ionization energy of the hydrogen atom (12.59844 eV) [18] and the bond energy of H_2^+ (2.651 eV) [22].

^b The experimental total energy of the deuterium molecule is given by adding the first (15.466 eV) [23] and second (16.294 eV) ionization energies where the second ionization energy is given by the addition of the ionization energy of the deuterium atom (12.603 eV) [19] and the bond energy of D_2^+ (2.692 eV) [23].

^c The experimental second ionization energy of the hydrogen molecule, IP_2 , is given by the sum of the ionization energy of the hydrogen atom (12.59844 eV) [18] and the bond energy of H_2^+ (2.651 eV) [22].

^d The experimental second ionization energy of the deuterium molecule, IP_2 , is given by the sum of the ionization energy of the deuterium atom (12.603 eV) [19] and the bond energy of D_2^+ (2.692 eV) [23].

^e The internuclear distances are not corrected for the reduction due to \bar{E}_{osc} .

^f The internuclear distances are not corrected for the increase due to \bar{E}_{osc} .

THE DIHYDRINO MOLECULAR ION $H_2 [2c' = a_0]^+$

15

FORCE BALANCE OF THE DIHYDRINO MOLECULAR ION

Force balance between the electric and centrifugal forces of $H_2^+(1/2)$ is given by Eq. (11.115) where $p = 2$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{2pe^2}{8\pi\epsilon_0 ab^2} D \quad (11.316)$$

20 which has the parametric solution given by Eq. (11.83) when

$$a = a_o \quad (11.317)$$

The semimajor axis, a , is also given by Eq. (11.116) where $p=2$. The internuclear distance, $2c'$, which is the distance between the foci is given by Eq. (11.111) where $p=2$.

$$2c' = a_o \quad (11.318)$$

5 The semiminor axis is given by Eq. (11.112) where $p=2$.

$$b = \frac{\sqrt{3}}{2} a_o \quad (11.319)$$

The eccentricity, e , is given by Eq. (11.113).

$$e = \frac{1}{2} \quad (11.320)$$

10 ENERGIES OF THE DIHYDRINO MOLECULAR ION

The potential energy, V_e , of the electron MO in the field of magnitude twice that of the protons at the foci ($\xi = 0$) is given by Eq. (11.117) where $p=2$.

$$V_e = \frac{-8e^2}{8\pi\epsilon_o\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \quad (11.321)$$

The potential energy, V_p , due to proton-proton repulsion in the field of magnitude twice that
15 of the protons at the foci ($\xi = 0$) is given by Eq. (11.120) where $p=2$

$$V_p = \frac{2e^2}{8\pi\epsilon_o\sqrt{a^2-b^2}} \quad (11.322)$$

The kinetic energy, T , of the electron MO is given by Eq. (11.119) where $p=2$

$$T = \frac{2\hbar^2}{m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \quad (11.323)$$

Substitution of a and b given by Eqs. (11.317) and (11.319), respectively, into Eqs. (11.321-
20 11.323) and using Eqs. (11.191-11.193) with $p=2$ gives

$$V_e = \frac{-16e^2}{8\pi\epsilon_o a_o} \ln 3 = -239.16 \text{ eV} \quad (11.324)$$

$$V_p = \frac{4e^2}{8\pi\epsilon_o a_o} = 54.42 \text{ eV} \quad (11.325)$$

$$T = \frac{8e^2}{8\pi\epsilon_o a_o} \ln 3 = 119.58 \text{ eV} \quad (11.326)$$

$$E_T = V_e + V_p + T + \bar{E}_{osc} \quad (11.327)$$

$$E_T = -2^2 \left\{ \frac{e^2}{8\pi\epsilon_0 a_H} (4 \ln 3 - 1 - 2 \ln 3) \left[1 + 2 \sqrt{\frac{2\hbar \sqrt{\frac{2e^2}{4\pi\epsilon_0 (2a_H)^3}}}{m_e c^2}} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right] \right\}$$

$$= -2^2 (16.13392 \text{ eV}) - 2^3 (0.118755 \text{ eV}) \quad (11.328)$$

$$= -65.49 \text{ eV}$$

where Eqs. (11.324-11.326) are equivalent to Eqs. (11.122-11.124) with $p = 2$. The bond dissociation energy, E_D , given by Eq. (11.198) with $p = 2$ is the difference between the total
5 energy of the corresponding hydrino atom and E_T given by Eq. (11.328):

$$E_D = E_T(H(1/p)) - E_T(H_2^+(1/p))$$

$$= 2^2 (2.535 \text{ eV}) + 2^3 (0.118755 \text{ eV}) \quad (11.329)$$

$$= 11.09 \text{ eV}$$

VIBRATION OF THE DIHYDRINO MOLECULAR ION

It can be shown that a perturbation of the orbit determined by an inverse-squared force results
10 in simple harmonic oscillatory motion of the orbit [11]. The resonant vibrational frequency for $H_2^+(1/2)$ from Eq. (11.160) is

$$\omega(0) = 2^2 \sqrt{\frac{165.51 \text{ Nm}^{-1}}{\mu}} = 1.78 \times 10^{15} \text{ radians/s} \quad (11.330)$$

wherein $p = 2$. The spring constant, $k(0)$, for $H_2^+(1/2)$ from Eq. (11.162) is

$$k(0) = 2^4 165.51 \text{ Nm}^{-1} = 2648 \text{ Nm}^{-1} \quad (11.331)$$

15 The amplitude of oscillation from Eq. (11.164) is

$$A(0) = \frac{\sqrt{\hbar}}{2^{3/2} (2^4 (165.51) \text{ Nm}^{-1} \mu)^{1/4}} = \frac{5.952 \times 10^{-12} \text{ m}}{2} = \frac{0.1125 a_0}{2} \quad (11.332)$$

The vibrational energy, $E_{vib}(1)$, for the $\nu = 1 \rightarrow \nu = 0$ transition given by Eq. (11.166) is

$$E_{vib}(1) = 2^2 (0.270 \text{ eV}) = 1.08 \text{ eV} \quad (11.333)$$

THE DIHYDRINO MOLECULE $H_2 \left[2c' = \frac{a_o}{\sqrt{2}} \right]$

FORCE BALANCE OF THE DIHYDRINO MOLECULE

The force balance equation for the dihydrino molecule $H_2(1/2)$ is given by Eq. (11.200)

5 where $p=2$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{2e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D \quad (11.334)$$

which has the parametric solution given by Eq. (11.83) when

$$a = \frac{a_o}{2} \quad (11.335)$$

The semimajor axis, a , is also given by Eq. (11.202) where $p=2$. The internuclear
10 distance, $2c'$, which is the distance between the foci is given by Eq. (11.204) where $p=2$.

$$2c' = \frac{1}{\sqrt{2}} a_o \quad (11.336)$$

The semiminor axis is given by Eq. (11.205) where $p=2$.

$$b = c = \frac{1}{2\sqrt{2}} a_o \quad (11.337)$$

The eccentricity, e , is given by Eq. (11.206).

$$15 \quad e = \frac{1}{\sqrt{2}} \quad (11.338)$$

ENERGIES OF THE DIHYDRINO MOLECULE

The energies of the dihydrino molecule $H_2(1/2)$ are given by Eqs. (11.207-11.210) and Eqs.
(11.239-11.241) with $p=2$

$$20 \quad V_e = \frac{-4e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -271.34 \text{ eV} \quad (11.339)$$

$$V_p = \frac{2}{8\pi\epsilon_0} \frac{e^2}{\sqrt{a^2 - b^2}} = 76.97 \text{ eV} \quad (11.340)$$

$$T = \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 135.67 \text{ eV} \quad (11.341)$$

The energy, V_m , of the magnetic force is

$$V_m = \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -67.84 \text{ eV} \quad (11.342)$$

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc} \quad (11.343)$$

$$E_T = -2^2 \left\{ \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] \left[1 + 2 \sqrt{\frac{2\hbar \sqrt{\frac{e^2}{4\pi\epsilon_0 a_0^3}}}{m_e c^2}} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right] \right\}$$

$$= -2^2 (31.351 \text{ eV}) - 2^3 (0.326469 \text{ eV}) \quad (11.344)$$

$$= -128.02 \text{ eV}$$

where Eqs. (11.339-11.342) are equivalent to Eqs. (11.207-11.210) with $p = 2$. The bond
5 dissociation energy, E_D , given by Eq. (11.252) with $p = 2$ is the difference between the total energy of the corresponding hydrino atoms and E_T given by Eq. (11.344).

$$E_D = E_T(2H(1/p)) - E_T(H_2(1/p))$$

$$= 2^2 (4.151 \text{ eV}) + 2^3 (0.326469 \text{ eV}) \quad (11.345)$$

$$= 19.22 \text{ eV}$$

VIBRATION OF THE DIHYDRINO MOLECULE

10 It can be shown that a perturbation of the orbit determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit [11]. The resonant vibrational frequency for the $H_2(1/2)$ from Eq. (11.217) is

$$\omega(0) = 2^2 \sqrt{\frac{k}{\mu}} = 2^2 \sqrt{\frac{621.98 \text{ Nm}^{-1}}{\mu}} = 3.45 \times 10^{15} \text{ radians/s} \quad (11.346)$$

wherein $p = 2$. The spring constant, $k(0)$, for $H_2(1/2)$ from Eq. (11.219) is

$$15 \quad k(0) = 2^4 621.98 \text{ Nm}^{-1} = 9952 \text{ Nm}^{-1} \quad (11.347)$$

The amplitude of oscillation from Eq. (11.221) is

$$A(0) = \frac{\sqrt{\hbar}}{2^{3/2} (2^4 (621.98) \text{ Nm}^{-1} \mu)^{1/4}} = \frac{4.275 \times 10^{-12} \text{ m}}{2} = \frac{0.08079 a_0}{2} \quad (11.348)$$

The vibrational energy, $E_{vib}(1)$, of $H_2(1/2)$ from Eq. (11.223) is

$$E_{vib}(1) = 2^2 (0.517) \text{ eV} = 2.07 \text{ eV} \quad (11.349)$$

GEOMETRY

The internuclear distance can also be determined geometrically. The spheroidal MO of the hydrogen molecule is an equipotential energy surface, which is an energy minimum surface. For the hydrogen molecule, the electric field is zero for $\xi > 0$. Consider two hydrogen atoms
 5 A and B approaching each other. Consider that the two electrons form a spheroidal MO as the two atoms overlap, and the charge is distributed such that an equipotential two-dimensional surface is formed. The electric fields of atoms A and B add vectorially as the atoms overlap. The energy at the point of intersection of the overlapping orbitspheres decreases to a minimum as they superimpose and then rises with further overlap. When this
 10 energy is a minimum the internuclear distance is determined. It can be demonstrated [33] that when two hydrogen orbitspheres superimpose such that the radial electric field vector from nucleus A and B makes a 45° angle with the point of intersection of the two original orbitspheres, the electric energy of interaction between orbitspheres given by

$$E_{\text{interaction}} = 2 \times \frac{1}{2} \epsilon_0 \int \Delta E^2 dv \quad (11.350)$$

15 is a minimum (Figure 7.1 of [33]). The MO is a minimum potential energy surface; therefore, a minimum of energy of one point on the surface is a minimum for the entire surface of the MO. Thus,

$$R_{H_2} = \sqrt{2} a_0 = 0.748 \text{ \AA} \quad (11.351)$$

The experimental internuclear bond distance is 0.746 \AA .

20

DIHYDRINO IONIZATION ENERGIES

The first ionization energy, IP_1 , of the dihydrino molecule



is given by Eq. (11.244) with $p = 2$.

$$25 \quad IP_1 = E_T(H_2^+(1/p)) - E_T(H_2(1/p)) \quad (11.353)$$

$$IP_1 = 2^2 (15.2171 \text{ eV}) + 2^3 (0.2077 \text{ eV}) = 62.53 \text{ eV} \quad (11.354)$$

The second ionization energy, IP_2 , is given by Eq. (11.245) with $p = 2$.

$$IP_2 = 2^2 (16.13392 \text{ eV}) + 2^3 (0.118755 \text{ eV}) = 65.49 \text{ eV} \quad (11.355)$$

A hydrino atom can react with a hydrogen, deuterium, or tritium nucleus to form a
 30 dihydrino molecular ion that further reacts with an electron to form a dihydrino molecule.



The energy released is

$$E = E(H(1/p)) - E_T \quad (11.357)$$

where E_T is given by Eq. (11.241).

- 5 A hydrino atom can react with a hydrogen, deuterium, or tritium atom to form a dihydrino molecule.



The energy released is

$$E = E(H(1/p)) + E(H) - E_T \quad (11.359)$$

- 10 where E_T is given by Eq. (11.241).

SIZES OF REPRESENTATIVE ATOMS AND MOLECULES

ATOMS

15

Helium Atom (He)

Helium comprises the nucleus at the origin and two electrons as a spherical shell at $r = 0.567a_0$.

20 Hydrogen Atom ($H[a_H]$)

Hydrogen comprises the nucleus at the origin and the electron as a spherical shell at $r = a_H$.

Hydrino Atom ($H\left[\frac{a_H}{2}\right]$)

Hydrino atom (1/2) comprises the nucleus at the origin and the electron as a spherical shell at

$$25 \quad r = \frac{a_H}{2}.$$

MOLECULES

Hydrogen Molecular Ion ($H_2[2c' = 2a_0]^+$)

$$a = 2a_0$$

$$b = c = \sqrt{3}a_0$$

$$c' = a_0$$

$$2c' = 2a_0$$

5 Hydrogen Molecule ($\text{H}_2 [2c' = \sqrt{2}a_0]$)

$$a = a_0$$

$$b = c = \frac{1}{\sqrt{2}}a_0$$

$$c' = \frac{1}{\sqrt{2}}a_0$$

$$2c' = \sqrt{2}a_0$$

10

Dihydrino Molecular Ion ($\text{H}_2 [2c' = a_0]^+$)

$$a = a_0$$

$$b = c = \frac{\sqrt{3}}{2}a_0$$

$$b = c = \frac{1}{2}a_0$$

15 $2c' = a_0$

Dihydrino Molecule ($\text{H}_2 [2c' = \frac{1}{\sqrt{2}}a_0]$)

$$a = \frac{1}{2}a_0$$

$$b = c = \frac{1}{2\sqrt{2}}a_0$$

20 $c' = \frac{1}{2\sqrt{2}}a_0$

$$2c' = \frac{1}{\sqrt{2}}a_0$$

ORTHO-PARA TRANSITION OF HYDROGEN-TYPE MOLECULES

Each proton of hydrogen-type molecules possesses a magnetic moment, which is derived in the Proton and Neutron section and is given by

$$\mu_p = \frac{\left(\frac{2}{3}\right)^2 e\hbar}{2 \frac{m_p}{2\pi}} \quad (11.360)$$

- 5 The magnetic moment, \mathbf{m} , of the proton is given by Eq. (11.360), and the magnetic field of the proton follows from the relationship between the magnetic dipole field and the magnetic moment, \mathbf{m} , as given by Jackson [34] where $\mathbf{m} = \mu_p \mathbf{i}_z$.

$$\mathbf{H} = \frac{\mu_p}{r^3} (\mathbf{i}_r 2 \cos \theta - \mathbf{i}_\theta \sin \theta) \quad (11.361)$$

Multiplication of Eq. (11.361) by the permeability of free space, μ_0 , gives the magnetic flux,

- 10 \mathbf{B} , due to proton one at proton two.

$$\mathbf{B} = \frac{\mu_0 \mu_p}{r^3} (\mathbf{i}_r 2 \cos \theta - \mathbf{i}_\theta \sin \theta) \quad (11.362)$$

$\Delta E_{mag}^{ortho/para}$, the energy to flip the orientation of proton two's magnetic moments, μ_p , from ortho (parallel magnetic moments) to para (antiparallel magnetic moments) with respect to the direction of the magnetic moment of proton one with corresponding magnetic flux \mathbf{B} is

$$15 \quad \Delta E_{mag}^{ortho/para} = -2\mu_p \mathbf{B} = \frac{-2\mu_0 \mu_p^2}{r^3} \quad (11.363)$$

where r is the internuclear distance $2c'$ where c' is given by Eq. (11.204). Substitution of the internuclear distance into Eq. (11.363) for r gives

$$\Delta E_{mag}^{ortho/para} = -2\mu_p \mathbf{B} = \frac{-2\mu_0 \mu_p^2 p^3}{(\sqrt{2}a_o)^3} \quad (11.364)$$

- The frequency, f , can be determined from the energy using the Planck relationship, Eq. 20 (2.18).

$$f = \frac{\Delta E_{mag}^{ortho/para}}{h} = \frac{\frac{-2\mu_0 \mu_p^2 p^3}{(\sqrt{2}a_o)^3}}{h} \quad (11.365)$$

From Eq. (11.365) with $p = 2$, the ortho-para transition energy of the dihydrino molecule is 14.4 MHz.

NUCLEAR MAGNETIC RESONANCE SHIFT

The proton gyromagnetic ratio, $\gamma_p / 2\pi$, is

$$\gamma_p / 2\pi = 42.57602 \text{ MHz } T^{-1} \quad (11.366)$$

The NMR frequency, f , is the product of the proton gyromagnetic ratio given by Eq. (11.366) and the magnetic flux, B .

$$f = \gamma_p / 2\pi B = 42.57602 \text{ MHz } T^{-1} B \quad (11.367)$$

A typical flux for a superconducting NMR magnet is 1.5 T . According to Eq. (11.367) this corresponds to a radio frequency (RF) of 63.86403 MHz . With a constant magnetic field, the frequency is scanned to yield the spectrum where the frequency scan is typically achieved using a Fourier transform on the free induction decay signal following a radio frequency pulse. Or, in a less common type of NMR spectrometer, the radiofrequency is held constant (e.g. 60 MHz), the applied magnetic field, H_0 ($H_0 = \frac{B}{\mu_0}$), is varied over a small range, and the frequency of energy absorption is recorded at the various values for H_0 . The spectrum is typically scanned and displayed as a function of increasing H_0 . The protons that absorb energy at a lower H_0 give rise to a downfield absorption peak; whereas, the protons that absorb energy at a higher H_0 give rise to an upfield absorption peak. The electrons of the compound of a sample influence the field at the nucleus such that it deviates slightly from the applied value. For the case that the chemical environment has no NMR effect, the value of H_0 at resonance with the radiofrequency held constant at 60 MHz is

$$\frac{2\pi f}{\mu_0 \gamma_p} = \frac{(2\pi)(60 \text{ MHz})}{\mu_0 42.57602 \text{ MHz } T^{-1}} = H_0 \quad (11.368)$$

In the case that the chemical environment has a NMR effect, a different value of H_0 is required for resonance. This chemical shift is proportional to the electronic magnetic flux charge at the nucleus due to the applied field, which in the case of each dihydrino molecule is a function of its semimajor and semiminor axes as shown *infra*.

Consider the application of a z-axis-directed uniform external magnetic flux, B_z , to a dihydrino molecule comprising prolate spheroidal electron MOs with two spin-paired electrons. The diamagnetic reaction current increases or decreases the MO current to counteract any applied flux according to Lenz's law as shown in the Hydrino Hydride Ion Nuclear Magnetic Resonance Shift section. The current of hydrogen-type molecules is along

elliptical orbits parallel to the semimajor axis. Thus, the electronic interaction with the nuclei requires that each nuclear magnetic moment is in the direction of the semiminor axis. Thus, the nuclei are NMR active towards B_z when the orientation of the semimajor axis, a , is along the x-axis, and the semiminor axes, $b = c$, are along the y-axis and z-axis, respectively.

- 5 The flux is applied over the time interval $\Delta t = t_i - t_f$ such that the field increases at a rate dB/dt . The electric field, E , along a perpendicular elliptic path of the dihydrino MO at the plane $z = 0$ is given by

$$\oint E \cdot ds = \int \frac{dB}{dt} \cdot dA \quad (11.369)$$

- The induced electric field must be constant along the path; otherwise, compensating currents
10 would flow until the electric field is constant. Thus, Eq. (11.369) becomes

$$E = \frac{\int \frac{dB}{dt} \cdot dA}{\oint ds} = \frac{\int \frac{dB}{dt} \cdot dA}{4aE(k)} = \frac{\pi ab \frac{dB}{dt}}{4aE(k)} \quad (11.370)$$

where $E(k)$ is the elliptic integral given by

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1 - k \sin^2 \phi} d\phi = 1.2375 \quad (11.371)$$

$$k = e = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{2}}{2} \quad (11.372)$$

- 15 the area of an ellipse, A , is

$$A = \pi ab \quad (11.373)$$

the perimeter of an ellipse, s , is

$$s = 4aE(k) \quad (11.374)$$

- a is the semimajor axis given by Eq. (11.202), b is the semiminor axis given by Eq.
20 (11.205), and e is the eccentricity given by Eq. (11.206). The acceleration along the path, dv/dt , during the application of the flux is determined by the electric force on the charge density of the electrons:

$$m_e \frac{dv}{dt} = eE = \frac{e\pi ab}{4aE(k)} \frac{dB}{dt} \quad (11.375)$$

Thus, the relationship between the change in velocity, v , and the change in B is

$$25 \quad dv = \frac{e\pi ab}{4aE(k)m_e} dB \quad (11.376)$$

Let Δv represent the net change in v over the time interval $\Delta t = t_i - t_f$ of the application of the flux. Then,

$$\Delta v = \int_{v_0}^{v_0 + \Delta v} dv = \frac{e\pi ab}{4aE(k)m_e} \int_0^B dB = \frac{e\pi ab B}{4aE(k)m_e} \quad (11.377)$$

The average current, I , of a charge moving time harmonically along an ellipse is

$$5 \quad I = ef = \frac{ev}{4aE(k)} \quad (11.378)$$

where f is the frequency. The corresponding magnetic moment is given by

$$m = AI = \pi ab I = \frac{\pi ab ev}{4aE(k)} \quad (11.379)$$

Thus, from Eqs. (11.377) and (11.379), the change in the magnetic moment, Δm , due to an applied magnetic flux, B , is [35]

$$10 \quad \Delta m = -\frac{(e\pi ab)^2 B}{(4aE(k))^2 m_e} \quad (11.380)$$

Next, the contribution from all plane cross sections of the prolate spheroid MO must be integrated along the z -axis. The spheroidal surface is given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} = 1 \quad (11.381)$$

The intersection of the plane $z = z'$ ($-b \leq z' \leq b$) with the spheroid determines the curve

$$15 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 - \frac{z'^2}{b^2} \quad (11.382)$$

or

$$\frac{x^2}{a^2 \left(1 - \frac{z'^2}{b^2}\right)} + \frac{y^2}{b^2 \left(1 - \frac{z'^2}{b^2}\right)} = 1 \quad (11.383)$$

Eq. (11.383) is an ellipse with semimajor axis, a' , and semiminor axis, b' , given by

$$a' = a \sqrt{1 - \frac{z'^2}{b^2}} \quad (11.384)$$

$$20 \quad b' = b \sqrt{1 - \frac{z'^2}{b^2}} \quad (11.385)$$

The eccentricity, e' , is given by

$$e' = \frac{\sqrt{a^2 \left(1 - \frac{z'^2}{b^2}\right) - b^2 \left(1 - \frac{z'^2}{b^2}\right)}}{a \sqrt{1 - \frac{z'^2}{b^2}}} = \frac{\sqrt{a^2 - b^2}}{a} = e \quad (11.386)$$

where e is given by Eq. (11.372). The area, A' , is given by

$$A' = \pi a' b' \quad (11.387)$$

and the perimeter, s' , is given by

$$5 \quad s' = 4a' E(k) = 4aE(k) \sqrt{1 - \frac{z'^2}{b^2}} = s \sqrt{1 - \frac{z'^2}{b^2}} \quad (11.388)$$

where s is given by Eq. (11.374). The differential magnetic moment change along the z -axis is

$$d\Delta\mathbf{m} = -\frac{1}{2b} \frac{(e\pi a' b')^2 \mathbf{B}}{(4a' E(k))^2 m_e} dz' \quad (11.389)$$

Using Eq. (11.385) for the parameter b' , the change in magnetic moment for the dihydrino
10 molecule is given by the integral over $-b \leq b' \leq b$:

$$\Delta\mathbf{m} = -\frac{1}{2b} \int_{-b}^b \frac{\left(e\pi a' b' \sqrt{1 - \frac{z'^2}{b^2}}\right)^2 \mathbf{B}}{(4a' E(k))^2 m_e} dz' = -C_1 \frac{1}{m_e} \left(\frac{\pi e}{4E(k)}\right)^2 \quad (11.390)$$

Then, integral to correct for the z -dependence of b' is

$$C_1 = \frac{\int_{-b}^b (b^2 - z^2) dz}{2b} = \frac{2}{3} b^2 = \frac{a_0^2}{3p} \quad (11.391)$$

where the semiminor axis, $b = \frac{a_0}{p\sqrt{2}}$, given by Eq. (11.205) was used.

15 The change in magnetic moment would be given by the substitution of Eq. (11.391) into Eq. (11.390), if the charge density were constant along the path of Eqs. (11.370) and (11.378), but it is not. The charge density of the MO in rectangular coordinates (Eq. (11.42)) is

$$\sigma = \frac{e}{4\pi abc} \frac{1}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \quad (11.392)$$

(The mass-density function of an MO is equivalent to its charge-density function where m replaces q of Eq. (11.42)). The equation of the plane tangent to the ellipsoid at the point x_0, y_0, z_0 is

$$X \frac{x_0}{a^2} + Y \frac{y_0}{b^2} + Z \frac{z_0}{c^2} = 1 \quad (11.393)$$

5 where X, Y, Z are running coordinates in the plane. After dividing through by the square root of the sum of the squares of the coefficients of X, Y , and Z , the right member is the distance D from the origin to the tangent plane. That is,

$$D = \frac{1}{\sqrt{\frac{x_0^2}{a^4} + \frac{y_0^2}{b^4} + \frac{z_0^2}{c^4}}} \quad (11.394)$$

so that

$$10 \quad \sigma = \frac{e}{4\pi abc} D \quad (11.395)$$

In other words, the surface density at any point on the ellipsoidal MO is proportional to the perpendicular distance from the center of the ellipsoid to the plane tangent to the ellipsoid at the point. The charge is thus greater on the more sharply rounded ends farther away from the origin. In order to maintain current continuity, the diamagnetic velocity of Eq. (11.377) must
15 be a constant along any given path integral corresponding to a constant electric field. Consequently, the charge density must be the minimum value of that given by Eq. (11.392). The minimum corresponds to $y = b$ and $x = z = 0$ such that the charge density is

$$\sigma = \frac{e}{4\pi ab^2} \frac{1}{\sqrt{\frac{0^2}{a^4} + \frac{b^2}{b^4} + \frac{0^2}{c^4}}} = \frac{e}{4\pi ab} \quad (11.396)$$

The MO is an equipotential surface, and the current must be continuous over the two-
20 dimensional surface. Continuity of the surface current density, K , due to the diamagnetic effect of the applied magnetic field on the MO and the equipotential boundary condition require that the current of each elliptical curve determined by the intersection of the plane $z = z'$ ($-b \leq z' \leq b$) with the spheroid be the same. The charge density is spheroidally symmetrical about the semimajor axis. Thus, λ , the charge density per unit length along
25 each elliptical path cross section of Eq. (11.383) is given by distributing the surface charge density of Eq. (11.396) uniformly along the z -axis for $-b \leq z' \leq b$. So, $\lambda(z' = 0)$, the linear charge density λ in the plane $z' = 0$, is

$$\lambda(z'=0) = \frac{\sigma}{\frac{1}{2b}} = \frac{e}{4\pi ab} 2b = \frac{e}{2\pi a} \quad (11.397)$$

And, the linear charge density must be equally distributed over each elliptical path cross section corresponding to each plane $z = z'$. The current is independent of z' when the linear charge density, $\lambda(z')$, is normalized for the path length:

$$5 \quad \lambda(z') = \frac{e}{2\pi a} \frac{4aE(k)}{4a'E(k')} = \frac{e}{2\pi a'} \quad (11.398)$$

where the equality of the eccentricities of each elliptical plane cross section given by Eq. (11.386) was used. Substitution of Eq. (11.388) for the corresponding charge density, $\frac{e}{4a'E(k)}$, of Eq. (11.390) and using Eq. (11.391) gives

$$\Delta \mathbf{m} = \frac{2}{3} \frac{e^2 b^2 \mathbf{B}}{4m_e} = \frac{e^2 a_0^2 \mathbf{B}}{12 p^2 m_e} \quad (11.399)$$

10 The two electrons are spin-paired and the velocities are mirror opposites. Thus, the change in velocity of each electron treated individually (Eq. (10.3)) due to the applied field would be equal and opposite. However, as shown in the Three Electron Atom section, the two paired electrons may be treated as one with twice the mass where m_e is replaced by $2m_e$ in Eq. (11.399). In this case, the paired electrons spin together about the applied field axis,
15 the z-axis, to cause a reduction in the applied field according to Lenz's law. Thus, from Eq. (11.399), the change in magnetic moment is given by

$$\Delta \mathbf{m} = \frac{e^2 a_0^2 \mathbf{B}}{24 p^2 m_e} \quad (11.400)$$

The opposing diamagnetic flux is uniform, parallel, and opposite the applied field as given by Stratton [36]. Specifically, the change in magnetic flux, $\Delta \mathbf{B}$, at the nucleus due to
20 the change in magnetic moment, $\Delta \mathbf{m}$, is

$$\Delta \mathbf{B} = \mu_0 A_2 \Delta \mathbf{m} \quad (11.401)$$

where μ_0 is the permeability of vacuum,

$$A_2 = \int_0^\infty \frac{ds}{(s+b^2)R_s} \quad (11.402)$$

is an elliptic integral of the second kind given by Whittaker and Watson [37], and

$$25 \quad R_s = (s+b^2)\sqrt{(s+a^2)} \quad (11.403)$$

Substitution of Eq. (11.403) into Eq. (11.402) gives

$$A_2 = \int_0^{\infty} \frac{ds}{(s+b^2)^2 (s+a^2)^{1/2}} \quad (11.404)$$

From integral 154 of Lide [38]:

$$A_2 = - \left\{ \frac{1}{a^2-b^2} \frac{\sqrt{s+a^2}}{s+b^2} \right\}_0^{\infty} - \frac{1}{2} \frac{1}{a^2-b^2} \int_0^{\infty} \frac{ds}{(s+b^2) \sqrt{s+a^2}} \quad (11.405)$$

5 The evaluation at the limits of the first integral is

$$- \left\{ \frac{1}{a^2-b^2} \frac{\sqrt{s+a^2}}{s+b^2} \right\}_0^{\infty} = \frac{a}{b^2(a^2-b^2)} \quad (11.406)$$

From integral #147 of Lide [9], the second integral is:

$$- \frac{1}{2} \frac{1}{a^2-b^2} \int_0^{\infty} \frac{ds}{(s+b^2) \sqrt{s+a^2}} = \left\{ \frac{1}{2(a^2-b^2)^{3/2}} \ln \frac{\sqrt{s+a^2} + \sqrt{a^2-b^2}}{\sqrt{s+a^2} - \sqrt{a^2-b^2}} \right\}_0^{\infty} \quad (11.407)$$

Evaluation at the limits of the second integral gives

$$10 \quad - \frac{1}{2} \frac{1}{(a^2-b^2)^{3/2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \quad (11.408)$$

Combining Eq. (11.406) and Eq. (11.408) gives

$$A_2 = \frac{a}{b^2(a^2-b^2)} - \frac{1}{2} \frac{1}{(a^2-b^2)^{3/2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = \frac{p^3 4}{a_0^3} - \frac{p^3 \sqrt{2}}{a_0^3} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} \quad (11.409)$$

where the semimajor axis, $a = \frac{a_0}{p}$, given by Eq. (11.202) and the semiminor axis, $b = \frac{a_0}{p\sqrt{2}}$,

given by Eq. (11.205) were used.

15 Substitution of Eq. (11.400) and Eq. (11.409) into Eq. (11.401) gives

$$\Delta B = -\mu_0 \left(\frac{p^3 4}{a_0^3} - \frac{p^3 \sqrt{2}}{a_0^3} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} \right) \frac{a_0^2 e^2 \mathbf{B}}{24 p^2 m_e} \quad (11.410)$$

Additionally, it is found both theoretically and experimentally that the dimensions, r^2 , of the molecule corresponding to the area in Eqs. (11.369) and (11.379) used to derived Eq. (11.410) must be replaced by an average, $\langle r^2 \rangle$, that takes into account averaging over the

20 orbits isotropically oriented. The correction of 2/3 is given by Purcell [35]. In the case of hydrogen-type molecules, the electronic interaction with the nuclei require that each nuclear magnetic moment is in the direction of the semiminor axis. But free rotation about each of

three axes results in an isotropic averaging of 2/3 where the rotational frequencies of hydrogen-type molecules are much greater than the corresponding NMR frequency (e.g. 10^{12} Hz versus 10^8 Hz). Thus, Eq. (11.410) gives the absolute upfield chemical shift, $\frac{\Delta B}{B}$, of H_2 relative to a bare proton:

$$\begin{aligned} \frac{\Delta B}{B} &= \frac{\Delta B}{B} = -\mu_0 \left(\frac{p^3 4}{a_0^3} - \frac{p^3 \sqrt{2}}{a_0^3} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} \right) \frac{a_0^2 e^2}{36 p^2 m_e} \\ &= -\mu_0 \left(4 - \sqrt{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} \right) \frac{p e^2}{36 a_0 m_e} \\ &= -p 28.01 \text{ ppm} \end{aligned} \quad (11.411)$$

where $p=1$ for H_2 .

It follows from Eqs. (11.202) and (11.411) that the diamagnetic flux (flux opposite to the applied field) at each nucleus is inversely proportional to the semimajor radius, $a = \frac{a_0}{p}$.

For resonance to occur, ΔH_0 , the change in applied field from that given by Eq. (11.368), must compensate by an equal and opposite amount as the field due to the electrons of the dihydrino molecule. According to Eq. (11.202), the ratio of the semimajor axis of the dihydrino molecule $H_2 (1/p)$ to that of the hydrogen molecule H_2 is the reciprocal of an integer p . Similarly it is shown in the Hydrino Hydride Ion Nuclear Magnetic Resonance Shift section and previously [39], that according to Eq. (7.87) the ratio of the radius of the hydrino hydride ion $H^- (1/p)$ to that of the hydride ion $H^- (1/1)$ is the reciprocal of an integer p . It follows from Eqs. (7.90-7.96) that compared to a proton with no chemical shift, the ratio of ΔH_0 for resonance of the proton of the hydrino hydride ion $H^- (1/p)$ to that of the hydride ion $H^- (1/1)$ is a positive integer. That is, if only the radius is considered, the absorption peak of the hydrino hydride ion occurs at a value of ΔH_0 that is a multiple of p times the value that is resonant for the hydride ion compared to that of a proton with no shift. However, a hydrino hydride ion is equivalent to the ordinary hydride ion except that it is in a lower energy state. The source current of the state must be considered in addition to the reduced radius.

As shown in the Stability of "Ground" and Hydrino States section, for the below "ground" (fractional quantum number) energy states of the hydrogen atom, σ_{photon} , the two-

dimensional surface charge due to the "trapped photon" at the electron orbitsphere and phase-locked with the electron orbitsphere current, is given by Eqs. (5.08) and (2.11).

$$\sigma_{\text{photon}} = \frac{e}{4\pi(r_n)^2} \left[Y_0^0(\theta, \phi) - \frac{1}{n} \left[Y_0^0(\theta, \phi) + \text{Re}\{Y_l^m(\theta, \phi)e^{i\omega_n t}\} \right] \right] \delta(r - r_n)$$

$$n = \frac{1}{p} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \quad (11.412)$$

5 And, σ_{electron} , the two-dimensional surface charge of the electron orbitsphere is

$$\sigma_{\text{electron}} = \frac{-e}{4\pi(r_n)^2} \left[Y_0^0(\theta, \phi) + \text{Re}\{Y_l^m(\theta, \phi)e^{i\omega_n t}\} \right] \delta(r - r_n) \quad (11.413)$$

The superposition of σ_{photon} (Eq. (11.412)) and σ_{electron} , (Eq. (11.413)) where the spherical harmonic functions satisfy the conditions given in the Angular Function section is

$$\sigma_{\text{photon}} + \sigma_{\text{electron}} = \frac{-e}{4\pi(r_n)^2} \left[\frac{1}{n} Y_0^0(\theta, \phi) + \left(1 + \frac{1}{n}\right) \text{Re}\{Y_l^m(\theta, \phi)e^{i\omega_n t}\} \right] \delta(r - r_n)$$

$$10 \quad n = \frac{1}{p} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \quad (11.414)$$

The ratio of the total charge distributed over the surface at the radius of the hydrino hydride ion $H^-(1/p)$ to that of the hydride ion $H^-(1/1)$ is an integer p , and the corresponding total source current of the hydrino hydride ion is equivalent to an integer p times that of an electron. The "trapped photon" obeys the phase-matching condition given in Excited States
15 of the One-Electron Atom (Quantization) section, but does not interact with the applied flux directly. Only each electron does; thus, $\Delta\nu$ of Eq. (11.377) must be corrected by a factor of $1/p$ corresponding to the normalization of the electron source current according to the invariance of charge under Gauss' Integral Law. As also shown by Eqs. (7.17-7.23) and (7.87), the "trapped photon" gives rise to a correction to the change in magnetic moment due
20 to the interaction of each electron with the applied flux. The correction factor of $1/p$ consequently cancels the NMR effect of the reduced radius which is consistent with general observations on diamagnetism [40]. It follows that the same result applies in the case of Eq. (11.411) for $H_2(1/p)$ wherein the coordinates are ellipsoidal rather than spherical.

The cancellation of the chemical shift due to the reduced radius or the reduced
25 semiminor and semimajor axes in the case of $H^-(1/p)$ and $H_2(1/p)$, respectively, by the corresponding source current is exact except for an additional relativistic effect. The relativistic effect for $H^-(1/p)$ arises due to the interaction of the currents corresponding to

the angular momenta of the "trapped photon" and the electrons and is analogous to that of the fine structure of the hydrogen atom involving the $^2P_{3/2} \rightarrow ^2P_{1/2}$ transition. The derivation follows that of the fine structure given in the Spin-Orbital Coupling section.

$\frac{e}{m_e}$ of the electron, the electron angular momentum of \hbar , and the electron magnetic

5 momentum of μ_B are invariant for any electronic state. The same applies for the paired electrons of dihydrogen hydride ions. The condition that flux must be linked by the electron in units of the magnetic flux quantum in order to conserve the invariant electron angular momentum of \hbar gives the additional chemical shift due to relativistic effects. Using Eqs. (2.159-2.160), Eq. (2.166) may be written as

$$10 \quad E_{s/o} = \frac{\alpha \pi \mu_0 e^2 \hbar^2}{m_e^2 r^3} \sqrt{\frac{3}{4}} = \alpha 2\pi 2 \frac{e\hbar}{2m_e} \frac{\mu_0 e\hbar}{2m_e a_0^3} \sqrt{\frac{3}{4}} = \alpha 2\pi 2 \mu_B B \quad (11.415)$$

From Eq. (11.415) and Eq. (1.205), the relativistic stored magnetic energy contributes a factor of $\alpha 2\pi$. In spherical coordinates, the relativistic change in flux ΔB_{SR} may be calculated using Eq. (7.95) and the relativistic factor of $\gamma_{SR} = 2\pi\alpha$ which is the same as that given by Eq. (1.229):

$$15 \quad \Delta B_{SR} = -\gamma_{SR} \mu_0 \frac{\Delta m}{r_n^3} (\mathbf{i}_r \cos \theta - \mathbf{i}_\theta \sin \theta) = -2\pi\alpha \mu_0 \frac{\Delta m}{r_n^3} (\mathbf{i}_r \cos \theta - \mathbf{i}_\theta \sin \theta) \quad (11.416)$$

for $r < r_n$.

The stored magnetic energy term of the electron g factor of each electron of a dihydrogen molecule is the same as that of a hydrogen atom since $\frac{e}{m_e}$ is invariant and the invariant angular momentum and magnetic moment of the former are also \hbar and μ_B ,
20 respectively, as given in the Magnetic Moment of an Ellipsoidal MO and Magnetic Field of an Ellipsoidal MO sections. Thus, the corresponding correction in ellipsoidal coordinates follows from Eq. (2.166) wherein the result of the length contraction for the circular path in spherical coordinates is replaced by that of the elliptical path.

The only position on the elliptical path at which the current is perpendicular to the
25 radial vector defined by the central force of the protons is at the semimajor axis. It was shown in the Special Relativistic Correction to the Ionization Energies section that when the condition that the electron's motion is tangential to the radius is met, the radius is Lorentzian invariant. That is, for the case that k is the lightlike k^0 , with $k = \omega_n / c$, a is invariant. In

the case of a spherically symmetrical MO such as the case of the hydrogen atom, it was also shown that this condition determines that the electron's angular momentum of \hbar , $\frac{e}{m_e}$ of Eq. (1.110), and the electron's magnetic moment of a Bohr magneton, μ_B , are invariant. The effect of the relativistic length contraction and time dilation for constant spherical motion is a change in the angle of motion with a corresponding decrease in the electron wavelength. The angular motion becomes projected onto the radial axis which contracts, and the extent of the decrease in the electron wavelength and radius due to the electron motion in the laboratory inertial frame are given by

$$\lambda = 2\pi r' \sqrt{1 - \left(\frac{v}{c}\right)^2} \sin \left[\frac{\pi}{2} \left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2} \right] + r' \cos \left[\frac{\pi}{2} \left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2} \right] \quad (11.417)$$

10 and

$$r = r' \left[\sqrt{1 - \left(\frac{v}{c}\right)^2} \sin \left[\frac{\pi}{2} \left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2} \right] + \frac{1}{2\pi} \cos \left[\frac{\pi}{2} \left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2} \right] \right] \quad (11.418)$$

respectively. Then, the relativistic factor γ^* is

$$\gamma^* = \frac{2\pi}{2\pi \sqrt{1 - \left(\frac{v}{c}\right)^2} \sin \left[\frac{\pi}{2} \left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2} \right] + \cos \left[\frac{\pi}{2} \left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2} \right]} \quad (11.419)$$

where the velocity is given by Eq. (1.56) with the radius given by Eq. (1.233).

15 Each point or coordinate position on the continuous two-dimensional electron MO of the dihydrino molecule defines an infinitesimal mass-density element which moves along an elliptical orbit of a spheroidal MO in such a way that its eccentric angle, θ , changes at a constant rate. That is $\theta = \omega t$ at time t where ω is a constant, and

$$r(t) = ia \cos \omega t + jb \sin \omega t \quad (11.420)$$

20 is the parametric equation of the ellipse. Next, special relativistic effects on distance and time are considered. The parametric radius, $r(t)$, is a minimum at the position of the semiminor axis of length b , and the motion is transverse to the radial vector. Since the angular momentum of \hbar is constant, the electron wavelength without relativistic correction is given by

$$25 \quad 2\pi b = \lambda = \frac{h}{mv} \quad (11.421)$$

such that the angular momentum, L , is given by

$$L = r \times mv = bmv = \hbar \quad (11.422)$$

The nonradiation and the \hbar , $\frac{e}{m_e}$, and μ_B invariance conditions require that the angular frequencies, ω_s and ω_e , for spherical and ellipsoidal motion, respectively, are

$$\omega_s = \frac{\hbar}{m_e r^2} = \frac{\frac{\pi L}{m_e}}{A} \quad (11.423)$$

5 and

$$\omega_e = \frac{\pi \hbar}{m_e A} = \frac{\hbar}{m_e ab} \quad (11.424)$$

where A is the area of the closed orbit, the area of an ellipse given by Eq. (11.373). Since the angular frequency ω_e has the form as ω_s , the time dilation corrections are equivalent, where the correction for ω_s is given in the Special Relativistic Correction to the Ionization Energies
10 section. Since the semimajor axis, a , is invariant, but b undergoes length contraction, the relationship between the velocity and the electron wavelength at the semiminor axis from Eq. (11.417) and Eq. (11.421) is

$$\lambda = 2\pi b \sqrt{1 - \left(\frac{v}{c}\right)^2} \sin \left[\frac{\pi}{2} \left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2} \right] + a \cos \left[\frac{\pi}{2} \left(1 - \left(\frac{v}{c}\right)^2 \right)^{3/2} \right] \quad (11.425)$$

where $\lambda \rightarrow a$ as $v \rightarrow c$ replaces the spherical coordinate result of $\lambda \rightarrow r'$ as $v \rightarrow c$. Thus, in
15 the electron frame at rest $v = 0$, and, Eq. (11.425) becomes

$$\lambda' = 2\pi b \quad (11.426)$$

In the laboratory inertial frame for the case that $v = c$ in Eq. (11.425), λ is

$$\lambda = a \quad (11.427)$$

Thus, using Eqs. (11.426) and (11.427), the relativistic factor, γ^* , is

$$20 \quad \gamma^* = \frac{\lambda}{\lambda'} = \frac{a}{2\pi b} \quad (11.428)$$

From Eqs. (11.417-11.419) and Eq. (11.428), the relativistic diamagnetic effect of the inverse integer radius of $H_2(1/p)$ compared to H_2 , each with ellipsoidal MOs, is equivalent to the ratio of the semiminor and semimajor axes times the correction for the spherical orbital case given in Eq. (11.416). From the mass (Eq. (2.165)) and radius corrections (Eq. (2.163))
25 in Eq. (2.166), the relativistic stored magnetic energy contributes a factor γ_{SR} of

$$\gamma_{SR} = 2\pi\alpha \left(\frac{b}{a}\right)^2 = \pi\alpha \quad (11.429)$$

Thus, from Eqs. (11.401), (11.416), and (11.429), the relativistic change in flux, ΔB_{SR} , for the dihydrino molecule $H_2(1/p)$ is

$$\Delta B_{SR} = -\gamma_{SR}\mu_0 A_2 \Delta m = -\pi\alpha\mu_0 A_2 \Delta m \quad (11.430)$$

5 Thus, using Eq. (11.411) and Eq. (11.430), the upfield chemical shift, $\frac{\Delta B_{SR}}{B}$, due to the relativistic effect of the molecule $H_2(1/p)$ corresponding to the lower-energy state with principal quantum energy state p is given by

$$\frac{\Delta B_{SR}}{B} = -\mu_0\pi\alpha \left(4 - \sqrt{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1}\right) \frac{pe^2}{36a_0m_e} \quad (11.431)$$

The total shift, $\frac{\Delta B_T}{B}$, for $H_2(1/p)$ is given by the sum of that of H_2 given by Eq. (11.411)

10 with $p=1$ plus that given by Eq. (11.431):

$$\frac{\Delta B_T}{B} = -\mu_0 \left(4 - \sqrt{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1}\right) \frac{e^2}{36a_0m_e} (1 + \pi\alpha p) \quad (11.432)$$

$$\frac{\Delta B_T}{B} = -(28.01 + 0.64p) \text{ ppm} \quad (11.433)$$

where $p = \text{integer} > 1$.

H_2 has been characterized by gas phase 1H NMR. The experimental absolute
15 resonance shift of gas-phase TMS relative to the proton's gyromagnetic frequency is -28.5 ppm [30]. H_2 was observed at 0.48 ppm compared to gas phase TMS set at 0.00 ppm [31]. Thus, the corresponding absolute H_2 gas-phase resonance shift of -28.0 ppm (-28.5 + 0.48) ppm was in excellent agreement with the predicted absolute gas-phase shift of -28.01 ppm given by Eq. (11.411).

20

References for Section I

1. G. R. Fowles, *Analytical Mechanics*, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 145-158.

2. J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Company, (1941), pp. 38-59; 195-267.
3. Jahnke-Emde, *Tables of Functions*, 2nd ed., Teubner, (1933).
4. J. D. Jackson, *Classical Electrodynamics*, Second Edition, John Wiley & Sons, New
5 York, (1975), pp. 17-22.
5. H. A. Haus, J. R. Melcher, "Electromagnetic Fields and Energy", Department of Electrical
engineering and Computer Science, Massachusetts Institute of Technology, (1985), Sec.
5.3.
6. J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Company, (1941), p. 195.
- 10 7. G. R. Fowles, *Analytical Mechanics*, Third Edition, Holt, Rinehart, and Winston, New
York, (1977), pp. 119-124.
8. J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Company, (1941), pp. 38-54;
207-209.
9. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca
15 Raton, Florida, (1998-9), p. A-29.
10. H. A. Haus, "On the radiation from point charges", *American Journal of Physics*, 54,
(1986), pp. 1126-1129.
11. G. R. Fowles, *Analytical Mechanics*, Third Edition, Holt, Rinehart, and Winston, New
York, (1977), pp. 161-164.
- 20 12. G. R. Fowles, *Analytical Mechanics*, Third Edition, Holt, Rinehart, and Winston, New
York, (1977), pp. 57-66.
13. M. Mizushima, *Quantum Mechanics of Atomic Spectra and Atomic Structure*, W.A.
Benjamin, Inc., New York, (1970), p.17.
14. M. Karplus, R. N. Porter, *Atoms and Molecules an Introduction for Students of Physical
25 Chemistry*, The Benjamin/Cummings Publishing Company, Menlo Park, California,
(1970), pp. 447-484.
15. J. D. Jackson, *Classical Electrodynamics*, Second Edition, John Wiley & Sons, New
York, (1975), p. 659.
16. J. D. Jackson, *Classical Electrodynamics*, Second Edition, John Wiley & Sons, New
30 York, (1975), pp. 780-786.
17. D. A. McQuarrie, *Quantum Chemistry*, University Science Books, Mill Valley, CA,
(1983), p. 172.
18. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca
Raton, Florida, (1998-9), p. 10-175.

19. NIST Atomic Spectra Database, www.physics.nist.gov/cgi-bin/AtData/display.ksh.
20. M. Abramowitz, I. A. Stegun (Editors), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc, New York, (1970), pp. 753-759
- 5 21. N. V. Sidgwick, *The Chemical Elements and Their Compounds*, Volume I, Oxford, Clarendon Press, (1950), p.17.
22. P. W. Atkins, *Physical Chemistry*, Second Edition, W. H. Freeman, San Francisco, (1982), p. 589.
23. K. P. Huber, G. Herzberg, *Molecular Spectra and Molecular Structure, IV. Constants of*
10 *Diatomic Molecules*, Van Nostrand Reinhold Company, New York, (1979).
24. W. J. Nellis, "Making Metallic Hydrogen", *Scientific American*, May, (2000), pp. 84-90.
25. H. Beutler, *Z. Physical Chem.*, "Die dissoziationswarne des wasserstoffmolekuls H_2 , aus einem neuen ultravioletten resonanzbandenzug bestimmt", Vol. 27B, (1934), pp. 287-302.
26. G. Herzberg, L. L. Howe, "The Lyman bands of molecular hydrogen", *Can. J. Phys.*, Vol.
15 37, (1959), pp. 636-659.
27. J. Itatani, J. Levesque, D. Zeidler, H. Niikura, H. Pepin, J. C. Kieffer, P. B. Corkum, D. M. Villeneuve, "Tomographic imaging of molecular orbitals", *Nature*, Vol. 432, (2004), pp. 867-871.
28. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca
20 Raton, Florida, (1998-9), p. 10-181.
29. R. Loch, R. Stengler, G. Werth, "Measurement of the electronic g factor of H_2^+ ", *Phys. Rev. A*, Vol. 38, No. 11, (1988), pp. 5484-5488.
30. C. Suarez, E. J. Nicholas, M. R. Bowman, "Gas-phase dynamic NMR study of the internal rotation in N-trifluoroacetlypyrrolidine", *J. Phys. Chem. A*, Vol. 107, (2003), pp. 3024-
25 3029.
31. C. Suarez, "Gas-phase NMR spectroscopy", *The Chemical Educator*, Vol. 3, No. 2, (1998).
32. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. 9-82.
- 30 33. R. L. Mills, J. J. Farrell, *The Grand Unified Theory*, Science Press, (1989), pp. 46-47; 117-119.
34. J. D. Jackson, *Classical Electrodynamics*, Second Edition, John Wiley & Sons, New York, (1975), p. 178.

35. E. Purcell, *Electricity and Magnetism*, McGraw-Hill, New York, (1965), pp. 370-389.
36. J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Company, (1941), pp. 211-215, 257-258.
37. Whittaker and Watson, *Modern Analysis*, 4th Edition, Cambridge University Press,
5 (1927), pp. 512ff.
38. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. A-30.
39. R. Mills, P. Ray, B. Dhandapani, W. Good, P. Jansson, M. Nansteel, J. He, A. Voigt,
10 "Spectroscopic and NMR Identification of Novel Hydride Ions in Fractional Quantum Energy States Formed by an Exothermic Reaction of Atomic Hydrogen with Certain Catalysts", *European Physical Journal-Applied Physics*, Vol. 28, (2004), pp. 83-104.
40. E. Purcell, *Electricity and Magnetism*, McGraw-Hill, New York, (1985), pp. 417-418.

Section II

GENERAL DIATOMIC AND POLYATOMIC MOLECULAR IONS AND MOLECULES

5 Non-hydrogen diatomic and polyatomic molecular ions and molecules can be solved using the same principles as those used to solve hydrogen molecular ions and molecules wherein the hydrogen molecular orbitals (MOs) and hydrogen atomic orbitals serve as basis functions for the MOs of the general diatomic and polyatomic molecular ions or molecules. The MO must (1) be a solution of Laplace's equation to give a equipotential energy surface, (2) correspond to an orbital solution of the Newtonian equation of motion in an inverse-radius-squared central field having a constant total energy, (3) be stable to radiation, and (4) conserve the electron angular momentum of \hbar . Energy of the MO must be matched to that of the outermost atomic orbital of a bonding heteroatom in the case where a minimum energy is achieved with a direct bond to the atomic orbital (AO). In the case that an independent

15 MO is formed, the AO force balance causes the remaining electrons to be at lower energy and a smaller radius. The atomic orbital may hybridize in order to achieve a bond at an energy minimum. At least one molecule or molecular ion representative of each of these cases was solved. Specifically, the results of the determination of bond parameters of H_3^+ , D_3^+ , OH , OD , H_2O , D_2O , NH , ND , NH_2 , ND_2 , NH_3 , ND_3 , CH , CD , CH_2 , CH_3 , CH_4 , N_2 , O_2 , F_2 , Cl_2 , CN , CO , and NO are given in Table 13.1. The calculated results for homo- and hetero-diatomic radicals and molecules, and polyatomic molecular ions and molecules are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

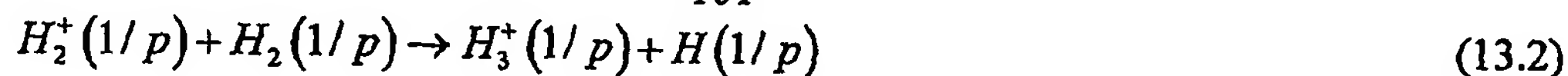
25

TRIATOMIC MOLECULAR HYDROGEN-TYPE ION (H_3^+)

The polyatomic molecular ion $H_3^+(1/p)$ is formed by the reaction of a proton with a hydrogen-type molecule



30 and by the exothermic reaction



FORCE BALANCE OF H_3^+ -TYPE MOLECULAR IONS

$H_3^+(1/p)$ -type molecular ions comprise two indistinguishable spin-paired electrons bound by three protons. The ellipsoidal molecular orbital (MO) satisfies the boundary constraints as shown in the Nature of the Chemical Bond of Hydrogen-Type Molecules section. Since the protons are indistinguishable, ellipsoidal MOs about each pair of protons taken one at a time are indistinguishable. $H_3^+(1/p)$ is then given by a superposition or linear combinations of three equivalent ellipsoidal MOs that form an equilateral triangle where the points of contact between the prolate spheroids are equivalent in energy and charge density. The outer perimeter of the superposition of three prolate spheroids is the $H_3^+(1/p)$ MO with the protons at the foci that bind and maintain the electron MO.

As in the case for $H_2^+(1/p)$ and $H_2(1/p)$ shown in the Nature of the Chemical Bond of Hydrogen-Type Molecules section, the stability of $H_3^+(1/p)$ is due to the dependence of the charge density of the distance D from the origin to the tangent plane. That is,

$$D = \frac{1}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}} \quad (13.3)$$

so that

$$\sigma = \frac{e}{4\pi ab^2} D \quad (13.4)$$

In other words, the surface density at any point on a charged ellipsoidal conductor is proportional to the perpendicular distance from the center of the ellipsoid to the plane tangent to the ellipsoid at the point. The charge is thus greater on the more sharply rounded ends farther away from the origin. This distribution places the charge closest to the protons to give a minimum energy.

The balanced forces also depend on D as shown in the Nature of the Chemical Bond of Hydrogen-Type Molecules section. The D -dependence of the charge density as well as the centrifugal and Coulombic central field of two nuclei at the foci of the ellipsoid applies to each ellipsoid which is given from any other by a rotation of $|\phi| = \frac{\pi}{3}$ about an axis at a focus

that is perpendicular to the plane of the equilateral triangle defined by the three foci. Since the centrifugal, Coulombic, and magnetic forces relate mass and charge densities which are interchangeable by the ratio e/m_e , the conditions at any point on any given ellipsoid is applicable to any other point on the ellipsoid. Furthermore, this condition can be generalized to any point of the other members of the set of three ellipsoids due to equivalence. As a further constraint to maintain the force balance between the three protons and the $H_3^+(1/p)$ MO comprising the superposition of the three $H_2(1/p)$ -type ellipsoidal MOs, the total charge of the two electrons must be normalized over the three basis set $H_2(1/p)$ -type ellipsoidal MOs. In this case, the parameters of each basis element $H_2(1/p)$ -type ellipsoidal MO is solved, and the energies are given by the electron charge where it appears multiplied by a factor of 3/2 (three MOs normalized by the total charge of two electrons).

Consider each $H_2(1/p)$ -type ellipsoidal MO. At each point on the $H_3^+(1/p)$ MO, the electron experiences a centrifugal force, and the balancing centripetal force (on each electron) is produced by the electric force between the electron and the ellipsoidal electric field and the radiation-reaction-type magnetic force between the two electrons causing the electrons to pair. The force balance equation derived in Force Balance of Hydrogen-Type Molecules section is given by Eq. (11.200):

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{pe^2}{8\pi\epsilon_0 ab^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.5)$$

$$\frac{2a_0}{pa} - \frac{a_0}{pa} = 1 \quad (13.6)$$

$$a = \frac{a_0}{p} \quad (13.7)$$

Substitution of Eq. (13.7) into Eq. (11.79) is

$$c' = \frac{1}{p\sqrt{2}} a_0 \quad (13.8)$$

The internuclear distance given by multiplying Eq. (13.8) by two is

$$2c' = \frac{a_0\sqrt{2}}{p} \quad (13.9)$$

Substitution of Eqs. (13.7-13.8) into Eq. (11.80) is

$$b = c = \frac{1}{p\sqrt{2}} a_0 \quad (13.10)$$

Substitution of Eqs. (13.7-13.8) into Eq. (11.67) is

$$e = \frac{1}{\sqrt{2}} \quad (13.11)$$

Using the parameters given by Eqs. (13.7-13.11), the resulting $H_3^+(1/p)$ MO comprising the superposition of three $H_2(1/p)$ -type ellipsoidal MOs is shown in Figure 6. The outer surface of the superposition comprises charge density of the MO. The equilateral triangular structure was confirmed experimentally [1]. The $H_3^+(1/p)$ MO having no distinguishable electrons is consistent with the absence of strong excited states observed for H_3^+ [1]. It is also consistent with the absence of a permanent dipole moment [1].

10 ENERGIES OF H_3^+ -TYPE MOLECULAR IONS

The due to the equivalence of the $H_2(1/p)$ -type ellipsoidal MOs and the linear superposition of their energies, the energy components defined previously for the molecule, Eqs. (11.207-11.212) apply in the case of the corresponding $H_3^+(1/p)$ molecular ion. And, each molecular energy component is given by the integral of corresponding force in Eq. (13.5). Each energy component is the total for the two equivalent electrons with the exception that the total charge of the two electrons is normalized over the three basis set $H_2(1/p)$ -type ellipsoidal MOs. Thus, the energies are those given for $H_2(1/p)$ in the Energies of Hydrogen-Type Molecules section with the electron charge, where it appears, multiplied by a factor of 3/2. In addition, the three sets of equivalent proton-proton pairs give rise to a factor of three times the proton-proton repulsion energy given by Eq. (11.208). The parameters a and b are given by Eqs. (13.7) and (13.10), respectively.

$$V_e = \frac{3}{2} \frac{-2pe^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \quad (13.12)$$

$$V_p = 3 \frac{p}{8\pi\epsilon_0} \frac{e^2}{\sqrt{a^2-b^2}} \quad (13.13)$$

$$T = \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \quad (13.14)$$

25 The energy, V_m , corresponding to the magnetic force of Eq. (13.5) is

$$V_m = \frac{3}{2} \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (13.15)$$

$$E_T = V_s + T + V_m + V_p \quad (13.16)$$

$$E_T = -\frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(3p^2\sqrt{2} - p^2\sqrt{2} + 3\frac{p^2\sqrt{2}}{4} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - 3p^2\sqrt{2} \right] = -p^2 35.54975 \text{ eV} \quad (13.17)$$

5 where the charge e appears in the magnetic energy V_m according to Eqs. (7.14-7.24) as discussed in the Force Balance of Hydrogen-Type Molecules section.

VIBRATION OF H_3^+ -TYPE MOLECULAR IONS

The vibrational energy levels of H_3^+ -type molecular ions may be solved as three equivalent
10 coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [2] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

15 THE DOPPLER ENERGY TERM OF H_3^+ -TYPE MOLECULAR IONS

As shown in the Vibration of Hydrogen-type Molecular Ions section, the electron orbiting the nuclei at the foci of an ellipse may be perturbed such that a stable reentrant orbit is established that gives rise to a vibrational state corresponding to time harmonic oscillation of the nuclei and electron. The perturbation is caused by a photon that is resonant with the
20 frequency of oscillation of the nuclei wherein the radiation is electric dipole with the corresponding selection rules.

Oscillation may also occur in the transition state. The perturbation arises from the decrease in internuclear distance as the molecular bond forms. Relative to the unperturbed case given in the Force Balance of Hydrogen-type Molecular Ions section, the reentrant orbit
25 may give rise to a decrease in the total energy while providing a transient kinetic energy to the vibrating nuclei. However, as an additional condition for stability, radiation must be considered. A nonradiative state must be achieved after the emission due to transient vibration wherein the nonradiative condition given by Eq. (11.24) must be satisfied. The radiation reaction force due to the vibration of $H_2^+(1/p)$ and $H_2(1/p)$ in the transition state

was derived in the Doppler Energy Term of Hydrogen-type Molecular Ions section and the Doppler Energy Term of Hydrogen-type Molecules section, respectively, and corresponds to a Doppler energy, E_D , that is dependent on the motion of the electron and the nuclei. The radiation reaction force in the case of the vibration of $H_3^+(1/p)$ in the transition state also
 5 corresponds to the Doppler energy, E_D , given by Eq. (11.181) that is dependent on the motion of the electrons and the nuclei. Here, a nonradiative state must also be achieved after the emission due to transient vibration wherein the nonradiative condition given by Eq. (11.24) must be satisfied. Typically, a third body is required to form H_3^+ -type molecular ions. For example, the exothermic chemical reaction of $H + H$ to form H_2 does not occur
 10 with the emission of a photon. Rather, the reaction requires a collision with a third body, M , to remove the bond energy- $H + H + M \rightarrow H_2 + M^*$ [3]. The third body distributes the energy from the exothermic reaction, and the end result is the H_2 molecule and an increase in the temperature of the system. Thus, a third body removes the energy corresponding to the additional force term given by Eq. (11.180).

15 The kinetic energy of the transient vibration is derived from the corresponding central forces. From Eqs. (13.5) and (13.12), the central force terms between the electron MO and the protons of each of the three $H_2(1/p)$ -type ellipsoidal MOs are

$$f(a) = -\frac{3}{2} \frac{pe^2}{4\pi\epsilon_0 a^2} \quad (13.18)$$

and

$$20 \quad f'(a) = \frac{3}{2} \frac{2pe^2}{4\pi\epsilon_0 a^3} \quad (13.19)$$

Thus, using Eqs. (11.136) and (13.18-13.19), the angular frequency of this oscillation is

$$\omega = \sqrt{\frac{\frac{3}{2} \frac{pe^2}{4\pi\epsilon_0 \left(\frac{a_0}{p}\right)^3}}{m_e}} = p^2 5.06326 \times 10^{16} \text{ rad/s} \quad (13.20)$$

where the semimajor axis, a , is $a = \frac{a_0}{p}$ according to Eq. (13.7). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$25 \quad \bar{E}_K = \hbar\omega = \hbar p^2 5.06326 \times 10^{16} \text{ rad/s} = p^2 33.3273 \text{ eV} \quad (13.21)$$

In Eq. (11.181), substitution of the total energy of the H_3^+ -type molecular ion, E_T , (Eq. (13.17)) for E_{in} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.21) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit.

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -35.54975 p^2 \text{ eV} \sqrt{\frac{2e(p^2 33.3273 \text{ eV})}{m_e c^2}} = -p^3 0.406013 \text{ eV} \quad (13.22)$$

5 The total energy of the H_3^+ -type molecular ion is decreased by \bar{E}_D .

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. On average, the total energy of vibration is equally distributed between kinetic energy and potential energy [4]. Thus, the average kinetic energy of vibration corresponding to the Doppler energy of the electrons, \bar{E}_{Kvib} , is 1/2
10 of the vibrational energy of the H_3^+ -type molecular ion given by Eq. (11.148). The decrease in the energy of the molecular ion due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D and \bar{E}_{Kvib} . Using Eq. (13.22) and the experimental vibrational energy H_3^+ of $E_{vib} = 2521.31 \text{ cm}^{-1} = 0.312605 \text{ eV}$ [1] gives

$$15 \quad \bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar p^2 \sqrt{\frac{k}{\mu}} \quad (13.23)$$

$$\bar{E}_{osc} = -p^3 0.406013 \text{ eV} + \frac{1}{2} p^2 (0.312605 \text{ eV}) \quad (13.24)$$

The reentrant orbit for the binding of a proton to $H_2(1/p)$ causes two bonds to oscillate by increasing and decreasing in length along opposite sides of the equilateral triangle at a relative phase angle of 180° . Since the vibration and reentrant oscillation is along two
20 lengths of the equilateral triangular MO with E symmetry, \bar{E}_{osc} for $H_3^+(1/p)$, $\bar{E}_{osc}(H_3^+(1/p))$, is:

$$\begin{aligned} \bar{E}_{osc}(H_3^+(1/p)) &= 2 \left(\bar{E}_D + \frac{1}{2} \hbar p^2 \sqrt{\frac{k}{\mu}} \right) \\ &= 2 \left(-p^3 0.406013 \text{ eV} + \frac{1}{2} p^2 (0.312605 \text{ eV}) \right) \end{aligned} \quad (13.25)$$

To the extent that the MO dimensions are the same, the electron reentrant orbital energies, \bar{E}_K , are the same independent of the isotope of hydrogen, but the vibrational

energies are related by Eq. (11.148). Thus, the differences in bond energies are essentially given by 1/2 the differences in vibrational energies per bond. Using Eq. (13.22), Eq. (13.25), and the experimental vibrational energy D_3^+ of $E_{vib} = 1834.67 \text{ cm}^{-1} = 0.227472 \text{ eV}$ [1], the corresponding $\bar{E}_{osc}(D_3^+(1/p))$ is

$$5 \quad \bar{E}_{osc}(D_3^+(1/p)) = 2 \left(-p^3 0.406013 \text{ eV} + \frac{1}{2} p^2 (0.227472 \text{ eV}) \right) \quad (13.26)$$

TOTAL AND BOND ENERGIES OF $H_3^+(1/p)$ - AND $D_3^+(1/p)$ -TYPE MOLECULAR IONS

The total energy of the $H_3^+(1/p)$ -type molecular ion is given by the sum of E_T (Eqs. (13.16-10 13.17)) and $\bar{E}_{osc}(H_3^+(1/p))$ given Eqs. (13.20-13.25). Thus, the total energy of $H_3^+(1/p)$ having a central field of $+pe$ at each focus of the prolate spheroid molecular orbital including the Doppler term is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}(H_3^+(1/p)) \quad (13.27)$$

$$15 \quad E_T = -p^2 \left\{ \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(3\sqrt{2} - \sqrt{2} + \frac{3\sqrt{2}}{4} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - 3\sqrt{2} \right] \left[1 + 2p \sqrt{\frac{2\hbar \sqrt{\frac{3}{2} \frac{e^2}{4\pi\epsilon_0 a_0^3}}}{m_e c^2}} \right] - 2 \left(\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\}$$

$$= -p^2 35.54975 - 2p^3 0.406013 \text{ eV} + 2p^2 \left(\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (13.28)$$

From Eqs. (13.24-13.25) and (13.27-13.28), the total energy of the H_3^+ -type molecular ion is

$$\begin{aligned} E_T &= -p^2 35.54975 \text{ eV} + \bar{E}_{osc}(H_3^+(1/p)) \\ &= -p^2 35.54975 - 2p^3 0.406013 \text{ eV} + 2 \left(\frac{1}{2} p^2 (0.312605 \text{ eV}) \right) \\ &= -p^2 35.23714 \text{ eV} - p^3 0.812025 \text{ eV} \end{aligned} \quad (13.29)$$

The total energy of the D_3^+ -type molecular ion is given by the sum of E_T (Eq. (13.17)) and 20 $\bar{E}_{osc}(D_3^+(1/p))$ given by Eq. (13.26):

$$\begin{aligned}
 E_T &= -p^2 35.54975 \text{ eV} + \bar{E}_{osc}^{108}(D_3^+(1/p)) \\
 &= -p^2 35.54975 - 2p^3 0.406013 \text{ eV} + 2\left(\frac{1}{2}p^2 (0.227472 \text{ eV})\right) \\
 &= -p^2 35.32227 \text{ eV} - p^3 0.812025 \text{ eV}
 \end{aligned} \tag{13.30}$$

The bond dissociation energy, E_D , is the difference between the total energy of the corresponding hydrogen molecule and E_T

$$E_D = E(H_2(1/p)) - E_T \tag{13.31}$$

5 where $E(H_2(1/p))$ is given by Eq. (11.241):

$$E(H_2(1/p)) = -p^2 31.351 \text{ eV} - p^3 0.326469 \text{ eV} \tag{13.32}$$

and $E(D_2(1/p))$ is given by Eq. (11.242):

$$E(D_2(1/p)) = -p^2 31.4345 \text{ eV} - p^3 0.326469 \text{ eV} \tag{13.33}$$

The H_3^+ bond dissociation energy, E_D , is given by Eqs. (13.31-13.32) and (13.29):

$$\begin{aligned}
 E_D &= -p^2 31.351 \text{ eV} - p^3 0.326469 \text{ eV} - E_T \\
 10 \quad &= -p^2 31.351 \text{ eV} - p^3 0.326469 \text{ eV} - (-p^2 35.23714 \text{ eV} - p^3 0.812025 \text{ eV}) \\
 &= p^2 3.88614 \text{ eV} + p^3 0.485556 \text{ eV}
 \end{aligned} \tag{13.34}$$

The D_3^+ bond dissociation energy, E_D , is given by Eqs. (13.31), (13.33), and (13.30):

$$\begin{aligned}
 E_D &= -p^2 31.4345 \text{ eV} - p^3 0.326469 \text{ eV} - E_T \\
 &= -p^2 31.4345 \text{ eV} - p^3 0.326469 \text{ eV} - (-p^2 35.32227 \text{ eV} - p^3 0.812025 \text{ eV}) \\
 &= p^2 3.88777 \text{ eV} + p^3 0.485556 \text{ eV}
 \end{aligned} \tag{13.35}$$

15 THE H_3^+ MOLECULAR ION

FORCE BALANCE OF THE H_3^+ MOLECULAR ION

The force balance equation for H_3^+ is given by Eq. (13.5) where $p=1$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D \tag{13.36}$$

20 which has the parametric solution given by Eq. (11.83) when

$$a = a_0 \tag{13.37}$$

The semimajor axis, a , is also given by Eq. (13.7) where $p=1$. The internuclear distance, $2c'$, which is the distance between the foci is given by Eq. (13.9) where $p=1$.

$$2c' = \sqrt{2}a_0 \quad (13.38)$$

The semiminor axis is given by Eq. (13.10) where $p=1$.

$$5 \quad b = \frac{1}{\sqrt{2}}a_0 \quad (13.39)$$

The eccentricity, e , is given by Eq. (13.11).

$$e = \frac{1}{\sqrt{2}} \quad (13.40)$$

ENERGIES OF THE H_3^+ MOLECULAR ION

10 The energies of H_3^+ are given by Eqs. (13.12-13.15) where $p=1$

$$V_e = \frac{3}{2} \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -101.7538 \text{ eV} \quad (13.41)$$

$$V_p = 3 \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 57.7245 \text{ eV} \quad (13.42)$$

$$T = \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 33.9179 \text{ eV} \quad (13.43)$$

The energy, V_m , of the magnetic force is

$$15 \quad V_m = \frac{3}{2} \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -25.4384 \text{ eV} \quad (13.44)$$

The Doppler terms, $\bar{E}_{osc}(H_3^+(1/p))$ and $\bar{E}_{osc}(D_3^+(1/p))$ are given by Eqs. (13.25) and (13.26), respectively, where $p=1$

$$\begin{aligned} \bar{E}_{osc}(H_2^+) &= 2(\bar{E}_D + \bar{E}_{Kvib}) \\ &= 2\left(-0.406013 \text{ eV} + \frac{1}{2}(0.312605 \text{ eV})\right) \\ &= -0.499420 \text{ eV} \end{aligned} \quad (13.45)$$

$$\begin{aligned} \bar{E}_{osc}(D_2) &= 2\left(-0.406013 \text{ eV} + \frac{1}{2}(0.227472 \text{ eV})\right) \\ &= -0.584553 \text{ eV} \end{aligned} \quad (13.46)$$

20 The total energy, E_T , for H_3^+ given by Eqs. (13.27-13.29) is

$$\begin{aligned}
 E_T &= - \left\{ \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(3\sqrt{2} - \sqrt{2} + \frac{3\sqrt{2}}{4} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - 3\sqrt{2} \right] \left[1 + 2 \sqrt{\frac{2\hbar \sqrt{\frac{3}{2} \frac{e^2}{4\pi\epsilon_0 a_0^3}}}{m_e c^2}} \right] - 2 \left(\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\} \\
 &= -35.54975 - 2(0.406013 \text{ eV}) + 2 \left(\frac{1}{2} (0.31260516 \text{ eV}) \right) \\
 &= -36.049167 \text{ eV}
 \end{aligned}
 \tag{13.47}$$

From Eqs. (13.27-13.28) and (13.30), the total energy, E_T , for D_3^+ is

$$\begin{aligned}
 E_T &= -35.54975 - 2(0.406013 \text{ eV}) + 2 \left(\frac{1}{2} (0.227472 \text{ eV}) \right) \\
 &= -36.134300 \text{ eV}
 \end{aligned}
 \tag{13.48}$$

- 5 The bond dissociation energy, E_D , is the difference between the total energy of H_2 or D_2 and E_T . The H_3^+ molecular bond dissociation energy, E_D , given by the difference between the experimental total energy of H_2 [5-7]¹ and the total energy of H_3^+ (Eqs. (13.29) where $p=1$ and (13.47)) is

$$\begin{aligned}
 E_D &= -31.675 \text{ eV} - (-36.049167 \text{ eV}) \\
 &= 4.374167 \text{ eV}
 \end{aligned}
 \tag{13.49}$$

- 10 The H_3^+ bond dissociation energy, E_D , given by Eq. (13.34) where $p=1$ is

$$\begin{aligned}
 E_D &= 3.88614 \text{ eV} + 0.485556 \text{ eV} \\
 &= 4.37170 \text{ eV}
 \end{aligned}
 \tag{13.50}$$

The experimental bond dissociation energy of H_3^+ [8] is

$$E_D = 4.373 \text{ eV}
 \tag{13.51}$$

- 15 The difference between the results of Eqs. (13.49) and (13.50) is within the experimental and propagated errors in the different calculations. The calculated results are based on first principles and given in closed-form equations containing fundamental constants only. The agreement between the experimental and calculated results for the H_3^+ bond dissociation energy is excellent.

¹ The experimental total energy of the hydrogen molecule is given by adding the first (15.42593 eV) [5] and second (16.2494 eV) ionization energies where the second ionization energy is given by the addition of the ionization energy of the hydrogen atom (13.59844 eV) [6] and the bond energy of H_2^+ (2.651 eV) [7].

The predicted D_3^+ molecular bond dissociation energy, E_D , given by the difference between the total energy of D_3^+ (Eqs. (13.30) where $p=1$ and (13.48)) and the experimental total energy of D_2 [9-10]² is

$$\begin{aligned} E_D &= -31.76 \text{ eV} - (-36.134300 \text{ eV}) \\ &= 4.374300 \text{ eV} \end{aligned} \quad (13.52)$$

5 The D_3^+ bond dissociation energy, E_D , given by Eq. (13.35) where $p=1$ is

$$\begin{aligned} E_D &= 3.88777 \text{ eV} + 0.485556 \text{ eV} \\ &= 4.373331 \text{ eV} \end{aligned} \quad (13.53)$$

The results of the determination of bond parameters of H_3^+ are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and
10 calculated results is excellent.

HYDROXYL RADICAL (OH)

The water molecule can be solved by first considering the solution of the hydroxyl radical which is formed by the reaction of a hydrogen atom and an oxygen atom:



The hydroxyl radical OH can be solved using the same principles as those used to solve the hydrogen molecule wherein the diatomic molecular orbital (MO) developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section serves as basis function in linear combination with an oxygen atomic orbital (AO) to form the MO of OH .
20 The MO must (1) be a solution of Laplace's equation to give a equipotential energy surface, (2) correspond to an orbital solution of the Newtonian equation of motion in an inverse-radius-squared central field having a constant total energy, (3) be stable to radiation, and (4) conserve the electron angular momentum of \hbar . A further constraint with the substitution of a heteroatom (O) for one of the hydrogen atoms is that the constant energy of the MO must
25 match the energy of the heteroatom.

² The experimental total energy of the deuterium molecule is given by adding the first (15.466 eV) [9] and second (16.294 eV) ionization energies where the second ionization energy is given by the addition of the ionization energy of the deuterium atom (13.603 eV) [10] and the bond energy of D_2^+ (2.692 eV) [9].

FORCE BALANCE OF OH

OH comprises two spin-paired electrons in a chemical bond between the oxygen atom and the hydrogen atom such that one electron on O remains unpaired. The OH radical MO is determined by considering properties of the binding atoms and the boundary constraints. The
 5 prolate spheroidal H_2 MO developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules section satisfies the boundary constraints; thus, the H -atom electron forms a H_2 -type ellipsoidal MO with one of the O -atom electrons. The O electron configuration given in the Eight-Electron Atoms section is $1s^2 2s^2 2p^4$, and the orbital arrangement is

$$\begin{array}{ccc} & 2p \text{ state} & \\ \uparrow \downarrow & \uparrow & \uparrow \\ \hline 1 & 0 & -1 \end{array} \quad (13.55)$$

10 corresponding to the ground state 3P_2 .

In determining the central forces for O in the Radius and Ionization Energy of the Outer Electron of the Oxygen Atom section, it was shown that the energy is minimized with conservation of angular momentum by the cancellation of the orbital angular momentum of a p_x electron by that of the p_y electron with the pairing of electron eight to fill the p_x orbital.
 15 Then, the diamagnetic force is given by Eq. (10.156) is that of atomic nitrogen (Eq. (10.136) corresponding to the p_z -orbital electron (Eq. (10.82) with $m=0$) as the source of diamagnetism with an additional contribution from the uncanceled p_x electron (Eq. (10.82) with $m=1$). From Eqs. (10.83) and (10.89), the paramagnetic force, $F_{mag\ 2}$, is given by Eq. (10.157) corresponding to the spin-angular-momentum contribution alone of the p_x electron
 20 and the orbital angular momentum of the p_z electron, respectively. The diamagnetic and paramagnetic forces cancel such that the central force is purely the Coulombic force. This central force is maintained with bond formation such that the energy of the $O2p$ shell is unchanged. Thus, the angular momentum of each electron of the $O2p$ shell is conserved with bond formation. The central paramagnetic force due to spin is provided by the spin-
 25 pairing force of the OH MO that has the symmetry of an s orbital that superimposes with the $2p$ orbitals such that the corresponding angular momenta of the $O2p$ orbitals are unchanged.

The $O2p_y$ electron combines with the $H1s$ electron to form a molecular orbital. The proton of the H atom is along the internuclear axis. Due to symmetry, the other O electrons

are equivalent to point charges at the origin. (See Eqs. (19-38) of Appendix IV.) Thus, the energies in the OH MO involve only the $O2p_y$ and $H1s$ electrons and the change in the magnetic energy of the $O2p_y$ electron with the other O electrons (Eq. (13.152)) with the formation of the OH MO. The forces are determined by these energies.

5 As in the case of H_2 , the MO is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into O atom for distances shorter than the radius of the $2p$ shell. Otherwise, the electric field of the other $O2p$ electrons would be perturbed, and the $2p$ shell would not be stable. The corresponding increase in energy of O would not be offset by any energy decrease in the OH MO based on the distance from the O nucleus to
10 the $H1s$ electron compared to those of the $O2p$ electrons. Thus, the MO surface comprises a prolate spheroid at the H proton that is continuous with the $2p$ shell at the O atom. The energy of the prolate spheroid is matched to that of the $O2p$ shell.

The orbital energy E for each elliptical cross section of the prolate spheroidal MO is given by the sum of the kinetic T and potential V energies. $E = T + V$ is constant, and the
15 closed orbits are those for which $T < |V|$, and the open orbits are those for which $T \geq |V|$. It can be shown that the time average of the kinetic energy, $\langle T \rangle$, for elliptic motion in an inverse-squared field is $1/2$ that of the time average of the magnitude of the potential energy, $|\langle V \rangle|$. $\langle T \rangle = 1/2 |\langle V \rangle|$ [11]. In the case of an atomic orbital (AO), $E = T + V$, and for all points on the AO, $|E| = T = 1/2 |V|$. As shown in the Hydrogen-type Molecular Ions section,
20 each point or coordinate position on the continuous two-dimensional electron MO defines an infinitesimal mass-density element which moves along an orbit comprising an elliptic plane cross section of the spheroidal MO through the foci. The motion is such that eccentric angle, θ , changes at a constant rate at each point. That is $\theta = \omega t$ at time t where ω is a constant, and

$$25 \quad r(t) = ia \cos \omega t + jb \sin \omega t \quad (13.56)$$

Consider the boundary condition that the MO of OH comprises a linear combination of an oxygen AO and a H_2 -type ellipsoidal MO. The charge density of H_2 -type ellipsoidal MO given by Eq. (13.4) maintains that the surface is an equipotential; however, the potential and kinetic energy of a point on the surface changes as it orbits the central field. The potential
30 energy is a maximum and the kinetic energy is a minimum at the semimajor axis, and the reverse occurs at the semiminor axis. Since the time average of the kinetic energy, $\langle T \rangle$, for

elliptic motion in an inverse-squared field is $1/2$ that of the time average of the magnitude of the potential energy, by symmetry, the $\langle T \rangle = 1/2 \langle |V| \rangle$ condition holds for $1/2$ of the H_2 -type ellipsoidal MO having the H focus and ending at the plane defined by the semiminor axes. The O nucleus comprises the other focus of the OH MO. The $O2p$ AO obeys the
 5 energy relationship for all points. Thus, the linear combination of the H_2 -type ellipsoidal MO with the $O2p$ AO must involve a 25% contribution from the H_2 -type ellipsoidal MO to the $O2p$ AO in order to match the energy relationships. Thus, the OH MO must comprise 75% of a H_2 -type ellipsoidal MO ($1/2 + 25\%$) and an oxygen AO:

$$10 \quad 1 O2p_y \text{ AO} + 0.75 H_2 \text{ MO} \rightarrow OH \text{ MO} \quad (13.57)$$

The force balance of the OH MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (13.57). The force constant k of a H_2 -type ellipsoidal MO due to the equivalent of two point charges of at the foci is given by
 15 Eq. (11.65):

$$k = \frac{2e^2}{4\pi\epsilon_0} \quad (13.58)$$

Since the H_2 -type ellipsoidal MO comprises 75% of the OH MO, the electron charge density in Eq. (13.58) is given by $-0.75e$. Thus, k' of the H_2 -type-ellipsoidal-MO component of the OH MO is

$$20 \quad k' = \frac{(0.75)2e^2}{4\pi\epsilon_0} \quad (13.59)$$

L for the electron equals \hbar ; thus, the distance from the origin of the OH MO to each focus c' is given by Eqs. (11.79) and (13.59):

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 1.5a}} = \sqrt{\frac{2aa_0}{3}} \quad (13.60)$$

The internuclear distance from Eq. (13.60) is

$$25 \quad 2c' = 2\sqrt{\frac{2aa_0}{3}} \quad (13.61)$$

The length of the semiminor axis of the prolate spheroidal OH MO $b = c$ given by Eq. (11.80) is

$$b = \sqrt{a^2 - c'^2} \quad (13.62)$$

The eccentricity, e , is

$$e = \frac{c'}{a} \quad (13.63)$$

Then, the solution of the semimajor axis a allows for the solution of the other axes of the
5 prolate spheroidal and eccentricity of the OH MO.

The general equation of the ellipsoidal MO having semiprincipal axes a, b, c given by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (13.64)$$

is also completely determined by the total energy E given by Eq. (11.18):

$$10 \quad r = \frac{m \frac{L^2}{m^2} k^{-1}}{1 + \left(1 + 2Em \frac{L^2}{m^2} k^{-2} \right)^{1/2} \cos \theta} \quad (13.65)$$

The energy of the oxygen $2p$ shell is the negative of the ionization energy of the oxygen atom given by Eq. (10.163). Experimentally, the energy is [12]

$$E(2p \text{ shell}) = -E(\text{ionization}; O) = -13.6181 \text{ eV} \quad (13.66)$$

Since the prolate spheroidal MO transitions to the O AO, the energy E in Eq. (13.66) adds
15 to that of the H_2 -type ellipsoidal MO to give the total energy of the OH MO. From the energy equation and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the OH MO are solved.

The energy components derived previously for the hydrogen molecule, Eqs. (11.207-11.212), apply in the case of the H_2 -type ellipsoidal MO. As in the case of the energies of
20 $H_3^+(1/p)$ given by Eqs. (13.12-13.16), each energy component of the H_2 -type ellipsoidal MO is the total for the two equivalent electrons with the exception that the total charge and energies of the two electrons is normalized by the percentage composition given by Eq. (13.57):

$$V_e = \left(\frac{3}{4} \right) \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (13.67)$$

$$25 \quad V_p = \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \quad (13.68)$$

$$T = \left(\frac{3}{4}\right) \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (13.69)$$

$$V_m = \left(\frac{3}{4}\right) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (13.70)$$

$$E_T = V_e + T + V_m + V_p \quad (13.71)$$

$$E_T = -\frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[\left(\frac{3}{2} - \frac{3}{4} \frac{a_0}{a} + \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] \quad (13.72)$$

$$5 \quad E_T = -\frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] \quad (13.73)$$

Since the prolate spheroidal MO transitions to the O AO and the energy of the $O2p$ shell must remain constant and equal to the negative of the ionization energy given by Eq. (13.66), the total energy $E_T(OH)$ of the OH MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the O AO and the H_2 -type ellipsoidal MO that forms the OH MO as given by Eq. (13.57):

$$\begin{aligned} E_T(OH) &= E_T + E(2p \text{ shell}) \\ &= E_T - E(\text{ionization}; O) \\ &= -\frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] - 13.6181 \text{ eV} \end{aligned} \quad (13.74)$$

To match the boundary condition that the total energy of the entire the H_2 -type ellipsoidal MO is given by Eq. (11.212):

$$E_T(H_2) = -\frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2} + 1}{\sqrt{2} - 1} - \sqrt{2} \right] = -31.63536831 \text{ eV} \quad (13.75)$$

15 $E_T(OH)$ given by Eq. (13.74) is set equal to Eq. (13.75):

$$\begin{aligned} E_T(OH) &= -\frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] - 13.6181 \text{ eV} = -31.63536831 \text{ eV} \\ (13.76) \end{aligned}$$

From the energy relationship given by Eq. (13.76) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the OH MO can be solved.

20 Substitution of Eq. (13.60) into Eq. (13.76) gives

$$\frac{e^2}{8\pi\epsilon_0\sqrt{\frac{2aa_0}{3}}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e18.01726831 \quad (13.77)$$

The most convenient way to solve Eq. (13.77) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.26430a_0 = 6.69039 \times 10^{-11} \text{ m} \quad (13.78)$$

5 Substitution of Eq. (13.78) into Eq. (13.60) gives

$$c' = 0.91808a_0 = 4.85826 \times 10^{-11} \text{ m} \quad (13.79)$$

The internuclear distance given by multiplying Eq. (13.79) by two is

$$2c' = 1.83616a_0 = 9.71651 \times 10^{-11} \text{ m} \quad (13.80)$$

The experimental bond distance is [13]

$$10 \quad 2c' = 9.71 \times 10^{-11} \text{ m} \quad (13.81)$$

Substitution of Eqs. (13.78-13.79) into Eq. (13.62) gives

$$b = c = 0.86925a_0 = 4.59985 \times 10^{-11} \text{ m} \quad (13.82)$$

Substitution of Eqs. (13.78-13.79) into Eq. (13.63) gives

$$e = 0.72615 \quad (13.83)$$

15 The nucleus of the *H* atom and the nucleus of the *O* atom comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $O2p_y$ AO can be determined from the polar equation of the ellipse (Eq. (11.10)):

$$r = r_0 \frac{1+e}{1+e \cos \theta'} \quad (13.84)$$

20 The radius of the $O2p_y$ AO given by Eq. (10.162) is $r_s = a_0$, and the polar radial coordinate of the ellipse and the radius of the $O2p_y$ AO are equal at the point of intersection. Thus, Eq. (13.84) becomes

$$a_0 = (a - c') \frac{1 + \frac{c'}{a}}{1 + \frac{c'}{a} \cos \theta'} \quad (13.85)$$

such that the polar angle θ' is given by

$$\theta' = \cos^{-1} \left(\frac{a}{c'} \left((a - c') \frac{1 + \frac{c'}{a_0}}{a_0} - 1 \right) \right) \quad (13.86)$$

Substitution of Eqs. (13.78-13.79) into Eq. (13.86) gives

$$\theta' = 123.65^\circ \quad (13.87)$$

Then, the angle θ_{O2p_yAO} the radial vector of the $O2p_y$ AO makes with the internuclear axis is

$$5 \quad \theta_{O2p_yAO} = 180^\circ - 123.65^\circ = 56.35^\circ \quad (13.88)$$

as shown in Figure 7.

The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{H_2MO}$ between the
10 internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the O radial vector obeys the following relationship:

$$a_0 \sin \theta_{O2p_yAO} = b \sin \theta_{H_2MO} \quad (13.89)$$

such that

$$\theta_{H_2MO} = \sin^{-1} \frac{a_0 \sin 56.35^\circ}{b} \quad (13.90)$$

15 with the use of Eq. (13.88). Substitution of Eq. (13.82) into Eq. (13.90) gives

$$\theta_{H_2MO} = 73.27^\circ \quad (13.91)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{H_2MO} = a \cos \theta_{H_2MO} \quad (13.92)$$

20 Substitution of Eqs. (13.78) and (13.91) into Eq. (13.92) gives

$$d_{H_2MO} = 0.36397a_0 = 1.92606 \times 10^{-11} \text{ m} \quad (13.93)$$

The distance d_{O2p_yAO} along the internuclear axis from the origin of the O atom to the point of intersection of the orbitals is given by

$$d_{O2p_yAO} = c' - d_{H_2MO} \quad (13.94)$$

25 Substitution of Eqs. (13.79) and (13.93) into Eq. (13.94) gives

$$d_{O2p_yAO} = 0.55411a_0 = 2.93220 \times 10^{-11} \text{ m} \quad (13.95)$$

As shown in Eq. (13.57), in addition to the p -orbital charge-density modulation, the uniform charge-density in the p_y orbital is increased by a factor of 0.25 and the H -atom density is decreased by a factor of 0.25. The internuclear axis of the $O-H$ bond is perpendicular to the bonding p_y orbital. Using the orbital composition of OH (Eq. (13.57)), the radii of $O1s = 0.12739a_0$ (Eq. (10.51)), $O2s = 0.59020a_0$ (Eq. (10.62)), and $O2p = a_0$ (Eq. (10.162)) shells, and the parameters of the OH MO given by Eqs. (13.3-13.4), (13.78-13.80), (13.82-13.83), and (13.87-13.95), the dimensional diagram and charge-density of the OH MO comprising the linear combination of the H_z -type ellipsoidal MO and the O AO according to Eq. (13.57) are shown in Figures 7 and 8, respectively.

10

ENERGIES OF OH

The energies of OH given by the substitution of the semiprincipal axes (Eqs. (13.78-13.80) and (13.82)) into the energy equations (Eqs. (13.67-13.73)) are

$$V_e = \left(\frac{3}{4}\right) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -40.92709 \text{ eV} \quad (13.96)$$

$$15 \quad V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 14.81988 \text{ eV} \quad (13.97)$$

$$T = \left(\frac{3}{4}\right) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 16.18567 \text{ eV} \quad (13.98)$$

$$V_m = \left(\frac{3}{4}\right) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -8.09284 \text{ eV} \quad (13.99)$$

$$E_T(OH) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 13.6181 \text{ eV} = -31.63247 \text{ eV} \quad (13.100)$$

where $E_T(OH)$ is given by Eq. (13.74) which is reiteratively matched to Eq. (13.75) within five-significant-figure round-off error.

20

VIBRATION AND ROTATION OF OH

The vibrational energy of OH may be solved in the same manner as that of hydrogen-type molecular ions and hydrogen molecules given in the Vibration of Hydrogen-type Molecular Ions section, and the Vibration of Hydrogen-type Molecules section, respectively, except that the orbital composition and the requirement that the $O2p$ shell remain at the same energy and

25

radius in the OH MO as it is in the O atom must be considered. Each p -orbital comprises the sum of a constant function and a spherical harmonic function as given by Eq. (1.65). In addition to the p -orbital charge-density modulation, the uniform charge-density in p_y orbital is increased by a factor of 0.25, and the H -atom electron density is decreased by a factor of 5 0.25. The force between the electron density of the H_2 -type ellipsoidal MO and the nuclei determines the vibrational energy. With the radius of the orbit at the oxygen atom fixed at

$$r_g = a_0 \quad (13.101)$$

according to Eq. (10.162), the central-force terms for the reentrant orbit between the electron density and the nuclei of the H_2 -type ellipsoidal MO are given by Eqs. (11.213-11.214), 10 except that the corresponding charge of $-0.75e$ replaces the charge of $-e$ of Eqs. (11.213-11.214). Furthermore, due to condition that the $O2p$ shell remain at the same energy and radius in the OH MO as it is in the O atom, the oscillation of H_2 -type ellipsoidal is along the semiminor axis with the apsidal angle of Eq. (11.140) given by $\psi = \pi$. Thus, the semimajor axis a of Eqs. (11.213-11.214) is replaced by the semiminor axis b :

$$15 \quad f(b) = -\frac{0.75e^2}{8\pi\epsilon_0 b^2} \quad (13.102)$$

and

$$f'(b) = \frac{0.75e^2}{4\pi\epsilon_0 b^3} \quad (13.103)$$

Here, the force factor of 0.75 is equal to the equivalent term of Eq. (13.59). As the H_2 -type ellipsoidal oscillates along b , the internuclear distance changes 180° out of phase. Thus, the 20 distance for the reactive nuclear-repulsive terms is given by internuclear distance $2c'$ (Eq. (13.80)). Similar to that of Eqs. (11.215-11.216), the contribution from the repulsive force between the two nuclei is

$$f(2c') = \frac{e^2}{8\pi\epsilon_0 (2c')^2} \quad (13.104)$$

and

$$25 \quad f'(2c') = -\frac{e^2}{4\pi\epsilon_0 (2c')^3} \quad (13.105)$$

Thus, from Eqs. (11.136), (11.213-11.217), and (13.102-13.105), the angular frequency of the oscillation is

$$\begin{aligned}
 \omega &= \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 b^3} - \frac{e^2}{8\pi\epsilon_0 (2c')^3}}{\mu}} \\
 &= \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 (0.86925a_0)^3} - \frac{e^2}{8\pi\epsilon_0 (1.83616a_0)^3}}{\frac{16}{17}m_p}} \\
 &= 6.96269 \times 10^{14} \text{ rad/s}
 \end{aligned} \tag{13.106}$$

where b is given by Eq. (13.82), $2c'$ is given by Eq. (13.80), and the reduced mass of ^{16}OH is given by:

$$\mu_{^{16}\text{OH}} = \frac{m_1 m_2}{m_1 + m_2} = \frac{(1)(16)}{1+16} m_p \tag{13.107}$$

5 where m_p is the proton mass. Thus, during bond formation, the perturbation of the orbit determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit, and the corresponding frequency, $\omega(0)$, for ^{16}OH given by Eqs. (11.136), (11.148), and (13.106) is

$$\omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{763.18 \text{ Nm}^{-1}}{\mu}} = 6.96269 \times 10^{14} \text{ radians/s} \tag{13.108}$$

10 where the reduced nuclear mass of ^{16}OH is given by Eq.(13.107) and the spring constant, $k(0)$, given by Eqs. (11.136) and (13.106) is

$$k(0) = 763.18 \text{ Nm}^{-1} \tag{13.109}$$

The ^{16}OH transition-state vibrational energy, $E_{\text{vib}}(0)$, given by Planck's equation (Eq. (11.127)) is:

$$15 \quad E_{\text{vib}}(0) = \hbar\omega = \hbar 6.96269 \times 10^{14} \text{ rad/s} = 0.4583 \text{ eV} = 3696.38 \text{ cm}^{-1} \tag{13.110}$$

Zero-order or zero-point vibration is not physical and is not observed experimentally as discussed in the Diatomic Molecular Vibration section; yet, there is a term ω_e of the old point-particle-probability-wave-mechanics that can be compared to $E_{\text{vib}}(0)$. From Herzberg [14], ω_e , from the experimental curve fit of the vibrational energies of ^{16}OH is

$$20 \quad \omega_e = 3735.21 \text{ cm}^{-1} \tag{13.111}$$

As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and

expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter (K&P) [15] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods. The energy $\tilde{\nu}_v$ of state v is

$$5 \quad \tilde{\nu}_v = v\omega_0 - v(v-1)\omega_0 x_0, \quad v = 0, 1, 2, 3 \dots \quad (13.112)$$

where

$$\omega_0 x_0 = \frac{hc\omega_0^2}{4D_0} \quad (13.113)$$

ω_0 is the frequency of the $v=1 \rightarrow v=0$ transition, and D_0 is the bond dissociation energy given by Eq. (13.162). From Eq. (13.112), ω_0 is given by

$$10 \quad \omega_0 = E_{vib}(0) - 2\omega_0 x_0 \quad (13.114)$$

Substitution of Eq. (13.113) into Eq. (13.114) gives

$$\omega_0 = E_{vib}(0) - 2\frac{hc\omega_0^2}{4D_0} \quad (13.115)$$

Eq. (13.115) can be expressed as

$$\omega_0^2 + \frac{2D_0}{hc}\omega_0 - \frac{2D_0}{hc}E_{vib}(0) = 0 \quad (13.116)$$

15 which can be solved by the quadratic formula:

$$\omega_0 = \frac{-\frac{2D_0}{hc} \pm \sqrt{\left(\frac{2D_0}{hc}\right)^2 + 4\frac{2D_0}{hc}E_{vib}(0)}}{2} \quad (13.117)$$

Only the positive root is real, physical; thus,

$$\begin{aligned} \omega_0 (cm^{-1}) &= \frac{-\frac{2D_0}{100hc} + \sqrt{\left(\frac{2D_0}{100hc}\right)^2 + 4\frac{2D_0}{100hc}E_{vib}(0)}}{2} \\ &= \frac{-\frac{2e(4.4104 eV)}{100hc} + \sqrt{\left(\frac{2e(4.4104 eV)}{100hc}\right)^2 + 4\frac{2e(4.4104 eV)}{100hc}(3696.37 cm^{-1})}}{2} \\ &= 3522.02 cm^{-1} \end{aligned} \quad (13.118)$$

where $E_{vib}(0)$ is given by Eq. (13.110) and D_0 is given by Eq. (13.156). The corresponding ^{16}OH $\nu=1 \rightarrow \nu=0$ vibrational energy, $E_{vib}(1)$, in electron volts is:

$$E_{vib}(1) = 0.43666 \text{ eV} \quad (13.119)$$

The experimental vibrational energy of ^{16}OH is [16-17]

$$5 \quad E_{vib}(1) = 0.4424 \text{ eV} \quad (3568 \text{ cm}^{-1}) \quad (13.120)$$

Using Eqs. (13.118-13.119) with Eq. (13.113), the anharmonic perturbation term, $\omega_0 x_0$, of ^{16}OH is

$$\omega_0 x_0 = \frac{100hc \left(8.06573 \times 10^3 \frac{\text{cm}^{-1}}{\text{eV}} 0.43666 \text{ eV} \right)^2}{4e(4.4104 \text{ eV})} \text{cm}^{-1} = 87.18 \text{ cm}^{-1} \quad (13.121)$$

The experimental anharmonic perturbation term, $\omega_0 x_0$, of ^{16}OH [14] is

$$10 \quad \omega_0 x_0 = 82.81 \text{ cm}^{-1} \quad (13.122)$$

The vibrational energies of successive states are given by Eqs. (13.110), (13.112), and (13.121).

Using the reduced nuclear mass of ^{16}OD given by

$$\mu_{^{16}OD} = \frac{m_1 m_2}{m_1 + m_2} = \frac{(2)(16)}{2+16} m_p \quad (13.123)$$

15 where m_p is the proton mass, the corresponding parameters for deuterated hydroxyl radical ^{16}OD (Eqs. (13.102-13.121) and (13.162)) are

$$\omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{763.18 \text{ Nm}^{-1}}{\mu}} = 5.06610 \times 10^{14} \text{ radians/s} \quad (13.124)$$

$$k(0) = 763.18 \text{ Nm}^{-1} \quad (13.125)$$

$$E_{vib}(0) = \hbar\omega = \hbar 5.06610 \times 10^{14} \text{ rad/s} = 0.33346 \text{ eV} = 2689.51 \text{ cm}^{-1} \quad (13.126)$$

$$\begin{aligned}
 \omega_0 (cm^{-1}) &= \frac{-\frac{2D_0}{100hc} + \sqrt{\left(\frac{2D_0}{100hc}\right)^2 + 4\frac{2D_0}{100hc} E_{vib}(0)}}{2} \\
 &= \frac{-\frac{2e(4.4687 eV)}{100hc} + \sqrt{\left(\frac{2e(4.4687 eV)}{100hc}\right)^2 + 4\frac{2e(4.4687 eV)}{100hc} (2689.51 cm^{-1})}}{2} \\
 &= 2596.02 cm^{-1}
 \end{aligned}
 \tag{13.127}$$

$$E_{vib}(1) = 0.3219 eV \tag{13.128}$$

$$\omega_0 x_0 = \frac{100hc \left(8.06573 \times 10^3 \frac{cm^{-1}}{eV} 0.3219 eV \right)^2}{4e(4.4687 eV)} cm^{-1} = 46.75 cm^{-1} \tag{13.129}$$

From Herzberg [14], ω_e , from the experimental curve fit of the vibrational energies of ^{16}OD is

$$\omega_e = 2720.9 cm^{-1} \tag{13.130}$$

The experimental vibrational energy of ^{16}OD is [16-17]

$$E_{vib}(1) = 0.3263 eV \quad (2632.1 cm^{-1}) \tag{13.131}$$

and the experimental anharmonic perturbation term, $\omega_0 x_0$, of ^{16}OD [14] is

$$\omega_0 x_0 = 44.2 cm^{-1} \tag{13.132}$$

which match the predictions given by Eqs. (13.126), (13.127-13.128), and (13.129), respectively.

15 The B_e rotational parameters for ^{16}OH and ^{16}OD are given by Eq. (12.65):

$$B_e = \frac{\hbar^2}{2I_e hc} \tag{13.133}$$

where

$$I = \mu r^2 \tag{13.134}$$

Using the internuclear distance, $r = 2c'$, and reduced mass of ^{16}OH given by Eqs. (13.80) and (13.107), respectively, the corresponding B_e is

$$B_e = 18.835 cm^{-1} \tag{13.135}$$

The experimental B_e rotational parameter of ^{16}OH is [14]

$$B_e = 18.871 \text{ cm}^{-1} \quad (13.136)$$

Using the internuclear distance, $r = 2c'$, and reduced mass of ^{16}OD given by Eqs. (13.80) and (13.123), respectively, the corresponding B_e is

$$B_e = 9.971 \text{ cm}^{-1} \quad (13.137)$$

5 The experimental B_e rotational parameter of ^{16}OD is [14]

$$B_e = 10.01 \text{ cm}^{-1} \quad (13.138)$$

THE DOPPLER ENERGY TERMS OF ^{16}OH AND ^{16}OD

The radiation reaction force in the case of the vibration of ^{16}OH in the transition state
10 corresponds to the Doppler energy, E_D , given by Eq. (11.181) and Eq. (13.22) that is
dependent on the motion of the electrons and the nuclei. The kinetic energy of the transient
vibration is derived from the corresponding central forces. Following the same consideration
as those used to derive Eqs. (13.102-13.103) and Eqs. (11.231-11.232), the central force
terms between the electron density and the nuclei of ^{16}OH MO with the radius of the orbit at
15 the oxygen atom fixed at

$$r_s = a_0 \quad (13.139)$$

according to Eq. (10.162) are

$$f(b) = -\frac{0.75e^2}{4\pi\epsilon_0 b^2} \quad (13.140)$$

and

$$20 \quad f'(b) = \frac{(0.75)2e^2}{4\pi\epsilon_0 b^3} \quad (13.141)$$

wherein the oscillation of H_2 -type ellipsoidal MO is along the semiminor axis b with the
apsidal angle of Eq. (11.140) given by $\psi = \pi$ due to condition that the $O2p$ shell remain at
the same energy and radius in the OH MO as it is in the O atom. Thus, using Eqs. (11.136)
and (13.140-13.141), the angular frequency of this oscillation is

$$25 \quad \omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 4.41776 \times 10^{16} \text{ rad/s} \quad (13.142)$$

The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

126

$$\bar{E}_K = \hbar\omega = \hbar 4.41776 \times 10^{16} \text{ rad/s} = 29.07844 \text{ eV} \quad (13.143)$$

In Eq. (11.181), substitution of the total energy of OH , $E_T(OH)$, (Eq. (13.76)) for E_{lv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.143) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit.

$$5 \quad \bar{E}_D \cong E_{lv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(29.07844 \text{ eV})}{m_e c^2}} = -0.33749 \text{ eV} \quad (13.144)$$

The total energy of OH is decreased by \bar{E}_D .

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. On average, the total energy of vibration is equally distributed between kinetic energy and potential energy [4]. Thus, the average
10 kinetic energy of vibration corresponding to the Doppler energy of the electrons, \bar{E}_{Kvib} , is 1/2 of the vibrational energy of OH given by Eq. (13.120). The decrease in the energy of the OH due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D and \bar{E}_{Kvib} . Using Eq. (13.144) and the experimental ^{16}OH ω_e of
15 3735.21 cm^{-1} (0.463111 eV) [16-17] gives

$$\bar{E}_{osc}(^{16}OH) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.145)$$

$$\bar{E}_{osc}(^{16}OH) = -0.33749 \text{ eV} + \frac{1}{2} (0.463111 \text{ eV}) = -0.10594 \text{ eV} \quad (13.146)$$

To the extent that the MO dimensions are the same, the electron reentrant orbital energies, \bar{E}_K , are the same independent of the isotope of hydrogen, but the vibrational
20 energies are related by Eq. (11.148). Thus, the differences in bond energies are essentially given by 1/2 the differences in vibrational energies per bond. Using Eq. (13.144), Eqs. (13.145-13.146), and the experimental ^{16}OD ω_e of 2720.9 cm^{-1} (0.33735 eV) [16-17], the corresponding $\bar{E}_{osc}(^{16}OD)$ is

$$\bar{E}_{osc}(^{16}OD) = -0.33749 \text{ eV} + \frac{1}{2} (0.33735 \text{ eV}) = -0.16881 \text{ eV} \quad (13.147)$$

TOTAL AND BOND ENERGIES OF ^{16}OH AND ^{16}OD RADICALS

$E_{T+osc}(^{16}\text{OH})$, the total energy of the ^{16}OH radical including the Doppler term, is given by the sum of $E_T(\text{OH})$ (Eq. (13.76)) and $\bar{E}_{osc}(^{16}\text{OH})$ given by Eqs. (13.142-13.146):

$$\begin{aligned} E_{T+osc}(^{16}\text{OH}) &= V_e + T + V_m + V_p + E(2p \text{ shell}) + \bar{E}_{osc}(^{16}\text{OH}) \\ &= E_T(\text{OH}) + \bar{E}_{osc}(^{16}\text{OH}) \end{aligned} \quad (13.148)$$

$$\begin{aligned} E_{T+osc}(^{16}\text{OH}) &= \left\{ \frac{-e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 13.6181 \text{ eV} \right\} \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} \right] + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \\ &= -31.63537 \text{ eV} - 0.33749 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \end{aligned} \quad (13.149)$$

From Eqs. (13.145-13.146) and (13.148-13.149), the total energy of ^{16}OH is

$$\begin{aligned} E_{T+osc}(^{16}\text{OH}) &= -31.63537 \text{ eV} + \bar{E}_{osc}(^{16}\text{OH}) \\ &= -31.63537 \text{ eV} - 0.33749 \text{ eV} + \frac{1}{2} (0.463111 \text{ eV}) \\ &= -31.74130 \text{ eV} \end{aligned} \quad (13.150)$$

where the experimental ω_e was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. $E_{T+osc}(^{16}\text{OD})$, the total energy of

^{16}OD including the Doppler term, is given by the sum of $E_T(\text{OD}) = E_T(\text{OH})$ (Eq. (13.76)) and $\bar{E}_{osc}(^{16}\text{OD})$ given by Eq. (13.147):

$$\begin{aligned} E_{T+osc}(^{16}\text{OD}) &= -31.63537 \text{ eV} + \bar{E}_{osc}(^{16}\text{OD}) \\ &= -31.63537 \text{ eV} - 0.33749 \text{ eV} + \frac{1}{2} (0.33735 \text{ eV}) \\ &= -31.80418 \text{ eV} \end{aligned} \quad (13.151)$$

where the experimental ω_e was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. The dissociation of the bond of the

hydroxyl radical forms a free hydrogen atom with one unpaired electron and an oxygen atom with two unpaired electrons as shown in Eq. (13.55) which interact to stabilize the atom as shown by Eq. (10.161-10.162). The lowering of the energy of the reactants decreases the bond energy. Thus, the total energy of oxygen is reduced by the energy in the field of the two magnetic dipoles given by Eq. (7.46) and Eq. (13.101):

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 a_0^3} = \frac{8\pi\mu_0 \mu_B^2}{a_0^3} = 0.114411 \text{ eV} \quad (13.152)$$

The corresponding bond dissociation energy, E_D , is given by the sum of the total energies of the oxygen atom and the corresponding hydrogen atom minus the sum of $E_{T+osc}({}^{16}\text{OH})$ and $E(\text{magnetic})$:

$$5 \quad E_D = E({}^{16}\text{O}) + E(\text{H}) - E_{T+osc}({}^{16}\text{OH}) - E(\text{magnetic}) \quad (13.153)$$

$E({}^{16}\text{O})$ is given by Eq. (13.66), $E_D(\text{H})$ [18] is

$$E(\text{H}) = -13.59844 \text{ eV} \quad (13.154)$$

and $E_D(\text{D})$ [19] is

$$E(\text{D}) = -13.603 \text{ eV} \quad (13.155)$$

10 The ${}^{16}\text{OH}$ bond dissociation energy, $E_D({}^{16}\text{OH})$, is given by Eqs. (13.150) and (13.152-13.155):

$$\begin{aligned} E_D({}^{16}\text{OH}) &= -(13.6181 \text{ eV} + 13.59844 \text{ eV}) - (E(\text{magnetic}) + E_{T+osc}({}^{16}\text{OH})) \\ &= -27.21654 \text{ eV} - (0.114411 \text{ eV} - 31.74130 \text{ eV}) \\ &= 4.4104 \text{ eV} \end{aligned} \quad (13.156)$$

The experimental ${}^{16}\text{OH}$ bond dissociation energy is [20]

$$E_D({}^{16}\text{OH}) = 4.41174 \text{ eV} \quad (13.157)$$

15 The ${}^{16}\text{OD}$ bond dissociation energy, $E_D({}^{16}\text{OD})$, is given by Eqs. (13.151-13.153):

$$\begin{aligned} E_D({}^{16}\text{OD}) &= -(13.6181 \text{ eV} + 13.603 \text{ eV}) - (E(\text{magnetic}) + E_{T+osc}({}^{16}\text{OD})) \\ &= -27.2211 \text{ eV} - (0.114411 \text{ eV} - 31.804183 \text{ eV}) \\ &= 4.4687 \text{ eV} \end{aligned} \quad (13.158)$$

The experimental ${}^{16}\text{OD}$ bond dissociation energy is [21-22]

$$E_D({}^{16}\text{OD}) = 4.454 \text{ eV} \quad (13.159)$$

The results of the determination of bond parameters of OH and OD are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

WATER MOLECULE (H_2O)

The water molecule H_2O is formed by the reaction of a hydrogen atom with a hydroxyl radical:



- 5 The water molecule can be solved using the same principles as those used to solve the hydrogen molecule, H_2 , and OH wherein the diatomic molecular orbital (MO) developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section serves as basis function in a linear combination with an oxygen atomic orbital (AO) to form the MO of H_2O . The solution is very similar to that of OH except that there are two OH
- 10 bonds in water.

FORCE BALANCE OF H_2O

H_2O comprises two chemical bonds between oxygen and hydrogen. Each $O-H$ bond comprises two spin-paired electrons with one from an initially unpaired electron of the

15 oxygen atom and the other from the hydrogen atom. The H_2O MO is determined by considering properties of the binding atoms and the boundary constraints. The H_2 prolate spheroidal MO satisfies the boundary constraints as shown in the Nature of the Chemical Bond of Hydrogen-Type Molecules section; thus, each H -atom electron forms a H_2 -type ellipsoidal MO with one of the initially unpaired O -atom electrons. The initial O electron

20 configuration given in the Eight-Electron Atoms section is $1s^2 2s^2 2p^4$, and the orbital arrangement is given by Eqs. (10.154) and Eq. (13.55).

As shown in the case of OH in the Force Balance of OH section, the forces that determine the radius and the energy of the $O2p$ shell are unchanged with bond formation. Thus, the angular momentum of each electron of the $O2p$ is conserved with bond formation.

25 The central paramagnetic force due to spin of each $O-H$ bond is provided by the spin-paring force of the H_2O MO that has the symmetry of an s orbital that superimposes with the $O2p$ orbitals such that the corresponding angular momenta are unchanged.

Each of the $O2p_z$ and $O2p_x$ electron combines with a $H1s$ electron to form a molecular orbital. The proton of the H atom is along the internuclear axis. Due to

30 symmetry, the other O electrons are equivalent to point charges at the origin. (See Eqs. (19-

38) of Appendix IV.) Thus, the energies in the H_2O MO involve only each $O2p$ and each $H1s$ electron with the formation of each $O-H$ bond. The forces are determined by these energies.

As in the case of H_2 , each of two $O-H$ -bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into the O atom for distances shorter than the radius of the $2p$ shell. Otherwise, the electric field of the other $O2p$ electrons would be perturbed, and the $2p$ shell would not be stable. The corresponding increase in energy of O would not be offset by any energy decrease in the $O-H$ -bond MO based on the distance from the O nucleus to the $H1s$ electron compared to those of the $O2p$ electrons. Thus, the MO surface comprises a prolate spheroid at each H proton that is continuous with the $2p$ shell at the O atom. The sum of the energies of the prolate spheroids is matched to that of the $2p$ shell.

The orbital energy E for each elliptical cross section of the prolate spheroidal MO is given by the sum of the kinetic T and potential V energies. $E = T + V$ is constant, and the closed orbits are those for which $T < |V|$, and the open orbits are those for which $T \geq |V|$. It can be shown that the time average of the kinetic energy, $\langle T \rangle$, for elliptic motion in an inverse-squared field is $1/2$ that of the time average of the magnitude of the potential energy, $\langle |V| \rangle$. $\langle T \rangle = 1/2 \langle |V| \rangle$ [11]. In the case of an atomic orbital (AO), $E = T + V$, and for all points on the AO, $|E| = T = 1/2|V|$. As shown in the Hydrogen-type Molecular Ions section, each point or coordinate position on the continuous two-dimensional electron MO defines an infinitesimal mass-density element which moves along an orbit comprising an elliptic plane cross section of the spheroidal MO through the foci. The motion is such that eccentric angle, θ , changes at a constant rate at each point. That is $\theta = \omega t$ at time t where ω is a constant, and

$$r(t) = ia \cos \omega t + jb \sin \omega t \quad (13.161)$$

Consider the boundary condition that the MO of H_2O comprises a linear combination of an oxygen AO and two H_2 -type ellipsoidal MOs, one for each $O-H$ -bond. The charge density of each H_2 -type ellipsoidal MO given by Eqs. (11.44-11.45) and (13.3-13.4) maintains that the surface is an equipotential; however, the potential and kinetic energy of a point on the surface changes as it orbits the central field. The potential energy is a maximum and the kinetic energy is a minimum at the semimajor axis, and the reverse occurs at the semiminor

axis. Since the time average of the kinetic energy, $\langle T \rangle$, for elliptic motion in an inverse-squared field is $1/2$ that of the time average of the magnitude of the potential energy, by symmetry, the $\langle T \rangle = 1/2 \langle |V| \rangle$ condition holds for $1/2$ of each H_2 -type ellipsoidal MO having the H focus and ending at the plane defined by the semiminor axes. The O nucleus
 5 comprises the other focus of each OH -MO component of the H_2O MO. The $O2p$ AO obeys the energy relationship for all points. Thus, the linear combination of the H_2 -type ellipsoidal MO with the $O2p$ AO must involve a 25% contribution from the H_2 -type ellipsoidal MO to the $O2p$ AO in order to match the energy relationships. Thus, the H_2O MO must comprise two $O-H$ -bonds with each comprising 75% of a H_2 -type ellipsoidal
 10 MO ($1/2 + 25\%$) and an oxygen AO:

$$[1 O2p_z AO + 0.75 H_2 MO] + [1 O2p_y AO + 0.75 H_2 MO] \rightarrow H_2O MO \quad (13.162)$$

The force balance of the H_2O MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (13.162). The force constant k
 15 of a H_2 -type ellipsoidal MO due to the equivalent of two point charges of at the foci is given by Eq. (11.65):

$$k = \frac{2e^2}{4\pi\epsilon_0} \quad (13.163)$$

Since the each H_2 -type ellipsoidal MO comprises 75% of the $O-H$ -bond MO, the electron charge density in Eq. (13.163) is given by $-0.75e$. Thus, k' of the each H_2 -type-ellipsoidal-
 20 MO component of the H_2O MO is

$$k' = \frac{(0.75)2e^2}{4\pi\epsilon_0} \quad (13.164)$$

L for the electron equals \hbar ; thus, the distance from the origin of each $O-H$ -bond MO to each focus c' is given by Eqs. (11.79) and (13.164):

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 1.5a}} = \sqrt{\frac{2aa_0}{3}} \quad (13.165)$$

25 The internuclear distance from Eq. (13.165) is

$$2c' = 2\sqrt{\frac{2aa_0}{3}} \quad (13.166)$$

The length of the semiminor axis of the prolate spheroidal $O-H$ -bond MO $b=c$ given by Eq. (11.80) is

$$b = \sqrt{a^2 - c'^2} \quad (13.167)$$

The eccentricity, e , is

$$5 \quad e = \frac{c'}{a} \quad (13.168)$$

The solution of the semimajor axis a then allows for the solution of the other axes of the prolate spheroid and eccentricity of the $O-H$ -bond MO.

The general equation of the ellipsoidal MO having semiprincipal axes a, b, c given by

$$10 \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad (13.169)$$

is also completely determined by the total energy E given by Eq. (11.18):

$$r = \frac{m \frac{L^2}{m^2} k^{-1}}{1 + \left(1 + 2Em \frac{L^2}{m^2} k^{-2} \right)^{1/2} \cos \theta} \quad (13.170)$$

The energy of the oxygen $2p$ shell is the negative of the ionization energy of the oxygen atom given by Eqs. (10.163) and (13.66). Experimentally, the energy is [12]

$$15 \quad E(2p \text{ shell}) = -E(\text{ionization}; O) = -13.6181 \text{ eV} \quad (13.171)$$

Since each of the two prolate spheroidal $O-H$ -bond MOs comprises a H_2 -type-ellipsoidal MO that transitions to the O AO, the energy E in Eq. (13.171) adds to that of the two corresponding H_2 -type ellipsoidal MOs to give the total energy of the H_2O MO. From the energy equation and the relationship between the axes given by Eqs. (13.165-13.168), the
20 dimensions of the H_2O MO are solved.

The energy components defined previously for the molecule, Eqs. (11.207-11.212), apply in the case of H_2O . Since the H_2O MO comprises two equivalent $O-H$ -bond MOs, each a linear combination of a H_2 -type-ellipsoidal MO and an $O2p$ AO, the corresponding energy component of the H_2O MO is given by the linear superposition of the component
25 energies. Thus, the energy scale factor is given as two times the force factor, the term in parentheses in Eq. (13.164). In addition to the equivalence and linearity principles, this factor also arises from the consideration of the nature of each bond and the linear combination that

forms the H_2O MO. Each $O-H$ -bond-energy component is the total for the two equivalent electrons with the exception that the total charge of the two electrons is normalized over the three basis set functions, two $O-H$ -bond MOs (OH -type ellipsoidal MOs given in the Energies of OH section) and one $O2p$ AO. Thus, the contribution of the $O-H$ -bond MOs to the H_2O MO energies are those given for $H_2(1/p)$ in the Energies of Hydrogen-Type Molecules multiplied by a factor of $3/2$ as in the case with H_3^+ (Eqs. (13.12), (13.15), 13.18-13.20)). In addition, the two sets of equivalent nuclear-point-charge pairs give rise to a factor of two times the proton-proton repulsion energy given by Eq. (11.208). Thus, the component energies of the H_2O MO are twice the corresponding energies of the OH MO given by Eqs. (13.67-13.73). The parameters a , b , and c' are given by Eqs. (13.165-13.167), respectively.

$$V_e = 2 \left(\frac{3}{4} \right) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = \left(\frac{3}{2} \right) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \quad (13.172)$$

$$V_p = 2 \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \quad (13.173)$$

$$T = 2 \left(\frac{3}{4} \right) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = \left(\frac{3}{2} \right) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \quad (13.174)$$

$$V_m = 2 \left(\frac{3}{4} \right) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = \left(\frac{3}{2} \right) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \quad (13.175)$$

$$E_T = V_e + T + V_m + V_p \quad (13.176)$$

$$E_T = -\frac{e^2}{4\pi\epsilon_0\sqrt{a^2-b^2}} \left[\left(\frac{3}{2} - \frac{3}{4} \frac{a_0}{a} + \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} - 1 \right] \quad (13.177)$$

$$E_T = -\frac{e^2}{4\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \quad (13.178)$$

Since the each prolate spheroidal H_2 -type MO transitions to the O AO and the energy of the $O2p$ shell must remain constant and equal to the negative of the ionization energy given by Eq. (13.171), the total energy $E_T(H_2O)$ of the H_2O MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the O AO and the two H_2 -type ellipsoidal MOs that forms the H_2O MO as given by Eq. (13.162):

$$\begin{aligned}
 E_T(H_2O) &= E_T + E(2p \text{ shell}) \\
 &= E_T - E(\text{ionization}; O) \\
 &= -\frac{e^2}{4\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 13.6181 \text{ eV}
 \end{aligned} \tag{13.179}$$

The two hydrogen atoms and the oxygen atom can achieve an energy minimum as a linear combination of two H_2 -type ellipsoidal MOs each having the proton and the oxygen nucleus as the foci. Each $O-H$ -bond MO comprises the same $O2p$ shell of constant energy given by Eq. (13.171). Thus, the energy of the H_2O MO is also given by the sum of that of the two H_2 -type ellipsoidal MOs given by Eq. (11.212) minus the energy of the redundant oxygen atom of the linear combination given by Eq. (13.171):

$$\begin{aligned}
 E_T(2H_2 - O) &= -2 \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] - E(2p \text{ shell}) \\
 &= 2(-31.63536831 \text{ eV}) - (-13.6181 \text{ eV}) \\
 &= -49.652637 \text{ eV}
 \end{aligned} \tag{13.180}$$

$E_T(H_2O)$ given by Eq. (13.179) is set equal to two times the energy of the H_2 -type ellipsoidal MO minus the energy of the $O2p$ shell given by Eq. (13.180):

$$E_T(H_2O) = -\frac{e^2}{4\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 13.6181 \text{ eV} = -49.652637 \text{ eV} \tag{13.181}$$

From the energy relationship given by Eq. (13.181) and the relationship between the axes given by Eqs. (13.165-13.167), the dimensions of the H_2O MO can be solved.

Substitution of Eq. (13.165) into Eq. (13.181) gives

$$\frac{e^2}{4\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e36.034537 \tag{13.182}$$

The most convenient way to solve Eq. (13.182) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.2641a_0 = 6.68933 \times 10^{-11} \text{ m} \tag{13.183}$$

Substitution of Eq. (13.183) into Eq. (13.165) gives

$$c' = 0.918005a_0 = 4.85787 \times 10^{-11} \text{ m} \tag{13.184}$$

The internuclear distance given by multiplying Eq. (13.184) by two is

$$2c' = 1.83601a_0 = 9.71574 \times 10^{-11} \text{ m} \quad (13.185)$$

The experimental bond distance is [23]

$$2c' = 9.70 \pm .005 \times 10^{-11} \text{ m} \quad (13.186)$$

Substitution of Eqs. (13.177-13.176) into Eq. (13.167) gives

$$5 \quad b = c = 0.869031a_0 = 4.59871 \times 10^{-11} \text{ m} \quad (13.187)$$

Substitution of Eqs. (13.177-13.176) into Eq. (13.168) gives

$$e = 0.726212 \quad (13.188)$$

The nucleus of the H atom and the nucleus of the O atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of each H_2 -type ellipsoidal MO and the $O2p_y$ AO or $O2p_z$ AO can be determined from the polar equation of the ellipse (Eq. (11.10)):

$$r = r_0 \frac{1+e}{1+e \cos \theta'} \quad (13.189)$$

The radius of the $O2p$ shell given by Eq. (10.162) is $r_0 = a_0$, and the polar radial coordinate of the ellipse and the radius of the $O2p$ shell are equal at the point of intersection. Thus, Eq. (13.189) becomes

$$a_0 = (a - c') \frac{1 + \frac{c'}{a}}{1 + \frac{c'}{a} \cos \theta'} \quad (13.190)$$

such that the polar angle θ' is given by

$$\theta' = \cos^{-1} \left(\frac{a}{c'} \left((a - c') \frac{1 + \frac{c'}{a}}{a_0} - 1 \right) \right) \quad (13.191)$$

Substitution of Eqs. (13.177-13.176) into Eq. (13.191) gives

$$20 \quad \theta' = 123.66^\circ \quad (13.192)$$

Then, the angle θ_{O2pAO} the radial vector of the $O2p$ AO makes with the internuclear axis is

$$\theta_{O2pAO} = 180^\circ - 123.66^\circ = 56.33^\circ \quad (13.193)$$

as shown in Figure 7. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{H_2MO}$ between the internuclear axis and the point of intersection of each H_2 -type ellipsoidal MO with the O radial vector obeys the following relationship:

$$a_0 \sin \theta_{O2pAO} = b \sin \theta_{H_2MO} \quad (13.194)$$

such that

$$\theta_{H_2MO} = \sin^{-1} \frac{a_0 \sin \theta_{O2pAO}}{b} = \sin^{-1} \frac{a_0 \sin 56.33^\circ}{b} \quad (13.195)$$

with the use of Eq. (13.193). Substitution of Eq. (13.188) into Eq. (13.195) gives

$$5 \quad \theta_{H_2MO} = 73.28^\circ \quad (13.196)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{H_2MO} = a \cos \theta_{H_2MO} \quad (13.197)$$

Substitution of Eqs. (13.183) and (13.196) into Eq. (13.197) gives

$$10 \quad d_{H_2MO} = 0.3637a_0 = 1.9244 \times 10^{-11} \text{ m} \quad (13.198)$$

The distance d_{O2pAO} along the internuclear axis from the origin of the O atom to the point of intersection of the orbitals is given by

$$d_{O2pAO} = c' - d_{H_2MO} \quad (13.199)$$

Substitution of Eqs. (13.184) and (13.198) into Eq. (13.199) gives

$$15 \quad d_{O2pAO} = 0.5543a_0 = 2.93343 \times 10^{-11} \text{ m} \quad (13.200)$$

In addition to the intersection of the H_2 -type MO with the $O2p$ shell, two adjoining ellipsoidal H_2 -type MOs intersect at points of equipotential. The angle and distance parameters are given by Eqs. (13.595-13.600) for the limiting methane case wherein four adjoining intersecting H_2 -type MOs have the possibility of forming a self-contained two-
 20 dimension equipotential surface of charge and current. Charge continuity can be obeyed for the H_2O MO if the current is continuous between the adjoining H_2 -type MOs. However, in the limiting case of methane, the existence of a separate linear combination of the H_2 -type MOs comprising four-spin paired electrons, not connected to the bonding carbon heteroatom requires that the electron be divisible. It is possible for an electron to form time-dependent
 25 singular points or nodes having no charge as shown by Eqs. (1.65a-1.65b), and two-dimensional charge distributions having Laplacian potentials and one-dimensional regions of zero charge are possible for macroscopic charge densities and currents as given in Haus and Melcher [24]. However, it is not possible for single electrons to have two dimensional discontinuities in charge based on internal forces and first principles discussed in Appendix

IV. Thus, at the points of intersection of the H_2 -type MOs of methane, symmetry, electron indivisibility, current continuity, and conservation of energy and angular momentum require that the current between the points of mutual contact and the carbon atom be projected onto and flow along the radial vector to the surface of the $C2sp^3$ shell. This current designed the bisector current (BC) meets the $C2sp^3$ surface and does not travel to distances shorter than its radius. The methane result must also apply in the case of other bonds including that of the water molecule. Here, the H_2 -type MOs intersect and the ellipsoidal current is projected onto the radial vector to the $O2p$ shell and does not travel to distances shorter than its radius as in the case of a single $O-H$ bond.

As shown in Eq. (13.162), in addition to the p -orbital charge-density modulation, the uniform charge-density in the p_z and p_y orbitals is increased by a factor of 0.25 and the H atoms are each decreased by a factor of 0.25. Using the orbital composition of H_2O (Eq. (13.162)), the radii of $O1s = 0.12739a_0$ (Eq. (10.51)), $O2s = 0.59020a_0$ (Eq. (10.62)), and $O2p = a_0$ (Eq. (10.162)) shells, and the parameters of the H_2O MO given by Eqs. (13.3-13.4), (13.183-13.185), (13.187-13.188), and (13.192-13.200), the charge-density of the H_2O MO comprising the linear combination of two $O-H$ -bond MOs (OH -type ellipsoidal MOs given in the Energies of OH section) according to Eq. (13.162) is shown in Figure 9. Each $O-H$ -bond MO comprises a H_2 -type ellipsoidal MO and an $O2p$ AO having the dimensional diagram shown in Figure 8.

ENERGIES OF H_2O

The energies of H_2O given by the substitution of the semiprincipal axes (Eqs. (13.183-13.185) and (13.187)) into the energy equations (Eqs. (13.172-13.180)) are

$$V_e = \left(\frac{3}{2}\right) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -81.8715 \text{ eV} \quad (13.201)$$

$$V_p = 2 \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 29.6421 \text{ eV} \quad (13.202)$$

$$T = \left(\frac{3}{2}\right) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 32.3833 \text{ eV} \quad (13.203)$$

$$V_m = \left(\frac{3}{2}\right) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -16.1917 \text{ eV} \quad (13.204)$$

$$E_T(H_2O) = -\frac{e^2}{4\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] - 13.6181 \text{ eV} = -49.6558 \text{ eV} \quad (13.205)$$

where $E_T(H_2O)$ is given by Eq. (13.179) which is reiteratively matched to Eq. (13.180) within five-significant-figure round-off error.

5

VIBRATION OF H_2O

The vibrational energy levels of H_2O may be solved as two equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [2] wherein the spring constants are derived from the central forces as
10 given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERM OF H_2O

The radiation reaction force in the case of the vibration of H_2O in the transition state
15 corresponds to the Doppler energy, E_D , given by Eq. (11.181) and Eqs. (13.22) and (13.144) that is dependent on the motion of the electrons and the nuclei. The kinetic energy of the transient vibration is derived from the corresponding central forces. As in the case of H_3^+ , the water molecule is a linear combination of three orbitals. The water MO comprises two H_2 -type ellipsoidal MOs and the O AO. Thus, the force factor of water in the determination
20 of the Doppler frequency is equivalent to that of the H_3^+ ion given in Eqs. (13.18-13.20) and given by Eq. (13.164). From Eqs. (11.231-11.232) and (13.18-13.20), the central force terms between the electron density and the nuclei of each $O-H$ -bond MO with the radius of the orbit at the oxygen atom fixed at

$$r_g = a_0 \quad (13.206)$$

25 according to Eq. (10.162) with the oscillation along the semiminor axis are

$$f(b) = -\left(\frac{3}{2}\right) \frac{e^2}{4\pi\epsilon_0 b^2} \quad (13.207)$$

and

$$f'(b) = \left(\frac{3}{2}\right) \frac{2e^2}{4\pi\epsilon_0 b^3} \quad (13.208)$$

Thus, using Eqs. (11.136) and (13.207-13.208), the angular frequency of this oscillation is

$$\omega = \sqrt{\frac{\left(\frac{3}{2}\right) \frac{e^2}{4\pi\epsilon_0 b^3}}{m_e}} = 6.24996 \times 10^{16} \text{ rad/s} \quad (13.209)$$

The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$5 \quad \bar{E}_K = \hbar\omega = \hbar 6.24996 \times 10^{16} \text{ rad/s} = 41.138334 \text{ eV} \quad (13.210)$$

The three basis elements of water, H , H , and O , all have the same Coulombic energy as given by Eqs. (1.243) and (10.163), respectively, such that the Doppler energy involves the total energy of the H_2O MO. Thus, in Eq. (11.181), substitution of the total energy of H_2O , $E_T(H_2O)$, (Eqs. (13.179-13.180) and Eq. (13.181)) for E_{hv} , the mass of the electron, m_e , for
10 M , and the kinetic energy given by Eq. (13.210) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -49.652637 \text{ eV} \sqrt{\frac{2e(41.138334 \text{ eV})}{m_e c^2}} = -0.630041 \text{ eV} \quad (13.211)$$

The total energy of H_2O is decreased by \bar{E}_D .

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the
15 transition state at their corresponding frequency. On average, the total energy of vibration is equally distributed between kinetic energy and potential energy [4]. Thus, the average kinetic energy of vibration corresponding to the Doppler energy of the electrons, \bar{E}_{Kvib} , is 1/2 of the vibrational energy of H_2O . The decrease in the energy of H_2O due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and
20 nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D and \bar{E}_{Kvib} . Using Eq. (13.211) and the experimental $H^{16}OH$ vibrational energy of $E_{vib} = 3755.93 \text{ cm}^{-1} = 0.465680 \text{ eV}$ [25] gives

$$\bar{E}'_{osc} = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.212)$$

$$\bar{E}'_{osc} = -0.630041 \text{ eV} + \frac{1}{2} (0.465680 \text{ eV}) = -0.397201 \text{ eV} \quad (13.213)$$

per bond. As in the case for $H_3^+(1/p)$ shown in the Doppler Energy Term of H_3^+ -type Molecular Ions section, the reentrant orbit for the binding of a hydrogen atom to a hydroxyl radical causes the bonds to oscillate by increasing and decreasing in length along the two $O-H$ bonds at a relative phase angle of 180° . Since the vibration and reentrant oscillation
 5 is along two bonds for the asymmetrical stretch (ν_3), \bar{E}_{osc} for $H^{16}OH$, $\bar{E}_{osc}(H^{16}OH)$, is:

$$\begin{aligned}\bar{E}_{osc}(H^{16}OH) &= 2\left(\bar{E}_D + \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}}\right) \\ &= 2\left(-0.630041 \text{ eV} + \frac{1}{2}(0.465680 \text{ eV})\right) \\ &= -0.794402 \text{ eV}\end{aligned}\tag{13.214}$$

To the extent that the MO dimensions are the same, the electron reentrant orbital energies, \bar{E}_K , are the same independent of the isotope of hydrogen, but the vibrational energies are related by Eq. (11.148). Thus, the differences in bond energies are essentially
 10 given by 1/2 the differences in vibrational energies per bond. Using Eq. (13.211), Eqs. (13.212-13.214), and the experimental $D^{16}OD$ vibrational energy of $E_{vib} = 2787.92 \text{ cm}^{-1} = 0.345661 \text{ eV}$ [25], the corresponding $\bar{E}_{osc}(D^{16}OD)$ is

$$\begin{aligned}\bar{E}_{osc}(D^{16}OD) &= 2\left(-0.630041 \text{ eV} + \frac{1}{2}(0.345661 \text{ eV})\right) \\ &= -0.914421 \text{ eV}\end{aligned}\tag{13.215}$$

15 TOTAL AND BOND ENERGIES OF $H^{16}OH$ AND $D^{16}OD$

$E_{T+osc}(H_2^{16}O)$, the total energy of the $H^{16}OH$ including the Doppler term, is given by the sum of $E_T(H_2O)$ (Eq. (13.181)) and $\bar{E}_{osc}(H^{16}OH)$ given Eqs. (13.207-13.214):

$$\begin{aligned}E_{T+osc}(H_2^{16}O) &= V_e + T + V_m + V_p + E(O2p) + \bar{E}_{osc}(H^{16}OH) \\ &= E_T(H_2O) + \bar{E}_{osc}(H^{16}OH)\end{aligned}\tag{13.216}$$

$$E_{T+osc}(H_2^{16}O) = \left\{ \left(\frac{-e^2}{4\pi\epsilon_0 c'} \left(\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right) - 13.6181 \text{ eV} \right) \left(1 + 2 \sqrt{\frac{2\hbar \sqrt{\frac{3}{2} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} \right) + 2 \left(\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\}$$

$$= -49.652637 \text{ eV} - 2 \left(0.630041 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (13.217)$$

From Eqs. (13.214) and (13.216-13.217), the total energy of $H^{16}OH$ is

$$\begin{aligned} E_{T+osc}(H_2^{16}O) &= -49.652637 \text{ eV} + \bar{E}_{osc}(H^{16}OH) \\ &= -49.652637 \text{ eV} - 2 \left(0.630041 \text{ eV} - \frac{1}{2} (0.465680 \text{ eV}) \right) \\ &= -50.447039 \text{ eV} \end{aligned} \quad (13.218)$$

5 where the experimental vibrational energy was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. $E_{T+osc}(D_2^{16}O)$, the total energy of $D^{16}OD$ including the Doppler term is given by the sum of $E_T(D_2O) = E_T(H_2O)$ (Eq. (13.181)) and $\bar{E}_{osc}(D^{16}OD)$ given by Eq. (13.215):

$$\begin{aligned} E_{T+osc}(D_2^{16}O) &= -49.652637 \text{ eV} + \bar{E}_{osc}(D^{16}OD) \\ &= -49.652637 \text{ eV} - 2 \left(0.630041 \text{ eV} - \frac{1}{2} (0.345661 \text{ eV}) \right) \\ &= -50.567058 \text{ eV} \end{aligned} \quad (13.219)$$

where the experimental vibrational energy was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. As in the case of the

10 hydroxyl radical, the dissociation of the bond of the water molecule forms a free hydrogen atom and a hydroxyl radical, with one unpaired electron each. The lowering of the energy of the reactants due to the magnetic dipoles decreases the bond energy. Thus, the total energy of oxygen is reduced by the energy in the field of the two magnetic dipoles given by Eq. (13.152). The corresponding bond dissociation energy, E_D , is given by the sum of the total

energies of the corresponding hydroxyl radical and hydrogen atom minus the total energy of water, $E_{T+osc}(H^{16}OH)$, and $E(magnetic)$.

Thus, E_D of $H^{16}OH$ is given by:

$$E_D(H^{16}OH) = E(H) + E(^{16}OH) - E_{T+osc}(H^{16}OH) - E(magnetic) \quad (13.220)$$

5 where $E_T(^{16}OH)$ is given by the of the sum of the experimental energies of ^{16}O (Eq. (13.171)), H (Eq. (13.154)), and the negative of the bond energy of ^{16}OH (Eq. (13.157)):

$$E(^{16}OH) = -13.59844 \text{ eV} - 13.6181 \text{ eV} - 4.41174 \text{ eV} = -31.62828 \text{ eV} \quad (13.221)$$

From Eqs. (13.154), (13.218), and (13.220-13.221), $E_D(H^{16}OH)$ is

$$\begin{aligned} E_D(H^{16}OH) &= E(H) + E(^{16}OH) - (E(magnetic) + E_{T+osc}(H^{16}OH)) \\ &= -13.59844 \text{ eV} - 31.62828 \text{ eV} - (0.114411 \text{ eV} - 50.447039 \text{ eV}) \\ &= 5.1059 \text{ eV} \end{aligned}$$

10 (13.222)

The experimental $H^{16}OH$ bond dissociation energy is [26]

$$E_D(H^{16}OH) = 5.0991 \text{ eV} \quad (13.223)$$

Similarly, E_D of $D^{16}OD$ is given by:

$$E_D(D^{16}OH) = E(D) + E(^{16}OD) - (E(magnetic) + E_{T+osc}(D^{16}OD)) \quad (13.224)$$

15 where $E_T(^{16}OD)$ is given by the of the sum of the experimental energies of ^{16}O (Eq. (13.171)), D (Eq. (13.155)), and the negative of the bond energy of ^{16}OD (Eq. (13.159)):

$$E(^{16}OD) = -13.603 \text{ eV} - 13.6181 \text{ eV} - 4.454 \text{ eV} = -31.6721 \text{ eV} \quad (13.225)$$

From Eqs. (13.155), (13.220), and (13.224-13.225), $E_D(D^{16}OD)$ is

$$\begin{aligned} E_D(D^{16}OD) &= -13.603 \text{ eV} - 31.6721 \text{ eV} - (0.114411 \text{ eV} - 50.567058 \text{ eV}) \\ &= 5.178 \text{ eV} \end{aligned} \quad (13.226)$$

20 The experimental $D^{16}OD$ bond dissociation energy is [27]

$$E_D(D^{16}OD) = 5.191 \text{ eV} \quad (13.227)$$

BOND ANGLE OF H_2O

The H_2O MO comprises a linear combination of two $O-H$ -bond MOs. Each $O-H$ -bond
25 MO comprises the superposition of a H_2 -type ellipsoidal MO and the $O2p_z$ AO or the

$O2p_y$ AO with a relative charge-density of 0.75 to 1.25; otherwise, the $O2p$ orbitals are the same as those of the oxygen atom. A bond is also possible between the two H atoms of the $O-H$ bonds. Such $H-H$ bonding would decrease the $O-H$ -bond strength since electron density would be shifted from the $O-H$ bonds to the $H-H$ bond. Thus, the bond angle
 5 between the two $O-H$ bonds is determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal H atoms of the $O-H$ bonds is zero. Since the two H_2 -type ellipsoidal MOs comprise 75% of the H electron density of H_2 ; the energies and the total energy E_T of the $H-H$ bond is given by Eqs. (13.67-13.73). From Eq. (11.79), the distance from the origin to each focus of the $H-H$ ellipsoidal MO is

$$10 \quad c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{aa_0}{2}} \quad (13.228)$$

The internuclear distance from Eq. (13.228) is

$$2c' = 2\sqrt{\frac{aa_0}{2}} \quad (13.229)$$

The length of the semiminor axis of the prolate spheroidal $H-H$ MO $b=c$ is given by Eq. (13.167). Substitution of Eq. (13.228) into Eq. (13.73) gives

$$15 \quad E_T = -\frac{e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \quad (13.230)$$

The radiation reaction force in the case of the vibration of $H-H$ in the transition state corresponds to the Doppler energy, E_D , given by Eq. (11.181) that is dependent on the motion of the electrons and the nuclei. The total energy E_T that includes the radiation reaction of the $H-H$ MO is given by the sum of E_T (Eq. (13.73)) and $\bar{E}_{osc}(H_2)$ given Eqs.
 20 (11.213-11.220), (11.231-11.236), and (11.239-11.240). Thus, the total energy $E_T(H-H)$ of the $H-H$ MO including the Doppler term is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}(H-H) \quad (13.231)$$

$$\begin{aligned}
 E_T &= \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \left[1 + \sqrt{\frac{2\hbar\sqrt{\frac{0.75e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \\
 &= \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \left[1 + \sqrt{\frac{2\hbar\sqrt{\frac{0.75e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] \\
 &\quad + \frac{1}{2} \hbar \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 (a+c')^3}}{0.5m_p}}
 \end{aligned} \tag{13.232}$$

To match the boundary condition that the total energy of the $H-H$ ellipsoidal MO is zero, $E_T(H-H)$ given by Eq. (13.232) is set equal to zero:

$$0 = \left[\frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \left[1 + \sqrt{\frac{2\hbar\sqrt{\frac{0.75e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] \right. \\
 \left. + \frac{1}{2} \hbar \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 (a+c')^3}}{0.5m_p}} \right] \tag{13.233}$$

- 5 From the energy relationship given by Eq. (13.233) and the relationship between the axes given by Eqs. (13.165-13.167), the dimensions of the $H-H$ MO can be solved.

The most convenient way to solve Eq. (13.233) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 4.300a_0 = 2.275 \times 10^{-10} \text{ m} \tag{13.234}$$

- 10 Substitution of Eq. (13.234) into Eq. (13.228) gives

$$c' = 1.466a_0 = 7.759 \times 10^{-11} \text{ m} \tag{13.235}$$

The internuclear distance given by multiplying Eq. (13.235) by two is

$$2c' = 2.933a_0 = 1.552 \times 10^{-10} \text{ m} \tag{13.236}$$

Substitution of Eqs. (13.234-13.235) into Eq. (13.167) gives

$$15 \quad b = c = 4.042a_0 = 2.139 \times 10^{-10} \text{ m} \tag{13.237}$$

Substitution of Eqs. (13.234-13.235) into Eq. (13.168) gives

$$e = 0.341 \quad (13.238)$$

Using, distance between the two H atoms when the total energy of the corresponding MO is zero, the corresponding bond angle can be determined from the law of cosines:

$$A^2 + B^2 - 2AB\cos\theta = C^2 \quad (13.239)$$

- 5 With $A = B = 2c'_{O-H}$, the internuclear distance of each $O-H$ bond given by Eq. (13.185), and $C = 2c'_{H-H}$, the internuclear distance of the two H atoms, the bond angle between the $O-H$ bonds is given by

$$(2c'_{O-H})^2 + (2c'_{O-H})^2 - 2(2c'_{O-H})^2 \cos\theta = (2c'_{H-H})^2 \quad (13.240)$$

$$\theta = \cos^{-1} \left(\frac{2(2c'_{O-H})^2 - (2c'_{H-H})^2}{2(2c'_{O-H})^2} \right) \quad (13.241)$$

- 10 Substitution of Eqs. (13.185) and (13.236) into Eq. (13.241) gives

$$\begin{aligned} \theta &= \cos^{-1} \left(\frac{2(1.836)^2 - (2.933)^2}{2(1.836)^2} \right) \\ &= \cos^{-1}(-0.2756) \\ &= 105.998^\circ \end{aligned} \quad (13.242)$$

The experimental internuclear distance of the two H atoms, $2c'_{H-H}$, is [23]

$$2c'_{H-H} = 1.55 \pm 0.01 \times 10^{-10} \text{ m} \quad (13.243)$$

- which matches Eq. (13.236) very well. The experimental angle between the $O-H$ bonds is
15 [23]

$$\theta = 106^\circ \quad (13.244)$$

which matches the predicted angle given by Eq. (13.242).

- The results of the determination of bond parameters of H_2O and D_2O are given in Table 13.1. The calculated results are based on first principles and given in closed-form,
20 exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

HYDROGEN NITRIDE (NH)

- The ammonia molecule can be solved by first considering the solution of the hydrogen and
25 dihydrogen nitride radicals. The former is formed by the reaction of a hydrogen atom and a nitrogen atom:



The hydrogen nitride radicals, NH and NH_2 , and ammonia, NH_3 , can be solved using the same principles as those used to solve OH and H_2O .

FORCE BALANCE OF NH

- 5 NH comprises two spin-paired electrons in a chemical bond between the nitrogen atom and the hydrogen atom such that two electrons on N remain unpaired. The NH radical molecular orbital (MO) is determined by considering properties of the binding atoms and the boundary constraints. The prolate spheroidal H_2 MO developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules section satisfies the boundary constraints; thus,
- 10 the H -atom electron forms a H_2 -type ellipsoidal MO with one of the N -atom electrons. The N electron configuration given in the Seven-Electron Atoms section is $1s^2 2s^2 2p^3$, and the orbital arrangement is

$$\begin{array}{ccc} & \text{2p state} & \\ \uparrow & \uparrow & \uparrow \\ \hline 1 & 0 & -1 \end{array} \quad (13.246)$$

- corresponding to the ground state $^4S_{3/2}$. The $N2p_x$ electron combines with the $H1s$
- 15 electron to form a molecular orbital. The proton of the H atom is along the internuclear axis. Due to symmetry, the other N electrons are equivalent to point charges at the origin. (See Eqs. (19-38) of Appendix IV.) Thus, the energies in the NH MO involve only the $N2p_x$ and $H1s$ electrons and the change in the magnetic energy of the $N2p_x$ electron with the other N electrons (Eq. (13.305)) with the formation of the NH MO. The forces are
- 20 determined by these energies.

As in the case of H_2 , the MO is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into N atom for distances shorter than the radius of the $2p$ shell. Thus, the MO surface comprises a prolate spheroid at the H proton that is continuous with the $2p$ shell at the N atom whose nucleus serves as the other focus. The

25 energy of the prolate spheroid is matched to that of the $N2p$ shell. As in the case with OH , the linear combination of the H_2 -type ellipsoidal MO with the $N2p$ AO must involve a 25% contribution from the H_2 -type ellipsoidal MO to the $N2p$ atomic orbital (AO) in order to match potential, kinetic, and orbital energy relationships. Thus, the NH MO must comprise 75% of a H_2 -type ellipsoidal MO and a nitrogen AO:

$$1\ N2p_x\ AO + 0.75\ H_2\ MO \rightarrow NH\ MO \quad (13.247)$$

The force balance of the NH MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (13.247) and the energy matching condition between the hydrogen and nitrogen components of the MO.

Similar to the OH case given by Eq. (13.59), the H_2 -type ellipsoidal MO comprises 75% of the NH MO; so, the electron charge density in Eq. (11.65) is given by $-0.75e$. Based on the condition that the electron MO is an equipotential energy surface, Eq. (11.79) gives the ellipsoidal parameter c' in terms of the central force of the foci, the electron angular momentum, and the ellipsoidal parameter a . To meet the equipotential condition of the union of the H_2 -type-ellipsoidal-MO and the N AO, the force constant used to determine the ellipsoidal parameter c' is normalized by the ratio of the ionization energy of N 14.53414 eV [6] and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). This normalizes the force to match that of the Coulombic force alone to meet the force matching condition of the NH MO under the influence of the proton and the N nucleus. Thus, k' of Eq. (11.79) to determine c' is

$$k' = \frac{(0.75)2e^2}{4\pi\epsilon_0 \frac{14.53414}{13.605804}} = (0.936127) \frac{(0.75)2e^2}{4\pi\epsilon_0} \quad (13.248)$$

L for the electron equals \hbar ; thus, the distance from the origin of the NH MO to each focus c' is given by Eqs. (11.79) and (13.248):

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 1.5a (0.936127)}} = \sqrt{\frac{2aa_0}{3(0.936127)}} = \sqrt{0.712154aa_0} \quad (13.249)$$

The internuclear distance from Eq. (13.249) is

$$2c' = 2\sqrt{0.712154aa_0} \quad (13.250)$$

The length of the semiminor axis of the prolate spheroidal NH MO $b = c$ is given by Eqs. (11.80) and (13.62). The eccentricity, e , is given by Eq. (13.63). Then, the solution of the semimajor axis a allows for the solution of the other axes of the prolate spheroidal and eccentricity of the NH MO.

The energy of the nitrogen $2p$ shell is the negative of the ionization energy of the nitrogen atom given by Eq. (10.143). Experimentally, the energy is [6]

$$E(2p \text{ shell}) = -E(\text{ionization}; N) = -14.53414 \text{ eV} \quad (13.251)$$

Since the prolate spheroidal MO transitions to the N AO, the energy E in Eq. (13.251) adds to that of the H_2 -type ellipsoidal MO to give the total energy of the NH MO. From the energy equation and the relationship between the axes given by Eqs. (13.249-13.250) and 5 (13.62-13.63), the dimensions of the NH MO are solved.

The energy components of V_e , V_p , T , V_m , and E_T are the same as those of OH given by Eqs. (13.67-13.73). Similarly to OH , the total energy $E_T(NH)$ of the NH MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the N AO and the H_2 -type ellipsoidal MO that forms the NH MO as given 10 by Eq. (13.247):

$$\begin{aligned} E_T(NH) &= E_T + E(2p \text{ shell}) \\ &= E_T - E(\text{ionization}; N) \\ &= -\frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} \end{aligned} \quad (13.252)$$

To match the boundary condition that the total energy of the entire the H_2 -type ellipsoidal MO is given by Eqs. (11.212) and (13.75), $E_T(NH)$ given by Eq. (13.252) is set equal to Eq. (13.75):

$$15 \quad E_T(NH) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} = -31.63536831 \text{ eV} \quad (13.253)$$

From the energy relationship given by Eq. (13.252) and the relationship between the axes given by Eqs. (13.249-13.250) and (13.62-13.63), the dimensions of the NH MO can be solved.

20 Substitution of Eq. (13.249) into Eq. (13.253) gives

$$\frac{e^2}{8\pi\epsilon_0 \sqrt{0.712154aa_0}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+\sqrt{0.712154aa_0}}{a-\sqrt{0.712154aa_0}} - 1 \right] = e17.10123 \quad (13.254)$$

The most convenient way to solve Eq. (13.254) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.36275a_0 = 7.21136 \times 10^{-11} \text{ m} \quad (13.255)$$

25 Substitution of Eq. (13.255) into Eq. (13.249) gives

$$c' = 0.98513a_0 = 5.21310 \times 10^{-11} \text{ m} \quad (13.256)$$

The internuclear distance given by multiplying Eq. (13.256) by two is

$$2c' = 1.97027a_0 = 1.04262 \times 10^{-10} \text{ m} \quad (13.257)$$

The experimental bond distance is [28]

$$2c' = 1.0362 \times 10^{-10} \text{ m} \quad (13.258)$$

5 Substitution of Eqs. (13.255-13.256) into Eq. (13.62) gives

$$b = c = 0.94159a_0 = 4.98270 \times 10^{-11} \text{ m} \quad (13.259)$$

Substitution of Eqs. (13.255-13.256) into Eq. (13.63) gives

$$e = 0.72290 \quad (13.260)$$

The nucleus of the H atom and the nucleus of the N atom comprise the foci of the H_2 -type
10 ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $N2p_x$ AO are given by Eqs. (13.84-13.95). The polar intersection angle θ' is given by

$$\theta' = \cos^{-1} \left(\frac{a}{c'} \left((a - c') \frac{1 + \frac{c'}{a}}{r_n} - 1 \right) \right) \quad (13.261)$$

where $r_n = r_7 = 0.93084a_0$ is the radius of the N atom. Substitution of Eqs. (13.255-13.256)

15 into Eq. (13.86) gives

$$\theta' = 114.61^\circ \quad (13.262)$$

Then, the angle θ_{N2p_xAO} the radial vector of the $N2p_x$ AO makes with the internuclear axis is

$$\theta_{N2p_xAO} = 180^\circ - 114.61^\circ = 65.39^\circ \quad (13.263)$$

as shown in Figure 10.

20 The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{H_2MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the N radial vector obeys the following relationship:

$$r_7 \sin \theta_{N2p_xAO} = 0.93084a_0 \sin \theta_{N2p_xAO} = b \sin \theta_{H_2MO} \quad (13.264)$$

25 such that

$$\theta_{H_2MO} = \sin^{-1} \frac{0.93084a_0 \sin \theta_{N2p_xAO}}{b} = \sin^{-1} \frac{0.93084a_0 \sin 65.39^\circ}{b} \quad (13.265)$$

with the use of Eq. (13.263). Substitution of Eq. (13.259) into Eq. (13.265) gives

$$\theta_{H_2MO} = 64.00^\circ \quad (13.266)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{H_2MO} = a \cos \theta_{H_2MO} \quad (13.267)$$

5 Substitution of Eqs. (13.255) and (13.266) into Eq. (13.267) gives

$$d_{H_2MO} = 0.59747a_0 = 3.16166 \times 10^{-11} \text{ m} \quad (13.268)$$

The distance d_{N2pAO} along the internuclear axis from the origin of the N atom to the point of intersection of the orbitals is given by

$$d_{N2pAO} = c' - d_{H_2MO} \quad (13.269)$$

10 Substitution of Eqs. (13.79) and (13.93) into Eq. (13.94) gives

$$d_{N2pAO} = 0.38767a_0 = 2.05144 \times 10^{-11} \text{ m} \quad (13.270)$$

As shown in Eq. (13.247), in addition to the p -orbital charge-density modulation, the uniform charge-density in the p_x orbital is increased by a factor of 0.25 and the H -atom density is decreased by a factor of 0.25. The internuclear axis of the $N-H$ bond is
 15 perpendicular to the bonding p_x orbital. Using the orbital composition of NH (Eq. (13.27)), the radii of $N1s = 0.14605a_0$ (Eq. (10.51)), $N2s = 0.69385a_0$ (Eq. (10.62)), and $N2p = 0.93084a_0$ (Eq. (10.142)) shells, and the parameters of the NH MO given by Eqs. (13.3-13.4) and (13.255-13.270), the dimensional diagram and charge-density of the NH MO comprising the linear combination of the H_2 -type ellipsoidal MO and the N AO
 20 according to Eq. (13.247) are shown in Figures 10 and 11, respectively.

ENERGIES OF NH

The energies of NH given by the substitution of the semiprincipal axes (Eqs. (13.255-13.256) and (13.259)) into the energy equations (Eqs. (13.67-13.73)) are

$$25 \quad V_e = \left(\frac{3}{4}\right) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -37.85748 \text{ eV} \quad (13.271)$$

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 13.81113 \text{ eV} \quad (13.272)$$

$$T = \left(\frac{3}{4}\right) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 13.89011 \text{ eV} \quad (13.273)$$

$$V_m = \left(\frac{3}{4}\right) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -6.94505 \text{ eV} \quad (13.274)$$

$$E_T(NH) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} = -31.63544 \text{ eV} \quad (13.275)$$

where $E_T(NH)$ is given by Eq. (13.253) which is reiteratively matched to Eq. (13.75) within 5 five-significant-figure round-off error.

VIBRATION AND ROTATION OF NH

The vibrational energy of NH may be solved in the same manner as that of OH . From Eqs. (13.102-13.106) with the substitution of the NH parameters, the angular frequency of the 10 oscillation is

$$\begin{aligned} \omega &= \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 b^3} - \frac{e^2}{8\pi\epsilon_0 (2c')^3}}{\mu}} \\ &= \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 (0.94159a_0)^3} - \frac{e^2}{8\pi\epsilon_0 (1.97027a_0)^3}}{\frac{14}{15}m_p}} \\ &= 6.18700 \times 10^{14} \text{ rad/s} \end{aligned} \quad (13.276)$$

where b is given by Eq. (13.259), $2c'$ is given by Eq. (13.257), and the reduced mass of ^{14}NH is given by:

$$\mu_{^{14}NH} = \frac{m_1 m_2}{m_1 + m_2} = \frac{(1)(14)}{1+14} m_p \quad (13.277)$$

15 where m_p is the proton mass. Thus, during bond formation, the perturbation of the orbit determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit, and the corresponding frequency, $\omega(0)$, for ^{14}NH given by Eqs. (11.136), (11.148), and (13.276) is

$$\omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{597.59 \text{ Nm}^{-1}}{\mu}} = 6.18700 \times 10^{14} \text{ radians/s} \quad (13.278)$$

20 where the reduced nuclear mass of ^{14}NH is given by Eq.(13.277) and the spring constant, $k(0)$, given by Eqs. (11.136) and (13.276) is

$$k(0) = 597.59 \text{ Nm}^{-1} \quad (13.279)$$

The ^{14}NH transition-state vibrational energy, $E_{\text{vib}}(0)$, given by Planck's equation (Eq. (11.127)) is:

$$E_{\text{vib}}(0) = \hbar\omega = \hbar 6.18700 \times 10^{14} \text{ rad/s} = 0.407239 \text{ eV} = 3284.58 \text{ cm}^{-1} \quad (13.280)$$

5 ω_e , from the experimental curve fit of the vibrational energies of ^{14}NH is [28]

$$\omega_e = 3282.3 \text{ cm}^{-1} \quad (13.281)$$

Using Eqs. (13.112-13.118) with $E_{\text{vib}}(0)$ given by Eq. (13.280) and D_0 given by Eq. (13.311), the ^{14}NH $\nu=1 \rightarrow \nu=0$ vibrational energy, $E_{\text{vib}}(1)$ is

$$E_{\text{vib}}(1) = 0.38581 \text{ eV} \quad (3111.84 \text{ cm}^{-1}) \quad (13.282)$$

10 The experimental vibrational energy of ^{14}NH using ω_e and $\omega_e x_e$ [28] according to K&P [15] is

$$E_{\text{vib}}(1) = 0.38752 \text{ eV} \quad (3125.5 \text{ cm}^{-1}) \quad (13.283)$$

Using Eq. (13.113) with $E_{\text{vib}}(1)$ given by Eq. (13.282) and D_0 given by Eq. (13.311), the anharmonic perturbation term, $\omega_0 x_0$, of ^{14}NH is

$$15 \quad \omega_0 x_0 = 86.37 \text{ cm}^{-1} \quad (13.284)$$

The experimental anharmonic perturbation term, $\omega_0 x_0$, of ^{14}NH [28] is

$$\omega_0 x_0 = 78.4 \text{ cm}^{-1} \quad (13.285)$$

The vibrational energies of successive states are given by Eqs. (13.280), (13.112), and (13.284).

20 Using b given by Eq. (13.259), $2c'$ given by Eq. (13.257), D_0 given by Eq. (13.314), and the reduced nuclear mass of ^{14}ND given by

$$\mu_{^{14}\text{ND}} = \frac{m_1 m_2}{m_1 + m_2} = \frac{(2)(14)}{2+14} m_p \quad (13.286)$$

where m_p is the proton mass, the corresponding parameters for deuterium nitride ^{14}ND (Eqs. (13.102-13.121)) are

$$25 \quad \omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{597.59 \text{ Nm}^{-1}}{\mu}} = 4.51835 \times 10^{14} \text{ radians/s} \quad (13.287)$$

$$k(0) = 579.59 \text{ Nm}^{-1} \quad (13.288)$$

$$E_{vib}(0) = \hbar\omega = \hbar 4.51835 \times 10^{14} \text{ rad/s} = 0.29741 \text{ eV} = 2398.72 \text{ cm}^{-1} \quad (13.289)$$

$$E_{vib}(1) = 0.28710 \text{ eV} \quad (2305.35 \text{ cm}^{-1}) \quad (13.290)$$

$$\omega_0 x_0 = 47.40 \text{ cm}^{-1} \quad (13.291)$$

ω_e , from the experimental curve fit of the vibrational energies of ^{14}ND is [28]

$$5 \quad \omega_e = 2398 \text{ cm}^{-1} \quad (13.292)$$

The experimental vibrational energy of ^{14}ND using ω_e and $\omega_e x_e$ [28] according to K&P [15] is

$$E_{vib}(1) = 0.2869 \text{ eV} \quad (2314 \text{ cm}^{-1}) \quad (13.293)$$

and the experimental anharmonic perturbation term, $\omega_0 x_0$, of ^{14}ND [28] is

$$10 \quad \omega_0 x_0 = 42 \text{ cm}^{-1} \quad (13.294)$$

which match the predictions given by Eqs. (13.289), (13.290) and (13.291), respectively.

Using Eqs. (13.133-13.134) and the internuclear distance, $r = 2c'$, and reduced mass of ^{14}NH given by Eqs. (13.257) and (13.277), respectively, the corresponding B_e is

$$B_e = 16.495 \text{ cm}^{-1} \quad (13.295)$$

15 The experimental B_e rotational parameter of ^{14}NH is [28]

$$B_e = 16.6993 \text{ cm}^{-1} \quad (13.296)$$

Using the internuclear distance, $r = 2c'$, and reduced mass of ^{14}ND given by Eqs. (13.257) and (13.286), respectively, the corresponding B_e is

$$B_e = 8.797 \text{ cm}^{-1} \quad (13.297)$$

20 The experimental B_e rotational parameter of ^{14}ND is [28]

$$B_e = 8.7913 \text{ cm}^{-1} \quad (13.298)$$

THE DOPPLER ENERGY TERMS OF ^{14}NH AND ^{14}ND

The equations of the radiation reaction force of hydrogen and deuterium nitride are the same
25 as those of the corresponding hydroxyl radicals with the substitution of the hydrogen and deuterium nitride parameters. Using Eqs. (11.136) and (13.140-13.141), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 3.91850 \times 10^{16} \text{ rad/s} \quad (13.299)$$

where b is given by Eq. (13.259). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 3.91850 \times 10^{16} \text{ rad/s} = 25.79224 \text{ eV} \quad (13.300)$$

5 In Eq. (11.181), substitution of the total energy of NH , $E_T(NH)$, (Eq. (13.253)) for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.300) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(25.79224 \text{ eV})}{m_e c^2}} = -0.31785 \text{ eV} \quad (13.301)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the
10 transition state at their corresponding frequency. The decrease in the energy of NH due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.301) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of NH . Using the experimental ^{14}NH ω_e of 3282.3 cm^{-1} (0.40696 eV) [28]

15 $\bar{E}_{osc}(^{14}NH)$ is

$$\bar{E}_{osc}(^{14}NH) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.302)$$

$$\bar{E}_{osc}(^{14}NH) = -0.31785 \text{ eV} + \frac{1}{2} (0.40696 \text{ eV}) = -0.11437 \text{ eV} \quad (13.303)$$

Using Eqs. (13.301) and the experimental ^{14}ND ω_e of 2398 cm^{-1} (0.29732 eV) [28]

$\bar{E}_{osc}(^{14}ND)$ is

$$20 \quad \bar{E}_{osc}(^{14}ND) = -0.31785 \text{ eV} + \frac{1}{2} (0.29732 \text{ eV}) = -0.16919 \text{ eV} \quad (13.304)$$

TOTAL AND BOND ENERGIES OF ^{14}NH AND ^{14}ND

$E_{T+osc}(NH)$, the total energy of the ^{14}NH radical including the Doppler term, is given by the sum of $E_T(NH)$ (Eq. (13.253)) and $\bar{E}_{osc}(^{14}NH)$ given by Eq. (13.303):

$$\begin{aligned}
 E_{T+osc}(NH) &= V_e + T + V_m + V_p + E(2p \text{ shell}) + \bar{E}_{osc}(^{14}NH) \\
 &= E_T(NH) + \bar{E}_{osc}(^{14}NH)
 \end{aligned}
 \tag{13.305}$$

$$\begin{aligned}
 E_{T+osc}(NH) &= \left\{ \frac{-e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} \right\} \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} \right] + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \\
 &= -31.63537 \text{ eV} - 0.31785 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}
 \end{aligned}
 \tag{13.306}$$

From Eqs. (13.302-13.303) and (13.305-13.306), the total energy of ^{14}NH is

$$\begin{aligned}
 E_{T+osc}(NH) &= -31.63537 \text{ eV} + \bar{E}_{osc}(^{14}NH) \\
 &= -31.63537 \text{ eV} - 0.31785 \text{ eV} + \frac{1}{2} (0.40696 \text{ eV}) \\
 &= -31.74974 \text{ eV}
 \end{aligned}
 \tag{13.307}$$

where the experimental ω_e was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. $E_{T+osc}(ND)$, the total energy of ^{14}ND including the Doppler term, is given by the sum of $E_T(ND) = E_T(NH)$ (Eq. (13.253)) and $\bar{E}_{osc}(^{14}ND)$ given by Eq. (13.304):

$$\begin{aligned}
 E_{T+osc}(ND) &= -31.63537 \text{ eV} + \bar{E}_{osc}(^{14}ND) \\
 &= -31.63537 \text{ eV} - 0.31785 \text{ eV} + \frac{1}{2} (0.29732 \text{ eV}) \\
 &= -31.80456 \text{ eV}
 \end{aligned}
 \tag{13.308}$$

10 where the experimental ω_e was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. The dissociation of the bond of the hydrogen nitride forms a free hydrogen atom with one unpaired electron and a nitrogen atom with three unpaired electrons as shown in Eq. (13.246). The p_x and p_y fields cancel and the magnetic energy (Eq. (7.46) with $r_f = 0.93084a_0$ is subtracted due to the one component of E_{mag} given by Eq. (10.137):

$$15 \quad E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (0.93084a_0)^3} = \frac{8\pi\mu_0 \mu_B^2}{(0.93084a_0)^3} = 0.14185 \text{ eV}
 \tag{13.309}$$

The corresponding bond dissociation energy, E_D , is given by the sum of the total energies of the nitrogen atom and the corresponding hydrogen atom minus the sum of $E_{T+osc}(NH)$ and $E(magnetic)$:

$$E_D = E(^{14}N) + E(H) - E_{T+osc}(NH) - E(magnetic) \quad (13.310)$$

5 $E(^{14}N)$ is given by Eq. (13.251), $E_D(H)$ is given by Eq. (13.154), and $E_D(D)$ is given by Eq. (13.155). The ^{14}NH bond dissociation energy, $E_D(^{14}NH)$, is given by Eqs. (13.154), (13.251), (13.307), and (13.309-13.310):

$$\begin{aligned} E_D(^{14}NH) &= -(14.53414 \text{ eV} + 13.59844 \text{ eV}) - (E(magnetic) + E_{T+osc}(NH)) \\ &= -28.13258 \text{ eV} - (0.14185 - 31.74974 \text{ eV}) \\ &= 3.47530 \text{ eV} \end{aligned} \quad (13.311)$$

The experimental ^{14}NH bond dissociation energy from Ref. [29] and Ref. [30] is

$$10 \quad E_D(^{14}NH) = 3.42 \text{ eV} \quad (13.312)$$

$$E_D(^{14}NH) \leq 3.47 \text{ eV} \quad (13.313)$$

The ^{14}ND bond dissociation energy, $E_D(^{14}ND)$, is given by Eqs. (13.155), (13.251), (13.308), and (13.309-13.310):

$$\begin{aligned} E_D(^{14}ND) &= -(14.53414 \text{ eV} + 13.603 \text{ eV}) - (E(magnetic) + E_{T+osc}(ND)) \\ &= -28.13714 \text{ eV} - (0.14185 - 31.80456 \text{ eV}) \\ &= 3.5256 \text{ eV} \end{aligned} \quad (13.314)$$

15 The experimental ^{14}ND bond dissociation energy from Ref. [31] and Ref. [30] is

$$E_{D_{298}}(^{14}ND) \leq 339 \text{ kJ/mol} = 3.513 \text{ eV} \quad (13.315)$$

$$E_D(^{14}ND) \leq 3.54 \text{ eV} \quad (13.316)$$

The results of the determination of bond parameters of NH and ND are given in Table 13.1. The calculated results are based on first principles and given in closed-form, 20 exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

DIHYDROGEN NITRIDE (NH_2)

The dihydrogen nitride radical NH_2 is formed by the reaction of a hydrogen atom with a 25 hydrogen nitride radical:



NH_2 can be solved using the same principles as those used to solve H_2O . Two diatomic molecular orbitals (MOs) developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section serve as basis functions in a linear combination with
 5 two nitrogen atomic orbitals (AOs) to form the MO of NH_2 . The solution is very similar to that of NH except that there are two NH bonds in NH_2 .

FORCE BALANCE OF NH_2

NH_2 comprises two chemical bonds between nitrogen and hydrogen. Each $N-H$
 10 bond comprises two spin-paired electrons with one from an initially unpaired electron of the nitrogen atom and the other from the hydrogen atom. Each H -atom electron forms a H_2 -type ellipsoidal MO with one of the initially unpaired N -atom electrons, $2p_x$ or $2p_y$, such that the proton and the N nucleus serve as the foci. The initial N electron configuration given in the Seven-Electron Atoms section is $1s^2 2s^2 2p^3$, and the orbital arrangement is
 15 given by Eqs. (10.134) and (13.246). The radius and the energy of the $N2p$ shell are unchanged with bond formation. The central paramagnetic force due to spin of each $N-H$ bond is provided by the spin-pairing force of the NH_2 MO that has the symmetry of an s orbital that superimposes with the $N2p$ orbitals such that the corresponding angular momenta are unchanged.

20 As in the case of H_2 , each of two $N-H$ -bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into N atom for distances shorter than the radius of the $2p$ shell since it is energetically unfavorable. Thus, the MO surface comprises a prolate spheroid at each H proton that is continuous with the $2p$ shell at the N atom. The energies in the NH_2 MO involve only each $N2p$ and each $H1s$ electron with the
 25 formation of each $N-H$ bond. The sum of the energies of the prolate spheroids is matched to that of the $2p$ shell. The forces are determined by these energies. As in the case of NH , the linear combination of each H_2 -type ellipsoidal MO with each $N2p$ AO must involve a 25% contribution from the H_2 -type ellipsoidal MO to the $N2p$ AO in order to match potential, kinetic, and orbital energy relationships. Thus, the NH_2 MO must comprise two

$N-H$ bonds with each comprising 75% of a H_2 -type ellipsoidal MO (1/2 +25%) and a nitrogen AO:

$$[1 N2p_x AO + 0.75 H_2 MO] + [1 N2p_y AO + 0.75 H_2 MO] \rightarrow NH_2 MO \quad (13.318)$$

5

The force constant k' of the each H_2 -type-ellipsoidal-MO component of the NH_2 MO is given by Eq. (13.248). The distance from the origin of each $N-H$ -bond MO to each focus c' is given by Eq. (13.249). The internuclear distance is given by Eq. (13.250). The length of the semiminor axis of the prolate spheroidal $N-H$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of each $N-H$ -bond MO. Since each of the two prolate spheroidal $N-H$ -bond MOs comprises a H_2 -type-ellipsoidal MO that transitions to the N AO, the energy E in Eq. (13.251) adds to that of the two corresponding H_2 -type ellipsoidal MOs to give the total energy of the NH_2 MO. From the energy equation and the relationship between the axes, the dimensions of the NH_2 MO are solved.

The energy components of V_e , V_p , T , V_m , and E_T are twice those of OH and NH given by Eqs. (13.67-13.73) and equal to those of H_2O given by Eqs. (13.172-13.178). Similarly to H_2O , since the each prolate spheroidal H_2 -type MO transitions to the N AO and the energy of the $N2p$ shell must remain constant and equal to the negative of the ionization energy given by Eq. (13.251), the total energy $E_T(NH_2)$ of the NH_2 MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the N AO and the two H_2 -type ellipsoidal MOs that forms the NH_2 MO as given by Eq. (13.318):

$$\begin{aligned} E_T(NH_2) &= E_T + E(2p \text{ shell}) \\ &= E_T - E(\text{ionization; } N) \\ &= -\frac{e^2}{4\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} \end{aligned} \quad (13.319)$$

25

The two hydrogen atoms and the nitrogen atom can achieve an energy minimum as a linear combination of two H_2 -type ellipsoidal MOs each having the proton and the nitrogen nucleus as the foci. Each $N-H$ -bond MO comprises the same $N2p$ shell of constant

energy given by Eq. (13.251). Thus, the energy of the NH_2 MO is also given by the sum of that of the two H_2 -type ellipsoidal MOs given by Eq. (11.212) minus the energy of the redundant nitrogen atom of the linear combination given by Eq. (13.251):

$$\begin{aligned} E_T(2H_2 - N) &= -2 \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] - E(2p \text{ shell}) \\ &= 2(-31.63536831 \text{ eV}) - (-14.53414 \text{ eV}) \\ &= -48.73660 \text{ eV} \end{aligned} \quad (13.320)$$

5 $E_T(NH_2)$ given by Eq. (13.319) is set equal to two times the energy of the H_2 -type ellipsoidal MO minus the energy of the $N2p$ shell given by Eq. (13.320):

$$E_T(NH_2) = -\frac{e^2}{4\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} = -48.73660 \text{ eV} \quad (13.321)$$

From the energy relationship given by Eq. (13.321) and the relationship between the axes
10 given by Eqs. (13.248-13.250) and (13.62-13.63), the dimensions of the NH_2 MO can be solved.

Substitution of Eq. (13.249) into Eq. (13.321) gives

$$\frac{e^2}{4\pi\epsilon_0 \sqrt{0.712154aa_0}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+\sqrt{0.712154aa_0}}{a-\sqrt{0.712154aa_0}} - 1 \right] = e34.20246 \quad (13.322)$$

The most convenient way to solve Eq. (13.322) is by the reiterative technique using a
15 computer. The result to within the round-off error with five-significant figures is

$$a = 1.36276a_0 = 7.21141 \times 10^{-11} \text{ m} \quad (13.323)$$

Substitution of Eq. (13.323) into Eq. (13.249) gives

$$c' = 0.98514a_0 = 5.21312 \times 10^{-11} \text{ m} \quad (13.324)$$

The internuclear distance given by multiplying Eq. (13.324) by two is

$$20 \quad 2c' = 1.97027a_0 = 1.04262 \times 10^{-10} \text{ m} \quad (13.325)$$

The experimental bond distance is [32]

$$2c' = 1.024 \times 10^{-10} \text{ m} \quad (13.326)$$

Substitution of Eqs. (13.323-13.324) into Eq. (13.62) gives

$$b = c = 0.94160a_0 = 4.98276 \times 10^{-11} \text{ m} \quad (13.327)$$

25 Substitution of Eqs. (13.323-13.324) into Eq. (13.63) gives

$$e = 0.72290 \quad (13.328)$$

The nucleus of the H atom and the nucleus of the N atom comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of each H_2 -type ellipsoidal MO and the $N2p_x$ AO or $N2p_y$ AO are given by Eqs. (13.84-13.95) and (13.261-13.270). Using Eqs. (13.323-13.325) and (13.327-13.328), the polar intersection angle θ' given by
 5 Eq. (13.261) with $r_n = r_7 = 0.93084a_0$ is

$$\theta' = 114.61^\circ \quad (13.329)$$

Then, the angle θ_{N2pAO} the radial vector of the $N2p_x$ AO or $N2p_y$ AO makes with the internuclear axis is

$$\theta_{N2pAO} = 180^\circ - 114.61^\circ = 65.39^\circ \quad (13.330)$$

10 as shown in Figure 10. The angle θ_{H_2MO} between the internuclear axis and the point of intersection of each H_2 -type ellipsoidal MO with the N radial vector given by Eqs. (13.264-13.265), (13.327), and (13.330) is

$$\theta_{H_2MO} = 64.00^\circ \quad (13.331)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal
 15 MO to the point of intersection of the orbitals given by Eqs. (13.267), (13.323), and (13.331) is

$$d_{H_2MO} = 0.59748a_0 = 3.16175 \times 10^{-11} \text{ m} \quad (13.332)$$

The distance d_{N2pAO} along the internuclear axis from the origin of the N atom to the point of intersection of the orbitals given by Eqs. (13.269), (13.324), and (13.332) is

$$20 \quad d_{N2pAO} = 0.38765a_0 = 2.05137 \times 10^{-11} \text{ m} \quad (13.333)$$

As shown in Eq. (13.318), in addition to the p -orbital charge-density modulation, the uniform charge-density in the p_x and p_y orbitals is increased by a factor of 0.25 and the H atoms are each decreased by a factor of 0.25. Using the orbital composition of NH_2 (Eq. (13.318)), the radii of $N1s = 0.14605a_0$ (Eq. (10.51)), $N2s = 0.69385a_0$ (Eq. (10.62)), and
 25 $N2p = 0.93084a_0$ (Eq. (10.142)) shells, and the parameters of the NH_2 MO given by Eqs. (13.3-13.4) and (13.323-13.333), the charge-density of the NH_2 MO comprising the linear combination of two $N-H$ -bond MOs (NH -type ellipsoidal MOs given in the Energies of NH section) according to Eq. (13.318) is shown in Figure 12. Each $N-H$ -bond MO

comprises a H_2 -type ellipsoidal MO and an $N2p$ AO having the dimensional diagram shown in Figure 10.

ENERGIES OF NH_2

- 5 The energies of NH_2 given by the substitution of the semiprincipal axes ((Eqs. (13.323-13.325) and (13.327)) into the energy equations (Eqs. (13.172-13.176)) are

$$V_e = \left(\frac{3}{2}\right) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -75.71422 \text{ eV} \quad (13.334)$$

$$V_p = 2 \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 27.62216 \text{ eV} \quad (13.335)$$

$$T = \left(\frac{3}{2}\right) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 27.77974 \text{ eV} \quad (13.336)$$

$$10 \quad V_m = \left(\frac{3}{2}\right) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -13.88987 \text{ eV} \quad (13.337)$$

$$E_T(NH_2) = -\frac{e^2}{4\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} = -48.73633 \text{ eV} \quad (13.338)$$

where $E_T(NH_2)$ is given by Eq. (13.319) which is reiteratively matched to Eq. (13.320) within five-significant-figure round-off error.

15

VIBRATION OF NH_2

The vibrational energy levels of NH_2 may be solved as two equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [2] wherein the spring constants are derived from the central forces as
20 given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERM OF NH_2

The radiation reaction force in the case of the vibration of NH_2 in the transition state
25 corresponds to the Doppler energy, E_D , given by Eq. (11.181) and Eqs. (13.22) and (13.144)

that is dependent on the motion of the electrons and the nuclei. The kinetic energy of the transient vibration is derived from the corresponding central forces. The equations of the radiation reaction force of dihydrogen and dideuterium nitride are the same as those of the corresponding water molecules with the substitution of the dihydrogen and dideuterium
5 nitride parameters. Using Eqs. (11.136) and (13.207-13.209), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{\left(\frac{3}{2}\right) \frac{e^2}{4\pi\epsilon_0 b^3}}{m_e}} = 5.54150 \times 10^{16} \text{ rad/s} \quad (13.339)$$

where b is given by Eq. (13.327). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$10 \quad \bar{E}_K = \hbar\omega = \hbar 5.54150 \times 10^{16} \text{ rad/s} = 36.47512 \text{ eV} \quad (13.340)$$

In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_{mv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.340) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit:

$$15 \quad \bar{E}_D \cong E_{mv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(36.47512 \text{ eV})}{m_e c^2}} = -0.37798 \text{ eV} \quad (13.341)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of NH_2 due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by
20 Eq. (13.341) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of NH_2 . Using the experimental $^{14}NH_2$ vibrational energy of $E_{vib} = 3301.110 \text{ cm}^{-1} = 0.40929 \text{ eV}$ [33] gives

$$\bar{E}'_{osc} = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.342)$$

$$\bar{E}'_{osc} = -0.37798 \text{ eV} + \frac{1}{2} (0.40929 \text{ eV}) = -0.17334 \text{ eV} \quad (13.343)$$

25 per bond. As in the case for H_2O , the reentrant orbit for the binding of a hydrogen atom to a NH radical causes the bonds to oscillate by increasing and decreasing in length along the

two $N-H$ bonds at a relative phase angle of 180° . Since the vibration and reentrant oscillation is along two bonds for the asymmetrical stretch (ν_3), \bar{E}_{osc} for $^{14}NH_2$, $\bar{E}_{osc}(^{14}NH_2)$, is:

$$\begin{aligned}\bar{E}_{osc}(^{14}NH_2) &= 2\left(\bar{E}_D + \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}}\right) \\ &= 2\left(-0.37798 \text{ eV} + \frac{1}{2}(0.40929 \text{ eV})\right) \\ &= -0.34668 \text{ eV}\end{aligned}\quad (13.344)$$

5 Using Eq. (13.341), Eqs. (13.342-13.344), and the $^{14}ND_2$ vibrational energy of $E_{vib} = 2410.79 \text{ cm}^{-1} = 0.29890 \text{ eV}$, calculated from the experimental $^{14}NH_2$ vibrational energy using Eq. (11.148), the corresponding $\bar{E}_{osc}(^{14}ND_2)$ is

$$\begin{aligned}\bar{E}_{osc}(^{14}ND_2) &= 2\left(-0.37798 \text{ eV} + \frac{1}{2}(0.29890 \text{ eV})\right) \\ &= -0.45707 \text{ eV}\end{aligned}\quad (13.345)$$

10 TOTAL AND BOND ENERGIES OF $^{14}NH_2$ AND $^{14}ND_2$

$E_{T+osc}(^{14}NH_2)$, the total energy of the $^{14}NH_2$ including the Doppler term, is given by the sum of $E_T(NH_2)$ (Eq. (13.321)) and $\bar{E}_{osc}(^{14}NH_2)$ given Eqs. (13.339-13.344):

$$\begin{aligned}E_{T+osc}(^{14}NH_2) &= V_e + T + V_m + V_p + E(N2p) + \bar{E}_{osc}(^{14}NH_2) \\ &= E_T(NH_2) + \bar{E}_{osc}(^{14}NH_2)\end{aligned}\quad (13.346)$$

$$\begin{aligned}E_{T+osc}(^{14}NH_2) &= \left\{ \left(\frac{-e^2}{4\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} \right) \right. \\ &\quad \left. - 2 \left((31.63536831 \text{ eV}) \sqrt{\frac{2\hbar \sqrt{\frac{3}{2} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\} \\ &= -48.73660 \text{ eV} - 2 \left(0.37798 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)\end{aligned}\quad (13.347)$$

15 From Eqs. (13.344) and (13.346-13.347), the total energy of $^{14}NH_2$ is

$$\begin{aligned}
 E_{T+osc}({}^{14}\text{NH}_2) &= -48.73660 \text{ eV} + \bar{E}_{osc}({}^{14}\text{NH}_2) \\
 &= -48.73660 \text{ eV} - 2 \left(0.37798 \text{ eV} - \frac{1}{2} (0.40929 \text{ eV}) \right) \\
 &= -49.08328 \text{ eV}
 \end{aligned} \tag{13.348}$$

where the experimental ${}^{14}\text{NH}_2$ vibrational energy was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

$E_{T+osc}({}^{14}\text{ND}_2)$, the total energy of ${}^{14}\text{ND}_2$ including the Doppler term is given by the sum of $E_T(\text{ND}_2) = E_T(\text{NH}_2)$ (Eq. (13.321)) and $\bar{E}_{osc}({}^{14}\text{ND}_2)$ given by Eq. (13.345):

$$\begin{aligned}
 E_{T+osc}({}^{14}\text{ND}_2) &= -48.73660 \text{ eV} + \bar{E}_{osc}({}^{14}\text{ND}_2) \\
 &= -48.73660 \text{ eV} - 2 \left(0.37798 \text{ eV} - \frac{1}{2} (0.29890 \text{ eV}) \right) \\
 &= -49.19366 \text{ eV}
 \end{aligned} \tag{13.349}$$

where the experimental ${}^{14}\text{NH}_2$ vibrational energy corrected for the reduced mass difference of hydrogen and deuterium was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. The corresponding bond dissociation energy, E_D , is given by the sum of the total energies of the corresponding hydrogen nitride radical and hydrogen atom minus the total energy of dihydrogen nitride,

$$E_{T+osc}({}^{14}\text{NH}_2).$$

Thus, E_D of ${}^{14}\text{NH}_2$ is given by:

$$E_D({}^{14}\text{NH}_2) = E(\text{H}) + E({}^{14}\text{NH}) - E_{T+osc}({}^{14}\text{NH}_2) \tag{13.350}$$

where $E_T({}^{14}\text{NH})$ is given by the of the sum of the experimental energies of ${}^{14}\text{N}$ (Eq. (13.251)), H (Eq. (13.154)), and the negative of the bond energy of ${}^{14}\text{NH}$ (Eq. (13.312)):

$$E({}^{14}\text{NH}) = -13.59844 \text{ eV} - 14.53414 \text{ eV} - 3.42 \text{ eV} = -31.55258 \text{ eV} \tag{13.351}$$

From Eqs. (13.154), (13.348), and (13.350-13.351), $E_D({}^{14}\text{NH}_2)$ is

$$\begin{aligned}
 E_D({}^{14}\text{NH}_2) &= E(\text{H}) + E({}^{14}\text{NH}) - E_{T+osc}({}^{14}\text{NH}_2) \\
 &= -13.59844 \text{ eV} - 31.55258 \text{ eV} - (-49.08328 \text{ eV}) \\
 &= 3.9323 \text{ eV}
 \end{aligned} \tag{13.352}$$

The experimental ${}^{14}\text{NH}_2$ bond dissociation energy from Ref. [34] and Ref. [35] is

$$E_D({}^{14}\text{NH}_2) = 88 \pm 4 \text{ kcal/mole} = 3.8160 \text{ eV} \tag{13.353}$$

165

$$E_D(^{14}\text{NH}_2) = 91.0 \pm 0.5 \text{ kcal/mole} = 3.9461 \text{ eV} \quad (13.354)$$

Similarly, E_D of $^{14}\text{ND}_2$ is given by:

$$E_D(^{14}\text{ND}_2) = E(D) + E(^{14}\text{ND}) - (E_{T+\text{osc}}(^{14}\text{ND}_2)) \quad (13.355)$$

where $E_T(^{14}\text{ND})$ is given by the of the sum of the experimental energies of ^{14}N (Eq. 5 (13.251)), D (Eq. (13.155)), and the negative of the bond energy of ^{14}ND (Eq. (13.315)):

$$E(^{14}\text{ND}) = -13.603 \text{ eV} - 14.53414 \text{ eV} - 3.513 \text{ eV} = -31.6506 \text{ eV} \quad (13.356)$$

From Eqs. (13.155), (13.349), and (13.355-13.356), $E_D(^{14}\text{ND}_2)$ is

$$\begin{aligned} E_D(^{14}\text{ND}_2) &= -13.603 \text{ eV} - 31.6506 \text{ eV} - (-49.19366 \text{ eV}) \\ &= 3.9401 \text{ eV} \end{aligned} \quad (13.357)$$

The $^{14}\text{ND}_2$ bond dissociation energy calculated from the average of the experimental bond 10 energies [34-35] and vibrational energy of $^{14}\text{NH}_2$ [33] is

$$\begin{aligned} E_D(^{14}\text{ND}_2) &= E_D(^{14}\text{NH}_2) + \frac{1}{2}(E_{\text{vib}}(^{14}\text{NH}_2) - E_{\text{vib}}(^{14}\text{ND}_2)) \\ &= \frac{1}{2}(3.8160 \text{ eV} + 3.9461 \text{ eV}) + \frac{1}{2}(0.40929 \text{ eV} - 0.29890 \text{ eV}) \\ &= 3.9362 \text{ eV} \end{aligned} \quad (13.358)$$

BOND ANGLE OF NH_2

The NH_2 MO comprises a linear combination of two $\text{N}-\text{H}$ -bond MOs. Each $\text{N}-\text{H}$ -bond 15 MO comprises the superposition of a H_2 -type ellipsoidal MO and the $\text{N}2p_x$ AO or the $\text{N}2p_y$ AO with a relative charge density of 0.75 to 1.25; otherwise, the $\text{N}2p$ AOs are the same as those of the nitrogen atom. A bond is also possible between the two H atoms of the $\text{N}-\text{H}$ bonds. Such $\text{H}-\text{H}$ bonding would decrease the $\text{N}-\text{H}$ bond strength since electron density would be shifted from the $\text{N}-\text{H}$ bonds to the $\text{H}-\text{H}$ bond. Thus, the bond 20 angle between the two $\text{N}-\text{H}$ bonds is determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal H atoms of the $\text{N}-\text{H}$ bonds is zero. From Eqs. (11.79) and (13.228), the distance from the origin to each focus of the $\text{H}-\text{H}$ ellipsoidal MO is

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{aa_0}{2}} \quad (13.359)$$

The internuclear distance from Eq. (13.229) is

$$2c' = 2\sqrt{\frac{aa_0}{2}} \quad (13.360)$$

The length of the semiminor axis of the prolate spheroidal $H-H$ MO $b=c$ is given by Eq. (13.167).

- 5 Since the two H_2 -type ellipsoidal MOs comprise 75% of the H electron density of H_2 and the energy of each H_2 -type ellipsoidal MO is matched to that of the $N2p$ AO; the component energies and the total energy E_T of the $H-H$ bond are given by Eqs. (13.67-13.73) except that V_e , T , and V_m are corrected for the energy matching factor of 0.93613 given in Eq. (13.248). Substitution of Eq. (13.359) into Eq. (13.233) with the energy-
- 10 matching factor gives

$$0 = \left[\frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left[(0.93613)^{-1} \left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \left[1 + \sqrt{\frac{2\hbar\sqrt{\frac{0.75e^2}{4\pi\epsilon_0 a^3}}}}{m_e c^2} \right] \right. \\ \left. + \frac{1}{2} \hbar \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 (a+c')^3}}{0.5m_p}} \right] \quad (13.361)$$

From the energy relationship given by Eq. (13.361) and the relationship between the axes given by Eqs. (13.359-13.360) and (13.167-13.168), the dimensions of the $H-H$ MO can

15 be solved.

The most convenient way to solve Eq. (13.361) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 4.9500a_0 = 2.6194 \times 10^{-10} \text{ m} \quad (13.362)$$

Substitution of Eq. (13.362) into Eq. (13.359) gives

$$20 \quad c' = 1.5732a_0 = 8.3251 \times 10^{-11} \text{ m} \quad (13.363)$$

The internuclear distance given by multiplying Eq. (13.363) by two is

$$2c' = 3.1464a_0 = 1.6650 \times 10^{-10} \text{ m} \quad (13.364)$$

Substitution of Eqs. (13.362-13.363) into Eq. (13.167) gives

$$b = c = 4.6933a_0 = 2.4836 \times 10^{-10} \text{ m} \quad (13.365)$$

25 Substitution of Eqs. (13.362-13.363) into Eq. (13.168) gives

$$e = 0.3178 \quad (13.366)$$

Using, $2c'_{H-H}$ (Eq. (13.364)), the distance between the two H atoms when the total energy of the corresponding MO is zero (Eq. (13.361)), and $2c'_{N-H}$ (Eq. (13.325)), the internuclear distance of each $N-H$ bond, the corresponding bond angle can be determined from the law of cosines. Using, Eq. (13.242), the bond angle θ between the $N-H$ bonds is

$$\begin{aligned} \theta &= \cos^{-1} \left(\frac{2(1.9703)^2 - (3.1464)^2}{2(1.9703)^2} \right) \\ &= \cos^{-1}(-0.2751) = 105.969^\circ \end{aligned} \quad (13.367)$$

The experimental angle between the $N-H$ bonds is [32]

$$\theta = 103.3^\circ \quad (13.368)$$

The results of the determination of bond parameters of NH_2 and ND_2 are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

AMMONIA (NH_3)

Ammonia (NH_3) is formed by the reaction of a hydrogen atom with a dihydrogen nitride radical:



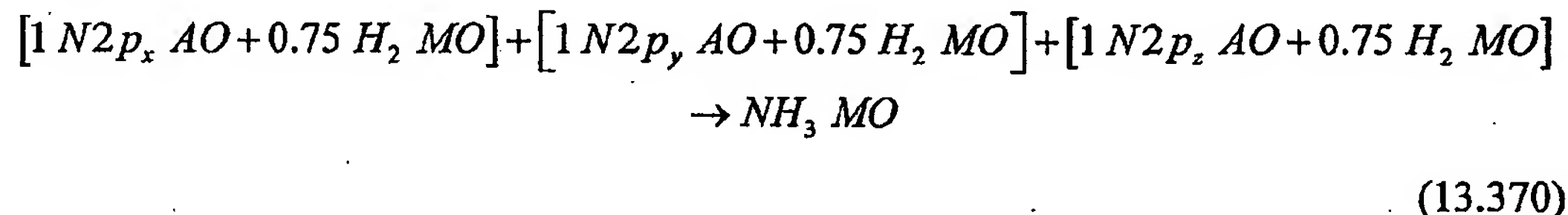
NH_3 can be solved using the same principles as those used to solve NH_2 except that three rather than two H_2 -type prolate spheroidal molecular orbitals (MOs) serve as basis functions in a linear combination with nitrogen atomic orbitals (AOs) to form the MO of NH_3 .

FORCE BALANCE OF NH_3

NH_3 comprises three chemical bonds between nitrogen and hydrogen. Each $N-H$ bond comprises two spin-paired electrons with one from an initially unpaired electron of the nitrogen atom and the other from the hydrogen atom. Each H -atom electron forms a H_2 -type ellipsoidal MO with one of the initially unpaired N -atom electrons, $2p_x$, $2p_y$, or $2p_z$, such that the proton and the N nucleus serve as the foci. The initial N electron configuration given in the Seven-Electron Atoms section is $1s^2 2s^2 2p^3$, and the orbital

arrangement is given by Eqs. (10.134) and (13.246). The radius and the energy of the $N2p$ shell are unchanged with bond formation. The central paramagnetic force due to spin of each $N-H$ bond is provided by the spin-pairing force of the NH_3 MO that has the symmetry of an s orbital that superimposes with the $N2p$ orbitals such that the corresponding angular
5 momenta are unchanged.

As in the case of H_2 , each of three $N-H$ -bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into the N atom for distances shorter than the radius of the $2p$ shell since it is energetically unfavorable. Thus, the MO surface comprises a prolate spheroid at each H proton that is continuous with the $2p$ shell at
10 the N atom. The energies in the NH_3 MO involve only each $N2p$ and each $H1s$ electron with the formation of each $N-H$ bond. The sum of the energies of the prolate spheroids is matched to that of the $2p$ shell. The forces are determined by these energies. As in the cases of NH and NH_2 , the linear combination of each H_2 -type ellipsoidal MO with each $N2p$ AO must involve a 25% contribution from the H_2 -type ellipsoidal MO to the $N2p$
15 AO in order to match potential, kinetic, and orbital energy relationships. Thus, the NH_3 MO must comprise three $N-H$ bonds with each comprising 75% of a H_2 -type ellipsoidal MO ($1/2 + 25\%$) and a nitrogen AO:



20 The force constant k' of the each H_2 -type-ellipsoidal-MO component of the NH_3 MO is given by Eq. (13.248). The distance from the origin of each $N-H$ -bond MO to each focus c' is given by Eq. (13.249). The internuclear distance is given by Eq. (13.250). The length of the semiminor axis of the prolate spheroidal $N-H$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a
25 then allows for the solution of the other axes of each prolate spheroid and eccentricity of each $N-H$ -bond MO. Since each of the three prolate spheroidal $N-H$ -bond MOs comprises a H_2 -type-ellipsoidal MO that transitions to the N AO, the energy E in Eq. (13.251) adds to that of the three corresponding H_2 -type ellipsoidal MOs to give the total energy of the NH_3

MO. From the energy equation and the relationship between the axes, the dimensions of the NH_3 MO are solved.

The energy components of V_e , V_p , T , V_m , and E_T are three times those of OH and NH given by Eqs. (13.67-13.73) and 1.5 times those of H_2O given by Eqs. (13.172-13.178).

5 Similarly to H_2O , since the each prolate spheroidal H_2 -type MO transitions to the N AO and the energy of the $N2p$ shell must remain constant and equal to the negative of the ionization energy given by Eq. (13.251), the total energy $E_T(NH_3)$ of the NH_3 MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the N AO and the three H_2 -type ellipsoidal MOs that forms the NH_3 MO as
10 given by Eq. (13.370):

$$\begin{aligned} E_T(NH_3) &= E_T + E(2p \text{ shell}) \\ &= E_T - E(\text{ionization; } N) \\ &= -3 \frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} \end{aligned} \quad (13.371)$$

The three hydrogen atoms and the nitrogen atom can achieve an energy minimum as a linear combination of three H_2 -type ellipsoidal MOs each having the proton and the nitrogen nucleus as the foci. Each $N-H$ -bond MO comprises the same $N2p$ shell of constant
15 energy given by Eq. (13.251). Thus, an energy term of the NH_3 MO is given by the sum of the three H_2 -type ellipsoidal MOs given by Eq. (11.212) minus the energy of the redundant nitrogen atom of the linear combination given by Eq. (13.251). The total sum is determined by the energy matching condition of the binding atoms.

In Eq. (13.248), the equipotential condition of the union of each H_2 -type-ellipsoidal-
20 MO and the N AO was met when the force constant used to determine the ellipsoidal parameter c' was normalized by the ratio of the ionization energy of N 14.53414 eV [6] and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). This normalized the force to match that of the Coulombic force alone to meet the force matching condition of the NH MO under the influence of the proton
25 and the N nucleus. The minimum total energy of the NH_3 MO from the sum of energies of a linear combination from four atoms is determined using the energy matching condition of Eq. (13.248). Since each of the three prolate spheroidal $N-H$ -bond MOs of NH_3 comprises a H_2 -type-ellipsoidal MO that transitions to the N AO and the energy matching

condition is met, the nitrogen energy E (Eq. (13.251)) and the energy (Eq. (1.243)) of a hydrogen atomic orbital (H AO), $E_{Coulomb}(H)$, corresponding to the Coulombic force of $+e$ from the nitrogen nucleus is subtracted from the sum of the energies of the three corresponding H_2 -type ellipsoidal MOs to given an energy minimum. From another
 5 perspective, the electron configuration of NH_2 is equivalent to that of OH and is given by Eq. (10.174). NH_2 serves as a one-electron atom that is energy matched by the H AO as a basis element to minimize the energy of NH_3 in the formation of the third $N-H$ bond.

$$E_T(3H_2 - N - H) = \left\{ -3 \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] \right. \\ \left. - E(N2p \text{ shell}) - E_{Coulomb}(H) \right\} \\ = 3(-31.63536831 \text{ eV}) - (-14.53414 \text{ eV} - 13.605804 \text{ eV}) \quad (13.372) \\ = -66.76616 \text{ eV}$$

$E_T(NH_3)$ given by Eq. (13.371) is set equal to Eq. (13.372), three times the energy of the
 10 H_2 -type ellipsoidal MO minus the energy of the $N2p$ shell and the H AO:

$$E_T(NH_3) = -3 \frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} = -66.76616 \text{ eV} \quad (13.373)$$

From the energy relationship given by Eq. (13.373) and the relationship between the axes given by Eqs. (13.248-13.250) and (13.62-13.63), the dimensions of the NH_3 MO can be
 15 solved.

Substitution of Eq. (13.249) into Eq. (13.373) gives

$$\frac{3e^2}{8\pi\epsilon_0 \sqrt{0.712154aa_0}} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{0.712154aa_0}}{a - \sqrt{0.712154aa_0}} - 1 \right] = e52.23202 \quad (13.374)$$

The most convenient way to solve Eq. (13.374) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$20 \quad a = 1.34750a_0 = 7.13066 \times 10^{-11} \text{ m} \quad (13.375)$$

Substitution of Eq. (13.375) into Eq. (13.249) gives

$$c' = 0.97961a_0 = 5.18385 \times 10^{-11} \text{ m} \quad (13.376)$$

The internuclear distance given by multiplying Eq. (13.376) by two is

$$2c' = 1.95921a_0 = 1.03677 \times 10^{-10} \text{ m} \quad (13.377)$$

The experimental bond distance is [32]

$$2c' = 1.012 \times 10^{-10} \text{ m} \quad (13.378)$$

Substitution of Eqs. (13.375-13.376) into Eq. (13.62) gives

$$b = c = 0.92527a_0 = 4.89633 \times 10^{-11} \text{ m} \quad (13.379)$$

5 Substitution of Eqs. (13.375-13.376) into Eq. (13.63) gives

$$e = 0.72698 \quad (13.380)$$

The nucleus of the H atom and the nucleus of the N atom comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of each H_2 -type ellipsoidal MO and the $N2p_x$, $N2p_y$, or $N2p_z$ AO are given by Eqs. (13.84-13.95), (13.261-13.270), and
10 (13.261-13.270). Using Eqs. (13.375-13.377) and (13.379-13.380), the polar intersection angle θ' given by Eq. (13.261) with $r_n = r_f = 0.93084a_0$ is

$$\theta' = 115.89^\circ \quad (13.381)$$

Then, the angle θ_{N2pAO} the radial vector of the $N2p_x$, $N2p_y$, or $N2p_z$ AO makes with the internuclear axis is

$$15 \quad \theta_{N2pAO} = 180^\circ - 115.89^\circ = 64.11^\circ \quad (13.382)$$

as shown in Figure 10. The angle θ_{H_2MO} between the internuclear axis and the point of intersection of each H_2 -type ellipsoidal MO with the N radial vector given by Eqs. (13.264-13.265), (13.379), and (13.382) is

$$\theta_{H_2MO} = 64.83^\circ \quad (13.383)$$

20 Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals given by Eqs. (13.267), (13.375), and (13.383) is

$$d_{H_2MO} = 0.57314a_0 = 3.03292 \times 10^{-11} \text{ m} \quad (13.384)$$

The distance d_{N2pAO} along the internuclear axis from the origin of the N atom to the point of
25 intersection of the orbitals given by Eqs. (13.269), (13.376), and (13.384) is

$$d_{N2pAO} = 0.40647a_0 = 2.15093 \times 10^{-11} \text{ m} \quad (13.385)$$

As shown in Eq. (13.370), in addition to the p -orbital charge-density modulation, the uniform charge-density in the p_x , p_y , and p_z orbitals is increased by a factor of 0.25 and the H atoms are each decreased by a factor of 0.25. Using the orbital composition of NH_3 (Eq.

(13.370)), the radii of $N1s = 0.14605a_0$ (Eq. (10.51)), $N2s = 0.69385a_0$ (Eq. (10.62)), and $N2p = 0.93084a_0$ (Eq. (10.142)) shells, and the parameters of the NH_3 MO given by Eqs. (13.3-13.4) and (13.375-13.385), the charge-density of the NH_3 MO comprising the linear combination of three $N-H$ -bond MOs (NH -type ellipsoidal MOs given in the Energies of 5 NH section) according to Eq. (13.370) is shown in Figure 13. Each $N-H$ -bond MO comprises a H_2 -type ellipsoidal MO and an $N2p$ AO having the dimensional diagram shown in Figure 10.

ENERGIES OF NH_3

10 The energies of NH_3 given by the substitution of the semiprincipal axes ((Eqs. (13.375-13.377) and (13.379)) into the energy equations (Eqs. (13.67-13.73)) multiplied by three are

$$V_e = 3 \left(\frac{3}{4} \right) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -115.28799 \text{ eV} \quad (13.386)$$

$$V_p = 3 \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 41.66718 \text{ eV} \quad (13.387)$$

$$T = 3 \left(\frac{3}{4} \right) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 42.77848 \text{ eV} \quad (13.388)$$

$$15 \quad V_m = 3 \left(\frac{3}{4} \right) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -21.38924 \text{ eV} \quad (13.389)$$

$$E_T(NH_3) = -3 \frac{e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} = -66.76571 \text{ eV} \quad (13.390)$$

where $E_T(NH_3)$ is given by Eq. (13.371) which is reiteratively matched to Eq. (13.372) within five-significant-figure round-off error.

20

VIBRATION OF NH_3

The vibrational energy levels of NH_3 may be solved as three equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [2] wherein the spring constants are derived from the central forces as

given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERM OF NH_3

- 5 The radiation reaction force in the case of the vibration of NH_3 in the transition state corresponds to the Doppler energy, E_D , given by Eq. (11.181) and Eqs. (13.22) and (13.144) that is dependent on the motion of the electrons and the nuclei. The kinetic energy of the transient vibration is derived from the corresponding central forces. The equations of the radiation reaction force of ammonia are the same as those of the corresponding water and
- 10 dihydrogen and dideuterium nitride radicals with the substitution of the ammonia parameters. Using Eqs. (11.136) and (13.207-13.209), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{\left(\frac{3}{2}\right) \frac{e^2}{4\pi\epsilon_0 b^3}}{m_e}} = 5.68887 \times 10^{16} \text{ rad/s} \quad (13.391)$$

- where b is given by Eq. (13.379). The kinetic energy, E_K , is given by Planck's equation
- 15 (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 5.68887 \times 10^{16} \text{ rad/s} = 37.44514 \text{ eV} \quad (13.392)$$

- In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO acting independently due to the D_{3h} symmetry point group, for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.392) for
- 20 \bar{E}_K gives the Doppler energy of the electrons of each of the three bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.6353683 \text{ eV} \sqrt{\frac{2e(37.44514 \text{ eV})}{m_e c^2}} = -0.38298 \text{ eV} \quad (13.393)$$

- In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of NH_3 due to
- 25 the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.393) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational

energy of NH_3 . Using the experimental $^{14}NH_3$ vibrational energy of $E_{vib} = 3443.59 \text{ cm}^{-1} = 0.426954 \text{ eV}$ [36] gives

$$\bar{E}'_{osc} = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.394)$$

$$\bar{E}'_{osc} = -0.38298 \text{ eV} + \frac{1}{2} (0.426954 \text{ eV}) = -0.16950 \text{ eV} \quad (13.395)$$

5 per bond. The reentrant orbit for the binding of a hydrogen atom to a NH_2 radical involves three $N-H$ bonds. Since the vibration and reentrant oscillation is along three bonds, \bar{E}_{osc} for $^{14}NH_3$, $\bar{E}_{osc}(^{14}NH_3)$, is:

$$\begin{aligned} \bar{E}_{osc}(^{14}NH_3) &= 3 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\ &= 3 \left(-0.38298 \text{ eV} + \frac{1}{2} (0.426954 \text{ eV}) \right) \\ &= -0.50850 \text{ eV} \end{aligned} \quad (13.396)$$

Using Eq. (13.393), Eqs. (13.394-13.396), and the $^{14}ND_3$ experimental vibrational
10 energy of $E_{vib} = 2563.96 \text{ cm}^{-1} = 0.317893 \text{ eV}$ [36], the corresponding $\bar{E}_{osc}(^{14}ND_3)$ is

$$\begin{aligned} \bar{E}_{osc}(^{14}ND_3) &= 3 \left(-0.38298 \text{ eV} + \frac{1}{2} (0.317893 \text{ eV}) \right) \\ &= -0.67209 \text{ eV} \end{aligned} \quad (13.397)$$

TOTAL AND BOND ENERGIES OF $^{14}NH_3$ AND $^{14}ND_3$

$E_{T+osc}(^{14}NH_3)$, the total energy of the $^{14}NH_3$ including the Doppler term, is given by the sum
15 of $E_T(NH_3)$ (Eq. (13.373)) and $\bar{E}_{osc}(^{14}NH_3)$ given Eqs. (13.391-13.396):

$$\begin{aligned} E_{T+osc}(^{14}NH_3) &= V_e + T + V_m + V_p + E(N2p) + \bar{E}_{osc}(^{14}NH_3) \\ &= E_T(NH_3) + \bar{E}_{osc}(^{14}NH_3) \end{aligned} \quad (13.398)$$

$$\begin{aligned}
 E_{T+osc}({}^{14}\text{NH}_3) &= \left\{ \begin{aligned} & \left(3 \frac{-e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.53414 \text{ eV} \right) \\ & - 3 \left((31.63536831 \text{ eV}) \sqrt{\frac{2\hbar \sqrt{\frac{3}{2} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}} \right) \end{aligned} \right\} \quad (13.399) \\
 &= -66.76616 \text{ eV} - 3 \left(0.38298 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)
 \end{aligned}$$

From Eqs. (13.396) and (13.398-13.399), the total energy of ${}^{14}\text{NH}_2$ is

$$\begin{aligned}
 E_{T+osc}({}^{14}\text{NH}_3) &= -66.76616 \text{ eV} + \bar{E}_{osc}({}^{14}\text{NH}_3) \\
 &= -66.76616 \text{ eV} - 3 \left(0.38298 \text{ eV} - \frac{1}{2} (0.426954 \text{ eV}) \right) \quad (13.400) \\
 &= -67.27466 \text{ eV}
 \end{aligned}$$

where the experimental ${}^{14}\text{NH}_3$ vibrational energy was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

5 $E_{T+osc}({}^{14}\text{ND}_3)$, the total energy of ${}^{14}\text{ND}_3$ including the Doppler term is given by the sum of

$E_T(\text{ND}_3) = E_T(\text{NH}_3)$ (Eq. (13.373)) and $\bar{E}_{osc}({}^{14}\text{ND}_3)$ given by Eq. (13.397):

$$\begin{aligned}
 E_{T+osc}({}^{14}\text{ND}_3) &= -66.76616 \text{ eV} + \bar{E}_{osc}({}^{14}\text{ND}_3) \\
 &= -66.76616 \text{ eV} - 3 \left(0.38298 \text{ eV} - \frac{1}{2} (0.317893 \text{ eV}) \right) \quad (13.401) \\
 &= -67.43780 \text{ eV}
 \end{aligned}$$

where the experimental ${}^{14}\text{ND}_3$ vibrational energy was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. The corresponding bond dissociation energy, E_D , is given by the sum of the total energies of the
 10 corresponding dihydrogen nitride radical and hydrogen atom minus the total energy of ammonia, $E_{T+osc}({}^{14}\text{NH}_3)$.

Thus, E_D of ${}^{14}\text{NH}_3$ is given by:

$$E_D({}^{14}\text{NH}_3) = E(\text{H}) + E({}^{14}\text{NH}_2) - E_{T+osc}({}^{14}\text{NH}_3) \quad (13.402)$$

where $E_T({}^{14}\text{NH}_2)$ is given by the of the sum of the experimental energies of ${}^{14}\text{N}$ (Eq.
 15 (13.251)), two H (Eq. (13.154)), and the negative of the bond energies of ${}^{14}\text{NH}$ (Eq. (13.312)) and ${}^{14}\text{NH}_2$ (Eq. (13.354)):

176

$$E(^{14}\text{NH}_2) = 2(-13.59844 \text{ eV}) - 14.53414 \text{ eV} - 3.42 \text{ eV} - 3.946 \text{ eV} = -49.09709 \text{ eV} \quad (13.403)$$

From Eqs. (13.154), (13.400), and (13.402-13.403), $E_D(^{14}\text{NH}_2)$ is

$$\begin{aligned} E_D(^{14}\text{NH}_3) &= E(\text{H}) + E(^{14}\text{NH}_2) - E_{T+\text{osc}}(^{14}\text{NH}_3) \\ &= -13.59844 \text{ eV} - 49.09709 \text{ eV} - (-67.27466 \text{ eV}) \\ &= 4.57913 \text{ eV} \end{aligned} \quad (13.404)$$

5 The experimental $^{14}\text{NH}_3$ bond dissociation energy [37] is

$$E_D(^{14}\text{NH}_3) = 4.60155 \text{ eV} \quad (13.405)$$

Similarly, E_D of $^{14}\text{ND}_3$ is given by:

$$E_D(^{14}\text{ND}_3) = E(\text{D}) + E(^{14}\text{ND}_2) - (E_{T+\text{osc}}(^{14}\text{ND}_3)) \quad (13.406)$$

where $E_T(^{14}\text{ND}_2)$ is given by the of the sum of the experimental energies of ^{14}N (Eq. (13.251)), two times the energy of D (Eq. (13.155)), and the negative of the bond energies of ^{14}ND (Eq. (13.315)) and $^{14}\text{ND}_2$ (Eq. (13.358)):

$$E(^{14}\text{ND}_2) = 2(-13.603 \text{ eV}) - 14.53414 \text{ eV} - 3.5134 \text{ eV} - 3.9362 \text{ eV} = -49.18981 \text{ eV} \quad (13.407)$$

From Eqs. (13.155), (13.401), and (13.406-13.407), $E_D(^{14}\text{ND}_3)$ is

$$15 \quad E_D(^{14}\text{ND}_3) = -13.603 \text{ eV} - 49.18981 \text{ eV} - (-67.43780 \text{ eV}) = 4.64499 \text{ eV} \quad (13.408)$$

The experimental $^{14}\text{ND}_3$ bond dissociation energy [37] is

$$E_D(^{14}\text{ND}_3) = 4.71252 \text{ eV} \quad (13.409)$$

BOND ANGLE OF NH_3

20 Using, $2c'_{\text{H-H}}$ (Eq. (13.364)), the distance between the two H atoms when the total energy of the corresponding MO is zero (Eq. (13.361)), and $2c'_{\text{N-H}}$, the internuclear distance of each N-H bond (Eq. (13.377)), the corresponding bond angle can be determined from the law of cosines. Using Eq. (13.367), the bond angle θ between the N-H bonds is

$$\theta = \cos^{-1} \left(\frac{2(1.95921)^2 - (3.14643)^2}{2(1.95921)^2} \right) = \cos^{-1}(-0.28956) = 106.67^\circ \quad (13.410)$$

25 The experimental angle between the N-H bonds is [36]

$$\theta = 106.67^\circ \quad (13.411)$$

The NH_3 molecule has a pyramidal structure with the nitrogen atom along the z-axis at the apex and the hydrogen atoms at the base in the xy-plane. Since any two $N-H$ bonds form an isosceles triangle, the distance $d_{origin-H}$ from the origin to the nucleus of a hydrogen atom is given by

$$d_{origin-H} = \frac{2c'_{H-H}}{2 \sin 60^\circ} \quad (13.412)$$

Substitution of Eq. (13.364) into Eq. (13.412) gives

$$d_{origin-H} = 1.81659a_0 \quad (13.413)$$

The height along the z-axis of the pyramid from the origin to N nucleus d_{height} is given by

$$d_{height} = \sqrt{(2c'_{N-H})^2 - (d_{origin-H})^2} \quad (13.414)$$

Substitution of Eqs. (13.377) and (13.413) into Eq. (13.414) gives

$$d_{height} = 0.73383a_0 \quad (13.415)$$

The angle θ_v of each $N-H$ bond from the z-axis is given by

$$\theta_v = \tan^{-1} \left(\frac{d_{origin-H}}{d_{height}} \right) \quad (13.416)$$

Substitution of Eqs. (13.413) and (13.415) into Eq. (13.417) gives

$$\theta_v = 68.00^\circ \quad (13.417)$$

The NH_3 MO shown in Figure 13 was rendered using these parameters.

The results of the determination of bond parameters of NH_3 and ND_3 are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

HYDROGEN CARBIDE (CH)

The methane molecule can be solved by first considering the solution of the hydrogen carbide, dihydrogen carbide, and methyl radicals. The former is formed by the reaction of a hydrogen atom and a carbon atom:



The hydrogen carbide radicals, CH and CH_2 , methyl radical, CH_3 , and methane, CH_4 , can be solved using the same principles as those used to solve OH , H_2O , NH , NH_2 , and NH_3 with the exception that the carbon $2s$ and $2p$ shells hybridize to form a single $2sp^3$ shell as an energy minimum.

5

FORCE BALANCE OF CH

CH comprises two spin-paired electrons in a chemical bond between the carbon atom and the hydrogen atom. The CH radical molecular orbital (MO) is determined by considering properties of the binding atoms and the boundary constraints. The prolate spheroidal H_2 MO developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules section satisfies the boundary constraints; thus, the H -atom electron forms a H_2 -type ellipsoidal MO with one of the C -atom electrons. However, such a bond is not possible with the outer C electrons in their ground state since the resulting H_2 -type ellipsoidal MO would have a shorter internuclear distance than the radius of the carbon $2p$ shell, which is not energetically stable. Thus, when bonding the carbon $2s$ and $2p$ shells hybridize to form a single $2sp^3$ shell as an energy minimum.

The C electron configuration given in the Six-Electron Atoms section is $1s^2 2s^2 2p^2$, and the orbital arrangement is

$$\begin{array}{ccc} & \text{2p state} & \\ \uparrow & \uparrow & \underline{\quad} \\ 1 & 0 & -1 \end{array} \quad (13.419)$$

corresponding to the ground state 3P_0 . The radius r_6 of the $2p$ shell given by Eq. (10.122) is

$$r_6 = 1.20654a_0 \quad (13.420)$$

The energy of the carbon $2p$ shell is the negative of the ionization energy of the carbon atom given by Eq. (10.123). Experimentally, the energy is [12]

$$E(C, 2p \text{ shell}) = -E(\text{ionization}; C) = -11.2603 \text{ eV} \quad (13.421)$$

The $C2s$ atomic orbital (AO) combines with the $C2p$ AOs to form a single $2sp^3$ hybridized orbital (HO) with the orbital arrangement

25

$$\begin{array}{cccc}
 & & 179 & \\
 & & 2sp^3 \text{ state} & \\
 \uparrow & \uparrow & \uparrow & \uparrow \\
 0,0 & 1,-1 & 1,0 & 1,1
 \end{array} \quad (13.422)$$

where the quantum numbers (ℓ, m_ℓ) are below each electron. The total energy of the state is given by the sum over the four electrons. The sum $E_T(C, 2sp^3)$ of calculated energies of C, C⁺, C²⁺, and C³⁺ from Eqs. (10.123), (10.113-10.114), (10.68), and (10.48), respectively, is

$$E_T(C, 2sp^3) = 64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + 11.27671 \text{ eV} = 148.25751 \text{ eV} \quad (13.423)$$

which agrees well with the sum of 148.02532 eV from the experimental [6] values. The orbital-angular-momentum interactions cancel such that the energy of the $E_T(C, 2sp^3)$ is purely Coulombic. By considering that the central field decreases by an integer for each successive electron of the shell, the radius r_{2sp^3} of the C2sp³ shell may be calculated from the Coulombic energy using Eq. (10.102):

$$r_{2sp^3} = \sum_{n=2}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0 (e148.25751 \text{ eV})} = \frac{10e^2}{8\pi\epsilon_0 (e148.25751 \text{ eV})} = 0.91771a_0 \quad (13.424)$$

Using Eqs. (10.102) and (13.424), the Coulombic energy $E_{Coulomb}(C, 2sp^3)$ of the outer electron of the C2sp³ shell is

$$E_{Coulomb}(C, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.91771a_0} = -14.82575 \text{ eV} \quad (13.425)$$

During hybridization, one of the spin-paired 2s electrons is promoted to C2sp³ shell as an unpaired electron. The energy for the promotion is the magnetic energy given by Eq. (13.152) at the initial radius of the 2s electrons. From Eq. (10.62) with $Z = 6$, the radius r_3 of C2s shell is

$$r_3 = 0.84317a_0 \quad (13.426)$$

Using Eqs. (13.152) and (13.426), the unpairing energy is

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} = \frac{8\pi\mu_0 \mu_B^2}{(0.84317a_0)^3} = 0.19086 \text{ eV} \quad (13.427)$$

Using Eqs. (13.425) and (13.427), the energy $E(C, 2sp^3)$ of the outer electron of the C2sp³

shell is

$$\begin{aligned}
 E(C, 2sp^3) &= \frac{-e^2}{8\pi\epsilon_0 r_{2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} \\
 &= -14.82575 \text{ eV} + 0.19086 \text{ eV} \\
 &= -14.63489 \text{ eV}
 \end{aligned}
 \tag{13.428}$$

The nitrogen atom's $2p$ -shell electron configuration given by Eq. (10.134) is the same as that of the $C2sp^3$ shell, and nitrogen's calculated energy of 14.61664 eV given by Eq. (10.143) is a close match with $E(C, 2sp^3)$. Thus, the binding should be very similar except that four bonds to hydrogen can occur with carbon.

The carbon $C2sp^3$ electron combines with the $H1s$ electron to form a molecular orbital. The proton of the H atom and the nucleus of the C atom are along the internuclear axis and serve as the foci. Due to symmetry, the other C electrons are equivalent to point charges at the origin. (See Eqs. (19-38) of Appendix IV.) Thus, the energies in the CH MO involve only the $C2sp^3$ and $H1s$ electrons. The forces are determined by these energies.

As in the case of H_2 , the MO is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into the $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell. Thus, the MO surface comprises a prolate spheroid at the H proton that is continuous with the $C2sp^3$ shell at the C atom whose nucleus serves as the other focus. The energy of the H_2 -type ellipsoidal MO is matched to that of the $C2sp^3$ shell. As in the case with OH and NH , the linear combination of the H_2 -type ellipsoidal MO with the $C2sp^3$ HO must involve a 25% contribution from the H_2 -type ellipsoidal MO to the $C2sp^3$ HO in order to match potential, kinetic, and orbital energy relationships. Thus, the CH MO must comprise 75% of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO:



The force balance of the CH MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (13.429) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

As in the case with OH (Eq. (13.57)), the H_2 -type ellipsoidal MO comprises 75% of the CH MO; so, the electron charge density in Eq. (11.65) is given by $-0.75e$. The force constant k' to determine the ellipsoidal parameter c' in terms of the central force of the foci is given by Eq. (13.59). The distance from the origin to each focus c' is given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b=c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of the CH MO. Since the CH MO comprises a H_2 -type-ellipsoidal MO that transitions to the $C2sp^3$ HO, the energy $E(C,2sp^3)$ in Eq. (13.428) adds to that of the H_2 -type ellipsoidal MO to give the total energy of the CH MO. From the energy equation and the relationship between the axes, the dimensions of the CH MO are solved.

The energy components of V_e , V_p , T , and V_m are those of H_2 (Eqs. (11.207-11.212)) except that they are corrected for electron hybridization. Hybridization gives rise to the $C2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C,2sp^3)$ given by Eq. (13.425). To meet the equipotential condition of the union of the H_2 -type-ellipsoidal-MO and the $C2sp^3$ HO, the electron energies are normalized by the ratio of $14.82575 eV$, the magnitude of $E_{Coulomb}(C,2sp^3)$ given by Eq. (13.425), and $13.605804 eV$, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). This normalizes the energies to match that of the Coulombic energy alone to meet the energy matching condition of the CH MO under the influence of the proton and the C nucleus. The hybridization energy factor C_{C2sp^3HO} is

$$C_{C2sp^3HO} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{2sp^3}}} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 0.91771a_0}} = \frac{13.605804 eV}{14.82575 eV} = 0.91771 \quad (13.430)$$

The total energy $E_T(CH)$ of the CH MO is given by the sum of the energies of the orbitals, the H_2 -type ellipsoidal MO and the $C2sp^3$ HO, that form the hybridized CH MO. $E_T(CH)$ follows from by Eq. (13.74) for OH , but the energy of the $C2sp^3$ HO given by Eq. (13.428) is substituted for the energy of O and the H_2 -type-ellipsoidal-MO energies are

those of H_2 (Eqs. (11.207-11.212)) multiplied by the electron hybridization factor rather than by the factor of 0.75:

$$E_T(CH) = E_T + E(C, 2sp^3) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} \quad (13.431)$$

To match the boundary condition that the total energy of the entire the H_2 -type ellipsoidal MO is given by Eqs. (11.212) and (13.75), $E_T(CH)$ given by Eq. (13.431) is set equal to Eq. (13.75):

$$E_T(CH) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} = -31.63536831 \text{ eV} \quad (13.432)$$

From the energy relationship given by Eq. (13.432) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH MO can be solved.

Substitution of Eq. (13.60) into Eq. (13.432) gives

$$\frac{e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e17.00048 \quad (13.433)$$

The most convenient way to solve Eq. (13.433) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.67465a_0 = 8.86186 \times 10^{-11} \text{ m} \quad (13.434)$$

Substitution of Eq. (13.434) into Eq. (13.60) gives

$$c' = 1.05661a_0 = 5.59136 \times 10^{-11} \text{ m} \quad (13.435)$$

The internuclear distance given by multiplying Eq. (13.435) by two is

$$2c' = 2.11323a_0 = 1.11827 \times 10^{-10} \text{ m} \quad (13.436)$$

The experimental bond distance is [14]

$$2c' = 1.1198 \times 10^{-10} \text{ m} \quad (13.437)$$

Substitution of Eqs. (13.434-13.435) into Eq. (13.62) gives

$$b = c = 1.29924a_0 = 6.87527 \times 10^{-11} \text{ m} \quad (13.438)$$

Substitution of Eqs. (13.434-13.435) into Eq. (13.63) gives

$$e = 0.63095 \quad (13.439)$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{2sp^3} = 0.91771a_0$ is the radius of the

5 $C2sp^3$ shell. Substitution of Eqs. (13.434-13.435) into Eq. (13.261) gives

$$\theta' = 81.03^\circ \quad (13.440)$$

Then, the angle θ_{C2sp^3HO} the radial vector of the $C2sp^3$ HO makes with the internuclear axis is

$$\theta_{C2sp^3HO} = 180^\circ - 81.03^\circ = 98.97^\circ \quad (13.441)$$

10 as shown in Figure 14.

The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{H_2MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C2sp^3$

15 radial vector obeys the following relationship:

$$r_{2sp^3} \sin \theta_{C2sp^3HO} = 0.91771a_0 \sin \theta_{C2sp^3HO} = b \sin \theta_{H_2MO} \quad (13.442)$$

such that

$$\theta_{H_2MO} = \sin^{-1} \frac{0.91771a_0 \sin \theta_{C2sp^3HO}}{b} = \sin^{-1} \frac{0.91771a_0 \sin 98.97^\circ}{b} \quad (13.443)$$

with the use of Eq. (13.441). Substitution of Eq. (13.438) into Eq. (13.443) gives

$$20 \quad \theta_{H_2MO} = 44.24^\circ \quad (13.444)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{H_2MO} = a \cos \theta_{H_2MO} \quad (13.445)$$

Substitution of Eqs. (13.434) and (13.444) into Eq. (13.445) gives

$$25 \quad d_{H_2MO} = 1.19968a_0 = 6.34845 \times 10^{-11} \text{ m} \quad (13.446)$$

The distance d_{C2sp^3HO} along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$d_{C2sp^3HO} = d_{H_2MO} - c' \quad (13.447)$$

Substitution of Eqs. (13.435) and (13.446) into Eq. (13.447) gives

$$d_{C2sp^3HO} = 0.14307a_0 = 7.57090 \times 10^{-12} \text{ m} \quad (13.448)$$

As shown in Eq. (13.429), the uniform charge-density in the $C2sp^3$ HO is increased by a factor of 0.25 and the H -atom density is decreased by a factor of 0.25. Using the orbital composition of CH (Eq. (13.429)), the radii of $C1s = 0.17113a_0$ (Eq. (10.51)) and

- 5 $C2sp^3 = 0.91771a_0$ (Eq. (10.424)) shells, and the parameters of the CH MO given by Eqs. (13.3-13.4), (13.434-13.436), and (13.438-13.448), the dimensional diagram and charge-density of the CH MO comprising the linear combination of the H_2 -type ellipsoidal MO and the $C2sp^3$ HO according to Eq. (13.429) are shown in Figures 14 and 15, respectively.

10

ENERGIES OF CH

The energies of CH are given by the substitution of the semiprincipal axes (Eqs. (13.434-13.435) and (13.438)) into the energy equations, (Eq. (13.431) and Eqs. (11.207-11.211) that are corrected for electron hybridization using Eq. (13.430):

$$15 \quad V_e = (0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -35.12015 \text{ eV} \quad (13.449)$$

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 12.87680 \text{ eV} \quad (13.450)$$

$$T = (0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 10.48582 \text{ eV} \quad (13.451)$$

$$V_m = (0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -5.24291 \text{ eV} \quad (13.452)$$

$$E_T(CH) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} = -31.63533 \text{ eV} \quad (13.453)$$

20

where $E_T(CH)$ is given by Eq. (13.431) which is reiteratively matched to Eq. (13.75) within five-significant-figure round-off error.

VIBRATION AND ROTATION OF CH

The vibrational energy of CH may be solved in the same manner as that of OH and NH except that the force between the electrons and the foci given by Eq. (13.102) is doubled due to electron hybridization of the two shells of carbon after Eq. (11.141). From Eqs. (13.102-5 13.106) with the substitution of the CH parameters, the angular frequency of the oscillation is

$$\begin{aligned}\omega &= \sqrt{\frac{\frac{0.75e^2}{4\pi\epsilon_0 b^3} - \frac{e^2}{8\pi\epsilon_0 (2c')^3}}{\mu}} \\ &= \sqrt{\frac{\frac{0.75e^2}{4\pi\epsilon_0 (1.29924a_0)^3} - \frac{e^2}{8\pi\epsilon_0 (2.11323a_0)^3}}{\frac{12}{13}m_p}} \quad (13.454) \\ &= 5.39828 \times 10^{14} \text{ rad/s}\end{aligned}$$

where b is given by Eq. (13.438), $2c'$ is given by Eq. (13.436), and the reduced mass of ^{12}CH is given by:

$$10 \quad \mu_{^{12}CH} = \frac{m_1 m_2}{m_1 + m_2} = \frac{(1)(12)}{1+12} m_p \quad (13.455)$$

where m_p is the proton mass. Thus, during bond formation, the perturbation of the orbit determined by an inverse-squared force results in simple harmonic oscillatory motion of the orbit, and the corresponding frequency, $\omega(0)$, for ^{12}CH given by Eqs. (11.136), (11.148), and (13.454) is

$$15 \quad \omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{449.94 \text{ Nm}^{-1}}{\mu}} = 5.39828 \times 10^{14} \text{ radians/s} \quad (13.456)$$

where the reduced nuclear mass of ^{12}CH is given by Eq.(13.455) and the spring constant, $k(0)$, given by Eqs. (11.136) and (13.454) is

$$k(0) = 449.94 \text{ Nm}^{-1} \quad (13.457)$$

The ^{12}CH transition-state vibrational energy, $E_{vib}(0)$, given by Planck's equation (Eq. 20 (11.127)) is:

$$E_{vib}(0) = \hbar\omega = \hbar 5.39828 \times 10^{14} \text{ rad/s} = 0.35532 \text{ eV} = 2865.86 \text{ cm}^{-1} \quad (13.458)$$

ω_e , from the experimental curve fit of the vibrational energies of ^{12}CH is [14]

$$\omega_e = 2861.6 \text{ cm}^{-1} \quad (13.459)$$

Using Eqs. (13.112-13.118) with $E_{vib}(0)$ given by Eq. (13.458) and D_0 given by Eq. (13.488), the ^{12}CH $\nu=1 \rightarrow \nu=0$ vibrational energy, $E_{vib}(1)$ is

$$E_{vib}(1) = 0.33879 \text{ eV} \quad (2732.61 \text{ cm}^{-1}) \quad (13.460)$$

5 The experimental vibrational energy of ^{12}CH using ω_e and $\omega_e x_e$ [14] according to K&P [15] is

$$E_{vib}(1) = 0.33885 \text{ eV} \quad (2733 \text{ cm}^{-1}) \quad (13.461)$$

Using Eq. (13.113) with $E_{vib}(1)$ given by Eq. (13.460) and D_0 given by Eq. (13.488), the anharmonic perturbation term, $\omega_0 x_0$, of ^{12}CH is

$$10 \quad \omega_0 x_0 = 66.624 \text{ cm}^{-1} \quad (13.462)$$

The experimental anharmonic perturbation term, $\omega_0 x_0$, of ^{12}CH [14] is

$$\omega_0 x_0 = 64.3 \text{ cm}^{-1} \quad (13.463)$$

The vibrational energies of successive states are given by Eqs. (13.458), (13.112), and (13.462).

15 Using b given by Eq. (13.438), $2c'$ given by Eq. (13.436), D_0 given by Eq. (13.490), and the reduced nuclear mass of ^{12}CD given by

$$\mu_{^{12}\text{CD}} = \frac{m_1 m_2}{m_1 + m_2} = \frac{(2)(12)}{2+12} m_p \quad (13.464)$$

where m_p is the proton mass, the corresponding parameters for deuterium carbide ^{12}CD (Eqs. (13.102-13.121)) are

$$20 \quad \omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{449.94 \text{ Nm}^{-1}}{\mu}} = 3.96126 \times 10^{14} \text{ radians/s} \quad (13.465)$$

$$k(0) = 449.94 \text{ Nm}^{-1} \quad (13.466)$$

$$E_{vib}(0) = \hbar\omega = \hbar 3.96126 \times 10^{14} \text{ rad/s} = 0.26074 \text{ eV} = 2102.97 \text{ cm}^{-1} \quad (13.467)$$

$$E_{vib}(1) = 0.25173 \text{ eV} \quad (2030.30 \text{ cm}^{-1}) \quad (13.468)$$

$$\omega_0 x_0 = 36.335 \text{ cm}^{-1} \quad (13.469)$$

25 ω_e , from the experimental curve fit of the vibrational energies of ^{12}CD is [14]

$$\omega_e = 2101.0 \text{ cm}^{-1} \quad (13.470)$$

The experimental vibrational energy of ^{12}CD using ω_e and $\omega_e x_e$ [14] according to K&P [15] is

$$E_{\text{vib}}(1) = 0.25189 \text{ eV} \quad (2031.6 \text{ cm}^{-1}) \quad (13.471)$$

and the experimental anharmonic perturbation term, $\omega_0 x_0$, of ^{12}CD is [14]

$$5 \quad \omega_0 x_0 = 34.7 \text{ cm}^{-1} \quad (13.472)$$

which match the predictions given by Eqs. (13.467), (13.468) and (13.469), respectively.

Using Eqs. (13.133-13.134) and the internuclear distance, $r = 2c'$, and reduced mass of ^{12}CH given by Eqs. (13.436) and (13.455), respectively, the corresponding B_e is

$$B_e = 14.498 \text{ cm}^{-1} \quad (13.473)$$

10 The experimental B_e rotational parameter of ^{12}CH is [14]

$$B_e = 14.457 \text{ cm}^{-1} \quad (13.474)$$

Using the internuclear distance, $r = 2c'$, and reduced mass of ^{12}CD given by Eqs. (13.436) and (13.464), respectively, the corresponding B_e is

$$B_e = 7.807 \text{ cm}^{-1} \quad (13.475)$$

15 The experimental B_e rotational parameter of ^{12}CD is [14]

$$B_e = 7.808 \text{ cm}^{-1} \quad (13.476)$$

THE DOPPLER ENERGY TERMS OF ^{12}CH AND ^{12}CD

The equations of the radiation reaction force of hydrogen and deuterium carbide are the same
20 as those of the corresponding hydroxyl and hydrogen nitride radicals with the substitution of the hydrogen and deuterium carbide parameters. Using Eqs. (11.136) and (13.140-13.142), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 2.41759 \times 10^{16} \text{ rad/s} \quad (13.477)$$

where b is given by Eq. (13.438). The kinetic energy, E_K , is given by Planck's equation
25 (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.41759 \times 10^{16} \text{ rad/s} = 15.91299 \text{ eV} \quad (13.478)$$

In Eq. (11.181), substitution of the total energy of CH , $E_T(CH)$, (Eq. (13.432)) for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.478) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(15.91299 \text{ eV})}{m_e c^2}} = -0.24966 \text{ eV} \quad (13.479)$$

5 In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.479) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational
10 energy of CH . The experimental ^{12}CH ω_e is 2861.6 cm^{-1} (0.35480 eV) [14] which matches the predicted ω_e of 2865.86 cm^{-1} (0.35532 eV) given by Eq. (13.458). Using the predicted ω_e for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(^{12}CH)$ is

$$\bar{E}_{osc}(^{12}CH) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.480)$$

$$\bar{E}_{osc}(^{12}CH) = -0.24966 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) = -0.07200 \text{ eV} \quad (13.481)$$

15 The experimental ^{12}CD ω_e is 2101.0 cm^{-1} (0.26049 eV) [14] which matches the predicted ω_e of 2102.97 cm^{-1} (0.26074 eV) given by Eq. (13.467). Using Eq. (13.479) and the predicted ω_e for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(^{12}CD)$ is

$$\bar{E}_{osc}(^{12}CD) = -0.24966 \text{ eV} + \frac{1}{2} (0.26074 \text{ eV}) = -0.11929 \text{ eV} \quad (13.482)$$

20 TOTAL AND BOND ENERGIES OF ^{12}CH AND ^{12}CD

$E_{T+osc}(^{12}CH)$, the total energy of the ^{12}CH radical including the Doppler term, is given by the sum of $E_T(CH)$ (Eq. (13.432)) and $\bar{E}_{osc}(^{12}CH)$ given by Eq. (13.481):

$$\begin{aligned} E_{T+osc}(^{12}CH) &= V_e + T + V_m + V_p + E(C, 2sp^3) + \bar{E}_{osc}(^{12}CH) \\ &= E_T(CH) + \bar{E}_{osc}(^{12}CH) \end{aligned} \quad (13.483)$$

$$E_{T+osc}({}^{12}\text{CH}) = \left\{ \begin{array}{l} \left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} \right) \\ \left(1 + \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} \right) + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \end{array} \right\}$$

$$= -31.63537 \text{ eV} - 0.24966 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}$$

(13.484)

From Eqs. (13.480-13.481) and (13.483-13.484), the total energy of ${}^{12}\text{CH}$ is

$$\begin{aligned} E_{T+osc}({}^{12}\text{CH}) &= -31.63537 \text{ eV} + \bar{E}_{osc}({}^{12}\text{CH}) \\ &= -31.63537 \text{ eV} - 0.24966 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) \\ &= -31.70737 \text{ eV} \end{aligned} \quad (13.485)$$

5 where the predicted ω_e (Eq. (13.458)) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term. $E_{T+osc}({}^{12}\text{CD})$, the total energy of ${}^{12}\text{CD}$ including the Doppler term, is given by the sum of $E_T(\text{CD}) = E_T(\text{CH})$ (Eq. (13.432)) and $\bar{E}_{osc}({}^{12}\text{CD})$ given by Eq. (13.482):

$$\begin{aligned} E_{T+osc}({}^{12}\text{CD}) &= -31.63537 \text{ eV} + \bar{E}_{osc}({}^{12}\text{CD}) \\ &= -31.63537 \text{ eV} - 0.24966 \text{ eV} + \frac{1}{2} (0.26074 \text{ eV}) \\ &= -31.75462 \text{ eV} \end{aligned} \quad (13.486)$$

where the predicted ω_e (Eq. (13.467)) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

10 The CH bond dissociation energy, $E_D({}^{12}\text{CH})$, is given by the sum of the total energies of the $\text{C}2sp^3$ HO and the hydrogen atom minus $E_{T+osc}({}^{12}\text{CH})$ ³:

$$E_D({}^{12}\text{CH}) = E(\text{C}, 2sp^3) + E(\text{H}) - E_{T+osc}({}^{12}\text{CH}) \quad (13.487)$$

³ The hybridization energy is the difference between $E(\text{C}, 2sp^3)$ given by Eq. (13.428) and $E(\text{C}, 2p \text{ shell})$ given by Eq. (13.421). Since this term adds to $E(\text{C}, 2p \text{ shell})$ to give the total energy from which $E_{T+osc}({}^{12}\text{CH})$ is subtracted to give $E_D({}^{12}\text{CH})$, it is more convenient to simply use $E(\text{C}, 2sp^3)$ directly in Eq. (13.487).

$E(C, 2sp^3)$ is given by Eq. (13.428), and $E_D(H)$ is given by Eq. (13.154). Thus, the ^{12}CH bond dissociation energy, $E_D(^{12}CH)$, given by Eqs. (13.154), (13.428), (13.485), and (13.487) is

$$\begin{aligned} E_D(^{12}CH) &= -(14.63489 \text{ eV} + 13.59844 \text{ eV}) - E_{T+osc}(CH) \\ &= -28.23333 \text{ eV} - (-31.70737 \text{ eV}) \\ &= 3.47404 \text{ eV} \end{aligned} \quad (13.488)$$

5 The experimental ^{12}CH bond dissociation energy is [14]

$$E_D(^{12}CH) = 3.47 \text{ eV} \quad (13.489)$$

which is a close match to that of NH as predicted based on the match between the N and $C2sp^3$ HO energies and electron configurations.

The ^{12}CD bond dissociation energy, $E_D(^{12}CD)$, is given by the sum of the total
10, energies of the $C2sp^3$ HO and the deuterium atom minus $E_{T+osc}(CD)$:

$$E_D(^{12}CD) = E(C, 2sp^3) + E(D) - E_{T+osc}(^{12}CD) \quad (13.490)$$

$E(C, 2sp^3)$ is given by Eq. (13.428), and $E_D(D)$ is given by Eq. (13.155). Thus, the ^{12}CD bond dissociation energy, $E_D(^{12}CD)$, given by Eqs. (13.155), (13.428), (13.486), and (13.490) is

$$\begin{aligned} E_D(^{12}CD) &= -(14.63489 \text{ eV} + 13.603 \text{ eV}) - E_{T+osc}(^{12}CD) \\ 15 \quad &= -28.23789 \text{ eV} - (-31.75462 \text{ eV}) \\ &= 3.51673 \text{ eV} \end{aligned} \quad (13.491)$$

The experimental ^{12}CD bond dissociation energy is [14]

$$E_D(^{12}CD) = 3.52 \text{ eV} \quad (13.492)$$

The results of the determination of bond parameters of CH and CD are given in Table 13.1. The calculated results are based on first principles and given in closed-form,
20 exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

DIHYDROGEN CARBIDE (CH_2)

The dihydrogen carbide radical CH_2 is formed by the reaction of a hydrogen atom with a
25 hydrogen carbide radical:



CH_2 can be solved using the same principles as those used to solve H_2O and NH_2 with the exception that the carbon $2s$ and $2p$ shells hybridize to form a single $2sp^3$ shell as an energy minimum. Two diatomic molecular orbitals (MOs) developed in the Nature of the
 5 Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section serve as basis functions in a linear combination with two carbon $2sp^3$ hybridized orbitals (HOs) to form the MO of CH_2 . The solution is very similar to that of CH except that there are two CH bonds in CH_2 .

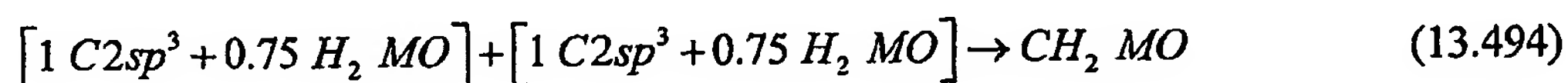
10 FORCE BALANCE OF CH_2

CH_2 comprises two chemical bonds between carbon and hydrogen atoms. Each $C-H$ bond comprises two spin-paired electrons with one from an initially unpaired electron of the carbon atom and the other from the hydrogen atom. Each H -atom electron forms a H_2 -type ellipsoidal MO with an unpaired C -atom electrons. However, such a bond is not
 15 possible with the outer two C electrons in their ground state since the resulting H_2 -type ellipsoidal MO would have a shorter internuclear distance than the radius of the carbon $2p$ shell, which is not energetically stable. Thus, when bonding the carbon $2s$ and $2p$ shells hybridize to form a single $2sp^3$ shell as an energy minimum. The electron configuration and the energy, $E(C, 2sp^3)$, of the $C2sp^3$ shell is given by Eqs. (13.422), and (13.428),
 20 respectively.

For each $C-H$ bond, a $C2sp^3$ electron combines with the $H1s$ electron to form a molecular orbital. The proton of the H atom and the nucleus of the C atom are along each internuclear axis and serve as the foci. As in the case of H_2 , each of the two $C-H$ -bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend
 25 into the $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell since it is energetically unfavorable. Thus, each MO surface comprises a prolate spheroid at the H proton that is continuous with the $C2sp^3$ shell at the C atom whose nucleus serves as the other focus. The radius and the energy of the $C2sp^3$ shell are unchanged with bond formation. The central paramagnetic force due to spin of each $C-H$ bond is provided by

the spin-pairing force of the CH_2 MO that has the symmetry of an s orbital that superimposes with the $C2sp^3$ orbitals such that the corresponding angular momenta are unchanged.

The energies in the CH_2 MO involve only each $C2sp^3$ and each $H1s$ electron with the formation of each $C-H$ bond. The sum of the energies of the H_2 -type ellipsoidal MOs is matched to that of the $C2sp^3$ shell. As in the cases with of OH , H_2O , NH , NH_2 , NH_3 , and CH the linear combination of each H_2 -type ellipsoidal MO with the $C2sp^3$ HO must involve a 25% contribution from the H_2 -type ellipsoidal MO to the $C2sp^3$ HO in order to match potential, kinetic, and orbital energy relationships. Thus, the CH_2 MO must comprise two $C-H$ bonds with each comprising 75% of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO:



The force balance of the CH_2 MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (13.494) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-ellipsoidal-MO component of the CH_2 MO in terms of the central force of the foci is given by Eq. (13.59). The distance from the origin of each $C-H$ -bond MO to each focus c' is given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of each $C-H$ -bond MO. Since each of the two prolate spheroidal $C-H$ -bond MOs comprises a H_2 -type-ellipsoidal MO that transitions to the $C2sp^3$ HO, the energy $E(C, 2sp^3)$ in Eq. (13.428) adds to that of the two corresponding H_2 -type ellipsoidal MOs to give the total energy of the CH_2 MO. From the energy equation and the relationship between the axes, the dimensions of the CH_2 MO are solved.

The energy components of V_e , V_p , T , and V_m are twice those of CH corresponding to the two $C-H$ bonds. Since the each prolate spheroidal H_2 -type MO transitions to the $C2sp^3$ HO and the energy of the $C2sp^3$ shell must remain constant and equal to the $E(C,2sp^3)$ given by Eq. (13.428), the total energy $E_T(CH_2)$ of the CH_2 MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the $C2sp^3$ HO and the two H_2 -type ellipsoidal MOs that forms the CH_2 MO as given by Eq. (13.494). Using Eq. (13.431), $E_T(CH_2)$ is given by

$$\begin{aligned} E_T(CH_2) &= E_T + E(C,2sp^3) \\ &= -\frac{e^2}{4\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} \end{aligned} \quad (13.495)$$

The two hydrogen atoms and the hybridized carbon atom can achieve an energy minimum as a linear combination of two H_2 -type ellipsoidal MOs each having the proton and the carbon nucleus as the foci. Hybridization gives rise to the $C2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C,2sp^3)$ given by Eq. (13.425). To meet the equipotential condition of the union of the H_2 -type-ellipsoidal-MO and the $C2sp^3$ HO, the electron energies in Eq. (13.495) were normalized by the ratio of 14.82575 eV, the magnitude of $E_{Coulomb}(C,2sp^3)$ given by Eq. (13.425), and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). The factor given by Eq. (13.430) normalized the energies to match that of the Coulombic energy alone to meet the energy matching condition of each $C-H$ -bond MO under the influence of the proton and the C nucleus. Each $C-H$ -bond MO comprises the same $C2sp^3$ shell having its energy normalized to that of the Coulombic energy between the electron and a charge of $+e$ at the carbon focus of the CH_2 MO. Thus, the energy of the CH_2 MO is also given by the sum of that of the two H_2 -type ellipsoidal MOs given by Eq. (11.212) minus the Coulombic energy, $E_{Coulomb}(H) = -13.605804 \text{ eV}$, of the redundant $+e$ of the linear combination:

$$\begin{aligned} E_T(2H_2 - H) &= -\frac{e^2}{4\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] - E_{Coulomb}(H) \\ &= 2(-31.63536831 \text{ eV}) - (-13.605804 \text{ eV}) \\ &= -49.66493 \text{ eV} \end{aligned} \quad (13.496)$$

$E_T(CH_2)$ given by Eq. (13.495) is set equal to two times the energy of the H_2 -type ellipsoidal MO minus the Coulombic energy of H given by Eq. (13.496):

$$E_T(CH_2) = -\frac{e^2}{4\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} = -49.66493 \text{ eV} \quad (13.497)$$

- 5 From the energy relationship given by Eq. (13.497) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH_2 MO can be solved.

Substitution of Eq. (13.60) into Eq. (13.497) gives

$$\frac{e^2}{4\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e35.03004 \quad (13.498)$$

- The most convenient way to solve Eq. (13.498) is by the reiterative technique using a
10 computer. The result to within the round-off error with five-significant figures is

$$a = 1.64010a_0 = 8.67903 \times 10^{-11} \text{ m} \quad (13.499)$$

Substitution of Eq. (13.499) into Eq. (13.60) gives

$$c' = 1.04566a_0 = 5.53338 \times 10^{-11} \text{ m} \quad (13.500)$$

The internuclear distance given by multiplying Eq. (13.500) by two is

$$15 \quad 2c' = 2.09132a_0 = 1.10668 \times 10^{-10} \text{ m} \quad (13.501)$$

The experimental bond distance is [38]

$$2c' = 1.111 \times 10^{-10} \text{ m} \quad (13.502)$$

Substitution of Eqs. (13.499-13.500) into Eq. (13.62) gives

$$b = c = 1.26354a_0 = 6.68635 \times 10^{-11} \text{ m} \quad (13.503)$$

- 20 Substitution of Eqs. (13.499-13.500) into Eq. (13.63) gives

$$e = 0.63756 \quad (13.504)$$

- The nucleus of the H atom and the nucleus of the C atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of each H_2 -type ellipsoidal MO and the $C2sp^3$ HO are given by Eqs. (13.84-13.95), (13.261-13.270), and (13.440-
25 13.448). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{2sp^3} = 0.91771a_0$ is the radius of the $C2sp^3$ shell. Substitution of Eqs. (13.499-13.500) into Eq. (13.261) gives

$$\theta' = 84.54^\circ \quad (13.505)$$

Then, the angle θ_{C2sp^3HO} the radial vector of the $C2sp^3$ HO makes with the internuclear axis is

$$\theta_{C2sp^3HO} = 180^\circ - 84.54^\circ = 95.46^\circ \quad (13.506)$$

5 as shown in Figure 14. The angle θ_{H_2MO} between the internuclear axis and the point of intersection of each H_2 -type ellipsoidal MO with the $C2sp^3$ radial vector given by Eqs. (13.442-13.443), (13.503), and (13.506) is

$$\theta_{H_2MO} = 46.30^\circ \quad (13.507)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal
10 MO to the point of intersection of the orbitals given by Eqs. (13.445), (13.499), and (13.507) is

$$d_{H_2MO} = 1.13305a_0 = 5.99585 \times 10^{-11} \text{ m} \quad (13.508)$$

The distance d_{C2sp^3HO} along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals given by Eqs. (13.447), (13.500), and (13.508) is

$$15 \quad d_{C2sp^3HO} = 0.08739a_0 = 4.62472 \times 10^{-12} \text{ m} \quad (13.509)$$

As shown in Eq. (13.494), the uniform charge-density in the $C2sp^3$ HO is increased by a factor of 0.25 and the H -atom density is decreased by a factor of 0.25 for by each $C-H$ bond. Using the orbital composition of CH_2 (Eq. (13.494)), the radii of $Cl s = 0.17113a_0$ (Eq. (10.51)) and $C2sp^3 = 0.91771a_0$ (Eq. (10.424)) shells, and the
20 parameters of the CH_2 MO given by Eqs. (13.3-13.4), (13.499-13.501), and (13.503-13.509), the charge-density of the CH_2 MO comprising the linear combination of two $C-H$ -bond MOs is shown in Figure 16. Each $C-H$ -bond MO comprises a H_2 -type ellipsoidal MO and a $C2sp^3$ HO having the dimensional diagram shown in Figure 14.

25 ENERGIES OF CH_2

The energies of CH_2 are two times those of CH and are given by the substitution of the semiprincipal axes (Eqs. (13.499-13.500) and (13.503)) into the energy equations Eq. (13.495) and (Eqs. (13.449-13.452)) that are multiplied by two:

$$V_e = (0.91771) \frac{-2e^2}{4\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -72.03287 \text{ eV} \quad (13.510)$$

$$V_p = \frac{e^2}{4\pi\epsilon_0\sqrt{a^2-b^2}} = 26.02344 \text{ eV} \quad (13.511)$$

$$T = (0.91771) \frac{\hbar^2}{m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 21.95990 \text{ eV} \quad (13.512)$$

$$V_m = (0.91771) \frac{-\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -10.97995 \text{ eV} \quad (13.513)$$

$$5 \quad E_T(CH_2) = -\frac{e^2}{4\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} = -49.66437 \text{ eV} \quad (13.514)$$

where $E_T(CH_2)$ is given by Eq. (13.495) which is reiteratively matched to Eq. (13.496) within five-significant-figure round-off error.

10 VIBRATION OF CH_2

The vibrational energy levels of CH_2 may be solved as two equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [2] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of
15 Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERMS OF $^{12}CH_2$

The reentrant oscillation of hybridized orbitals in the transition state is not coupled. Therefore, the equations of the radiation reaction force of dihydrogen and dideuterium
20 carbide are the same as those of the corresponding hydrogen carbide radicals with the substitution of the dihydrogen and dideuterium carbide parameters. Using Eqs. (11.136) and (13.140-13.142), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 2.52077 \times 10^{16} \text{ rad/s} \quad (13.515)$$

where b is given by Eq. (13.503). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.52077 \times 10^{16} \text{ rad/s} = 16.59214 \text{ eV} \quad (13.516)$$

In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.516) for \bar{E}_K gives the Doppler energy of the electrons of each of the two bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(16.59214 \text{ eV})}{m_e c^2}} = -0.25493 \text{ eV} \quad (13.517)$$

10 In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH_2 due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.517) and \bar{E}_{Kvib} , the average kinetic energy of vibration which
15 is 1/2 of the vibrational energy of each $C-H$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition state having two independent bonds, $\bar{E}'_{osc}(^{12}CH_2)$ per bond is

$$\bar{E}'_{osc}(^{12}CH_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.518)$$

$$\bar{E}'_{osc}(^{12}CH_2) = -0.25493 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) = -0.07727 \text{ eV} \quad (13.519)$$

Given that the vibration and reentrant oscillation is for two $C-H$ bonds, $\bar{E}_{osc}(^{12}CH_2)$, is:

$$\begin{aligned} \bar{E}_{osc}(^{12}CH_2) &= 2 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\ &= 2 \left(-0.25493 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -0.15454 \text{ eV} \end{aligned} \quad (13.520)$$

TOTAL AND BOND ENERGIES OF $^{12}\text{CH}_2$

$E_{T+osc}(^{12}\text{CH}_2)$, the total energy of the $^{12}\text{CH}_2$ radical including the Doppler term, is given by the sum of $E_T(\text{CH}_2)$ (Eq. (13.497)) and $\bar{E}_{osc}(^{12}\text{CH}_2)$ given by Eq. (13.520):

$$\begin{aligned} E_{T+osc}(\text{CH}_2) &= V_e + T + V_m + V_p + E(C, 2sp^3) + \bar{E}_{osc}(^{12}\text{CH}_2) \\ &= E_T(\text{CH}_2) + \bar{E}_{osc}(^{12}\text{CH}_2) \end{aligned} \quad (13.521)$$

$$\begin{aligned} E_{T+osc}(^{12}\text{CH}_2) &= \left\{ \left(\frac{-e^2}{4\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} \right) \right. \\ &\quad \left. - 2 \left((31.63536831 \text{ eV}) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}} \right) \right\} \quad (13.522) \\ &= -49.66493 \text{ eV} - 2 \left(0.25493 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \end{aligned}$$

From Eqs. (13.518-13.522), the total energy of $^{12}\text{CH}_2$ is

$$\begin{aligned} E_{T+osc}(^{12}\text{CH}_2) &= -49.66493 \text{ eV} + \bar{E}_{osc}(^{12}\text{CH}_2) \\ &= -49.66493 \text{ eV} - 2 \left(0.25493 \text{ eV} - \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -49.81948 \text{ eV} \end{aligned} \quad (13.523)$$

where ω_e given by Eq. (13.458) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

$^{12}\text{CH}_2$ has the same electronic configuration as ^{14}NH . The dissociation of the bond of the dihydrogen carbide radical forms a free hydrogen atom with one unpaired electron and a $\text{C}2sp^3$ HO with three unpaired electrons as shown in Eq. (13.422) wherein the magnetic moments cannot all cancel. Thus, the bond dissociation of $^{12}\text{CH}_2$ gives rise to ^{12}CH with the same electronic configuration as N as given by Eq. (10.134). The N configuration is more stable than H as shown in Eqs. (10.141-10.143). The lowering of the energy of the reactants decreases the bond energy. The total energy of carbon is reduced by the energy in the field of the two magnetic dipoles given by Eq. (7.46) and Eq. (13.424):

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_{2sp^3})^3} = \frac{8\pi\mu_0 \mu_B^2}{(0.91771a_0)^3} = 0.14803 \text{ eV} \quad (13.524)$$

The CH_2 bond dissociation energy, $E_D(^{12}CH_2)$, is given by the sum of the total energies of the CH radical and the hydrogen atom minus the sum of $E_{T+osc}(^{12}CH_2)$ and $E(magnetic)$:

$$E_D(^{12}CH_2) = E(^{12}CH) + E(H) - E_{T+osc}(^{12}CH_2) - E(magnetic) \quad (13.525)$$

where $E_T(^{12}CH)$ is given by the sum of the energies of the $C2sp^3$ HO, $E(C, 2sp^3)$ given by Eq. (13.428), $E_D(H)$ given by Eq. (13.154), and the negative of the bond energy of ^{12}CH given by Eq. (13.489):

$$E(^{12}CH) = -13.59844 \text{ eV} - 14.63489 \text{ eV} - 3.47 \text{ eV} = -31.70333 \text{ eV} \quad (13.526)$$

Thus, the $^{12}CH_2$ bond dissociation energy, $E_D(^{12}CH_2)$, given by Eqs. (13.154), and (13.523-13.526) is

$$\begin{aligned} E_D(^{12}CH_2) &= -(31.70333 \text{ eV} + 13.59844 \text{ eV}) - (E_{T+osc}(^{12}CH_2) + E(magnetic)) \\ &= -45.30177 \text{ eV} - (-49.81948 \text{ eV} + 0.14803 \text{ eV}) \\ &= 4.36968 \text{ eV} \end{aligned} \quad (13.527)$$

The experimental $^{12}CH_2$ bond dissociation energy is [39]

$$E_D(^{12}CH_2) = 4.33064 \text{ eV} \quad (13.528)$$

15 BOND ANGLE OF $^{12}CH_2$

The CH_2 MO comprises a linear combination of two $C-H$ -bond MOs. Each $C-H$ -bond MO comprises the superposition of a H_2 -type ellipsoidal MO and the $C2sp^3$ HO with a relative charge density of 0.75 to 1.25; otherwise, the $C2sp^3$ shell is unchanged. A bond is also possible between the two H atoms of the $C-H$ bonds. Such $H-H$ bonding would decrease the $C-H$ bond strength since electron density would be shifted from the $C-H$ bonds to the $H-H$ bond. Thus, the bond angle between the two $C-H$ bonds is determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal H atoms of the $C-H$ bonds is zero. From Eqs. (11.79) and (13.228), the distance from the origin to each focus of the $H-H$ ellipsoidal MO is

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{aa_0}{2}} \quad (13.529)$$

The internuclear distance from Eq. (13.229) is

$$2c' = 2\sqrt{\frac{aa_0}{2}} \quad (13.530)$$

The length of the semiminor axis of the prolate spheroidal $H-H$ MO $b=c$ is given by Eq. (13.62).

The bond angle of CH_2 is derived by using the orbital composition and an energy matching factor as in the case with NH_2 and NH_3 . Since the two H_2 -type ellipsoidal MOs comprise 75% of the H electron density of H_2 and the energy of each H_2 -type ellipsoidal MO is matched to that of the $C2sp^3$ HO; the component energies and the total energy E_T of the $H-H$ bond are given by Eqs. (13.67-13.73) except that V_e , T , and V_m are corrected for the hybridization-energy-matching factor of 0.91771 given by Eq. (13.430). Substitution of Eq. (13.529) into Eq. (13.233) with the hybridization factor gives

$$0 = \left[\frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left[(0.91771)^{-1} \left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \left[1 + \sqrt{\frac{\frac{0.75e^2}{4\pi\epsilon_0 a^3}}{\frac{m_e}{m_e c^2}}} \right] \right. \\ \left. + \frac{1}{2} \hbar \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 (a+c')^3}}{0.5m_p}} \right] \quad (13.531)$$

From the energy relationship given by Eq. (13.531) and the relationship between the axes given by Eqs. (13.529-13.530) and (13.62-13.63), the dimensions of the $H-H$ MO can be solved.

The most convenient way to solve Eq. (13.531) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 5.1500a_0 = 2.7253 \times 10^{-10} \text{ m} \quad (13.532)$$

Substitution of Eq. (13.532) into Eq. (13.529) gives

$$c' = 1.6047a_0 = 8.4916 \times 10^{-11} \text{ m} \quad (13.533)$$

The internuclear distance given by multiplying Eq. (13.533) by two is

$$2c' = 3.2094a_0 = 1.6983 \times 10^{-10} \text{ m} \quad (13.534)$$

Substitution of Eqs. (13.532-13.533) into Eq. (13.62) gives

$$b = c = 4.8936a_0 = 2.5896 \times 10^{-10} \text{ m} \quad (13.535)$$

Substitution of Eqs. (13.532-13.533) into Eq. (13.63) gives

$$e = 0.3116 \quad (13.536)$$

Using, $2c'_{H-H}$ (Eq. (13.534)), the distance between the two H atoms when the total energy of the corresponding MO is zero (Eq. (13.531)), and $2c'_{C-H}$ (Eq. (13.501)), the internuclear distance of each $C-H$ bond, the corresponding bond angle can be determined from the law of cosines. Using, Eq. (13.242), the bond angle θ between the $C-H$ bonds is

$$\theta = \cos^{-1} \left(\frac{2(2.09132)^2 - (3.2094)^2}{2(2.09132)^2} \right) = \cos^{-1}(-0.1775) = 100.22^\circ \quad (13.537)$$

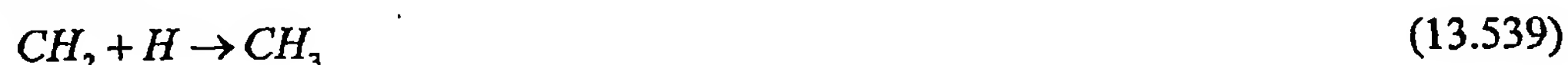
The experimental angle between the $C-H$ bonds is [38]

$$\theta = 102.4^\circ \quad (13.538)$$

- 10 The results of the determination of bond parameters of CH_2 are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

15 METHYL RADICAL (CH_3)

The methyl radical CH_3 is formed by the reaction of a hydrogen atom with a dihydrogen carbide radical:



- 20 CH_3 can be solved using the same principles as those used to solve and NH_3 with the exception that the carbon $2s$ and $2p$ shells hybridize to form a single $2sp^3$ shell as an energy minimum. Three diatomic molecular orbitals (MOs) developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section serve as basis functions in a linear combination with three carbon $2sp^3$ hybridized orbitals (HOs) to form
- 25 the MO of CH_3 . The solution is very similar to that of CH_2 except that there are three CH bonds in CH_3 .

FORCE BALANCE OF CH_3

CH_3 comprises three chemical bonds between carbon and hydrogen atoms. Each $C-H$ bond comprises two spin-paired electrons with one from an initially unpaired electron of the carbon atom and the other from the hydrogen atom. Each H -atom electron forms a
 5 H_2 -type ellipsoidal MO with an unpaired C -atom electrons. However, such a bond is not possible with the outer two C electrons in their ground state since the resulting H_2 -type ellipsoidal MO would have a shorter internuclear distance than the radius of the carbon $2p$ shell which is not energetically stable, and only two electrons are unpaired. Thus, when bonding the carbon $2s$ and $2p$ shells hybridize to form a single $2sp^3$ shell as an energy
 10 minimum. The electron configuration and the energy, $E(C, 2sp^3)$, of the $C2sp^3$ shell is given by Eqs. (13.422), and (13.428), respectively.

For each $C-H$ bond, a $C2sp^3$ electron combines with the $H1s$ electron to form a molecular orbital. The proton of the H atom and the nucleus of the C atom are along each internuclear axis and serve as the foci. As in the case of H_2 , each of the three $C-H$ -bond
 15 MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell since it is energetically unfavorable. Thus, each MO surface comprises a prolate spheroid at the H proton that is continuous with the $C2sp^3$ shell at the C atom whose nucleus serves as the other focus. The radius and the energy of the $C2sp^3$ shell are unchanged with bond
 20 formation. The central paramagnetic force due to spin of each $C-H$ bond is provided by the spin-pairing force of the CH_3 MO that has the symmetry of an s orbital that superimposes with the $C2sp^3$ orbitals such that the corresponding angular momenta are unchanged.

The energies in the CH_3 MO involve only each $C2sp^3$ and each $H1s$ electron with
 25 the formation of each $C-H$ bond. The sum of the energies of the H_2 -type ellipsoidal MOs is matched to that of the $C2sp^3$ shell. As in the cases with of OH , H_2O , NH , NH_2 , NH_3 , CH , and CH_2 the linear combination of each H_2 -type ellipsoidal MO with the $C2sp^3$ HO must involve a 25% contribution from the H_2 -type ellipsoidal MO to the $C2sp^3$ HO in order to match potential, kinetic, and orbital energy relationships. Thus, the CH_3 MO must

comprise three $C-H$ bonds with each comprising 75% of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO:



5

The force balance of the CH_3 MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (13.540) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-
10 ellipsoidal-MO component of the CH_3 MO in terms of the central force of the foci is given by Eq. (13.59). The distance from the origin of each $C-H$ -bond MO to each focus c' is given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b=c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for
15 the solution of the other axes of each prolate spheroid and eccentricity of each $C-H$ -bond MO. Since each of the three prolate spheroidal $C-H$ -bond MOs comprises a H_2 -type-ellipsoidal MO that transitions to the $C2sp^3$ HO, the energy $E(C,2sp^3)$ in Eq. (13.428) adds to that of the three corresponding H_2 -type ellipsoidal MOs to give the total energy of the CH_3 MO. From the energy equation and the relationship between the axes, the dimensions
20 of the CH_3 MO are solved.

The energy components of V_e , V_p , T , and V_m are three times those of CH corresponding to the three $C-H$ bonds. Since the each prolate spheroidal H_2 -type MO transitions to the $C2sp^3$ HO and the energy of the $C2sp^3$ shell must remain constant and equal to the $E(C,2sp^3)$ given by Eq. (13.428), the total energy $E_T(CH_3)$ of the CH_3 MO is
25 given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the $C2sp^3$ HO and the three H_2 -type ellipsoidal MOs that forms the CH_3 MO as given by Eq. (13.540). Using Eq. (13.431), $E_T(CH_3)$ is given by

$$\begin{aligned}
 E_T(CH_3) &= E_T + E(C, 2sp^3) \\
 &= -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV}
 \end{aligned} \tag{13.541}$$

The three hydrogen atoms and the hybridized carbon atom can achieve an energy minimum as a linear combination of three H_2 -type ellipsoidal MOs each having the proton and the carbon nucleus as the foci. Hybridization gives rise to the $C2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C, 2sp^3)$ given by Eq. (13.435). To meet the equipotential condition of the union of the H_2 -type-ellipsoidal-MO and the $C2sp^3$ HO, the electron energies in Eqs. (13.431), (13.495), and (13.541) were normalized by the ratio of 14.82575 eV, the magnitude of $E_{Coulomb}(C, 2sp^3)$ given by Eq. (13.425), and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). The factor given by Eq. (13.430) normalized the energies to match that of the Coulombic energy alone to meet the energy matching condition of each $C-H$ -bond MO under the influence of the proton and the C nucleus. Each $C-H$ -bond MO comprises the same $C2sp^3$ shell having its energy normalized to that of the Coulombic energy between the electron and a charge of $+e$ at the carbon focus of the CH_3 MO. Thus, the energy of the CH_3 MO is also given by the sum of that of the three H_2 -type ellipsoidal MOs given by Eq. (11.212) minus two times the Coulombic energy, $E_{Coulomb}(H) = -13.605804 \text{ eV}$, of the two redundant $+e$'s of the linear combination:

$$\begin{aligned}
 E_T(3H_2 - 2H) &= -\frac{3e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] - 2E_{Coulomb}(H) \\
 &= 3(-31.63536831 \text{ eV}) - 2(-13.605804 \text{ eV}) \\
 &= -67.69450 \text{ eV}
 \end{aligned} \tag{13.542}$$

$E_T(CH_3)$ given by Eq. (13.541) is set equal to three times the energy of the H_2 -type ellipsoidal MO minus two times the Coulombic energy of H given by Eq. (13.542):

$$E_T(CH_3) = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} = -67.69450 \text{ eV} \tag{13.543}$$

From the energy relationship given by Eq. (13.543) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH_3 MO can be solved.

Substitution of Eq. (13.60) into Eq. (13.543) gives

$$\frac{3e^2}{8\pi\epsilon_0\sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e53.05961 \quad (13.544)$$

- 5 The most convenient way to solve Eq. (13.544) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.62893a_0 = 8.61990 \times 10^{-11} \text{ m} \quad (13.545)$$

Substitution of Eq. (13.545) into Eq. (13.60) gives

$$c' = 1.04209a_0 = 5.51450 \times 10^{-11} \text{ m} \quad (13.546)$$

- 10 The internuclear distance given by multiplying Eq. (13.546) by two is

$$2c' = 2.08418a_0 = 1.10290 \times 10^{-10} \text{ m} \quad (13.547)$$

The experimental bond distance is [38]

$$2c' = 1.079 \times 10^{-10} \text{ m} \quad (13.548)$$

Substitution of Eqs. (13.545-13.546) into Eq. (13.62) gives

$$15 \quad b = c = 1.25198a_0 = 6.62518 \times 10^{-11} \text{ m} \quad (13.549)$$

Substitution of Eqs. (13.545-13.546) into Eq. (13.63) gives

$$e = 0.63974 \quad (13.550)$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of each H_2 -type ellipsoidal

- 20 MO and the $C2sp^3$ HO are given by Eqs. (13.84-13.95), (13.261-13.270), and (13.434-13.442). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{2sp^3} = 0.91771a_0$ is the radius of the $C2sp^3$ shell. Substitution of Eqs. (13.545-13.546) into Eq. (13.261) gives

$$\theta' = 85.65^\circ \quad (13.551)$$

- 25 Then, the angle θ_{C2sp^3HO} the radial vector of the $C2sp^3$ HO makes with the internuclear axis is

$$\theta_{C2sp^3HO} = 180^\circ - 85.65^\circ = 94.35^\circ \quad (13.552)$$

as shown in Figure 14. The angle θ_{H_2MO} between the internuclear axis and the point of intersection of each H_2 -type ellipsoidal MO with the $C2sp^3$ radial vector given by Eqs. (13.442-13.443), (13.549), and (13.552) is

$$\theta_{H_2MO} = 46.96^\circ \quad (13.553)$$

5 Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals given by Eqs. (13.445), (13.545), and (13.553) is

$$d_{H_2MO} = 1.11172a_0 = 5.88295 \times 10^{-11} \text{ m} \quad (13.554)$$

The distance d_{C2sp^3HO} along the internuclear axis from the origin of the C atom to the point
10 of intersection of the orbitals given by Eqs. (13.447), (13.546), and (13.554) is

$$d_{C2sp^3HO} = 0.06963a_0 = 3.68457 \times 10^{-12} \text{ m} \quad (13.555)$$

As shown in Eq. (13.540), the uniform charge-density in the $C2sp^3$ HO is increased by a factor of 0.25 and the H -atom density is decreased by a factor of 0.25 for by each $C-H$ bond. Using the orbital composition of CH_3 (Eq. (13.540)), the radii of
15 $Cl s = 0.17113a_0$ (Eq. (10.51)) and $C2sp^3 = 0.91771a_0$ (Eq. (10.424)) shells, and the parameters of the CH_3 MO given by Eqs. (13.3-13.4), (13.545-13.547), and (13.549-13.555), the charge-density of the CH_3 MO comprising the linear combination of three $C-H$ -bond MOs is shown in Figure 17. Each $C-H$ -bond MO comprises a H_2 -type ellipsoidal MO and a $C2sp^3$ HO having the dimensional diagram shown in Figure 14.

20

ENERGIES OF CH_3

The energies of CH_3 are three times those of CH and are given by the substitution of the semiprincipal axes (Eqs. (13.545-13.546) and (13.549)) into the energy equations Eq. (13.541) and (Eqs. (13.449-13.452)) that are multiplied by three:

$$25 \quad V_e = 3(0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -108.94944 \text{ eV} \quad (13.556)$$

$$V_p = \frac{3e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 39.16883 \text{ eV} \quad (13.557)$$

$$T = 3(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 33.44213 \text{ eV} \quad (13.558)$$

$$V_m = 3(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -16.72107 \text{ eV} \quad (13.559)$$

$$E_T(CH_3) = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] - 14.63489 \text{ eV} = -67.69444 \text{ eV} \quad (13.560)$$

5 where $E_T(CH_3)$ is given by Eq. (13.541) which is reiteratively matched to Eq. (13.542) within five-significant-figure-round-off-error.

VIBRATION OF CH_3

The vibrational energy levels of CH_3 may be solved as three equivalent coupled harmonic
10 oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [2] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

15 THE DOPPLER ENERGY TERMS OF $^{12}CH_3$

The reentrant oscillation of hybridized orbitals in the transition state is not coupled. Therefore, the equations of the radiation reaction force of methyl radical are the same as those of the corresponding hydrogen carbide radicals with the substitution of the methyl radical parameters. Using Eqs. (11.136) and (13.140-13.142), the angular frequency of the
20 reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} \frac{1}{m_e} = 2.55577 \times 10^{16} \text{ rad/s} \quad (13.561)$$

where b is given by Eq. (13.549). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.55577 \times 10^{16} \text{ rad/s} = 16.82249 \text{ eV} \quad (13.562)$$

25 In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for $E_{\mu\nu}$, the mass of the electron, m_e , for M , and the kinetic

energy given by Eq. (13.562) for \bar{E}_K gives the Doppler energy of the electrons of each of the three bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(16.82249 \text{ eV})}{m_e c^2}} = -0.25670 \text{ eV} \quad (13.563)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH_3 due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.563) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of each $C-H$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition state having three independent bonds, $\bar{E}'_{osc} (^{12}CH_3)$ per bond is

$$\bar{E}'_{osc} (^{12}CH_3) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.564)$$

$$\bar{E}'_{osc} (^{12}CH_3) = -0.25670 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) = -0.07904 \text{ eV} \quad (13.565)$$

Given that the vibration and reentrant oscillation is for three $C-H$ bonds, $\bar{E}_{osc} (^{12}CH_3)$, is:

$$\begin{aligned} \bar{E}_{osc} (^{12}CH_3) &= 3 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\ &= 3 \left(-0.25670 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -0.23711 \text{ eV} \end{aligned} \quad (13.566)$$

15

TOTAL AND BOND ENERGIES OF $^{12}CH_3$

$E_{T+osc} (^{12}CH_3)$, the total energy of the $^{12}CH_3$ radical including the Doppler term, is given by the sum of $E_T (CH_3)$ (Eq. (13.543)) and $\bar{E}_{osc} (^{12}CH_3)$ given by Eq. (13.566):

$$\begin{aligned} E_{T+osc} (CH_3) &= V_e + T + V_m + V_p + E(C, 2sp^3) + \bar{E}_{osc} (^{12}CH_3) \\ &= E_T (CH_3) + \bar{E}_{osc} (^{12}CH_3) \end{aligned} \quad (13.567)$$

$$E_{T+osc}({}^{12}\text{CH}_3) = \left\{ \begin{aligned} & \left(\frac{-3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} \right) \\ & - 3 \left((31.63536831 \text{ eV}) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}} \right) \end{aligned} \right\} \quad (13.568)$$

$$= -67.69450 \text{ eV} - 3 \left(0.25670 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)$$

From Eqs. (13.564-13.568), the total energy of ${}^{12}\text{CH}_3$ is

$$\begin{aligned} E_{T+osc}({}^{12}\text{CH}_3) &= -67.69450 \text{ eV} + \bar{E}_{osc}({}^{12}\text{CH}_3) \\ &= -67.69450 \text{ eV} - 3 \left(0.25670 \text{ eV} - \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -67.93160 \text{ eV} \end{aligned} \quad (13.569)$$

where ω_e given by Eq. (13.458) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

- 5 The CH_3 bond dissociation energy, $E_D({}^{12}\text{CH}_3)$, is given by the sum of the total energies of the CH_2 radical and the hydrogen atom minus $E_{T+osc}({}^{12}\text{CH}_3)$:

$$E_D({}^{12}\text{CH}_3) = E({}^{12}\text{CH}_2) + E(\text{H}) - E_{T+osc}({}^{12}\text{CH}_3) \quad (13.570)$$

- where $E_T({}^{12}\text{CH}_2)$ is given by the sum of the energies of the $\text{C}2sp^3$ HO, $E(\text{C}, 2sp^3)$ given by Eq. (13.428), $2E_D(\text{H})$ given by Eq. (13.154), and the negative of the bond energies of ${}^{12}\text{CH}$ given by Eq. (13.489) and ${}^{12}\text{CH}_2$ given by Eq. (13.528):

$$E({}^{12}\text{CH}_2) = 2(-13.59844 \text{ eV}) - 14.63489 \text{ eV} - 3.47 \text{ eV} - 4.33064 \text{ eV} = -49.63241 \text{ eV} \quad (13.571)$$

Thus, the ${}^{12}\text{CH}_3$ bond dissociation energy, $E_D({}^{12}\text{CH}_3)$, given by Eqs. (13.154), and (13.569-13.571) is

$$\begin{aligned} E_D({}^{12}\text{CH}_3) &= -(-49.63241 \text{ eV} - 13.59844 \text{ eV}) - E_{T+osc}({}^{12}\text{CH}_3) \\ &= -63.23085 \text{ eV} - (-67.93160 \text{ eV}) \\ &= 4.70075 \text{ eV} \end{aligned} \quad (13.572)$$

The experimental ${}^{12}\text{CH}_3$ bond dissociation energy is [40]

$$E_D({}^{12}\text{CH}_3) = 4.72444 \text{ eV} \quad (13.573)$$

BOND ANGLE OF $^{12}\text{CH}_3$

Using, $2c'_{H-H}$ (Eq. (13.534)), the distance between the two H atoms when the total energy of the corresponding MO is zero (Eq. (13.531)), and $2c'_{C-H}$, the internuclear distance of each $C-H$ bond (Eq. (13.547)), the corresponding bond angle can be determined from the law of cosines. Using Eq. (13.537), the bond angle θ between the $C-H$ bonds is

$$\begin{aligned}\theta &= \cos^{-1} \left(\frac{2(2.08418)^2 - (3.2094)^2}{2(2.08418)^2} \right) \\ &= \cos^{-1}(-0.18560) \\ &= 100.70^\circ\end{aligned}\tag{13.574}$$

The CH_3 radical has a pyramidal structure with the carbon atom along the z -axis at the apex and the hydrogen atoms at the base in the xy -plane. The distance $d_{\text{origin}-H}$ from the origin to the nucleus of a hydrogen atom given by Eqs. (13.534) and (13.412) is

$$d_{\text{origin}-H} = 1.85293a_0\tag{13.575}$$

The height along the z -axis of the pyramid from the origin to C nucleus d_{height} given by Eqs. (13.414), (13.547), and (13.575) is

$$d_{\text{height}} = 0.95418a_0\tag{13.576}$$

The angle θ_v of each $C-H$ bond from the z -axis given by Eqs. (13.416), (13.575), and (13.576) is

$$\theta_v = 62.75^\circ\tag{13.577}$$

The CH_3 MO shown in Figure 17 was rendered using these parameters.

The results of the determination of bond parameters of CH_3 are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

METHANE MOLECULE (CH_4)

The methane molecule CH_4 is formed by the reaction of a hydrogen atom with a methyl radical:



CH_4 can be solved using the same principles as those used to solve and CH_3 wherein the carbon $2s$ and $2p$ shells hybridize to form a single $2sp^3$ shell as an energy minimum. Four diatomic molecular orbitals (MOs) developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section serve as basis functions in a linear combination with four carbon $2sp^3$ hybridized orbitals (HOs) to form the MO of CH_4 . The solution is very similar to that of CH_3 except that there are four CH bonds in CH_4 . Methane is the simplest hydrocarbon that can be solved using the results for CH_3 . From the solution of CH_2 as well as CH_3 , more complex hydrocarbons can be solved using these radical as basis elements with bonding between the $C2sp^3$ hybridized carbons.

10

FORCE BALANCE OF CH_4

CH_4 comprises four chemical bonds between carbon and hydrogen atoms. Each $C-H$ bond comprises two spin-paired electrons with one from an initially unpaired electron of the carbon atom and the other from the hydrogen atom. Each H -atom electron forms a H_2 -type ellipsoidal MO with an unpaired C -atom electrons. However, such a bond is not possible with the outer two C electrons in their ground state since the resulting H_2 -type ellipsoidal MO would have a shorter internuclear distance than the radius of the carbon $2p$ shell which is not energetically stable, and only two electrons are unpaired. Thus, when bonding the carbon $2s$ and $2p$ shells hybridize to form a single $2sp^3$ shell as an energy minimum. The electron configuration and the energy, $E(C, 2sp^3)$, of the $C2sp^3$ shell is given by Eqs. (13.422), and (13.428), respectively.

For each $C-H$ bond, a $C2sp^3$ electron combines with the $H1s$ electron to form a molecular orbital. The proton of the H atom and the nucleus of the C atom are along each internuclear axis and serve as the foci. As in the case of H_2 , each of the four $C-H$ -bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into the $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell since it is energetically unfavorable. Thus, each MO surface comprises a prolate spheroid at the H proton that can be solve as being continuous with the $C2sp^3$ shell at the C atom whose nucleus serves as the other focus. The radius and the energy of the $C2sp^3$ shell are

unchanged with bond formation. The central paramagnetic force due to spin of each $C-H$ bond is provided by the spin-pairing force of the CH_4 MO that has the symmetry of an s orbital that superimposes with the $C2sp^3$ orbitals such that the corresponding angular momenta are unchanged.

5 The energies in the CH_4 MO involve only each $C2sp^3$ and each $H1s$ electron with the formation of each $C-H$ bond. The sum of the energies of the H_2 -type ellipsoidal MOs is matched to that of the $C2sp^3$ shell. As in the cases with of OH , H_2O , NH , NH_2 , NH_3 , CH , CH_2 , and CH_3 the CH_4 , the CH_4 MO must comprise four $C-H$ bonds with each having 75% of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO in a linear combination in order to
10 match potential, kinetic, and orbital energy relationships:



The force balance of the CH_4 MO is determined by the boundary conditions that arise from
15 the linear combination of orbitals according to Eq. (13.579) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-ellipsoidal-MO component of the CH_4 MO in terms of the central force of the foci is given by Eq. (13.59). The distance from the origin of each $C-H$ -bond MO to each focus c' is
20 given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of each $C-H$ -bond MO. Since each of the four prolate spheroidal $C-H$ -bond MOs comprises a H_2 -type-
25 ellipsoidal MO that transitions to the $C2sp^3$ HO, the energy $E(C, 2sp^3)$ in Eq. (13.428) adds to that of the four corresponding H_2 -type ellipsoidal MOs to give the total energy of the CH_4 MO. From the energy equation and the relationship between the axes, the dimensions of the CH_4 MO are solved.

The energy components of V_e , V_p , T , and V_m are four times those of CH corresponding to the four $C-H$ bonds. Since the each prolate spheroidal H_2 -type MO transitions to the $C2sp^3$ HO and the energy of the $C2sp^3$ shell must remain constant and equal to the $E(C,2sp^3)$ given by Eq. (13.428), the total energy $E_T(CH_4)$ of the CH_4 MO is
 5 given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the $C2sp^3$ HO and the four H_2 -type ellipsoidal MOs that forms the CH_4 MO as given by Eq. (13.579). Using Eq. (13.431), $E_T(CH_4)$ is given by

$$\begin{aligned} E_T(CH_4) &= E_T + E(C,2sp^3) \\ &= -\frac{4e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} \end{aligned} \quad (13.580)$$

The four hydrogen atoms and the hybridized carbon atom can achieve an energy minimum as
 10 a linear combination of four H_2 -type ellipsoidal MOs each having the proton and the carbon nucleus as the foci. Hybridization gives rise to the $C2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C,2sp^3)$ given by Eq. (13.435). To meet the equipotential condition of the union of the H_2 -type-ellipsoidal-MO and the $C2sp^3$ HO, the electron energies in Eqs. (13.431), (13.495), (13.541), and (13.580) were normalized by the ratio of 14.82575 eV, the
 15 magnitude of $E_{Coulomb}(C,2sp^3)$ given by Eq. (13.425), and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). The factor given by Eq. (13.430) normalized the energies to match that of the Coulombic energy alone to meet the energy matching condition of each $C-H$ -bond MO under the influence of the proton and the C nucleus. Each $C-H$ -bond MO comprises the same $C2sp^3$ shell
 20 having its energy normalized to that of the Coulombic energy between the electron and a charge of $+e$ at the carbon focus of the CH_4 MO. Thus, the energy of the CH_4 MO is also given by the sum of that of the four H_2 -type ellipsoidal MOs given by Eq. (11.212) minus three times the Coulombic energy, $E_{Coulomb}(H) = -13.605804 \text{ eV}$, of the three redundant $+e$'s of the linear combination:

$$\begin{aligned}
 E_T(4H_2-3H) &= -\frac{4e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] - 3E_{Coulomb}(H) \\
 &= 4(-31.63536831 \text{ eV}) - 3(-13.605804 \text{ eV}) \\
 &= -85.72406 \text{ eV}
 \end{aligned}
 \tag{13.581}$$

$E_T(CH_4)$ given by Eq. (13.580) is set equal to four times the energy of the H_2 -type ellipsoidal MO minus three times the Coulombic energy of H given by Eq. (13.581):

$$E_T(CH_4) = -\frac{4e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} = -85.72406 \text{ eV}
 \tag{13.582}$$

From the energy relationship given by Eq. (13.582) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH_4 MO can be solved.

Substitution of Eq. (13.60) into Eq. (13.543) gives

$$\frac{4e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e71.08917
 \tag{13.583}$$

10 The most convenient way to solve Eq. (13.583) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.62340a_0 = 8.59066 \times 10^{-11} \text{ m}
 \tag{13.584}$$

Substitution of Eq. (13.584) into Eq. (13.60) gives

$$c' = 1.04032a_0 = 5.50514 \times 10^{-11} \text{ m}
 \tag{13.585}$$

15 The internuclear distance given by multiplying Eq. (13.585) by two is

$$2c' = 2.08064a_0 = 1.10103 \times 10^{-10} \text{ m}
 \tag{13.586}$$

The experimental bond distance is [41]

$$2c' = 1.087 \times 10^{-10} \text{ m}
 \tag{13.587}$$

Substitution of Eqs. (13.584-13.585) into Eq. (13.62) gives

$$20 \quad b = c = 1.24626a_0 = 6.59492 \times 10^{-11} \text{ m}
 \tag{13.588}$$

Substitution of Eqs. (13.584-13.585) into Eq. (13.63) gives

$$e = 0.64083
 \tag{13.589}$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of each H_2 -type ellipsoidal

MO and the $C2sp^3$ HO in the absence of the other three are given by Eqs. (13.84-13.95), (13.261-13.270), (13.434-13.442), and (13.551-13.555). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{2sp^3} = 0.91771a_0$ is the radius of the $C2sp^3$ shell.

Substitution of Eqs. (13.584-13.585) into Eq. (13.261) gives

$$\theta' = 86.20^\circ \quad (13.590)$$

Then, the angle θ_{C2sp^3HO} the radial vector of the $C2sp^3$ HO makes with the internuclear axis is

$$\theta_{C2sp^3HO} = 180^\circ - 86.20^\circ = 93.80^\circ \quad (13.591)$$

as shown in Figure 14. The angle θ_{H_2MO} between the internuclear axis and the point of intersection of each H_2 -type ellipsoidal MO with the $C2sp^3$ radial vector given by Eqs. (13.442-13.443); (13.588), and (13.591) is

$$\theta_{H_2MO} = 47.29^\circ \quad (13.592)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals given by Eqs. (13.445), (13.584), and (13.592) is

$$d_{H_2MO} = 1.10121a_0 = 5.82734 \times 10^{-11} \text{ m} \quad (13.593)$$

The distance d_{C2sp^3HO} along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals given by Eqs. (13.447), (13.585), and (13.593) is

$$d_{C2sp^3HO} = 0.06089a_0 = 3.22208 \times 10^{-12} \text{ m} \quad (13.594)$$

The H_2 -type ellipsoidal MOs do not actually directly contact the $C2sp^3$ HO. As discussed in the Force Balance of H_2O section, with the addition of the fourth C-H bond, the H_2 -type ellipsoidal MOs may linearly combine to form a continuous two-dimensional surface of equipotential equivalent to that of the MOs if they did contact the $C2sp^3$ HO. However, Eqs. (13.579-13.580) must hold based on conservation of momentum and the potential, kinetic, and total energy relationships. In order that there is current continuity given the constraints of Eqs. (13.579-13.580), the existence of a self-contained, continuous-current, linear-combination of the H_2 -type ellipsoidal MOs requires that electrons are divisible between the combination H_2 -type MO and the $C2sp^3$ HO. This is not possible. Thus, at the points of intersection of the H_2 -type MOs of methane, symmetry, electron

indivisibility, current continuity, and conservation of energy and angular momentum require that the current between the $C2sp^3$ shell and points of mutual contact is projected onto and flows along the radial vector to the surface of $C2sp^3$ shell. This current designated the bisector current (BC) meets the $C2sp^3$ surface and does not travel to distances shorter than
 5 its radius. Moreover, an energy minimum is obtained when the H -atom charge-density of each $C-H$ -bond MO is decreased by a factor of 0.25 with a corresponding 0.25 increase in that of the three other $C-H$ -bond MOs. In this case, the angular momentum components of the transferred current mutually cancel. The geometry of the equivalent bonds is tetrahedral. The symmetry point group is T_d . This geometry is equivalent to the indistinguishable bonds
 10 positioned uniformly on a spherical surface or also at the apexes of a cube. The predicted angle θ between the $C-H$ bonds is

$$\theta = 109.5^\circ \quad (13.595)$$

The experimental bond angle is [41]

$$\theta = 109.5^\circ \quad (13.596)$$

15 The polar angle ϕ at which the H_2 -type ellipsoidal MOs intersect is given by the bisector of the angle θ between the $C-H$ bonds:

$$\phi = \frac{109.5}{2} = 54.75^\circ \quad (13.597)$$

With the carbon nucleus defined as the origin and one of the $C-H$ bonds defined as the positive x-axis, the polar-coordinate angle of the intersection occurs at

$$20 \quad \phi' = 54.75^\circ + 180^\circ = 234.75^\circ \quad (13.598)$$

The polar radius r_i at this angle is given by Eqs. (13.84-13.85):

$$r_i = (a - c') \frac{1 + \frac{c'}{a}}{1 + \frac{c'}{a} \cos \phi'} \quad (13.599)$$

Substitution of Eqs. (13.584-13.585) and (13.589) into Eq. (13.599) gives

$$r_i = 1.52223a_0 = 8.05530 \times 10^{-11} \text{ m} \quad (13.600)$$

25 Using the orbital composition of CH_4 (Eq. (13.579)), the radii of $Cl s = 0.17113a_0$ (Eq. (10.51)) and $C2sp^3 = 0.91771a_0$ (Eq. (10.424)) shells, and the parameters of the CH_4 MO given by Eqs. (13.3-13.4), (13.584-13.586), and (13.588-13.600), the charge-density of the CH_4 MO comprising the linear combination of four $C-H$ -bond MOs is shown in

Figure 18. Each $C-H$ -bond MO having the dimensional diagram shown in Figure 14 comprises a H_2 -type ellipsoidal MO and a $C2sp^3$ HO according to Eq. (13.579). But, based on the T_d symmetry of the H_2 -type MOs, the charge is distributed 1:1 between the H_2 -type MOs and the $C2sp^3$ shell.

5

ENERGIES OF CH_4

The energies of CH_4 are four times those of CH and are given by the substitution of the semiprincipal axes (Eqs. (13.584-13.585) and (13.588)) into the energy equations Eq. (13.580) and (Eqs. (13.449-13.452)) that are multiplied by four:

$$10 \quad V_s = 4(0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -145.86691 \text{ eV} \quad (13.601)$$

$$V_p = \frac{4e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 52.31390 \text{ eV} \quad (13.602)$$

$$T = 4(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 44.92637 \text{ eV} \quad (13.603)$$

$$V_m = 4(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -22.46318 \text{ eV} \quad (13.604)$$

$$E_T(CH_4) = -\frac{4e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} = -85.72472 \text{ eV} \quad (13.605)$$

15

where $E_T(CH_4)$ is given by Eq. (13.580) which is reiteratively matched to Eq. (13.581) within five-significant-figure round-off error.

VIBRATION OF CH_4

20 The vibrational energy levels of CH_4 may be solved as four equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [2] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

25

THE DOPPLER ENERGY TERMS OF $^{12}\text{CH}_4$

The reentrant oscillation of hybridized orbitals in the transition state is not coupled. Therefore, the equations of the radiation reaction force of methane are the same as those of OH , CH , CH_2 , and CH_3 with the substitution of the methane parameters. Using Eqs. 5 (11.136) and (13.140-13.142), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 2.57338 \times 10^{16} \text{ rad/s} \quad (13.606)$$

where b is given by Eq. (13.588). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$10 \quad \bar{E}_K = \hbar\omega = \hbar 2.57338 \times 10^{16} \text{ rad/s} = 16.93841 \text{ eV} \quad (13.607)$$

In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.607) for \bar{E}_K gives the Doppler energy of the electrons of each of the four bonds for the reentrant orbit:

$$15 \quad \bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(16.93841 \text{ eV})}{m_e c^2}} = -0.25758 \text{ eV} \quad (13.608)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH_4 due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding 20 energies, \bar{E}_D given by Eq. (13.608) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of each $\text{C}-\text{H}$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition state having four independent bonds, $\bar{E}'_{osc} (^{12}\text{CH}_4)$ per bond is

$$\bar{E}'_{osc} (^{12}\text{CH}_4) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.609)$$

$$\bar{E}'_{osc} (^{12}\text{CH}_4) = -0.25758 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) = -0.07992 \text{ eV} \quad (13.610)$$

The reentrant orbit for the binding of a hydrogen atom to a CH_3 radical involves four $C-H$ bonds. Since the vibration and reentrant oscillation is along four bonds, \bar{E}_{osc} for $^{12}CH_4$, $\bar{E}_{osc}(^{12}CH_4)$, is:

$$\begin{aligned}\bar{E}_{osc}(^{12}CH_4) &= 4 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\ &= 4 \left(-0.25758 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -0.31967 \text{ eV}\end{aligned}\quad (13.611)$$

5

TOTAL AND BOND ENERGIES OF $^{12}CH_4$

$E_{T+osc}(^{12}CH_4)$, the total energy of the $^{12}CH_4$ radical including the Doppler term, is given by the sum of $E_T(CH_4)$ (Eq. (13.582)) and $\bar{E}_{osc}(^{12}CH_4)$ given by Eq. (13.611):

$$\begin{aligned}E_{T+osc}(CH_4) &= V_e + T + V_m + V_p + E(C, 2sp^3) + \bar{E}_{osc}(^{12}CH_4) \\ &= E_T(CH_4) + \bar{E}_{osc}(^{12}CH_4)\end{aligned}\quad (13.612)$$

10

$$\begin{aligned}E_{T+osc}(^{12}CH_4) &= \left\{ \left(\frac{-4e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.63489 \text{ eV} \right) \right. \\ &\quad \left. - 4 \left((31.63536831 \text{ eV}) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}} \right) \right\} \\ &= -85.72406 \text{ eV} - 4 \left(0.25758 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)\end{aligned}\quad (13.613)$$

From Eqs. (13.609-13.613), the total energy of $^{12}CH_4$ is

$$\begin{aligned}E_{T+osc}(^{12}CH_4) &= -85.72406 \text{ eV} + \bar{E}_{osc}(^{12}CH_4) \\ &= -85.72406 \text{ eV} - 4 \left(0.25758 \text{ eV} - \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -86.04373 \text{ eV}\end{aligned}\quad (13.614)$$

where ω_e given by Eq. (13.458) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

The CH_4 bond dissociation energy, $E_D(^{12}CH_4)$, is given by the sum of the total energies of the CH_3 radical and the hydrogen atom minus $E_{T+osc}(^{12}CH_4)$:

$$E_D(^{12}CH_4) = E(^{12}CH_3) + E(H) - E_{T+osc}(^{12}CH_4) \quad (13.615)$$

where $E_T(^{12}CH_3)$ is given by the sum of the energies of the $C2sp^3$ HO, $E(C, 2sp^3)$ given by Eq. (13.428), $3E_D(H)$ given by Eq. (13.154), and the negative of the bond energies of ^{12}CH given by Eq. (13.489), $^{12}CH_2$ given by Eq. (13.528), and $^{12}CH_3$ given by Eq. (13.573):

$$E(^{12}CH_3) = \left(\begin{array}{l} 3(-13.59844 \text{ eV}) - 14.63489 \text{ eV} \\ -3.47 \text{ eV} - 4.33064 \text{ eV} - 4.72444 \text{ eV} \end{array} \right) = -67.95529 \text{ eV} \quad (13.616)$$

Thus, the $^{12}CH_4$ bond dissociation energy, $E_D(^{12}CH_4)$, given by Eqs. (13.154), and (13.614-13.616) is

$$\begin{aligned} E_D(^{12}CH_4) &= -(67.95529 \text{ eV} + 13.59844 \text{ eV}) - E_{T+osc}(^{12}CH_4) \\ &= -81.55373 \text{ eV} - (-86.04373 \text{ eV}) \\ &= 4.4900 \text{ eV} \end{aligned} \quad (13.617)$$

The experimental $^{12}CH_4$ bond dissociation energy is [40]

$$E_D(^{12}CH_4) = 4.48464 \text{ eV} \quad (13.618)$$

The results of the determination of bond parameters of CH_4 are given in Table 13.1.

The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

NITROGEN MOLECULE

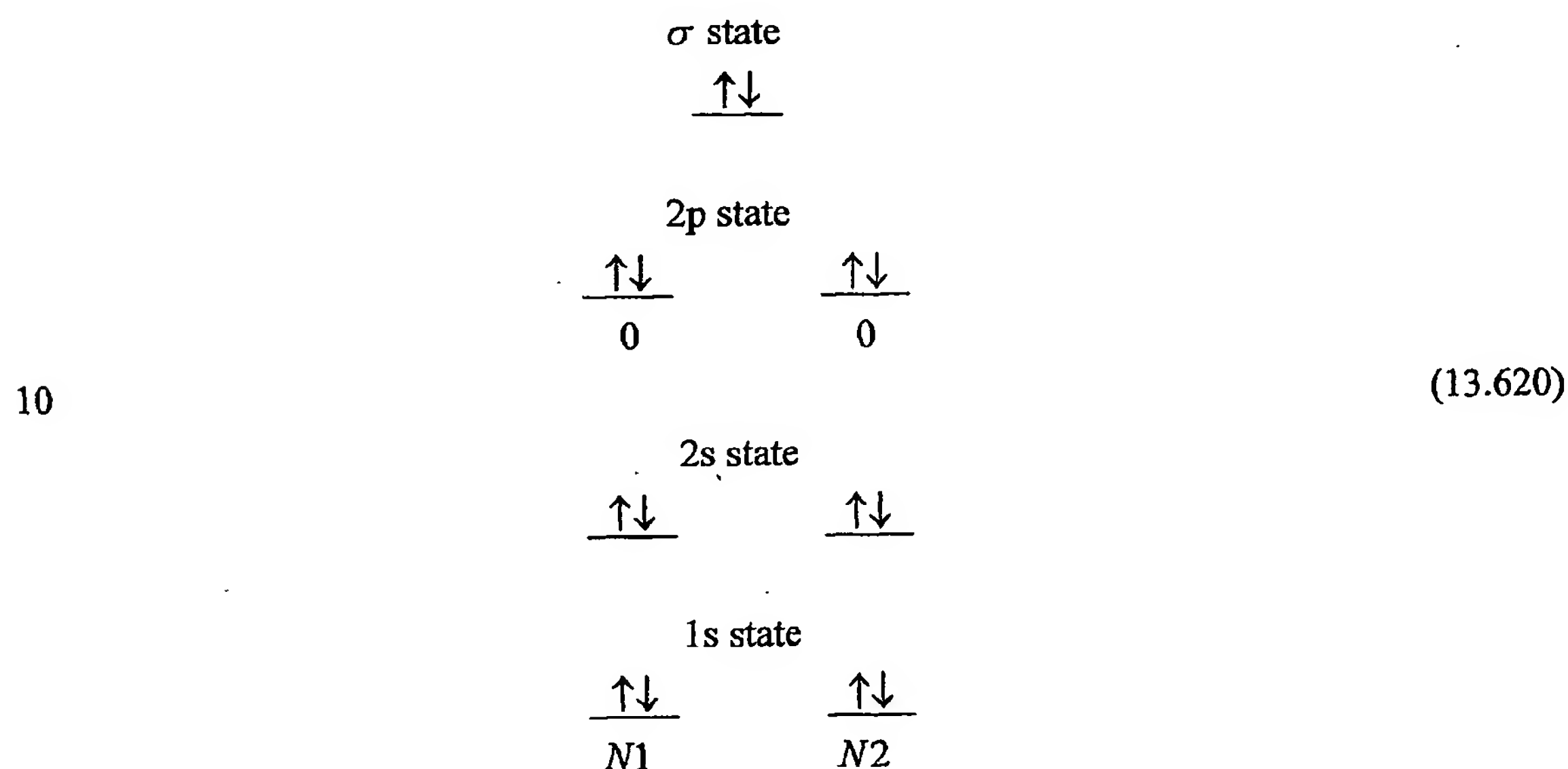
The nitrogen molecule can be formed by the reaction of two nitrogen atoms:



The bond in the nitrogen molecule comprises a H_2 -type molecular orbital (MO) with two paired electrons. The force balance equation and radius r_f of the $2p$ shell of N is derived in the Seven-Electron Atoms section. With the formation of the H_2 -type MO by the contribution of a $2p$ electron from each N atom, a diamagnetic force arises between the

remaining $2p$ electrons and the H_2 -type MO. This force from each N causes the H_2 -type MO to move to greater principal axes than would result with the Coulombic force alone. But, the integer increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining $2p$ electrons of each N decrease the radius of the

5 corresponding shell such that the energy minimum is achieved that is lower than that of the reactant atoms. The resulting electron configuration of N_2 is $1s_1^2 1s_2^2 2s_1^2 2s_2^2 2p_1^2 2p_2^2 \sigma_{1,2}^2$ where the subscript designates the N atom, 1 or 2, σ designates the H_2 -type MO, and the orbital arrangement is



Nitrogen is predicted to be diamagnetic in agreement with observations [42].

15 FORCE BALANCE OF THE $2p$ SHELL OF THE NITROGEN ATOMS OF THE NITROGEN MOLECULE

For each N atom, force balance for the outermost $2p$ electron of N_2 (electron 6) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to interactions between electron 6 and the other $2p$ -shell as well as the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Seven-

20 Electron Atoms section. The central Coulomb force on the outer-most $2p$ shell electron of N_2 (electron 6) due to the nucleus and the inner five electrons is given by Eq. (10.70) with

the appropriate charge and radius:

$$\mathbf{F}_{els} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} \mathbf{i}_r \quad (13.621)$$

for $r > r_5$. The $2p$ shell possess an external electric field given by Eq. (10.92) for $r > r_6$.

The energy is minimized with conservation of angular momentum. This condition is met
5 when the diamagnetic force, $\mathbf{F}_{diamagnetic}$, of Eq. (10.82) due to the p -orbital contribution is the same as that of the reactant nitrogen atoms given by Eq. (10.136) with r_6 replacing r_7 :

$$\mathbf{F}_{diamagnetic} = -\left(\frac{1}{3}\right) \frac{\hbar^2}{4m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (13.622)$$

And, $\mathbf{F}_{mag\ 2}$ corresponding to the conserved orbital angular momentum of the three orbitals is given by Eq. (10.89):

$$10 \quad \mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{3\hbar^2}{m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.623)$$

The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_6$ gives rise to a second diamagnetic force, $\mathbf{F}_{diamagnetic\ 2}$, given by Eq. (10.93). $\mathbf{F}_{diamagnetic\ 2}$ due to the binding of the p -orbital electron having an electric field of +1 outside of its radius is:

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_6^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (13.624)$$

15 In addition, the contribution of a $2p$ electron from each N atom in the formation of the σ MO gives rise to a paramagnetic force on the remaining two $2p$ electrons that pair. The force, $\mathbf{F}_{mag\ 3}$, follows from Eq. (10.11) wherein the two radii are equal to r_6 and the direction is positive, central:

$$\mathbf{F}_{mag\ 3} = \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.625)$$

20 $\mathbf{F}_{mag\ 3}$ is present in additional diatomic molecules where its contribution minimizes the energy. This AO spin-pairing force reduces the radius directly to reduce the energy, and it can also cancel the contribution of the corresponding electron to $\mathbf{F}_{diamagnetic}$ to further reduce the energy.

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to
25 the sum of the electric (Eq. (13.621)) and diamagnetic (Eqs. (13.622) and (13.624)), and paramagnetic (Eqs. (13.623) and (13.625)) forces as follows:

$$\frac{m_e v_6^2}{r_6} = \left(\begin{aligned} & \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{\hbar^2}{12m_e r_6^2 r_3} \sqrt{s(s+1)} + \frac{3\hbar^2}{Zm_e r_6^2 r_3} \sqrt{s(s+1)} \\ & - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{s(s+1)} + \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \end{aligned} \right) \quad (13.626)$$

Substitution of $v_6 = \frac{\hbar}{m_e r_6}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (13.626) gives:

$$\frac{\hbar^2}{m_e r_6^3} - \frac{\hbar^2}{4m_e r_6^3} \sqrt{\frac{3}{4}} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} + \frac{3\hbar^2}{Zm_e r_6^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{\frac{3}{4}} \quad (13.627)$$

5 The quadratic equation corresponding to Eq. (13.627) is

$$r_6^2 - \frac{\frac{\hbar^2}{m_e} \left(1 - \frac{\sqrt{3}}{8} \right)}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} r_6 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} = 0 \quad (13.628)$$

The solution of Eq. (13.628) using the quadratic formula is:

$$r_6 = \frac{a_0 \left(1 - \frac{\sqrt{3}}{8} \right)}{\left((Z-5) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \pm a_0 \sqrt{\frac{\left(\frac{1 - \frac{\sqrt{3}}{8}}{\left((Z-5) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \right)^2}{\left((Z-5) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} + \frac{20\sqrt{3} \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{\left((Z-5) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)}} \quad (13.629)$$

r_3 in units of a_0

10 The positive root of Eq. (13.629) must be taken in order that $r_6 > 0$. Substitution of

$\frac{r_3}{a_0} = 0.69385$ (Eq. (10.62) with $Z = 7$) into Eq. (13.629) gives

$$r_6 = 0.78402a_0 \quad (13.630)$$

ENERGIES OF THE $2p$ SHELL OF THE NITROGEN ATOMS OF THE NITROGEN MOLECULE

The central forces on the $2p$ shell of each N are increased with the formation of the σ MO, which reduces the shell's radius and increases its total energy. The Coulombic energy terms
5 of the total energy of the two N atoms at the new radius are calculated and added to the energy of the σ MO to give the total energy of N_2 . Then, the bond energy is determined from the total N_2 energy.

The radius r_7 of each nitrogen atom before bonding is given by Eq. (10.142):

$$r_7 = 0.93084a_0 \quad (13.631)$$

10 Using the initial radius r_7 of each N atom and the final radius r_6 of the $N2p$ shell of N_2 (Eq. (13.630)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(N_2, 2p)$ of the Coulombic energy change of the $N2p$ electrons of both atoms is determined using Eq. (10.102):

$$\begin{aligned} E_T(N_2, 2p) &= -2 \sum_{n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_7} \right) \\ &= -2(13.60580 \text{ eV})(0.20118)(2+3) \\ &= -27.37174 \text{ eV} \end{aligned} \quad (13.632)$$

15

FORCE BALANCE OF THE σ MO OF THE NITROGEN MOLECULE

The $2p$ shell gives rise to two diamagnetic forces on the σ MO. As given for the hydrogen molecule in the Hydrogen-Type Molecules section, the σ MO comprises two electrons, σ electron 1 and σ electron 2, that are bound at $\xi = 0$ as a equipotential prolate spheroidal MO
20 by the central Coulombic field due to the nitrogen atoms at the foci and the spin pairing force on σ electron 2 due to σ electron 1 that initially has smaller semiprincipal axes. The spin-pairing force given in Eq. (11.200) is equal to one half the centrifugal force of the two electrons. The spin-pairing electron of the σ MO is also repelled by the remaining $2p$ electrons of each N according to Lenz law, and the force is based on the total number of
25 these electrons n_e that interact with the binding σ -MO electron. This diamagnetic force

$F_{\text{diamagneticMO1}}$ is of the same form as the molecular spin-pairing force but in the opposite direction. The force follows from the derivations of Eqs. (10.219) and (11.200) which gives:

$$\mathbf{F}_{\text{diamagneticMO1}} = \frac{n_e \hbar^2}{4m_e a^2 b^2} D \mathbf{i}_z \quad (13.633)$$

In addition, there is a relativistically corrected Lorentzian force $\mathbf{F}_{\text{diamagneticMO2}}$ on the pairing electron of the σ MO that follows from Eqs. (7.15) and (11.200):

$$\mathbf{F}_{\text{diamagneticMO2}} = \frac{1}{Z} \frac{|L| \hbar}{2m_e a^2 b^2} D \mathbf{i}_z \quad (13.634)$$

5 where $|L|$ is the magnitude of the angular momentum of each N atom at a focus that is the source of the diamagnetism at the σ -MO.

The force balance equation for the σ -MO of the nitrogen molecule given by Eq. (11.200) and Eqs. (13.633-13.634) with $n_e = 2$ and $|L| = \hbar$ is

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z}\right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.635)$$

$$10 \quad \frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D - \frac{1}{Z} \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.636)$$

$$\left(2 + \frac{1}{Z}\right) \frac{\hbar^2}{2m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D \quad (13.637)$$

$$a = \left(2 + \frac{1}{Z}\right) a_0 \quad (13.638)$$

Substitution of $Z = 7$ into Eq. (13.638) gives

$$a = 2.14286 a_0 = 1.13395 \times 10^{-10} \text{ m} \quad (13.639)$$

15 Substitution of Eq. (13.639) into Eq. (11.79) is

$$c' = 1.03510 a_0 = 5.47750 \times 10^{-11} \text{ m} \quad (13.640)$$

The internuclear distance given by multiplying Eq. (13.640) by two is

$$2c' = 2.07020 a_0 = 1.09550 \times 10^{-10} \text{ m} \quad (13.641)$$

The experimental bond distance from Ref. [28] and Ref. [43] is

$$20 \quad 2c' = 1.09769 \times 10^{-10} \text{ m} \quad (13.642)$$

$$2c' = 1.094 \times 10^{-10} \text{ m} \quad (13.643)$$

Substitution of Eqs. (13.639-13.640) into Eq. (11.80) is

$$b = c = 1.87628 a_0 = 9.92882 \times 10^{-11} \text{ m} \quad (13.644)$$

Substitution of Eqs. (13.639-13.640) into Eq. (11.67) is

$$25 \quad e = 0.48305 \quad (13.645)$$

Using the electron configuration of N_2 (Eq. (13.620)), the radii of the $N1s = 0.14605a_0$ (Eq. (10.51)), $N2s = 0.69385a_0$ (Eq. (10.62)), and $N2p = 0.78402a_0$ (Eq. (13.630)) shells and the parameters of the σ MO of N_2 given by Eqs. (13.3-13.4), (13.639-13.641), and (13.644-13.645), the dimensional diagram and charge-density of the N_2 MO are shown in Figures 19 and 20, respectively.

Despite the predictions of standard quantum mechanics that preclude the imaging of a molecular orbital, the full three-dimensional structure of the outer molecular orbital of N_2 has been recently tomographically reconstructed [44]. The charge-density surface observed is consistent with that shown in Figure 20. This result constitutes direct evidence that 10 electrons are not point-particle probability waves that have no form until they are "collapsed to a point" by measurement. Rather they are physical, two-dimensional equipotential charge density surfaces.

SUM OF THE ENERGIES OF THE σ MO AND THE AOs OF THE 15 NITROGEN MOLECULE

The energies of the N_2 σ MO are given by the substitution of the semiprincipal axes (Eqs. (13.639-13.640) and (13.644)) into the energy equations (Eqs. (11.207-11.212)) of H_2 :

$$V_e = \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -27.70586 \text{ eV} \quad (13.646)$$

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 13.14446 \text{ eV} \quad (13.647)$$

$$20 \quad T = \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 6.46470 \text{ eV} \quad (13.648)$$

$$V_m = \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -3.23235 \text{ eV} \quad (13.649)$$

$$E_T = V_e + T + V_m + V_p \quad (13.650)$$

Substitution of Eqs. (11.79) and (13.646-13.649) into Eq. (13.650) gives

$$E_T(N_2, \sigma) = \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(\left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right) = -11.32906 \text{ eV} \quad (13.651)$$

where $E_T(N_2, \sigma)$ is the total energy of the σ MO of N_2 . The sum, $E_T(N_2)$, of $E_T(N_2, 2p)$, the $2p$ (AO) contribution given by Eq. (13.632), and $E_T(N_2, \sigma)$, the σ MO contribution given by Eq. (13.651) is:

$$\begin{aligned} E_T(N_2) &= E_T(N_2, 2p) + E_T(N_2, \sigma) \\ &= -27.37174 \text{ eV} - 11.32906 \text{ eV} \\ &= -38.70080 \text{ eV} \end{aligned} \quad (13.652)$$

5

VIBRATION OF N_2

The vibrational energy levels of N_2 may be solved by determining the Morse potential curve from the energy relationships for the transition from two N atoms whose parameters are given by Eqs. (10.134-10.143) to the two N atoms whose parameter r_e is given by Eq. (13.630) and the σ MO whose parameters are given by Eqs. (13.639-13.641) and (13.644-13.645). As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter (K&P) [15] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods.

THE DOPPLER ENERGY TERMS OF THE NITROGEN MOLECULE

The equations of the radiation reaction force of nitrogen are the same as those of H_2 with the substitution of the nitrogen parameters. Using Eqs. (11.231-11.233), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{e^2}{4\pi\epsilon_0 a^3 m_e}} = 1.31794 \times 10^{16} \text{ rad/s} \quad (13.653)$$

where a is given by Eq. (13.639). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 1.31794 \times 10^{16} \text{ rad/s} = 8.67490 \text{ eV} \quad (13.654)$$

25

In Eq. (11.181), substitution of $E_T(N_2)$ for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.654) for \bar{E}_K gives the Doppler energy of the electrons of the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -38.70080 \text{ eV} \sqrt{\frac{2e(8.67490 \text{ eV})}{m_e c^2}} = -0.22550 \text{ eV} \quad (13.655)$$

5 In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of the N_2 MO due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.655) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the
10 vibrational energy. Using the experimental N_2 ω_e of 2358.57 cm^{-1} (0.29243 eV) [28] for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(N_2)$ is

$$\bar{E}_{osc}(N_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.656)$$

$$\bar{E}_{osc}(N_2) = -0.22550 \text{ eV} + \frac{1}{2} (0.29243 \text{ eV}) = -0.07929 \text{ eV} \quad (13.657)$$

15 TOTAL AND BOND ENERGIES OF THE NITROGEN MOLECULE

$E_{T+osc}(N_2)$, the total energy of N_2 including the Doppler term, is given by the sum of $E_T(N_2)$ (Eq. (13.652)) and $\bar{E}_{osc}(N_2)$ given by Eq. (13.657):

$$\begin{aligned} E_{T+osc}(N_2) &= V_e + T + V_m + V_p + E_T(N_2, 2p) + \bar{E}_{osc}(N_2) \\ &= E_T(N_2, \sigma) + E_T(N_2, 2p) + \bar{E}_{osc}(N_2) \\ &= E_T(N_2) + \bar{E}_{osc}(N_2) \end{aligned} \quad (13.658)$$

$$\begin{aligned}
E_{T+osc}(N_2) &= \left[\left(\frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(\left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right) - 2 \sum_{n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_7} \right) \right) \right. \\
&\quad \left. \left(1 + \sqrt{\frac{\frac{e^2}{4\pi\epsilon_0 a^3}}{m_e c^2}} \right) + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right] \quad (13.659) \\
&= -38.70080 \text{ eV} - 0.22550 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}
\end{aligned}$$

From Eqs. (13.656-13.659), the total energy of the N_2 MO is

$$\begin{aligned}
E_{T+osc}(N_2) &= -38.70080 \text{ eV} + \bar{E}_{osc}(N_2) \\
&= -38.70080 \text{ eV} - 0.22550 \text{ eV} + \frac{1}{2} (0.29243 \text{ eV}) \\
&= -38.78009 \text{ eV}
\end{aligned} \quad (13.660)$$

where the experimental ω_e was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

- 5 The N_2 bond dissociation energy, $E_D(N_2)$, is given by the difference in the total energies of the two N atoms and $E_{T+osc}(N_2)$:

$$E_D(N_2) = 2E(N) - E_{T+osc}(N_2) \quad (13.661)$$

where the energy of a nitrogen atom is [6]

$$E(N) = -14.53414 \text{ eV} \quad (13.662)$$

- 10 Thus, the N_2 bond dissociation energy, $E_D(N_2)$, given by Eqs. (13.660-13.662) is

$$\begin{aligned}
E_D(N_2) &= -2(14.53414 \text{ eV}) - E_{T+osc}(N_2) \\
&= -29.06828 \text{ eV} - (-38.78009 \text{ eV}) \\
&= 9.71181 \text{ eV}
\end{aligned} \quad (13.663)$$

The experimental N_2 bond dissociation energy from Ref. [43] and Ref. [45] is

$$E_D(N_2) = 9.756 \text{ eV} \quad (13.664)$$

$$E_D(N_2) = 9.764 \text{ eV} \quad (13.665)$$

- 15 The results of the determination of bond parameters of N_2 are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations

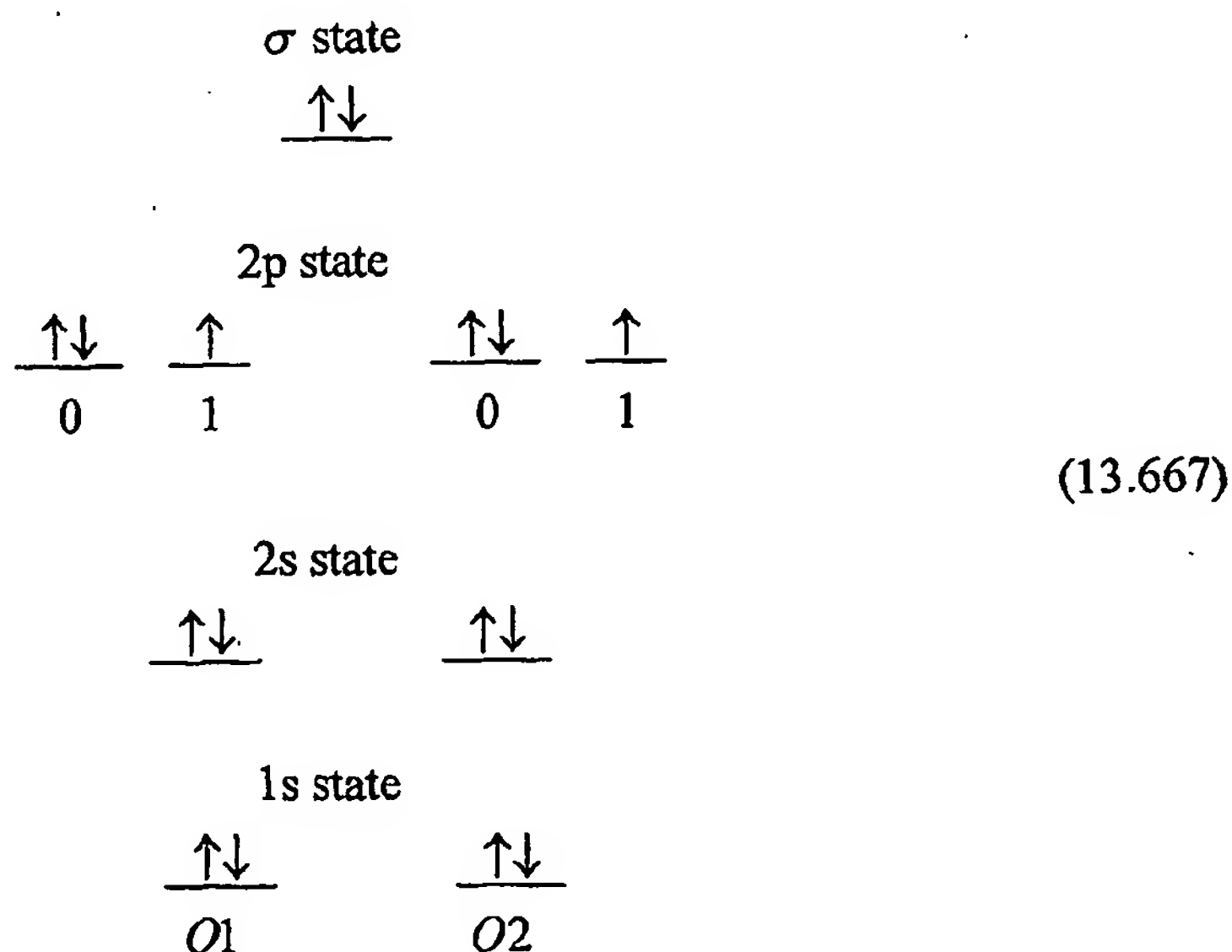
containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

OXYGEN MOLECULE

5 The oxygen molecule can be formed by the reaction of two oxygen atoms:



The bond in the oxygen molecule comprises a H_2 -type molecular orbital (MO) with two paired electrons. The force balance equation and radius r_b of the $2p$ shell of O is derived in the Eight-Electron Atoms section. With the formation of the H_2 -type MO by the contribution of a $2p$ electron from each O atom, a diamagnetic force arises between the remaining $2p$ electrons and the H_2 -type MO. This force from each O causes the H_2 -type MO to move to greater principal axes than would result with the Coulombic force alone. But, the integer increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining $2p$ electrons of each O decrease the radius of the corresponding shell such that the energy minimum is achieved that is lower than that of the reactant atoms. The resulting electron configuration of O_2 is $1s_1^2 1s_2^2 2s_1^2 2s_2^2 2p_1^3 2p_2^3 \sigma_{1,2}^2$ where the subscript designates the O atom, 1 or 2, σ designates the H_2 -type MO, and the orbital arrangement is



Oxygen is predicted to be paramagnetic in agreement with observations [42].

FORCE BALANCE OF THE $2p$ SHELL OF THE OXYGEN ATOMS OF THE OXYGEN MOLECULE

5 For each O atom, force balance for the outermost $2p$ electron of O_2 (electron 7) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to interactions between electron 7 and the other $2p$ -shell as well as the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Eight-Electron Atoms section. The central Coulomb force on the outer-most $2p$ shell electron of O_2
10 (electron 7) due to the nucleus and the inner six electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$\mathbf{F}_{ele} = \frac{(Z-6)e^2}{4\pi\epsilon_0 r_7^2} \mathbf{i}_r \quad (13.668)$$

for $r > r_6$. The $2p$ shell possess an external electric field given by Eq. (10.92) for $r > r_7$. The energy is minimized with conservation of angular momentum. This condition is met
15 when the magnetic forces are the same as those of the reactant oxygen atoms with r_7 replacing r_8 . The diamagnetic force, $\mathbf{F}_{diamagnetic}$, of Eq. (10.82) due to the p -orbital contributions is given by Eq. (10.156):

$$\mathbf{F}_{diamagnetic} = -\left(\frac{1}{3} + \frac{2}{3}\right) \frac{\hbar^2}{4m_e r_7^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{3\hbar^2}{12m_e r_7^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (13.669)$$

And, $\mathbf{F}_{mag\ 2}$ corresponding to the conserved spin and orbital angular momentum given by Eq.
20 (10.157) is

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{2\hbar^2}{m_e r_7^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.670)$$

The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_7$ gives rise to a second diamagnetic force, $\mathbf{F}_{diamagnetic\ 2}$, given by Eq. (10.93). $\mathbf{F}_{diamagnetic\ 2}$ due to the binding of the p -orbital electron having an electric field of +1 outside of its radius is :

$$25 \quad \mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-7}{Z-6}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_7^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (13.671)$$

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (13.688)) and diamagnetic (Eqs. (13.669) and (13.671)), and

paramagnetic (Eq. (13.670)) forces as follows:

$$\frac{m_e v_7^2}{r_7} = \left(\frac{(Z-6)e^2}{4\pi\epsilon_0 r_7^2} - \frac{3\hbar^2}{12m_e r_7^2 r_3} \sqrt{s(s+1)} + \frac{2\hbar^2}{Zm_e r_7^2 r_3} \sqrt{s(s+1)} - \left[\frac{Z-7}{Z-6} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_7^4 m_e} 10 \sqrt{s(s+1)} \right) \quad (13.672)$$

Substitution of $v_7 = \frac{\hbar}{m_e r_7}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (13.672) gives:

$$\frac{\hbar^2}{m_e r_7^3} = \frac{(Z-6)e^2}{4\pi\epsilon_0 r_7^2} - \frac{3\hbar^2}{12m_e r_7^2 r_3} \sqrt{\frac{3}{4}} + \frac{2\hbar^2}{Zm_e r_7^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-7}{Z-6} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_7^4 m_e} 10 \sqrt{\frac{3}{4}} \quad (13.673)$$

5

The quadratic equation corresponding to Eq. (13.673) is

$$r_7^2 - \frac{\frac{\hbar^2}{m_e}}{\left(\frac{(Z-6)e^2}{4\pi\epsilon_0} - \left(\frac{3}{12} - \frac{2}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} r_7 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-7}{Z-6} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-6)e^2}{4\pi\epsilon_0} - \left(\frac{3}{12} - \frac{2}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} = 0 \quad (13.674)$$

The solution of Eq. (13.674) using the quadratic formula is:

$$r_7 = \frac{\frac{a_0}{\left((Z-6) - \left(\frac{3}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \pm a_0 \sqrt{\frac{1}{\left((Z-6) - \left(\frac{3}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)^2} + \frac{20\sqrt{3} \left[\frac{Z-7}{Z-6} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{\left((Z-6) - \left(\frac{3}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)}}}{2} \quad (13.675)$$

r_3 in units of a_0

The positive root of Eq. (13.675) must be taken in order that $r_7 > 0$. Substitution of

$\frac{r_3}{a_0} = 0.59020$ (Eq. (10.62) with $Z = 8$) into Eq. (13.675) gives

$$r_7 = 0.91088a_0 \quad (13.676)$$

15 ENERGIES OF THE $2p$ SHELL OF THE OXYGEN ATOMS OF THE

OXYGEN MOLECULE

The central forces on the $2p$ shell of each O are increased with the formation of the σ MO, which reduces the shell's radius and increases its total energy. The Coulombic energy terms of the total energy of the two O atoms at the new radius are calculated and added to the energy of the σ MO to give the total energy of O_2 . Then, the bond energy is determined from the total O_2 energy.

The radius r_8 of each oxygen atom before bonding is given by Eq. (10.162):

$$r_8 = a_0 \quad (13.677)$$

Using the initial radius r_8 of each O atom and the final radius r_7 of the $O2p$ shell of O_2 (Eq. (13.676)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(O_2, 2p)$ of the Coulombic energy change of the $O2p$ electrons of both atoms is determined using Eq. (10.102):

$$\begin{aligned} E_T(O_2, 2p) &= -2 \sum_{n=4}^6 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_7} - \frac{1}{r_8} \right) \\ &= -2(13.60580 \text{ eV})(0.09784)(2+3+4) \\ &= -23.96074 \text{ eV} \end{aligned} \quad (13.678)$$

15 FORCE BALANCE OF THE σ MO OF THE OXYGEN MOLECULE

The force balance equation for the σ -MO of the oxygen molecule given by Eq. (11.200) and

Eqs. (13.633-13.634) with $n_e = 2$ and $|L| = \sqrt{\frac{3}{4}}\hbar$ is

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(\frac{3}{2} + \frac{\sqrt{\frac{3}{4}}}{Z} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.679)$$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D - \left(\frac{1}{2} + \frac{\sqrt{\frac{3}{4}}}{Z} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.680)$$

$$\left(\frac{5}{2} + \frac{\sqrt{\frac{3}{4}}}{Z} \right) \frac{\hbar^2}{2m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D \quad (13.681)$$

$$a = \left(\frac{5}{2} + \frac{\sqrt{3}}{Z} \right) a_0 \quad (13.682)$$

Substitution of $Z = 8$ into Eq. (13.682) gives

$$a = 2.60825a_0 = 1.38023 \times 10^{-10} \text{ m} \quad (13.683)$$

Substitution of Eq. (13.683) into Eq. (11.79) is

$$5 \quad c' = 1.14198a_0 = 6.04312 \times 10^{-11} \text{ m} \quad (13.684)$$

The internuclear distance given by multiplying Eq. (13.684) by two is

$$2c' = 2.28397a_0 = 1.20862 \times 10^{-10} \text{ m} \quad (13.685)$$

The experimental bond distance is [28]

$$2c' = 1.20752 \times 10^{-10} \text{ m} \quad (13.686)$$

10 Substitution of Eqs. (13.683-13.684) into Eq. (11.80) is

$$b = c = 2.34496a_0 = 1.24090 \times 10^{-10} \text{ m} \quad (13.687)$$

Substitution of Eqs. (13.683-13.684) into Eq. (11.67) is

$$e = 0.43783 \quad (13.688)$$

Using the electron configuration of O_2 (Eq. (13.667)), the radii of the $O1s = 0.12739a_0$ (Eq. (10.51)), $O2s = 0.59020a_0$ (Eq. (10.62)), and $O2p = 0.91088a_0$ (Eq. (13.676)) shells and the parameters of the σ MO of O_2 given by Eqs. (13.3-13.4), (13.683-13.685), and (13.687-13.688), the dimensional diagram and charge-density of the O_2 MO are shown in Figures 21 and 22, respectively.

20 SUM OF THE ENERGIES OF THE σ MO AND THE AOs OF THE OXYGEN MOLECULE

The energies of the O_2 σ MO are given by the substitution of the semiprincipal axes (Eqs. (13.683-13.684) and (13.687)) into the energy equations (Eqs. (11.207-11.212)) of H_2 :

$$V_e = \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -22.37716 \text{ eV} \quad (13.689)$$

$$25 \quad V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 11.91418 \text{ eV} \quad (13.690)$$

$$T = \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 4.28968 \text{ eV} \quad (13.691)$$

$$V_m = \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -2.14484 \text{ eV} \quad (13.692)$$

$$E_T = V_e + T + V_m + V_p \quad (13.693)$$

Substitution of Eqs. (11.79) and (13.689-13.692) into Eq. (13.693) gives

$$5 \quad E_T(O_2, \sigma) = \frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left(\left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right) = -8.31814 \text{ eV} \quad (13.694)$$

where $E_T(O_2, \sigma)$ is the total energy of the σ MO of O_2 . The sum, $E_T(O_2)$, of $E_T(O_2, 2p)$, the $2p$ AO contribution given by Eq. (13.678), and $E_T(O_2, \sigma)$, the σ MO contribution given by Eq. (13.694) is:

$$\begin{aligned} E_T(O_2) &= E_T(O_2, 2p) + E_T(O_2, \sigma) \\ &= -23.96074 \text{ eV} - 8.31814 \text{ eV} \\ &= -32.27888 \text{ eV} \end{aligned} \quad (13.695)$$

10

VIBRATION OF O_2

The vibrational energy levels of O_2 may be solved by determining the Morse potential curve from the energy relationships for the transition from two O atoms whose parameters are given by Eqs. (10.154-10.163) to the two O atoms whose parameter r_e is given by Eq. (13.676) and the σ MO whose parameters are given by Eqs. (13.683-13.685) and (13.687-13.688). As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter (K&P) [15] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic

20 perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods.

THE DOPPLER ENERGY TERMS OF THE OXYGEN MOLECULE

The equations of the radiation reaction force of oxygen are the same as those of H_2 with the substitution of the oxygen parameters. Using Eqs. (11.231-11.233), the angular frequency of the reentrant oscillation in the transition state is

$$5 \quad \omega = \sqrt{\frac{e^2}{4\pi\epsilon_0 a^3}} = 9.81432 \times 10^{16} \text{ rad/s} \quad (13.696)$$

where a is given by Eq. (13.683). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 9.81432 \times 10^{16} \text{ rad/s} = 6.45996 \text{ eV} \quad (13.697)$$

In Eq. (11.181), substitution of $E_T(O_2)$ for E_{hv} , the mass of the electron, m_e , for M , and the
10 kinetic energy given by Eq. (13.697) for \bar{E}_K gives the Doppler energy of the electrons of the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -32.27888 \text{ eV} \sqrt{\frac{2e(6.45996 \text{ eV})}{m_e c^2}} = -0.16231 \text{ eV} \quad (13.698)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of the O_2 MO
15 due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.698) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy. Using the experimental O_2 ω_e of 1580.19 cm^{-1} (0.19592 eV) [28] for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(O_2)$ is

$$20 \quad \bar{E}_{osc}(O_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.699)$$

$$\bar{E}_{osc}(O_2) = -0.16231 \text{ eV} + \frac{1}{2} (0.19592 \text{ eV}) = -0.06435 \text{ eV} \quad (13.700)$$

TOTAL AND BOND ENERGIES OF THE OXYGEN MOLECULE

$E_{T+osc}(O_2)$, the total energy of O_2 including the Doppler term, is given by the sum of
25 $E_T(O_2)$ (Eq. (13.695)) and $\bar{E}_{osc}(O_2)$ given by Eq. (13.700):

237

$$\begin{aligned}
 E_{T+osc}(O_2) &= V_e + T + V_m + V_p + E_T(O_2, 2p) + \bar{E}_{osc}(O_2) \\
 &= E_T(O_2, \sigma) + E_T(O_2, 2p) + \bar{E}_{osc}(O_2) \\
 &= E_T(O_2) + \bar{E}_{osc}(O_2)
 \end{aligned}
 \tag{13.701}$$

$$\begin{aligned}
 E_{T+osc}(O_2) &= \left[\left(\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left(\left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right) - 2 \sum_{n=4}^6 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_7} - \frac{1}{r_8} \right) \right) \right. \\
 &\quad \left. \left(1 + \sqrt{\frac{2\hbar \sqrt{\frac{e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right) + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right] \\
 &= -32.27888 \text{ eV} - 0.16231 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}
 \end{aligned}
 \tag{13.702}$$

From Eqs. (13.699-13.702), the total energy of the O_2 MO is

$$\begin{aligned}
 E_{T+osc}(O_2) &= -32.27888 \text{ eV} + \bar{E}_{osc}(O_2) \\
 &= -32.27888 \text{ eV} - 0.16231 \text{ eV} + \frac{1}{2} (0.19592 \text{ eV}) \\
 &= -32.34323 \text{ eV}
 \end{aligned}
 \tag{13.703}$$

5 where the experimental ω_e was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

The O_2 bond dissociation energy, $E_D(O_2)$, is given by the difference in the total energies of the two O atoms and $E_{T+osc}(O_2)$:

$$E_D(O_2) = 2E(O) - E_{T+osc}(O_2) \tag{13.704}$$

where the energy of an oxygen atom is [6]

$$10 \quad E(O) = -13.61806 \text{ eV} \tag{13.705}$$

Thus, the O_2 bond dissociation energy, $E_D(O_2)$, given by Eqs. (13.703-13.705) is

$$\begin{aligned}
 E_D(O_2) &= -2(13.61806 \text{ eV}) - E_{T+osc}(O_2) \\
 &= -27.23612 \text{ eV} - (-32.34323 \text{ eV}) \\
 &= 5.10711 \text{ eV}
 \end{aligned}
 \tag{13.706}$$

The experimental O_2 bond dissociation energy from Ref. [46] and Ref. [47] is

$$E_D(O_2) = 5.11665 \text{ eV} \tag{13.707}$$

$$15 \quad E_D(O_2) = 5.116696 \text{ eV} \tag{13.708}$$

The results of the determination of bond parameters of O_2 are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

5

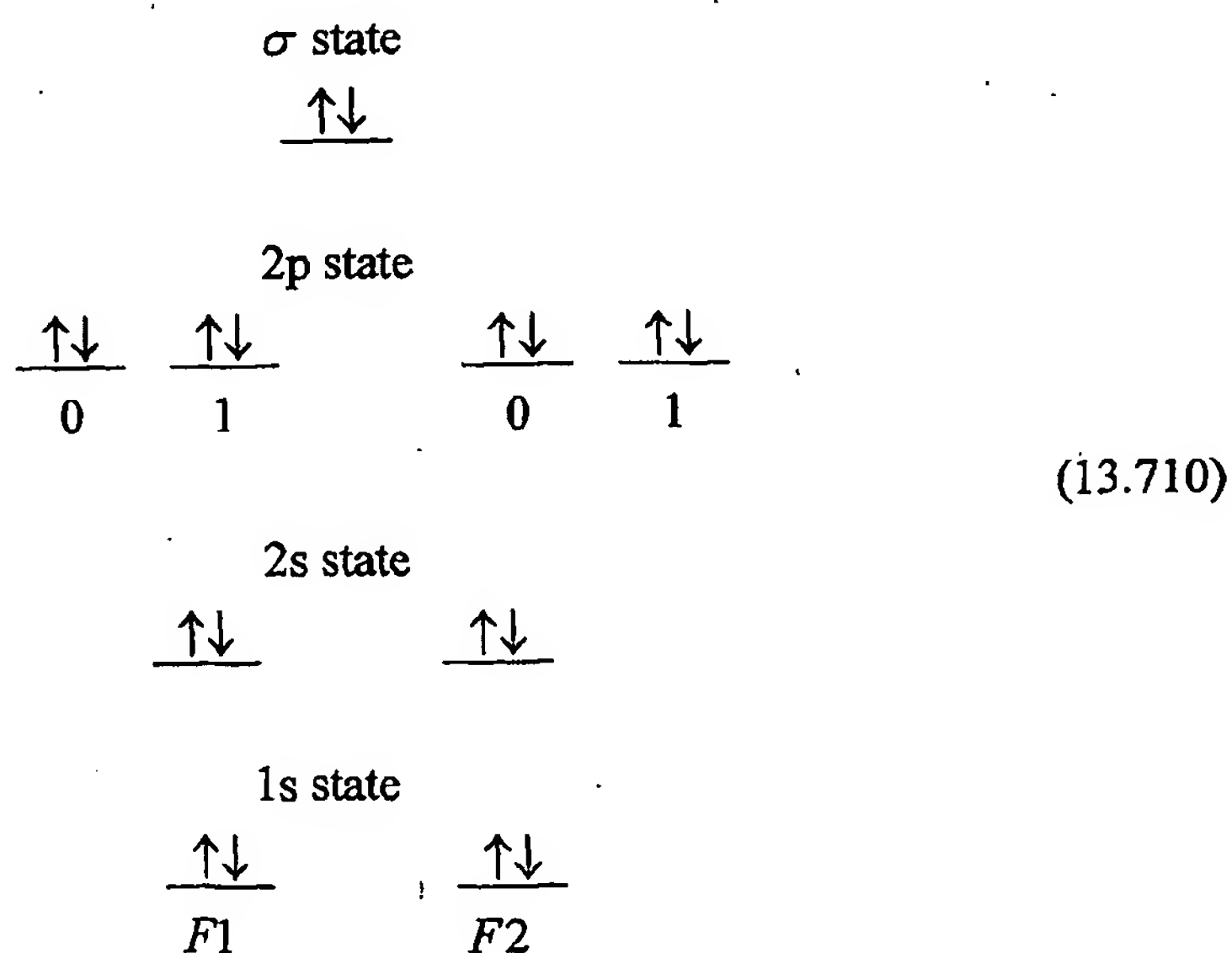
FLUORINE MOLECULE

The fluorine molecule can be formed by the reaction of two fluorine atoms:



The bond in the fluorine molecule comprises a H_2 -type molecular orbital (MO) with two
 10 paired electrons. The force balance equation and radius r_0 of the $2p$ shell of F is derived in the Nine-Electron Atoms section. With the formation of the H_2 -type MO by the contribution of a $2p$ electron from each F atom, a diamagnetic force arises between the remaining $2p$ electrons and the H_2 -type MO. This force from each F causes the H_2 -type MO to move to
 15 greater principal axes than would result with the Coulombic force alone. But, the integer increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining $2p$ electrons of each F decrease the radius of the corresponding shell such that the energy minimum is achieved that is lower than that of the reactant atoms. The resulting electron configuration of F_2 is $1s_1^2 1s_2^2 2s_1^2 2s_2^2 2p_1^4 2p_2^4 \sigma_{1,2}^2$ where the subscript designates the F atom, 1 or 2, σ designates the H_2 -type MO, and the orbital arrangement is

20



Fluorine is predicted to be diamagnetic in agreement with observations [42].

FORCE BALANCE OF THE $2p$ SHELL OF THE FLUORINE ATOMS OF 5 THE FLUORINE MOLECULE

For each F atom, force balance for the outermost $2p$ electron of F_2 (electron 8) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to interactions between electron 8 and the other $2p$ -shell as well as the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Nine-Electron
10 Atoms section. The central Coulomb force on the outer-most $2p$ shell electron of F_2 (electron 8) due to the nucleus and the inner seven electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$\mathbf{F}_{ele} = \frac{(Z-7)e^2}{4\pi\epsilon_0 r_8^2} \mathbf{i}_r \quad (13.711)$$

for $r > r_7$. The $2p$ shell possess an external electric field given by Eq. (10.92) for $r > r_8$.
15 The energy is minimized with conservation of angular momentum. This condition is met when the diamagnetic force, $\mathbf{F}_{diamagnetic}$, of Eq. (10.82) due to the p -orbital contributions is the same as that of the reactant fluorine atoms given by Eq. (10.176) with r_8 replacing r_9 :

$$\mathbf{F}_{diamagnetic} = -\left(\frac{2}{3}\right) \frac{\hbar^2}{4m_e r_8^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{2\hbar^2}{12m_e r_8^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (13.712)$$

Thus, $\mathbf{F}_{diamagnetic}$ due to the two filled $2p$ orbitals per F atom is twice that of N_2 given by
20 Eq. (13.622) having one filled $2p$ orbital per N atom. $\mathbf{F}_{mag 2}$ corresponding to the conserved spin and orbital angular momentum is also the same as that of the reactant fluorine atoms given by Eq. (10.177) and that of N_2 given by Eq. (13.623) where the outer radius of the $2p$ shell of the F atoms of F_2 is r_8 .

$$\mathbf{F}_{mag 2} = \frac{1}{Z} \frac{3\hbar^2}{m_e r_8^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.713)$$

25 The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_8$ gives rise to a second diamagnetic force, $\mathbf{F}_{diamagnetic 2}$, given by Eq. (10.93). $\mathbf{F}_{diamagnetic 2}$ due to the binding of the p -orbital electron having an electric field of +1 outside of its radius is :

$$\mathbf{F}_{\text{diamagnetic } 2} = -\left[\frac{Z-8}{Z-7}\right]\left(1-\frac{\sqrt{2}}{2}\right)\frac{r_3\hbar^2}{m_e r_8^4}10\sqrt{s(s+1)}\mathbf{i}_r \quad (13.714)$$

In addition, the contribution of a $2p$ electron from each F atom in the formation of the σ MO gives rise to a paramagnetic force on the remaining paired $2p$ electrons. The force $\mathbf{F}_{\text{mag } 3}$ is given by Eq. (13.625) wherein the radius is r_8 :

$$5 \quad \mathbf{F}_{\text{mag } 3} = \frac{\hbar^2}{4m_e r_8^3} \sqrt{s(s+1)}\mathbf{i}_r \quad (13.715)$$

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (13.711)) and diamagnetic (Eqs. (13.712) and (13.714)), and paramagnetic (Eqs. (13.713) and (13.715)) forces as follows:

$$\frac{m_e v_8^2}{r_8} = \left(\frac{(Z-7)e^2}{4\pi\epsilon_0 r_8^2} - \frac{2\hbar^2}{12m_e r_8^2 r_3} \sqrt{s(s+1)} + \frac{3\hbar^2}{Zm_e r_8^2 r_3} \sqrt{s(s+1)} - \left[\frac{Z-8}{Z-7} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_8^4 m_e} 10\sqrt{s(s+1)} + \frac{\hbar^2}{4m_e r_8^3} \sqrt{s(s+1)} \right) \quad (13.716)$$

10 Substitution of $v_8 = \frac{\hbar}{m_e r_8}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (13.716) gives:

$$\frac{\hbar^2}{m_e r_8^3} - \frac{\hbar^2}{4m_e r_8^3} \sqrt{\frac{3}{4}} = \frac{(Z-7)e^2}{4\pi\epsilon_0 r_8^2} - \frac{2\hbar^2}{12m_e r_8^2 r_3} \sqrt{\frac{3}{4}} + \frac{3\hbar^2}{Zm_e r_8^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-8}{Z-7} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_8^4 m_e} 10\sqrt{\frac{3}{4}} \quad (13.717)$$

The quadratic equation corresponding to Eq. (13.717) is

$$r_8^2 - \frac{\frac{\hbar^2}{m_e} \left(1 - \frac{\sqrt{3}}{8} \right)}{\left(\frac{(Z-7)e^2}{4\pi\epsilon_0} - \left(\frac{2}{12} - \frac{3}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} r_8 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-8}{Z-7} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 10\sqrt{\frac{3}{4}}}{\left(\frac{(Z-7)e^2}{4\pi\epsilon_0} - \left(\frac{2}{12} - \frac{3}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} = 0$$

$$15 \quad (13.718)$$

The solution of Eq. (13.718) using the quadratic formula is:

$$r_8 = \frac{a_0 \left(1 - \frac{\sqrt{3}}{8}\right)}{\left((Z-7) - \left(\frac{2}{12} - \frac{3}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)} \pm a_0 \sqrt{\frac{\left(\frac{1 - \frac{\sqrt{3}}{8}}{\left((Z-7) - \left(\frac{2}{12} - \frac{3}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)}\right)^2}{20\sqrt{3} \left[\frac{Z-8}{Z-7}\right] \left(1 - \frac{\sqrt{2}}{2}\right) r_3} + \frac{\left((Z-7) - \left(\frac{2}{12} - \frac{3}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)}{\left((Z-7) - \left(\frac{2}{12} - \frac{3}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)}} \quad (13.719)$$

r_3 in units of a_0

The positive root of Eq. (13.719) must be taken in order that $r_8 > 0$. Substitution of

$\frac{r_3}{a_0} = 0.51382$ (Eq. (10.62) with $Z = 9$) into Eq. (13.719) gives

$$r_8 = 0.73318a_0 \quad (13.720)$$

5

ENERGIES OF THE $2p$ SHELL OF THE FLUORINE ATOMS OF THE FLUORINE MOLECULE

The central forces on the $2p$ shell of each F are increased with the formation of the σ MO, which reduces the shell's radius and increases its total energy. The Coulombic energy terms of the total energy of the two F atoms at the new radius are calculated and added to the energy of the σ MO to give the total energy of F_2 . Then, the bond energy is determined from the total F_2 energy.

The radius r_9 of each fluorine atom before bonding is given by Eq. (10.182):

$$r_9 = 0.78069a_0 \quad (13.721)$$

15 Using the initial radius r_9 of each F atom and the final radius r_8 of the $F2p$ shell of F_2 (Eq. (13.720)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(F_2, 2p)$ of the Coulombic energy change of the $F2p$ electrons of both atoms is determined using Eq. (10.102):

$$\begin{aligned} E_T(F_2, 2p) &= -2 \sum_{n=4}^7 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_8} - \frac{1}{r_9} \right) \\ &= -2(13.60580 \text{ eV})(0.08301)(2+3+4+5) \\ &= -31.62353 \text{ eV} \end{aligned} \quad (13.722)$$

FORCE BALANCE OF THE σ MO OF THE FLUORINE MOLECULE

The relativistic diamagnetic force $F_{\text{diamagneticMO2}}$ of F_2 is one half that of N_2 due to the two versus one filled $2p$ orbitals per atom at the focus. The force balance equation for the σ -
5 MO of the fluorine molecule is given by Eq. (11.200) and Eqs. (13.633-13.634) with the correction of 1/2 due the two $2p$ orbitals per F after Eqs. (10.2-10.11), $n_e = 2$, and $|L| = \hbar$:

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(\frac{5}{2} + \frac{1}{2Z}\right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.723)$$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D - \left(\frac{3}{2} + \frac{1}{2Z}\right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.724)$$

$$\left(\frac{7}{2} + \frac{1}{2Z}\right) \frac{\hbar^2}{2m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D \quad (13.725)$$

$$10 \quad a = \left(\frac{7}{2} + \frac{1}{2Z}\right) a_0 \quad (13.726)$$

Substitution of $Z = 9$ into Eq. (13.726) gives

$$a = 3.55556a_0 = 1.88152 \times 10^{-10} \text{ m} \quad (13.727)$$

Substitution of Eq. (13.727) into Eq. (11.79) is

$$c' = 1.33333a_0 = 7.05569 \times 10^{-11} \text{ m} \quad (13.728)$$

15 The internuclear distance given by multiplying Eq. (13.728) by two is

$$2c' = 2.66667a_0 = 1.41114 \times 10^{-10} \text{ m} \quad (13.729)$$

The experimental bond distance is [28]

$$2c' = 1.41193 \times 10^{-10} \text{ m} \quad (13.730)$$

Substitution of Eqs. (13.727-13.728) into Eq. (11.80) is

$$20 \quad b = c = 3.29609a_0 = 1.74421 \times 10^{-10} \text{ m} \quad (13.731)$$

Substitution of Eqs. (13.727-13.728) into Eq. (11.67) is

$$e = 0.37500 \quad (13.732)$$

Using the electron configuration of F_2 (Eq. (13.710)), the radii of the $F1s = 0.11297a_0$ (Eq. (10.51)), $F2s = 0.51382a_0$ (Eq. (10.62)), and $F2p = 0.73318a_0$ (Eq. (13.720)) shells and the
25 parameters of the σ MO of F_2 given by Eqs. (13.3-13.4), (13.727-13.728), and (13.731-13.732), the dimensional diagram and charge-density of the F_2 MO are shown in Figures 23 and 24, respectively.

SUM OF THE ENERGIES OF THE σ MO AND THE AOs OF THE FLUORINE MOLECULE

- 5 The energies of the F_2 σ MO are given by the substitution of the semiprincipal axes (Eqs. (13.683-13.684) and (13.687)) into the energy equations (Eqs. (11.207-11.212)) of H_2 :

$$V_e = \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -16.09139 \text{ eV} \quad (13.733)$$

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 10.20435 \text{ eV} \quad (13.734)$$

$$T = \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 2.26285 \text{ eV} \quad (13.735)$$

$$10 \quad V_m = \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -1.13143 \text{ eV} \quad (13.736)$$

$$E_T = V_e + T + V_m + V_p \quad (13.737)$$

Substitution of Eqs. (11.79) and (13.733-13.736) into Eq. (13.737) gives

$$E_T(F_2, \sigma) = \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(\left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+\sqrt{\frac{aa_0}{2}}}{a-\sqrt{\frac{aa_0}{2}}} - 1 \right) = -4.75562 \text{ eV} \quad (13.738)$$

- where $E_T(F_2, \sigma)$ is the total energy of the σ MO of F_2 . The sum, $E_T(F_2)$, of $E_T(F_2, 2p)$,
 15 the $2p$ AO contribution given by Eq. (13.722), and $E_T(F_2, \sigma)$, the σ MO contribution given by Eq. (13.738) is:

$$\begin{aligned} E_T(F_2) &= E_T(F_2, 2p) + E_T(F_2, \sigma) \\ &= -31.62353 \text{ eV} - 4.75562 \text{ eV} \\ &= -36.37915 \text{ eV} \end{aligned} \quad (13.739)$$

VIBRATION OF F_2

- 20 The vibrational energy levels of F_2 may be solved by determining the Morse potential curve from the energy relationships for the transition from two F atoms whose parameters are given by Eqs. (10.174-10.183) to the two F atoms whose parameter r_8 is given by Eq.

(13.720) and the σ MO whose parameters are given by Eqs. (13.727-13.729) and (13.731-13.732). As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter 5 (K&P) [15] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods.

THE DOPPLER ENERGY TERMS OF THE FLUORINE MOLECULE

10 The equations of the radiation reaction force of fluorine are the same as those of H_2 with the substitution of the fluorine parameters. Using Eqs. (11.231-11.233), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{e^2}{4\pi\epsilon_0 a^3}} = 6.16629 \times 10^{15} \text{ rad/s} \quad (13.740)$$

where a is given by Eq. (13.727). The kinetic energy, E_K , is given by Planck's equation 15 (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 6.16629 \times 10^{15} \text{ rad/s} = 4.05876 \text{ eV} \quad (13.741)$$

In Eq. (11.181), substitution of $E_T(F_2)$ for E_{nv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.741) for \bar{E}_K gives the Doppler energy of the electrons of the reentrant orbit:

$$20 \quad \bar{E}_D \cong E_{nv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -36.37915 \text{ eV} \sqrt{\frac{2e(4.05876 \text{ eV})}{m_e c^2}} = -0.14499 \text{ eV} \quad (13.742)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of the F_2 MO due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given 25 by Eq. (13.742) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy. Using the experimental F_2 ω_e of 916.64 cm^{-1} (0.11365 eV) [28] for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(F_2)$ is

$$\bar{E}_{osc}(F_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.743)$$

$$\bar{E}_{osc}(F_2) = -0.14499 \text{ eV} + \frac{1}{2} (0.11365 \text{ eV}) = -0.08817 \text{ eV} \quad (13.744)$$

TOTAL AND BOND ENERGIES OF THE FLUORINE MOLECULE

5 $E_{T+osc}(F_2)$, the total energy of F_2 including the Doppler term, is given by the sum of $E_T(F_2)$ (Eq. (13.739)) and $\bar{E}_{osc}(F_2)$ given by Eq. (13.744):

$$\begin{aligned} E_{T+osc}(F_2) &= V_e + T + V_m + V_p + E_T(F_2, 2p) + \bar{E}_{osc}(F_2) \\ &= E_T(F_2, \sigma) + E_T(F_2, 2p) + \bar{E}_{osc}(F_2) \\ &= E_T(F_2) + \bar{E}_{osc}(F_2) \end{aligned} \quad (13.745)$$

$$\begin{aligned} E_{T+osc}(F_2) &= \left\{ \left[\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left(\left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right) - 2 \sum_{n=4}^7 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_8} - \frac{1}{r_9} \right) \right] \right. \\ &\quad \left. \left[1 + \sqrt{\frac{\frac{e^2}{4\pi\epsilon_0 a^3}}{m_e c^2}} \right] + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} \quad (13.746) \\ &= -36.37915 \text{ eV} - 0.14499 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \end{aligned}$$

From Eqs. (13.743-13.746), the total energy of the F_2 MO is

$$\begin{aligned} E_{T+osc}(F_2) &= -36.37915 \text{ eV} + \bar{E}_{osc}(F_2) \\ 10 \quad &= -36.37915 \text{ eV} - 0.14499 \text{ eV} + \frac{1}{2} (0.11365 \text{ eV}) \\ &= -36.46732 \text{ eV} \end{aligned} \quad (13.747)$$

where the experimental ω_e was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

The F_2 bond dissociation energy, $E_D(F_2)$, is given by the difference in the total energies of the two F atoms and $E_{T+osc}(F_2)$:

$$E_D(F_2) = 2E(F) - E_{T+osc}(F_2) \quad (13.748)$$

15 where the energy of a fluorine atom is [6]

$$E(F) = -17.42282 \text{ eV} \quad (13.749)$$

Thus, the F_2 bond dissociation energy, $E_D(F_2)$, given by Eqs. (13.747-13.749) is

$$\begin{aligned} E_D(F_2) &= -2(17.42282 \text{ eV}) - E_{T+osc}(F_2) \\ &= -34.84564 \text{ eV} - (-36.46732 \text{ eV}) \\ &= 1.62168 \text{ eV} \end{aligned} \quad (13.750)$$

The experimental F_2 bond dissociation energy is [48]

$$E_D(F_2) = 1.606 \text{ eV} \quad (13.751)$$

The results of the determination of bond parameters of F_2 are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

10

CHLORINE MOLECULE

The chlorine molecule can be formed by the reaction of two chlorine atoms:



The chlorine molecule can be solved by using the hybridization approach used to solve the methane series $CH_{n=1,2,3,4}$. In the methane series, the $2s$ and $2p$ shells of carbon hybridize to form a single $2sp^3$ shell to achieve an energy minimum, and in a likewise manner, the $3s$ and $3p$ shells of chlorine hybridize to form a single $3sp^3$ shell which forms the bonding orbital of Cl_2 .

20 FORCE BALANCE OF Cl_2

Cl_2 has two spin-paired electrons in a chemical bond between the chlorine atoms. The Cl_2 molecular orbital (MO) is determined by considering properties of the binding atoms and the boundary constraints. The prolate spheroidal H_2 MO developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules section satisfies the boundary constraints; thus, each Cl atom could contribute a $3p$ electron to form a σ MO (H_2 -type ellipsoidal MO) as in the case of N_2 , O_2 , and F_2 . However, such a bond is not possible with the outer Cl electrons in their ground state since the resulting $3p$ shells of chlorine atoms would overlap which is not energetically stable. Thus, when bonding, the chlorine $3s$ and $3p$ shells

25

hybridize to form a single $3sp^3$ shell to achieve an energy minimum.

The Cl electron configuration given in the Seventeen-Electron Atoms section is $1s^2 2s^2 2p^6 3s^2 3p^5$, and the orbital arrangement is

$$\begin{array}{c} 3p \text{ state} \\ \begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ \hline 1 & 0 & -1 \end{array} \end{array} \quad (13.753)$$

5 corresponding to the ground state $^2P_{3/2}^0$. The radius r_{17} of the $3p$ shell given by Eq. (10.363) is

$$r_{17} = 1.05158a_0 \quad (13.754)$$

The energy of the chlorine $3p$ shell is the negative of the ionization energy of the chlorine atom given by Eq. (10.364). Experimentally, the energy is [6]

$$10 \quad E(3p \text{ shell}) = -E(\text{ionization}; Cl) = -12.96764 \text{ eV} \quad (13.755)$$

The $Cl3s$ atomic orbital (AO) combines with the $Cl3p$ AOs to form a single $3sp^3$ hybridized orbital (HO) with the orbital arrangement

$$\begin{array}{c} 2sp^3 \text{ state} \\ \begin{array}{cccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ \hline 0,0 & 1,-1 & 1,0 & 1,1 \end{array} \end{array} \quad (13.756)$$

where the quantum numbers (ℓ, m_ℓ) are below each electron. The total energy of the state is
 15 given by the sum over the seven electrons. Using only the largest-force terms of the outer most and next inner shell, the calculated energies for the chlorine atom and the ions: Cl , Cl^+ , Cl^{2+} , Cl^{3+} , Cl^{4+} , Cl^{5+} and Cl^{6+} are given in Eqs. (10.363-10.364), (10.353-10.354), (10.331-10.332), (10.309-10.310), (10.288-10.289), (10.255-10.256), and (10.235-10.236), respectively. The sum $E_T(Cl, 3sp^3)$ of the experimental energies of Cl and these ions is [6]

$$\begin{aligned} 20 \quad E_T(Cl, 3sp^3) &= \left(\begin{array}{l} 12.96764 \text{ eV} + 23.814 \text{ eV} + 39.61 \text{ eV} + 53.4652 \text{ eV} \\ +67.8 \text{ eV} + 97.03 \text{ eV} + 114.1958 \text{ eV} \end{array} \right) \quad (13.757) \\ &= 408.88264 \text{ eV} \end{aligned}$$

The spin and orbital-angular-momentum interactions cancel such that the energy of the $E_T(Cl, 3sp^3)$ is purely Coulombic. By considering that the central field decreases by an integer for each successive electron of the shell, the radius r_{3sp^3} of the $Cl3sp^3$ shell may be calculated from the Coulombic energy using Eq. (10.102):

$$r_{3sp^3} = \sum_{n=10}^{16} \frac{(Z-n)e^2}{8\pi\epsilon_0 (e408.8826 \text{ eV})} = \frac{248}{8\pi\epsilon_0 (e408.8826 \text{ eV})} = 0.93172a_0 \quad (13.758)$$

where $Z=17$. Using Eqs. (10.102) and (13.758), the Coulombic energy $E_{Coulomb}(Cl, 3sp^3)$ of the outer electron of the $Cl3sp^3$ shell is

$$E_{Coulomb}(Cl, 3sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{3sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.93172a_0} = -14.60295 \text{ eV} \quad (13.759)$$

- 5 The calculated energy of the $C2sp^3$ shell of 14.63489 eV given by Eq. (13.428), and nitrogen's calculated energy of 14.61664 eV given by Eq. (10.143) is a close match with $E_{Coulomb}(Cl, 3sp^3)$.

The unpaired $Cl3sp^3$ electron from each of two chlorine atoms combine to form a molecular orbital. The nuclei of the Cl atoms are along the internuclear axis and serve as the
10 foci. Due to symmetry, the other Cl electrons are equivalent to point charges at the origin. (See Eqs. (19-38) of Appendix IV.) Thus, the energies in the Cl MO involve only the two $Cl3sp^3$ electrons. The forces are determined by these energies.

As in the case of H_2 , the MO is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into $Cl3sp^3$ HO for distances shorter than the radius of
15 the $Cl3sp^3$ shell of each atom. Thus, the MO surface comprises a partial prolate spheroid in between the nuclei and is continuous with the $Cl3sp^3$ shell at each Cl atom. The energy of the H_2 -type ellipsoidal MO is matched to that of the $Cl3sp^3$ shell. As in the case with OH , NH , and CH (where the latter also demonstrates sp^3 hybridization) the linear combination of the H_2 -type ellipsoidal MO with each $Cl3sp^3$ HO must involve a 25% contribution from
20 the H_2 -type ellipsoidal MO to the $Cl3sp^3$ HO in order to match potential, kinetic, and orbital energy relationships. Thus, the Cl_2 MO must comprise two $Cl3sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the two $Cl3sp^3$ HOs:



The force balance of the Cl_2 MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (13.760) and the energy matching condition between the H_2 -type-ellipsoidal-MO and $Cl3sp^3$ -HO components of the MO.

As in the case with OH (Eq. (13.57)), NH (Eq. (13.247)), and CH (Eq. 13.429)), the H_2 -
 5 type ellipsoidal MO comprises 75% of the Cl_2 MO; so, the electron charge density in Eq. (11.65) is given by $-0.75e$. Since the chlorine atoms of Cl_2 are hybridized and the k parameter is different from unity in order to meet the boundary constraints, both k and k' must comprise the corresponding hybridization factors. (In contrast, the chlorine atom of a $C-Cl$ bond of an alkyl chloride is not hybridized, and only k' must comprise the
 10 corresponding hybridization factor.) The force constant k' to determine the ellipsoidal parameter c' in terms of the central force of the foci is given by Eq. (13.59), except that k' is divided by two since the H_2 -type-ellipsoidal-MO is physically divided between two $Cl3sp^3$ HOs. In addition, the energy matching at both $Cl3sp^3$ HOs further requires that k' be corrected the hybridization factor given by Eq. (13.762). Thus, k' of the H_2 -type-
 15 ellipsoidal-MO component of the Cl_2 MO is

$$k' = C_{Cl3sp^3HO} \frac{(0.75)}{2} \frac{2e^2}{4\pi\epsilon_0} = 0.93172 \frac{(0.75)}{2} \frac{2e^2}{4\pi\epsilon_0} \quad (13.761)$$

The distance from the origin to each focus c' is given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $Cl-Cl$ -bond $b=c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq.
 20 (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of the Cl_2 MO. Since the Cl_2 MO comprises a H_2 -type-ellipsoidal MO that transitions to the $Cl3sp^3$ HOs at each end of the molecule, the energy $E(Cl,3sp^3)$ in Eq. (13.759) adds to that of the H_2 -type ellipsoidal MO to give the total energy of the Cl_2 MO. From the energy equation and the relationship between the axes,
 25 the dimensions of the Cl_2 MO are solved.

The energy components of V_e , V_p , T , and V_m are those of H_2 (Eqs. (11.207-11.211)) except that they are corrected for electron hybridization. Hybridization gives rise to the $Cl3sp^3$ HO-shell Coulombic energy $E_{Coulomb}(Cl,3sp^3)$ given by Eq. (13.759). To meet the

equipotential condition of the union of the H_2 -type-ellipsoidal-MO with each $Cl3sp^3$ HO, the electron energies are normalized by the ratio of 14.60295 eV , the magnitude of $E_{Coulomb}(Cl,3sp^3)$ given by Eq. (13.759), and 13.605804 eV , the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). This
 5 normalizes the energies to match that of the Coulombic energy alone to meet the energy matching condition of the Cl_2 MO under the influence of the two $Cl3sp^3$ HOs bridged by the H_2 -type-ellipsoidal MO. The hybridization energy factor C_{Cl3sp^3HO} is

$$C_{Cl3sp^3HO} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{3sp^3}}} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 0.93172 a_0}} = \frac{13.605804 \text{ eV}}{14.60295 \text{ eV}} = 0.93172 \quad (13.762)$$

The total energy $E_T(Cl_2)$ of the Cl_2 MO is given by the sum of the energies of the orbitals,
 10 the H_2 -type ellipsoidal MO and the two $Cl3sp^3$ HOs, that form the hybridized Cl_2 MO. $E_T(Cl_2)$ follows from by Eq. (13.74) for OH , but the energy of the $Cl3sp^3$ HO given by Eq. (13.759) is substituted for the energy of O and the H_2 -type-ellipsoidal-MO energies are those of H_2 (Eqs. (11.207-11.212)) multiplied by the electron hybridization factor rather than by the factor of 0.75:

$$E_T(Cl_2) = E_T + E_{Coulomb}(Cl,3sp^3) \\
 15 \quad = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.93172) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.60295 \text{ eV} \quad (13.763)$$

To match the boundary condition that the total energy of the entire the H_2 -type ellipsoidal MO is given by Eqs. (11.212) and (13.75), $E_T(Cl_2)$ given by Eq. (13.763) is set equal to Eq. (13.75):

$$E_T(Cl_2) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.93172) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.60295 \text{ eV} = -31.63537 \text{ eV} \\
 20 \quad (13.764)$$

From the energy relationship given by Eq. (13.764) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the Cl_2 MO can be solved.

Substitution of Eqs. (13.60) and (13.761) into Eq. (13.764) gives

$$\frac{e^2}{8\pi\epsilon_0\sqrt{\frac{4aa_0}{3(0.93172)}}} \left[(0.93172) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{4aa_0}{3(0.93172)}}}{a - \sqrt{\frac{4aa_0}{3(0.93172)}}} - 1 \right] = e17.03242$$

(13.765)

The most convenient way to solve Eq. (13.765) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$5 \quad a = 2.46500a_0 = 1.30442 \times 10^{-10} \text{ m} \quad (13.766)$$

Substitution of Eq. (13.766) into Eq. (13.60) gives

$$c' = 1.87817a_0 = 9.93887 \times 10^{-11} \text{ m} \quad (13.767)$$

The internuclear distance given by multiplying Eq. (13.767) by two is

$$2c' = 3.75635a_0 = 1.98777 \times 10^{-10} \text{ m} \quad (13.768)$$

10 The experimental bond distance is [28]

$$2c' = 1.988 \times 10^{-10} \text{ m} \quad (13.769)$$

Substitution of Eqs. (13.766-13.767) into Eq. (13.62) gives

$$b = c = 1.59646a_0 = 8.44810 \times 10^{-11} \text{ m} \quad (13.770)$$

Substitution of Eqs. (13.766-13.767) into Eq. (13.63) gives

$$15 \quad e = 0.76194 \quad (13.771)$$

The Cl nuclei comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $Cl3sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{3sp^3} = 0.93172a_0$ is the radius of the $Cl3sp^3$ shell. Substitution of Eqs. (13.766-13.767)

20 into Eq. (13.261) gives

$$\theta' = 81.72^\circ \quad (13.772)$$

Then, the angle θ_{Cl3sp^3HO} the radial vector of the $Cl3sp^3$ HO makes with the internuclear axis is

$$\theta_{Cl3sp^3HO} = 180^\circ - 81.72^\circ = 98.28^\circ \quad (13.773)$$

25 as shown in Figure 25. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{H_2MO}$

between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $Cl3sp^3$ radial vector obeys the following relationship:

$$r_{3sp^3} \sin \theta_{Cl3sp^3 HO} = 0.93172a_0 \sin \theta_{Cl3sp^3 HO} = b \sin \theta_{H_2 MO} \quad (13.774)$$

such that

$$5 \quad \theta_{H_2 MO} = \sin^{-1} \frac{0.93172a_0 \sin \theta_{Cl3sp^3 HO}}{b} = \sin^{-1} \frac{0.93172a_0 \sin 98.28^\circ}{b} \quad (13.775)$$

with the use of Eq. (13.773). Substitution of Eq. (13.770) into Eq. (13.775) gives

$$\theta_{H_2 MO} = 35.28^\circ \quad (13.776)$$

Then, the distance $d_{H_2 MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$10 \quad d_{H_2 MO} = a \cos \theta_{H_2 MO} \quad (13.777)$$

Substitution of Eqs. (13.766) and (13.776) into Eq. (13.777) gives

$$d_{H_2 MO} = 2.01235a_0 = 1.06489 \times 10^{-10} \text{ m} \quad (13.778)$$

The distance $d_{Cl3sp^3 HO}$ along the internuclear axis from the origin of each Cl atom to the point of intersection of the orbitals is given by

$$15 \quad d_{Cl3sp^3 HO} = d_{H_2 MO} - c' \quad (13.779)$$

Substitution of Eqs. (13.768) and (13.778) into Eq. (13.779) gives

$$d_{Cl3sp^3 HO} = 0.13417a_0 = 7.10022 \times 10^{-12} \text{ m} \quad (13.780)$$

As shown in Eq. (13.760), a factor of 0.25 of the charge-density of the H_2 -type ellipsoidal MO is distributed on each $Cl3sp^3$ HO. Using the orbital composition of Cl_2 (Eq. (13.760)), the radii of the $Cl1s = 0.05932a_0$ (Eq. (10.51)), $Cl2s = 0.25344a_0$ (Eq. (10.62)), $Cl2p = 0.31190a_0$ (Eq. (10.212)), and $Cl3sp^3 = 0.93172a_0$ (Eq. (13.758)) shells, and the parameters of the Cl_2 MO given by Eqs. (13.3-13.4), (13.766-13.768), and (13.770-13.771), the dimensional diagram and charge-density of the Cl_2 MO comprising the linear combination of the H_2 -type ellipsoidal MO and two $Cl3sp^3$ HOs according to Eq. (13.760) are shown in Figures 25 and 26, respectively.

ENERGIES OF Cl_2

The energies of Cl_2 are given by the substitution of the semiprincipal axes (Eqs. (13.766-13.767) and (13.770)) into the energy equations, (Eq. (13.763) and Eqs. (11.207-11.211) of 5 H_2) that are corrected for electron hybridization using Eq. (13.762):

$$V_e = (0.93172) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -27.02007 \text{ eV} \quad (13.781)$$

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 7.24416 \text{ eV} \quad (13.782)$$

$$T = (0.93172) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 5.48074 \text{ eV} \quad (13.783)$$

$$V_m = (0.93172) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -2.74037 \text{ eV} \quad (13.784)$$

$$10 \quad E_T(^{35}Cl_2) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.93172) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.60295 \text{ eV} = -31.63849 \text{ eV} \quad (13.785)$$

where $E_T(Cl_2)$ is given by Eq. (13.763) which is reiteratively matched to Eq. (13.75) within five-significant-figure round-off error.

15 VIBRATION AND ROTATION OF Cl_2

In Cl_2 , the division of the H_2 -type ellipsoidal MO between the two $Cl3sp^3$ HOs and the hybridization must be considered in determining the vibrational parameters. One approach is to use Eq. (13.761) for the force constant and r_{3sp^3} given by Eq. (13.758) for the distance parameter of the central force in Eq. (11.213) since the H_2 -type ellipsoidal MO is energy 20 matched to the $Cl3sp^3$ HOs. With the substitution of the Cl_2 parameters in Eqs. (11.213-11.217), the angular frequency of the oscillation is

$$\begin{aligned}
 \omega &= \sqrt{\frac{0.93172 \frac{(0.75)}{2} \frac{e^2}{8\pi\epsilon_0 (r_{3sp})^3} - \frac{e^2}{8\pi\epsilon_0 (r_{3sp} + c')^3}}{\mu}} \\
 &= \sqrt{\frac{0.93172 \frac{(0.75)}{2} \frac{e^2}{8\pi\epsilon_0 (0.93172)^3} - \frac{e^2}{8\pi\epsilon_0 (0.93172a_0 + 1.87817a_0)^3}}{\frac{35}{2}m_p}} \quad (13.786) \\
 &= 1.01438 \times 10^{14} \text{ rad/s}
 \end{aligned}$$

where c' is given by Eq. (13.767), and the reduced mass of $^{35}\text{Cl}_2$ is given by:

$$\mu_{^{35}\text{Cl}_2} = \frac{m_1 m_2}{m_1 + m_2} = \frac{(35)(35)}{35 + 35} m_p \quad (13.787)$$

where m_p is the proton mass. Thus, during bond formation, the perturbation of the orbit
 5 determined by an inverse-squared force results in simple harmonic oscillatory motion of the
 orbit, and the corresponding frequency, $\omega(0)$, for $^{35}\text{Cl}_2$ given by Eqs. (11.136), (11.148),
 and (13.786) is

$$\omega(0) = \sqrt{\frac{k(0)}{\mu}} = \sqrt{\frac{301.19 \text{ Nm}^{-1}}{\mu}} = 1.01438 \times 10^{14} \text{ radians/s} \quad (13.788)$$

where the reduced nuclear mass of $^{35}\text{Cl}_2$ is given by Eq.(13.787) and the spring constant,
 10 $k(0)$, given by Eqs. (11.136) and (13.786) is

$$k(0) = 301.19 \text{ Nm}^{-1} \quad (13.789)$$

The $^{35}\text{Cl}_2$ transition-state vibrational energy, $E_{vib}(0)$ or ω_e , given by Planck's equation (Eq.
 (11.127)) is:

$$E_{vib}(0) = \omega_e = \hbar\omega = \hbar 1.01438 \times 10^{14} \text{ rad/s} = 0.06677 \text{ eV} = 538.52 \text{ cm}^{-1} \quad (13.790)$$

15 ω_e , from the experimental curve fit of the vibrational energies of $^{35}\text{Cl}_2$ is [28]

$$\omega_e = 559.7 \text{ cm}^{-1} \quad (13.791)$$

Using Eqs. (13.112-13.118) with $E_{vib}(0)$ given by Eq. (13.790) and D_0 given by Eq.
 (13.807), the $^{35}\text{Cl}_2$ $\nu=1 \rightarrow \nu=0$ vibrational energy, $E_{vib}(1)$ is

$$E_{vib}(1) = 0.0659 \text{ eV} \quad (531.70 \text{ cm}^{-1}) \quad (13.792)$$

20 The experimental vibrational energy of $^{35}\text{Cl}_2$ using ω_e and $\omega_e x_e$ [28] according to K&P [15]
 is

$$E_{vb}(1) = 0.0664 \text{ eV} \quad \left(535.55 \text{ cm}^{-1} \right) \quad (13.793)$$

Using Eq. (13.113) with $E_{vb}(1)$ given by Eq. (13.792) and D_0 given by Eq. (13.807), the anharmonic perturbation term, $\omega_0 x_0$, of $^{35}\text{Cl}_2$ is

$$\omega_0 x_0 = 3.41 \text{ cm}^{-1} \quad (13.794)$$

5 The experimental anharmonic perturbation term, $\omega_0 x_0$, of $^{35}\text{Cl}_2$ [28] is

$$\omega_0 x_0 = 2.68 \text{ cm}^{-1} \quad (13.795)$$

The vibrational energies of successive states are given by Eqs. (13.790), (13.112), and (13.794).

Using Eqs. (13.133-13.134) and the internuclear distance, $r = 2c'$, and reduced mass of $^{35}\text{Cl}_2$ given by Eqs. (13.768) and (13.787), respectively, the corresponding B_e is

$$B_e = 0.2420 \text{ cm}^{-1} \quad (13.796)$$

The experimental B_e rotational parameter of $^{35}\text{Cl}_2$ is [28]

$$B_e = 0.2440 \text{ cm}^{-1} \quad (13.797)$$

15 THE DOPPLER ENERGY TERMS OF Cl_2

The equations of the radiation reaction force of the symmetrical Cl_2 MO are the given by Eqs. (11.231-11.233) with the substitution of the Cl_2 parameters and the substitution of the force factor of Eq. (13.761). The angular frequency of the reentrant oscillation in the transition state is

$$20 \quad \omega = \sqrt{\frac{0.93172 \frac{(0.75)}{2} \frac{e^2}{4\pi\epsilon_0 a^3}}{m_e}} = 6.31418 \times 10^{15} \text{ rad/s} \quad (13.798)$$

where a is given by Eq. (13.766). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 6.31418 \times 10^{15} \text{ rad/s} = 4.15610 \text{ eV} \quad (13.799)$$

In Eq. (11.181), substitution of the total energy of Cl_2 , $E_T(\text{Cl}_2)$, (Eq. (13.764)) for E_m , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.799) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit:

$$\bar{E}_D \cong E_{lv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{256 \cdot 2e(4.15610 \text{ eV})}{m_e c^2}} = -0.12759 \text{ eV} \quad (13.800)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of Cl_2 due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the 5 electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.800) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of Cl_2 . Using the experimental $^{35}Cl_2$ ω_e of 559.7 cm^{-1} (0.06939 eV) [28] for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(^{35}Cl_2)$ is

$$\bar{E}_{osc}(^{35}Cl_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.801)$$

$$10 \quad \bar{E}_{osc}(^{35}Cl_2) = -0.12759 \text{ eV} + \frac{1}{2} (0.06939 \text{ eV}) = -0.09289 \text{ eV} \quad (13.802)$$

TOTAL AND BOND ENERGIES OF Cl_2

$E_{T+osc}(^{35}Cl_2)$, the total energy of the $^{35}Cl_2$ radical including the Doppler term, is given by the sum of $E_T(Cl_2)$ (Eq. (13.764)) and $\bar{E}_{osc}(^{35}Cl_2)$ given by Eq. (13.802):

$$15 \quad \begin{aligned} E_{T+osc}(^{35}Cl_2) &= V_e + T + V_m + V_p + E_{Coulomb}(Cl, 3sp^3) + \bar{E}_{osc}(^{35}Cl_2) \\ &= E_T(Cl_2) + \bar{E}_{osc}(^{35}Cl_2) \end{aligned} \quad (13.803)$$

$$E_{T+osc}(^{35}Cl_2) = \left\{ \left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[(0.93172) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 14.60295 \text{ eV} \right) \right. \\ \left. \left(1 + \sqrt{\frac{2\hbar \sqrt{\frac{0.93172(0.75)}{2} \frac{e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\}$$

$$= -31.63537 \text{ eV} - 0.12759 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}$$

(13.804)

From Eqs. (13.801-13.804), the total energy of $^{35}Cl_2$ is

$$\begin{aligned}
 E_{T+osc}({}^{35}\text{Cl}_2) &= -31.63537 \text{ eV} + \bar{E}_{osc}({}^{35}\text{Cl}_2) \\
 &= -31.63537 \text{ eV} - 0.12759 \text{ eV} + \frac{1}{2}(0.06939 \text{ eV}) \\
 &= -31.72826 \text{ eV}
 \end{aligned}
 \tag{13.805}$$

where the experimental ω_e (Eq. (13.791)) was used for the $\hbar\sqrt{\frac{k}{\mu}}$ term.

The Cl_2 bond dissociation energy, $E_D({}^{35}\text{Cl}_2)$, is given by the difference between the total energies of the two $\text{Cl}3sp^3$ HOs and $E_{T+osc}({}^{35}\text{Cl}_2)$:

$$5 \quad E_D({}^{35}\text{Cl}_2) = 2E_{\text{Coulomb}}(\text{Cl}, 3sp^3) - E_{T+osc}({}^{35}\text{Cl}_2) \tag{13.806}$$

$E_{\text{Coulomb}}(\text{Cl}, 3sp^3)$ is given by Eq. (13.759); thus, the ${}^{35}\text{Cl}_2$ bond dissociation energy, $E_D({}^{35}\text{Cl}_2)$, given by Eqs. (13.759) and (13.805-13.806) is

$$\begin{aligned}
 E_D({}^{35}\text{Cl}_2) &= -2(14.60295 \text{ eV}) - E_{T+osc}({}^{35}\text{Cl}_2) \\
 &= -29.20590 \text{ eV} - (-31.72826 \text{ eV}) \\
 &= 2.52236 \text{ eV}
 \end{aligned}
 \tag{13.807}$$

The experimental ${}^{35}\text{Cl}_2$ bond dissociation energy is [49]

$$10 \quad E_D({}^{35}\text{Cl}_2) = 2.51412 \text{ eV} \tag{13.808}$$

The results of the determination of bond parameters of Cl_2 are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

15

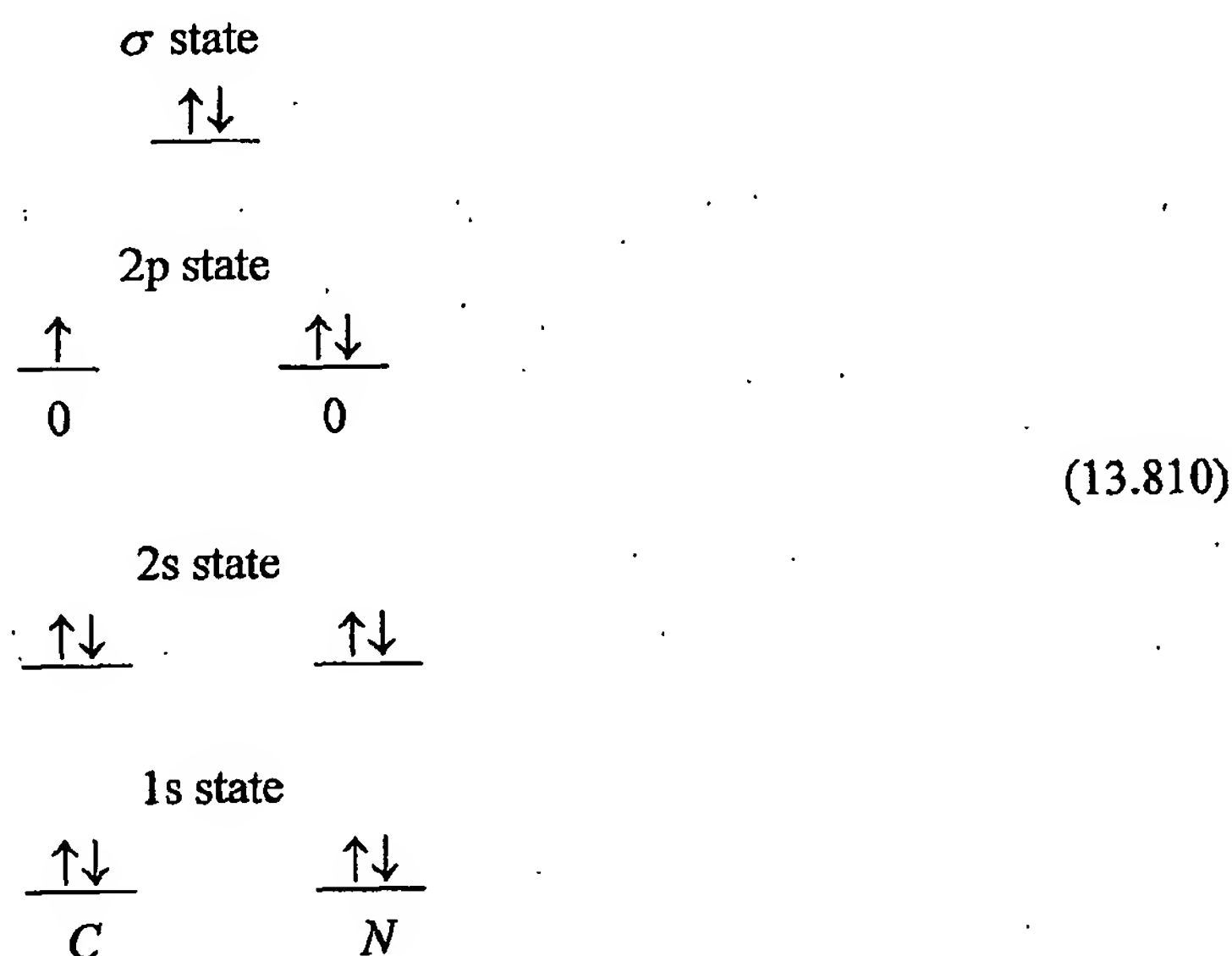
CARBON NITRIDE RADICAL

The carbon nitride radical can be formed by the reaction of carbon and nitrogen atoms:



The bond in carbon nitride radical comprises a H_2 -type molecular orbital (MO) with two
20 paired electrons. The force balance equations and radii, r_6 and r_7 , of the $2p$ shell of C and N are derived in the Six-Electron Atoms section and Seven-Electron Atoms section; respectively. With the formation of the H_2 -type MO by the contribution of a $2p$ electron from each of the C and N atoms, a diamagnetic force arises between the remaining $2p$ electrons of each atom and the H_2 -type MO. This force from each atom causes the H_2 -type

MO to move to greater principal axes than would result with the Coulombic force alone. But, the integer increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining $2p$ electrons of each atom decrease the radii of the corresponding shells such that the energy minimum is achieved that is lower than that of the 5 reactant atoms. The resulting electron configuration of CN is $C1s^2N1s^2C2s^2N2s^2C2p^1N2p^2\sigma_{C,N}^2$ where σ designates the H_2 -type MO, and the orbital arrangement is



10

The carbon nitride radical is predicted to be weakly paramagnetic .

FORCE BALANCE OF THE $2p$ SHELL OF THE CARBON ATOM OF THE CARBON NITRIDE RADICAL

15 For the C atom, force balance for the outermost $2p$ electron of CN (electron 5) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to interactions between electron 5 and the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Six-Electron Atoms section. The central Coulomb force on the outer-most $2p$ shell electron of CN (electron 5) due to the nucleus
20 and the inner four electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$F_{ele} = \frac{(Z-4)e^2}{4\pi\epsilon_0 r_5^2} \mathbf{i}_r \quad (13.811)$$

for $r > r_4$. The $2p$ shell possess an external electric field given by Eq. (10.92) for $r > r_5$.

The single unpaired carbon $2p$ electron gives rise to a diamagnetic force on the σ -MO as given by Eqs. (13.835-13.839). The corresponding Newtonian reaction force cancels $F_{diamagnetic}$, of Eq. (10.82). The energy is minimized with conservation of angular momentum.

5 This condition is met when

$$F_{diamagnetic} = 0 \quad (13.812)$$

And, $F_{mag\ 2}$ corresponding to the maximum orbital angular momentum of the three $2p$ orbitals given by Eq. (10.89) is

$$F_{mag\ 2} = \frac{1}{Z} \frac{3\hbar^2}{m_e r_5^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.813)$$

10 The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_5$ gives rise to a second diamagnetic force, $F_{diamagnetic\ 2}$, given by Eq. (10.93). $F_{diamagnetic\ 2}$ due to the binding of the p-orbital electron having an electric field of +1 outside of its radius is :

$$F_{diamagnetic\ 2} = - \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{m_e r_5^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (13.814)$$

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to
15 the sum of the electric (Eq. (13.811)) and diamagnetic (Eqs. (13.812) and (13.814)), and paramagnetic (Eq. (13.813)) forces as follows:

$$\frac{m_e v_5^2}{r_5} = \left(\frac{(Z-4)e^2}{4\pi\epsilon_0 r_5^2} + \frac{3\hbar^2}{Z m_e r_5^2 r_3} \sqrt{s(s+1)} - \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_5^4 m_e} 10 \sqrt{s(s+1)} \right) \quad (13.815)$$

Substitution of $v_5 = \frac{\hbar}{m_e r_5}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (13.815) gives:

$$\frac{\hbar^2}{m_e r_5^3} = \frac{(Z-4)e^2}{4\pi\epsilon_0 r_5^2} + \frac{3\hbar^2}{Z m_e r_5^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_5^4 m_e} 10 \sqrt{\frac{3}{4}} \quad (13.816)$$

20 The quadratic equation corresponding to Eq. (13.816) is

$$r_5^2 - \frac{\frac{\hbar^2}{m_e}}{\left(\frac{(Z-4)e^2}{4\pi\epsilon_0} + \frac{3\hbar^2}{Z m_e r_3} \sqrt{\frac{3}{4}} \right)} r_5 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-4)e^2}{4\pi\epsilon_0} + \frac{3\hbar^2}{Z m_e r_3} \sqrt{\frac{3}{4}} \right)} = 0$$

(13.817)

The solution of Eq. (13.817) using the quadratic formula is:

$$r_3 = \frac{a_0}{\left((Z-4) + \frac{3\sqrt{3}}{Z2r_3} \right)} \pm a_0 \frac{\left(\frac{1}{\left((Z-4) + \frac{3\sqrt{3}}{Z2r_3} \right)} \right)^2 + \frac{20\sqrt{3} \left(\left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)}{\left((Z-4) + \frac{3\sqrt{3}}{Z2r_3} \right)}}{2} \quad (13.818)$$

r_3 in units of a_0

The positive root of Eq. (13.818) must be taken in order that $r_3 > 0$. Substitution of

5 $\frac{r_3}{a_0} = 0.84317$ (Eq. (10.62) with $Z = 6$) into Eq. (13.818) gives

$$r_3 = 0.88084a_0 \quad (13.819)$$

FORCE BALANCE OF THE $2p$ SHELL OF THE NITROGEN ATOM OF THE CARBON NITRIDE RADICAL

- 10 For the N atom, force balance for the outermost $2p$ electron of CN (electron 6) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to interactions between electron 6 and the other $2p$ -shell as well as the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Seven-Electron Atoms section. The central Coulomb force on the outer-most $2p$ shell electron of CN
- 15 (electron 6) due to the nucleus and the inner five electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$\mathbf{F}_{ele} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} \mathbf{i}_r \quad (13.820)$$

for $r > r_5$. The $2p$ shell possess an external electric field given by Eq. (10.92) for $r > r_6$.

The forces to determine the radius of the $N2p$ shell of N in CN are the same as
 20 those of N in N_2 except that in CN there is a contribution from the Newtonian reaction force that arises from the single unpaired carbon $2p$ electron. The energy is minimized with

conservation of angular momentum. This condition is met when $F_{diamagnetic}$ of N in CN is canceled by the σ -MO-reaction force. Eq. (13.622) becomes

$$F_{diamagnetic} = 0 \quad (13.821)$$

And, $F_{mag\ 2}$ corresponding to the conserved orbital angular momentum of the three orbitals
5 given by Eq. (10.89) is

$$F_{mag\ 2} = \frac{1}{Z} \frac{3\hbar^2}{m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.822)$$

The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_6$ gives rise to a second diamagnetic force, $F_{diamagnetic\ 2}$, given by Eq. (10.93). $F_{diamagnetic\ 2}$ due to the binding of the p -orbital electron having an electric field of $+1$ outside of its radius is

$$10 \quad F_{diamagnetic\ 2} = - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{m_e r_6^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (13.823)$$

The N forces F_{ele} , $F_{mag\ 2}$, $F_{diamagnetic\ 2}$, and $F_{mag\ 3}$ of CN are the same as those of N_2 given by Eqs. (13.621) and (13.623-13.624), respectively. In both cases, the contribution of a $2p$ electron from the N atom in the formation of the σ MO gives rise to a paramagnetic force on the remaining two $2p$ electrons that pair. Thus, the force, $F_{mag\ 3}$ of CN , given by
15 Eq. (13.625) is

$$F_{mag\ 3} = \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.824)$$

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (13.820)) and diamagnetic (Eqs. (13.821) and (13.823)), and paramagnetic (Eqs. (13.822) and (13.824)) forces as follows:

$$20 \quad \frac{m_e v_6^2}{r_6} = \left(\frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} + \frac{3\hbar^2}{Zm_e r_6^2 r_3} \sqrt{s(s+1)} - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{s(s+1)} + \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \right) \quad (13.825)$$

Substitution of $v_6 = \frac{\hbar}{m_e r_6}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (13.626) gives:

$$\frac{\hbar^2}{m_e r_6^3} - \frac{\hbar^2}{4m_e r_6^3} \sqrt{\frac{3}{4}} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} + \frac{3\hbar^2}{Zm_e r_6^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{\frac{3}{4}} \quad (13.826)$$

The quadratic equation corresponding to Eq. (13.826) is

$$r_6^2 - \frac{\frac{\hbar^2}{m_e} \left(1 - \frac{\sqrt{3}}{8}\right)}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} + \frac{3\hbar^2}{Zm_e r_3} \sqrt{\frac{3}{4}}\right)} r_6 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) r_3 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} + \frac{3\hbar^2}{Zm_e r_3} \sqrt{\frac{3}{4}}\right)} = 0 \quad (13.827)$$

The solution of Eq. (13.827) using the quadratic formula is:

$$r_6 = \frac{a_0 \left(1 - \frac{\sqrt{3}}{8}\right)}{\left((Z-5) + \frac{3\sqrt{3}}{Z2r_3}\right)} \pm a_0 \sqrt{\left(\frac{1 - \frac{\sqrt{3}}{8}}{\left((Z-5) + \frac{3\sqrt{3}}{Z2r_3}\right)}\right)^2 + \frac{20\sqrt{3} \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) r_3}{\left((Z-5) + \frac{3\sqrt{3}}{Z2r_3}\right)}} \quad (13.828)$$

r_3 in units of a_0

The positive root of Eq. (13.828) must be taken in order that $r_6 > 0$. Substitution of

$\frac{r_3}{a_0} = 0.69385$ (Eq. (10.62) with $Z = 7$) into Eq. (13.828) gives

$$r_6 = 0.76366a_0 \quad (13.829)$$

10 ENERGIES OF THE $2p$ SHELLS OF THE CARBON AND NITROGEN ATOMS OF THE CARBON NITRIDE RADICAL

The central forces on the $2p$ shell of the C and N atoms are increased with the formation of the σ MO which reduces each shell's radius and increases its total energy. The Coulombic energy terms of the total energy of the C and N atoms at the new radii are calculated and added to the energy of the σ MO to give the total energy of CN . Then, the bond energy is determined from the total CN energy.

The radius r_6 of the carbon atom before bonding is given by Eq. (10.122):

$$r_6 = 1.20654a_0 \quad (13.830)$$

Using the initial radius r_6 of the C atom and the final radius r_3 of the $C2p$ shell of CN (Eq.

(13.819)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(CN, C2p)$ of the Coulombic energy change of the $C2p$ electron is determined using Eq. (10.102):

$$E_T(CN, C2p) = -\sum_{n=4}^4 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_5} - \frac{1}{r_6} \right) = -(13.60580 \text{ eV})(0.30647)(2) = -8.33948 \text{ eV}$$

5 (13.831)

The radius r_7 of the nitrogen atom before bonding is given by Eq. (10.142):

$$r_7 = 0.93084a_0 \quad (13.832)$$

Using the initial radius r_7 of the N atom and the final radius r_6 of the $N2p$ shell of CN (Eq. (13.829)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(CN, N2p)$ of the Coulombic energy change of the $N2p$ electron is determined using Eq. (10.102):

$$\begin{aligned} E_T(CN, N2p) &= -\sum_{n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_7} \right) \\ &= -(13.60580 \text{ eV})(0.23518)(2+3) \\ &= -15.99929 \text{ eV} \end{aligned} \quad (13.833)$$

FORCE BALANCE OF THE σ MO OF THE CARBON NITRIDE RADICAL

15 The diamagnetic force $F_{\text{diamagneticMO1}}$ for the σ -MO of the CN molecule due to the two paired electrons in the $N2p$ shell given by Eq. (13.633) with $n_e = 2$ is:

$$F_{\text{diamagneticMO1}} = \frac{\hbar^2}{2m_e a^2 b^2} D i_\xi \quad (13.834)$$

The force $F_{\text{diamagneticMO2}}$ is given by Eq. (13.634) except that the force is the summed over the individual diamagnetic-force terms due to each component of angular momentum $|L_i|$ acting on the electrons of the σ -MO from each atom having a nucleus of charge Z_j at one of the foci of the σ -MO:

$$F_{\text{diamagneticMO2}} = \sum_{i,j} \frac{|L_i| \hbar}{Z_j 2m_e a^2 b^2} D i_\xi \quad (13.835)$$

Using Eqs. (11.200), (13.633-13.634), and (13.834-13.835), the force balance for the σ -MO of the carbon nitride radical comprising carbon with charge $Z_1 = 6$ and $|L_1| = \hbar$ and

$|L_2| = \sqrt{\frac{3}{4}}\hbar$ and nitrogen with $Z_2 = 7$ and $|L_3| = \hbar$ is

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.836)$$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D - \left(\frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.837)$$

$$\left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D \quad (13.838)$$

$$5 \quad a = \left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) a_0 \quad (13.839)$$

Substitution of $Z_1 = 6$ and $Z_2 = 7$ into Eq. (13.839) gives

$$a = 2.45386a_0 = 1.29853 \times 10^{-10} \text{ m} \quad (13.840)$$

Substitution of Eq. (13.840) into Eq. (11.79) is

$$c' = 1.10767a_0 = 5.86153 \times 10^{-11} \text{ m} \quad (13.841)$$

10 The internuclear distance given by multiplying Eq. (13.841) by two is

$$2c' = 2.21534a_0 = 1.17231 \times 10^{-10} \text{ m} \quad (13.842)$$

The experimental bond distance from Ref. [28] is

$$2c' = 1.17181 \times 10^{-10} \text{ m} \quad (13.843)$$

Substitution of Eqs. (13.840-13.841) into Eq. (11.80) is

$$15 \quad b = c = 2.18964a_0 = 1.15871 \times 10^{-10} \text{ m} \quad (13.844)$$

Substitution of Eqs. (13.840-13.841) into Eq. (11.67) is

$$e = 0.45140 \quad (13.845)$$

Using the electron configuration of CN (Eq. (13.810)), the radii of the $C1s = 0.17113a_0$ (Eq.

(10.51)), $C2s = 0.84317a_0$ (Eq. (10.62)), $C2p = 0.88084a_0$ (Eq. (13.819)), $N1s = 0.14605a_0$

20 (Eq. (10.51)), $N2s = 0.69385a_0$ (Eq. (10.62)), and $N2p = 0.76366a_0$ (Eq. (13.829)) shells

and the parameters of the σ MO of CN given by Eqs. (13.3-13.4), (13.840-13.842), and (13.844-13.845), the dimensional diagram and charge-density of the CN MO are shown in Figures 27 and 28, respectively.

5 SUM OF THE ENERGIES OF THE σ MO AND THE AOs OF THE CARBON NITRIDE RADICAL

The energies of the CN σ MO are given by the substitution of the semiprincipal axes (Eqs. (13.840-13.841) and (13.844)) into the energy equations (Eqs. (11.207-11.212)) of H_2 :

$$V_e = \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -23.90105 \text{ eV} \quad (13.846)$$

$$10 \quad V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 12.28328 \text{ eV} \quad (13.847)$$

$$T = \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 4.87009 \text{ eV} \quad (13.848)$$

$$V_m = \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -2.43504 \text{ eV} \quad (13.849)$$

$$E_T = V_e + T + V_m + V_p \quad (13.850)$$

Substitution of Eqs. (11.79) and (13.846-13.849) into Eq. (13.850) gives

$$15 \quad E_T(CN, \sigma) = \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(\left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right) = -9.18273 \text{ eV} \quad (13.851)$$

where $E_T(CN, \sigma)$ is the total energy of the σ MO of CN. The sum, $E_T(CN)$, of $E_T(CN, C2p)$, the $C2p$ AO contribution given by Eq. (13.831), $E_T(CN, N2p)$, the $N2p$ AO contribution given by Eq. (13.833), and $E_T(CN, \sigma)$, the σ MO contribution given by Eq. (13.851) is:

$$\begin{aligned} E_T(CN) &= E_T(CN, C2p) + E_T(CN, N2p) + E_T(CN, \sigma) \\ 20 \quad &= -8.33948 \text{ eV} - 15.99929 \text{ eV} - 9.18273 \text{ eV} \\ &= -33.52149 \text{ eV} \end{aligned} \quad (13.852)$$

VIBRATION OF *CN*

The vibrational energy levels of *CN* may be solved by determining the Morse potential curve from the energy relationships for the transition from a *C* atom and *N* atom whose parameters are given by Eqs. (10.115-10.123) and (10.134-10.143), respectively, to a *C* atom
 5 whose parameter r_s is given by Eq. (10.819), a *N* atom whose parameter r_g is given by Eq. (13.829), and the σ MO whose parameters are given by Eqs. (13.840-13.842) and (13.844-13.845). As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter
 10 (K&P) [15] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods.

THE DOPPLER ENERGY TERMS OF THE CARBON NITRIDE RADICAL

15 The equations of the radiation reaction force of *CN* are the same as those of H_2 with the substitution of the *CN* parameters. Using Eqs. (11.231-11.233), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{e^2}{4\pi\epsilon_0 a^3}} = 1.07550 \times 10^{16} \text{ rad/s} \quad (13.853)$$

where a is given by Eq. (13.840). The kinetic energy, E_K , is given by Planck's equation
 20 (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 1.07550 \times 10^{16} \text{ rad/s} = 7.07912 \text{ eV} \quad (13.854)$$

In Eq. (11.181), substitution of $E_T(CN)$ for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.854) for \bar{E}_K gives the Doppler energy of the electrons of the reentrant orbit:

$$25 \quad \bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -33.59603 \text{ eV} \sqrt{\frac{2e(7.07912 \text{ eV})}{m_e c^2}} = -0.17684 \text{ eV} \quad (13.855)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of the *CN* MO due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation

of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.855) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy. Using the experimental CN ω_e of 2068.59 cm^{-1} (0.25647 eV) [28] for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(CN)$ is

$$5 \quad \bar{E}_{osc}(CN) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.856)$$

$$\bar{E}_{osc}(CN) = -0.17684 \text{ eV} + \frac{1}{2} (0.25647 \text{ eV}) = -0.04860 \text{ eV} \quad (13.857)$$

TOTAL AND BOND ENERGIES OF THE CARBON NITRIDE RADICAL

$E_{T+osc}(CN)$, the total energy of CN including the Doppler term, is given by the sum of

10 $E_T(CN)$ (Eq. (13.852)) and $\bar{E}_{osc}(CN)$ given by Eq. (13.857):

$$\begin{aligned} E_{T+osc}(CN) &= V_e + T + V_m + V_p + E_T(CN, C2p) + E_T(CN, N2p) + \bar{E}_{osc}(CN) \\ &= E_T(CN, \sigma) + E_T(CN, C2p) + E_T(CN, N2p) + \bar{E}_{osc}(CN) \\ &= E_T(CN) + \bar{E}_{osc}(CN) \end{aligned} \quad (13.858)$$

$$\begin{aligned} E_{T+osc}(CN) &= \left\{ \left[\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left(\left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right) \right. \right. \\ &\quad \left. \left. - \sum_{C,n=4}^4 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_5} - \frac{1}{r_6} \right) - \sum_{N,n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_7} \right) \right] \right. \\ &\quad \left. \left[1 + \sqrt{\frac{\frac{e^2}{4\pi\epsilon_0 a^3}}{m_e c^2}} \right] + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} \\ &= -33.52149 \text{ eV} - 0.17684 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \end{aligned} \quad (13.859)$$

From Eqs. (13.856-13.859), the total energy of the CN MO is

$$\begin{aligned} E_{T+osc}(CN) &= -33.52149 \text{ eV} + \bar{E}_{osc}(CN) \\ &= -33.52149 \text{ eV} - 0.17684 \text{ eV} + \frac{1}{2} (0.25647 \text{ eV}) \\ &= -33.56970 \text{ eV} \end{aligned} \quad (13.860)$$

268

where the experimental ω_e was used for the $\hbar\sqrt{\frac{k}{\mu}}$ term.

The CN bond dissociation energy, $E_D(CN)$, is given by the difference between the sum of the energies of the C and N atoms and $E_{T+osc}(CN)$:

$$E_D(CN) = E(C) + E(N) - E_{T+osc}(CN) \quad (13.861)$$

5 where the energy of a carbon atom is [6]

$$E(C) = -11.26030 \text{ eV} \quad (13.862)$$

and the energy of a nitrogen atom is [6]

$$E(N) = -14.53414 \text{ eV} \quad (13.863)$$

Thus, the CN bond dissociation energy, $E_D(CN)$, given by Eqs. (13.860-13.863) is

$$\begin{aligned} E_D(CN) &= -(11.26030 \text{ eV} + 14.53414 \text{ eV}) - E_{T+osc}(CN) \\ 10 \quad &= -25.79444 \text{ eV} - (-33.56970 \text{ eV}) \\ &= 7.77526 \text{ eV} \end{aligned} \quad (13.864)$$

The experimental CN bond dissociation energy is [50]

$$E_{D298}(CN) = 7.7731 \text{ eV} \quad (13.865)$$

The results of the determination of bond parameters of CN are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations
15 containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

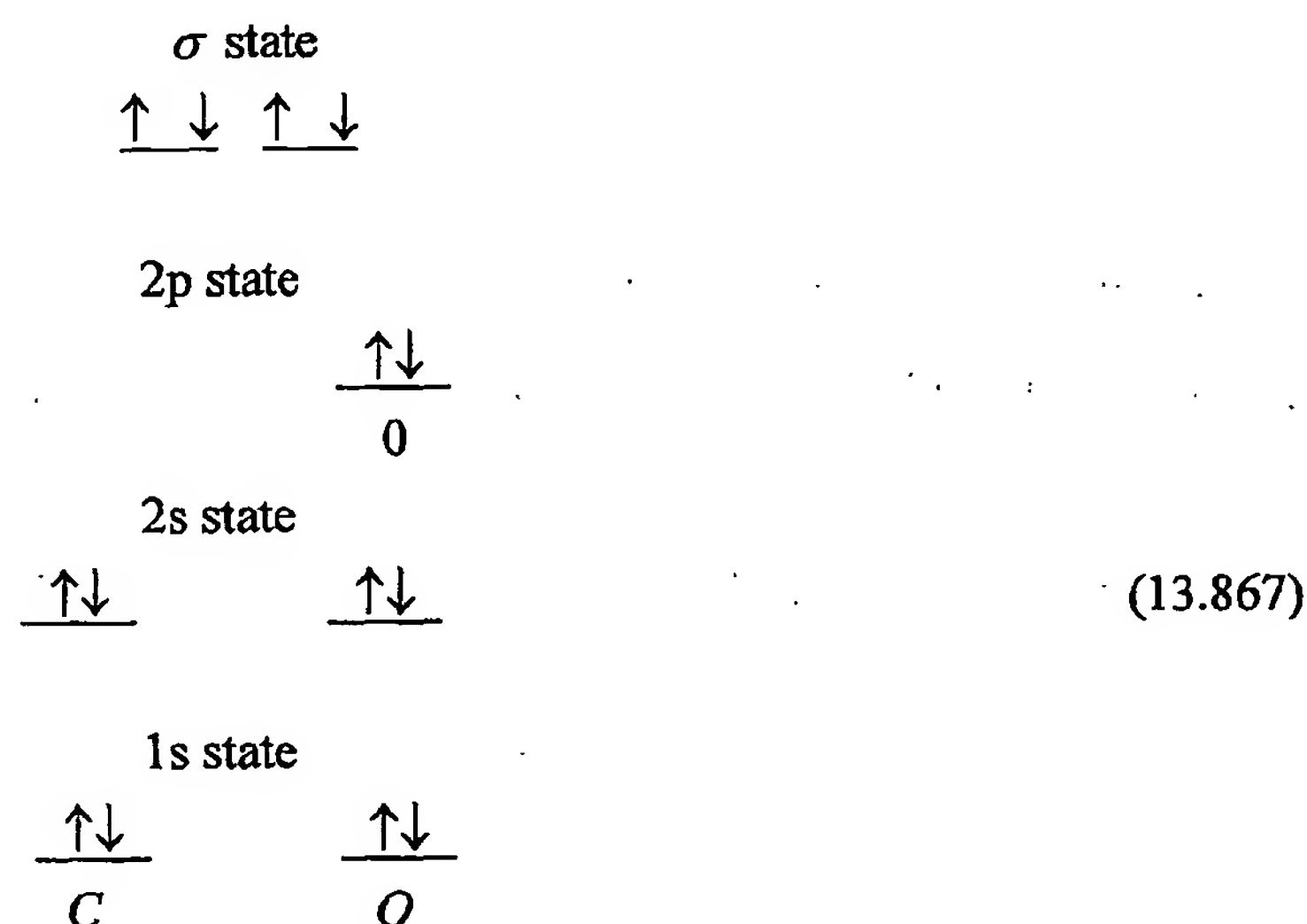
CARBON MONOXIDE MOLECULE

The carbon monoxide molecule can be formed by the reaction of carbon and oxygen atoms:



The bond in the carbon monoxide molecule comprises a double bond, a H_2 -type molecular orbital (MO) with four paired electrons. The force balance equation and radius r_6 of the $2p$ shell of C is derived in the Six-Electron Atoms section. The force balance equation and radius r_8 of the $2p$ shell of O is derived in the Eight-Electron Atoms section. With the
25 formation of the H_2 -type MO by the contribution of two $2p$ electrons from each of the C and O atoms, a diamagnetic force arises between the remaining outer shell atomic electrons, the $2s$ electrons of C and the $2p$ electrons of O , and the H_2 -type MO. This force from C

and O causes the H_2 -type MO to move to greater principal axes than would result with the Coulombic force alone. But, the factor of two increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining $O2p$ electrons decrease the radius of the corresponding shell such that the energy minimum is achieved that is lower than that of the reactant atoms. The resulting electron configuration of CO is $C1s^2O1s^2C2s^2O2s^2O2p^2\sigma_{C,O}^4$ where σ designates the H_2 -type MO, and the orbital arrangement is



Carbon monoxide is predicted to be diamagnetic in agreement with observations [42].

10

FORCE BALANCE OF THE $2p$ SHELL OF THE OXYGEN ATOM OF THE CARBON MONOXIDE MOLECULE

For the O atom, force balance for the outermost $2p$ electron of CO (electron 6) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to interactions between electron 6 and the other $2p$ electron as well as the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Eight-Electron Atoms section. The central Coulomb force on the outer-most $2p$ shell electron of CO (electron 6) due to the nucleus and the inner five electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$20 \quad F_{ele} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} \mathbf{i}_r \quad (13.868)$$

for $r > r_5$. The $2p$ shell possess a +2 external electric field given by Eq. (10.92) for $r > r_6$. The energy is minimized with conservation of angular momentum. This condition is met when the diamagnetic force, $F_{\text{diamagnetic}}$, of Eq. (10.82) due to the p -orbital contribution is given by:

$$5 \quad F_{\text{diamagnetic}} = -\left(\frac{1}{3}\right) \frac{\hbar^2}{4m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (13.869)$$

And, $F_{\text{mag } 2}$ corresponding to the conserved spin and orbital angular momentum given by Eq. (10.157) is

$$F_{\text{mag } 2} = \frac{1}{Z} \frac{2\hbar^2}{m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.870)$$

The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_6$ gives rise to a second diamagnetic force, $F_{\text{diamagnetic } 2}$, given by Eq. (10.93). $F_{\text{diamagnetic } 2}$ due to the binding of the p -orbital electron having an electric field of +2 outside of its radius is :

$$F_{\text{diamagnetic } 2} = -\left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_6^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (13.871)$$

In addition, the contribution of two $2p$ electrons in the formation of the σ molecular orbital (MO) gives rise to a paramagnetic force on the remaining paired $2p$ electrons. The force $F_{\text{mag } 3}$ is given by Eq. (13.625) wherein the radius is r_6 :

$$F_{\text{mag } 3} = \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.872)$$

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (13.868)) and diamagnetic (Eqs. (13.869) and (13.871)), and paramagnetic (Eqs. (13.870) and (13.872)) forces as follows:

$$20 \quad \frac{m_e v_6^2}{r_6} = \left[\frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{\hbar^2}{12m_e r_6^2 r_3} \sqrt{s(s+1)} + \frac{2\hbar^2}{Zm_e r_6^2 r_3} \sqrt{s(s+1)} \right. \\ \left. - \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{s(s+1)} + \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \right] \quad (13.873)$$

Substitution of $v_6 = \frac{\hbar}{m_e r_6}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (13.873) gives:

$$\frac{\hbar^2}{m_e r_6^3} - \frac{\hbar^2}{4m_e r_6^3} \sqrt{\frac{3}{4}} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} + \frac{2\hbar^2}{Zm_e r_6^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{\frac{3}{4}}$$

(13.874)

The quadratic equation corresponding to Eq. (13.874) is

$$r_6^2 - \frac{\frac{\hbar^2}{m_e} \left(1 - \frac{\sqrt{3}}{8}\right)}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{1}{12} - \frac{2}{Z}\right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}}\right)} r_6 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{1}{12} - \frac{2}{Z}\right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}}\right)} = 0 \quad (13.875)$$

The solution of Eq. (13.875) using the quadratic formula is:

$$r_6 = \frac{a_0 \left(1 - \frac{\sqrt{3}}{8}\right)}{\left((Z-5) - \left(\frac{1}{12} - \frac{2}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)} \pm a_0 \sqrt{\frac{\left(\frac{1 - \frac{\sqrt{3}}{8}}{\left((Z-5) - \left(\frac{1}{12} - \frac{2}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)}\right)^2 - 20\sqrt{3} \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) r_3}{\left((Z-5) - \left(\frac{1}{12} - \frac{2}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)^2}} \quad (13.876)$$

r_3 in units of a_0

The positive root of Eq. (13.876) must be taken in order that $r_6 > 0$. Substitution of

$\frac{r_3}{a_0} = 0.59020$ (Eq. (10.62) with $Z = 8$) into Eq. (13.876) gives

$$r_6 = 0.68835a_0 \quad (13.877)$$

10 ENERGIES OF THE $2s$ AND $2p$ SHELLS OF THE CARBON ATOM AND THE $2p$ SHELL OF THE OXYGEN ATOM OF THE CARBON MONOXIDE MOLECULE

With the formation of the H_2 -type MO by the contribution of two $2p$ electrons from the C atom, the remaining outer-shell atomic electrons comprise the $2s$ electrons, which are unchanged by bonding with oxygen. However, the total energy of the CO molecule, which is subtracted from the sum of the energies of the carbon and oxygen atoms to determine the bond energy, is increased by the ionization energies of C^+ and O^+ given by Eqs. (10.113-10.114) and (10.152-10.153), respectively. Experimentally, the energies are [6]

$$E(\text{ionization}; C^+) = 24.38332 \text{ eV} \quad (13.878)$$

$$E(\text{ionization}; O^+) = 35.11730 \text{ eV} \quad (13.879)$$

In addition, the central forces on the $2p$ shell of the O atom are increased with the formation of the σ MO, which reduces the shell's radius and increases its total energy. The Coulombic energy terms of the total energy of the O atom at the new radius are calculated and added to the ionization energies of C^+ and O^+ , and the energy of the σ MO to give the total energy of CO . Then, the bond energy is determined from the total CO energy.

The radius r_8 of the oxygen atom before bonding is given by Eq. (10.162):

$$r_8 = a_0 \quad (13.880)$$

Using the initial radius r_8 of the O atom and the final radius r_6 of the $O2p$ shell (Eq. (13.877)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(O, 2p)$ of the Coulombic energy change of the $O2p$ electrons of the O atom is determined using Eq. (10.102):

$$\begin{aligned} E_T(O, 2p) &= -\sum_{n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_8} \right) \\ &= -(13.60580 \text{ eV})(0.45275)(3+4) \\ &= -43.11996 \text{ eV} \end{aligned} \quad (13.881)$$

15 FORCE BALANCE OF THE σ MO OF THE CARBON MONOXIDE MOLECULE

The force balance can be considered due to a second pair of two electrons binding to a molecular ion having $+2e$ at each focus and a first bound pair. Then, the forces are the same as those of a molecule ion having $+e$ at each focus. The diamagnetic force $F_{\text{diamagneticMO1}}$ for the σ -MO of the CO molecule due to the two paired electrons in each of the $C2s$ and $O2p$ shells is given by Eq. (13.633) with $n_e = 2$:

$$F_{\text{diamagneticMO1}} = \frac{\hbar^2}{2m_e a^2 b^2} D i_\xi \quad (13.882)$$

The force $F_{\text{diamagneticMO2}}$ is given by Eqs. (13.634) and (13.835) as the sum of the contributions due to carbon with $Z = Z_1$ and oxygen with $Z = Z_2$. $F_{\text{diamagneticMO1}}$ for CO with $|L_i| = \hbar$ is

$$F_{\text{diamagneticMO2}} = \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D i_\xi \quad (13.883)$$

The force balance equation for the σ -MO of the carbon monoxide molecule given by Eqs. (11.200), (13.633-13.634), and (13.882-13.883) is

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)\right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.884)$$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D - \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.885)$$

$$5 \quad \left(2 + \frac{1}{Z_1} + \frac{1}{Z_2}\right) \frac{\hbar^2}{2m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D \quad (13.886)$$

$$a = \left(2 + \frac{1}{Z_1} + \frac{1}{Z_2}\right) a_0 \quad (13.887)$$

Substitution of $Z_1 = 6$ and $Z_2 = 8$ into Eq. (13.887) gives

$$a = 2.29167 a_0 = 1.21270 \times 10^{-10} \text{ m} \quad (13.888)$$

Substitution of Eq. (13.888) into Eq. (11.79) is

$$10 \quad c' = 1.07044 a_0 = 5.66450 \times 10^{-11} \text{ m} \quad (13.889)$$

The internuclear distance given by multiplying Eq. (13.889) by two is

$$2c' = 2.14087 a_0 = 1.13290 \times 10^{-10} \text{ m} \quad (13.890)$$

The experimental bond distance is [28]

$$2c' = 1.12823 \times 10^{-10} \text{ m} \quad (13.891)$$

15 Substitution of Eqs. (13.888-13.889) into Eq. (11.80) is

$$b = c = 2.02630 a_0 = 1.07227 \times 10^{-10} \text{ m} \quad (13.892)$$

Substitution of Eqs. (13.888-13.889) into Eq. (11.67) is

$$e = 0.46710 \quad (13.893)$$

Using the electron configuration of CO (Eq. (13.867)), the radii of the $Cl_s = 0.17113 a_0$ (Eq. (10.51)), $C2s = 0.84317 a_0$ (Eq. (10.62)), $O1s = 0.12739 a_0$ (Eq. (10.51)), $O2s = 0.59020 a_0$ (Eq. (10.62)), and $O2p = 0.68835 a_0$ (Eq. (13.877)) shells and the parameters of the σ MO of CO given by Eqs. (13.3-13.4), (13.888-13.890), and (13.892-13.893), the dimensional diagram and charge-density of the CO MO are shown in Figures 29 and 30, respectively.

SUM OF THE ENERGIES OF THE σ MO AND THE AOs OF THE CARBON MONOXIDE MOLECULE

The energies of the CO σ MO are given by the substitution of the semiprincipal axes (Eqs. 5 (13.888-13.889) and (13.892)) into the energy equations (Eqs. (11.207-11.212)) of H_2 except that the terms based on charge are multiplied by four and the kinetic energy term is multiplied by two due to the σ -MO double bond with two pairs of paired electrons:

$$V_e = 2^2 \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -102.97635 \text{ eV} \quad (13.894)$$

$$V_p = 2^2 \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 50.84210 \text{ eV} \quad (13.895)$$

$$10 \quad T = 2 \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 11.23379 \text{ eV} \quad (13.896)$$

$$V_m = 2^2 \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -11.23379 \text{ eV} \quad (13.897)$$

$$E_T = V_e + T + V_m + V_p \quad (13.898)$$

Substitution of Eqs. (11.79) and (13.894-13.897) into Eq. (13.898) gives

$$E_T(CO, \sigma) = \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(8 \ln \frac{a+\sqrt{\frac{aa_0}{2}}}{a-\sqrt{\frac{aa_0}{2}}} - 4 \right) = -52.13425 \text{ eV} \quad (13.899)$$

15 where $E_T(CO, \sigma)$ is the total energy of the σ MO of CO . The total energy of CO , $E_T(CO)$, is given by the sum of $E(\text{ionization}; C^+)$, the energy of the second electron of carbon (Eq. (13.878)) donated to the double bond, $E(\text{ionization}; O^+)$, the energy of the second electron of oxygen (Eq. (13.879)) donated to the double bond, $E_T(O, 2p)$, the $O2p$ AO contribution due to the decrease in radius with bond formation (Eq. (13.881)), and
20 $E_T(CO, \sigma)$, the σ MO contribution given by Eq. (13.899):

$$\begin{aligned} E_T(CO) &= E(\text{ionization}; C^+) + E(\text{ionization}; O^+) + E_T(O, 2p) + E_T(CO, \sigma) \\ &= 24.38332 \text{ eV} + 35.11730 \text{ eV} - 43.11996 \text{ eV} - 52.13425 \text{ eV} \\ &= -35.75359 \text{ eV} \end{aligned} \quad (13.900)$$

VIBRATION OF CO

The vibrational energy levels of CO may be solved by determining the Morse potential curve from the energy relationships for the transition from a C atom and O atom whose parameters are given by Eqs. (10.115-10.123) and (10.154-10.163), respectively, to a C atom
 5 whose parameter r_c is given by Eq. (10.61), an O atom whose parameter r_o is given by Eq. (13.877), and the σ MO whose parameters are given by Eqs. (13.888-13.890) and (13.892-13.893). As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter
 10 (K&P) [15] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods.

THE DOPPLER ENERGY TERMS OF THE CARBON MONOXIDE 15 MOLECULE

The equations of the radiation reaction force of carbon monoxide are the same as those of H_2 with the substitution of the CO parameters except that there is a factor of four increase in the central force in Eq. (11.231) due to the double bond. Using Eqs. (11.231-11.233), the angular frequency of the reentrant oscillation in the transition state is

$$20 \quad \omega = \sqrt{\frac{4e^2}{4\pi\epsilon_0 a^3}} = 2.38335 \times 10^{16} \text{ rad/s} \quad (13.901)$$

where a is given by Eq. (13.888). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.38335 \times 10^{16} \text{ rad/s} = 15.68762 \text{ eV} \quad (13.902)$$

In Eq. (11.181), substitution of $E_T(CO)$ for E_{hv} , the mass of the electron, m_e , for M , and
 25 the kinetic energy given by Eq. (13.902) for \bar{E}_K gives the Doppler energy of the electrons of the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -35.75359 \text{ eV} \sqrt{\frac{2e(15.68762 \text{ eV})}{m_e c^2}} = -0.28016 \text{ eV} \quad (13.903)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of the CO MO due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.903) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy. Using the experimental CO ω_e of 2169.81 cm^{-1} (0.26902 eV) [28] for \bar{E}_{Kvib} of the transition state, $\bar{E}'_{osc}(CO)$ per bond is

$$\bar{E}'_{osc}(CO) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.904)$$

$$\bar{E}'_{osc}(CO) = -0.28016\text{ eV} + \frac{1}{2}(0.26902\text{ eV}) = -0.14564\text{ eV} \quad (13.905)$$

10 Since the σ MO bond is a double bond with twice as many electrons as a single bond, $\bar{E}'_{osc}(CO)$ is multiplied by two to give

$$\bar{E}_{osc}(CO) = -0.29129\text{ eV} \quad (13.906)$$

TOTAL AND BOND ENERGIES OF THE CARBON MONOXIDE 15 MOLECULE

$E_{T+osc}(CO)$, the total energy of CO including the Doppler term, is given by the sum of $E_T(CO)$ (Eq. (13.900)) and $\bar{E}_{osc}(CO)$ given by Eq. (13.906):

$$\begin{aligned} E_{T+osc}(CO) &= \left(V_e + T + V_m + V_p + E(\text{ionization}; C^+) \right. \\ &\quad \left. + E(\text{ionization}; O^+) + E_T(O, 2p) + \bar{E}_{osc}(CO) \right) \\ &= \left(E_T(CO, \sigma) + E(\text{ionization}; C^+) + E(\text{ionization}; O^+) \right. \\ &\quad \left. + E_T(O, 2p) + \bar{E}_{osc}(CO) \right) \\ &= E_T(CO) + \bar{E}_{osc}(CO) \end{aligned} \quad (13.907)$$

$$\begin{aligned}
E_{T+osc}(CO) &= \left\{ \begin{aligned} & \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(8 \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 4 \right) + E(\text{ionization}; C^+) \\ & + E(\text{ionization}; O^+) - \sum_{O,n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_8} \right) \\ & \left(1 + 2 \sqrt{\frac{2\hbar\sqrt{\frac{4e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right) + 2 \left(\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \end{aligned} \right\} \quad (13.908) \\
&= -35.75359 \text{ eV} - 2(0.28016 \text{ eV}) + 2 \left(\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)
\end{aligned}$$

From Eqs. (13.906-13.908), the total energy of the CO MO is

$$\begin{aligned}
E_{T+osc}(CO) &= -35.75359 \text{ eV} + \bar{E}_{osc}(CO) \\
&= -35.75359 \text{ eV} + (-0.29129 \text{ eV}) \\
&= -36.04488 \text{ eV}
\end{aligned} \quad (13.909)$$

where the experimental ω_e was used for the $\hbar\sqrt{\frac{k}{\mu}}$ term.

- 5 The CO bond dissociation energy, $E_D(CO)$, is given by the difference between the sum of the energies of the C and O atoms and $E_{T+osc}(CO)$:

$$E_D(CO) = E(C) + E(O) - E_{T+osc}(CO) \quad (13.910)$$

where the energy of a carbon atom is [6]

$$E(C) = -11.26030 \text{ eV} \quad (13.911)$$

- 10 and the energy of an oxygen atom is [6]

$$E(O) = -13.61806 \text{ eV} \quad (13.912)$$

Thus, the CO bond dissociation energy, $E_D(CO)$, given by Eqs. (13.909-13.912) is

$$\begin{aligned}
E_D(CO) &= -(11.26030 \text{ eV} + 13.61806 \text{ eV}) - E_{T+osc}(CO) \\
&= -24.87836 \text{ eV} - (-36.04488 \text{ eV}) \\
&= 11.16652 \text{ eV}
\end{aligned} \quad (13.913)$$

The experimental CO bond dissociation energy is [49]

$$15 \quad E_{D298}(CO) = 11.15696 \text{ eV} \quad (13.914)$$

The results of the determination of bond parameters of CO are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

5

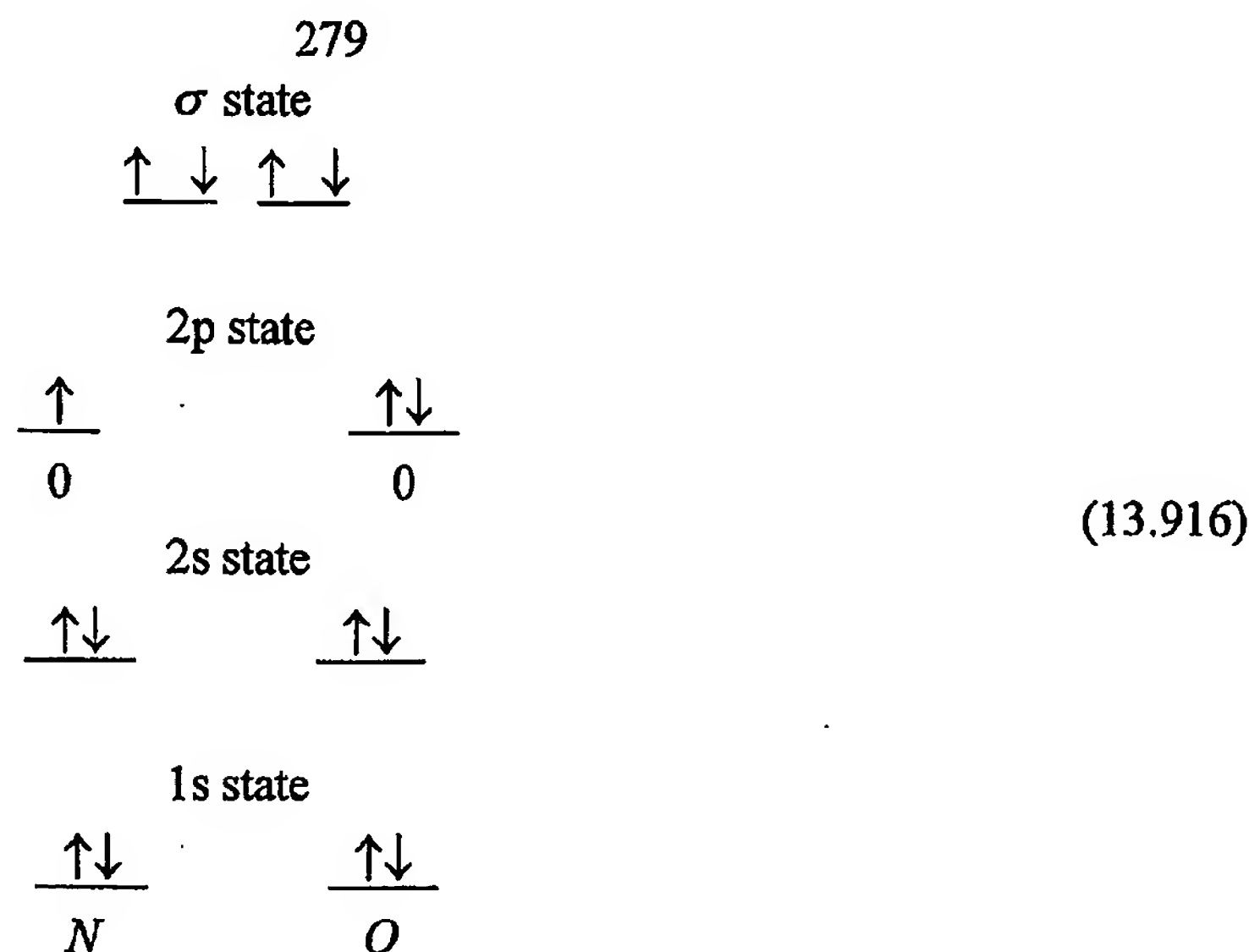
NITRIC OXIDE RADICAL

The nitric oxide radical can be formed by the reaction of nitrogen and oxygen atoms:



The bond in the nitric oxide radical comprises a double bond, a H_2 -type molecular orbital (MO) with four paired electrons. The force balance equation and radius r_7 of the $2p$ shell of N is derived in the Seven-Electron Atoms section. The force balance equation and radius r_8 of the $2p$ shell of O is derived in the Eight-Electron Atoms section. With the formation of the H_2 -type MO by the contribution of two $2p$ electrons from each of the N and O atoms, a diamagnetic force arises between the remaining outer shell atomic electrons, the $2s$ and $2p$ electrons of N and O , and the H_2 -type MO. This force from N and O causes the H_2 -type MO to move to greater principal axes than would result with the Coulombic force alone. But, the factor of two increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining N and O electrons decrease the radii of the corresponding shells such that the energy minimum is achieved that is lower than that of the reactant atoms. The resulting electron configuration of NO is $N1s^2O1s^2N2s^2O2s^2N2p^1O2p^2\sigma_{N,O}^4$ where σ designates the H_2 -type MO, and the orbital arrangement is

10
15
20



Nitric oxide is predicted to be weakly paramagnetic in agreement with observations [42].

FORCE BALANCE OF THE $2p$ SHELL OF THE NITROGEN ATOM OF 5 THE NITRIC OXIDE RADICAL

For the N atom, force balance for the outermost $2p$ electron of NO (electron 5) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to interactions between electron 5 and the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Seven-Electron Atoms section. The central
10 Coulomb force on the outer-most $2p$ shell electron of NO (electron 5) due to the nucleus and the inner four electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$\mathbf{F}_{ele} = \frac{(Z-4)e^2}{4\pi\epsilon_0 r_5^2} \mathbf{i}_r \quad (13.917)$$

for $r > r_4$. The $2p$ shell possess a +2 external electric field given by Eq. (10.92) for $r > r_5$. The energy is minimized with conservation of angular momentum. This condition is met
15 when the magnetic forces of N in NO are the same as those of N in the nitrogen molecule with r_5 replacing r_6 and with an increase of the central field by an integer. The diamagnetic force, $\mathbf{F}_{diamagnetic}$, of Eq. (10.82) due to the p -orbital contribution is given by Eq. (13.622) with r_5 replacing r_6 :

$$\mathbf{F}_{diamagnetic} = -\left(\frac{1}{3}\right) \frac{\hbar^2}{4m_e r_5^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{\hbar^2}{12m_e r_5^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (13.918)$$

20 And, $\mathbf{F}_{mag\ 2}$ corresponding to the conserved orbital angular momentum of the three orbitals is

also the same as that of N_2 given by Eq. (13.623) with r_5 replacing r_6 :

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{3\hbar^2}{m_e r_5^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.919)$$

The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_5$ gives rise to a second diamagnetic force, $\mathbf{F}_{diamagnetic\ 2}$, given by Eq. (10.93). $\mathbf{F}_{diamagnetic\ 2}$ due to the binding of the p -orbital electron having an electric field of $+2$ outside of its radius follows from Eq. (13.624):

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-5}{Z-4}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_5^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (13.920)$$

In addition to the N forces \mathbf{F}_{ele} , $\mathbf{F}_{diamagnetic}$, $\mathbf{F}_{mag\ 2}$, and $\mathbf{F}_{diamagnetic\ 2}$ of NO being the same as N_2 given by Eqs. (13.621-13.624), respectively, \mathbf{F}_{ele} , $\mathbf{F}_{mag\ 2}$, and $\mathbf{F}_{diamagnetic\ 2}$ are also the same as those of CN (Eqs. (13.820) and (13.822-13.823)). In the N_2 and CN cases, the contribution of a $2p$ electron from the N atom in the formation of the σ MO gives rise to an additional paramagnetic force on the remaining two $2p$ electrons that pair. However, the force, $\mathbf{F}_{mag\ 3}$, is absent in NO since the single outer electron is unpaired.

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (13.917)) and diamagnetic (Eqs. (13.918) and (13.920)), and paramagnetic (Eq. (13.919)) forces as follows:

$$\frac{m_e v_5^2}{r_5} = \left(\frac{(Z-4)e^2}{4\pi\epsilon_0 r_5^2} - \frac{\hbar^2}{12m_e r_5^2 r_3} \sqrt{s(s+1)} + \frac{3\hbar^2}{Zm_e r_5^2 r_3} \sqrt{s(s+1)} - \left[\frac{Z-5}{Z-4}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{r_5^4 m_e} 10 \sqrt{s(s+1)} \right) \quad (13.921)$$

Substitution of $v_5 = \frac{\hbar}{m_e r_5}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (13.921) gives:

$$\frac{\hbar^2}{m_e r_5^3} = \frac{(Z-4)e^2}{4\pi\epsilon_0 r_5^2} - \frac{\hbar^2}{12m_e r_5^2 r_3} \sqrt{\frac{3}{4}} + \frac{3\hbar^2}{Zm_e r_5^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-5}{Z-4}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{r_5^4 m_e} 10 \sqrt{\frac{3}{4}} \quad (13.922)$$

The quadratic equation corresponding to Eq. (13.922) is

$$r_s^2 - \frac{\frac{\hbar^2}{m_e}}{\left(\frac{(Z-4)e^2}{4\pi\epsilon_0} - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} r_s - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-4)e^2}{4\pi\epsilon_0} - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} = 0 \quad (13.923)$$

The solution of Eq. (13.923) using the quadratic formula is:

$$r_s = \frac{\frac{a_0}{\left((Z-4) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \pm a_0 \sqrt{\frac{1}{\left((Z-4) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)^2} + \frac{20\sqrt{3} \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{\left((Z-4) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)}}}{2} \quad (13.924)$$

r_3 in units of a_0

5 The positive root of Eq. (13.924) must be taken in order that $r_s > 0$. Substitution of

$\frac{r_3}{a_0} = 0.69385$ (Eq. (10.62) with $Z = 7$) into Eq. (13.924) gives

$$r_s = 0.74841 a_0 \quad (13.925)$$

FORCE BALANCE OF THE $2p$ SHELL OF THE OXYGEN ATOM OF THE 10 NITRIC OXIDE RADICAL

For the O atom, force balance for the outermost $2p$ electron of NO (electron 6) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to interactions between electron 6 and the other $2p$ electron as well as the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Eight-Electron
15 Atoms section. The central Coulomb force on the outer-most $2p$ shell electron of NO (electron 6) due to the nucleus and the inner five electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$\mathbf{F}_{ele} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} \mathbf{i}_r \quad (13.926)$$

for $r > r_5$. The $2p$ shell possess an external electric field of +2 given by Eq. (10.92) for $r > r_6$. The energy is minimized with conservation of angular momentum. This condition is met when the diamagnetic force, $F_{diamagnetic}$, of Eq. (10.82) due to the p -orbital contribution is given by:

$$5 \quad F_{diamagnetic} = -\left(\frac{2}{3}\right) \frac{\hbar^2}{4m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (13.927)$$

And, $F_{mag 2}$ corresponding to the conserved spin and orbital angular momentum given by Eqs. (10.157) and (13.670) is

$$F_{mag 2} = \frac{1}{Z} \frac{2\hbar^2}{m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.928)$$

The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_6$ gives rise to a second diamagnetic force, $F_{diamagnetic 2}$, given by Eq. (10.93). $F_{diamagnetic 2}$ due to the binding of the p -orbital electron having an electric field of +2 outside of its radius is :

$$F_{diamagnetic 2} = -\left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_6^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (13.929)$$

In addition, the contribution of two $2p$ electrons in the formation of the σ MO gives rise to a paramagnetic force on the remaining paired $2p$ electrons. The force $F_{mag 3}$ is given by Eq. (13.625) wherein the radius is r_6 :

$$F_{mag 3} = \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \mathbf{i}_r \quad (13.930)$$

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (13.926)) and diamagnetic (Eqs. (13.927) and (13.929)), and paramagnetic (Eqs. (13.928) and (13.930)) forces as follows:

$$20 \quad \frac{m_e v_6^2}{r_6} = \left(\frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{s(s+1)} + \frac{2\hbar^2}{Zm_e r_6^2 r_3} \sqrt{s(s+1)} - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{s(s+1)} + \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \right) \quad (13.931)$$

Substitution of $v_6 = \frac{\hbar}{m_e r_6}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (13.931) gives:

$$\frac{\hbar^2}{m_e r_6^3} - \frac{\hbar^2}{4m_e r_6^3} \sqrt{\frac{3}{4}} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} + \frac{2\hbar^2}{Zm_e r_6^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{\frac{3}{4}}$$

(13.932)

The quadratic equation corresponding to Eq. (13.932) is

$$r_6^2 - \frac{\frac{\hbar^2}{m_e} \left(1 - \frac{\sqrt{3}}{8}\right)}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{2}{12} - \frac{2}{Z}\right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}}\right)} r_6 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{2}{12} - \frac{2}{Z}\right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}}\right)} = 0 \quad (13.933)$$

The solution of Eq. (13.933) using the quadratic formula is:

$$r_6 = \frac{a_0 \left(1 - \frac{\sqrt{3}}{8}\right)}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)} \pm a_0 \sqrt{\frac{\left(\frac{1 - \frac{\sqrt{3}}{8}}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)}\right)^2 - \frac{20\sqrt{3} \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) r_3}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)}}}{2} \quad (13.934)$$

r_3 in units of a_0

The positive root of Eq. (13.934) must be taken in order that $r_6 > 0$. Substitution of

$\frac{r_3}{a_0} = 0.59020$ (Eq. (10.62) with $Z = 8$) into Eq. (13.934) gives

$$r_6 = 0.70460 a_0 \quad (13.935)$$

10 ENERGIES OF THE $2p$ SHELLS OF THE NITROGEN ATOM AND OXYGEN ATOM OF THE NITRIC OXIDE RADICAL

With the formation of the H_2 -type MO by the contribution of two $2p$ electrons from each of the N and O atoms, the total energy of the NO molecule, which is subtracted from the sum of the energies of the nitrogen and oxygen atoms to determine the bond energy, is increased by the ionization energies of N^+ and O^+ given by Eqs. (10.132-10.133) and (10.152-10.153), respectively. Experimentally, the energies are [6]

$$E(\text{ionization}; N^+) = 29.6013 \text{ eV} \quad (13.936)$$

$$E(\text{ionization}; O^+) = 35.11730 \text{ eV} \quad (13.937)$$

In addition, the central forces on the $2p$ shells of the N and O atoms are increased

with the formation of the σ MO which reduces each shell's radius and increases its total energy. The Coulombic energy terms of the total energy of the N and O atoms at the new radii are calculated and added to the ionization energies of N^+ and O^+ , and the energy of the σ MO to give the total energy of NO . Then, the bond energy is determined from the total

5 NO energy.

The radius r_7 of the nitrogen atom before bonding is given by Eq. (10.142):

$$r_7 = 0.93084a_0 \quad (13.938)$$

Using the initial radius r_7 of the N atom and the final radius r_5 of the $N2p$ shell (Eq. (13.925)) and by considering that the central Coulombic field decreases by an integer for

10 each successive electron of the shell, the sum $E_T(N, 2p)$ of the Coulombic energy change of the $N2p$ electrons of the N atom is determined using Eq. (10.102):

$$\begin{aligned} E_T(N, 2p) &= -\sum_{n=4}^4 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_5} - \frac{1}{r_7} \right) \\ &= -(13.60580 \text{ eV})(0.26186)(3) \\ &= -10.68853 \text{ eV} \end{aligned} \quad (13.939)$$

The radius r_8 of the oxygen atom before bonding is given by Eq. (10.162):

$$r_8 = a_0 \quad (13.940)$$

15 Using the initial radius r_8 of the O atom and the final radius r_6 of the $O2p$ shell (Eq. (13.935)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(O, 2p)$ of the Coulombic energy change of the $O2p$ electrons of the O atom is determined using Eq. (10.102):

$$\begin{aligned} E_T(O, 2p) &= -\sum_{n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_8} \right) \\ &= -(13.60580 \text{ eV})(0.41925)(3+4) \\ &= -39.92918 \text{ eV} \end{aligned} \quad (13.941)$$

20

FORCE BALANCE OF THE σ MO OF THE NITRIC OXIDE RADICAL

The force balance can be considered due to a second pair of two electrons binding to a molecular ion having $+2e$ at each focus and a first bound pair. Then, the forces are the same as those of a molecule ion having $+e$ at each focus. The diamagnetic force $F_{\text{diamagneticMO1}}$ for

25 the σ -MO of the NO molecule due to the two paired electrons in the $O2p$ shell is given by

Eq. (13.633) with $n_e = 2$:

$$F_{\text{diamagneticMO1}} = \frac{\hbar^2}{2m_e a^2 b^2} D i_\xi \quad (13.942)$$

$F_{\text{diamagneticMO2}}$ of the nitric oxide radical comprising nitrogen with charge $Z_1 = 7$ and $|L_1| = \hbar$ and $|L_2| = \sqrt{\frac{3}{4}}\hbar$ and oxygen with $Z_2 = 8$ and $|L_3| = \hbar$ is given by the corresponding sum of the 5 contributions. Using Eq. (13.835), $F_{\text{diamagneticMO2}}$ for NO is

$$F_{\text{diamagneticMO2}} = \left(\frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D i_\xi \quad (13.943)$$

The general force balance equation for the σ -MO of the nitric oxide radical given by Eqs. (11.200), (13.633-13.634), and (13.942-13.943) is the same as that of CN (Eq. (13.836)):

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 ab^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.944)$$

$$10 \quad \frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 ab^2} D - \left(\frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (13.945)$$

$$\left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 ab^2} D \quad (13.946)$$

$$a = \left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) a_0 \quad (13.947)$$

Substitution of $Z_1 = 7$ and $Z_2 = 8$ into Eq. (13.947) gives

$$a = 2.39158a_0 = 1.26557 \times 10^{-10} \text{ m} \quad (13.948)$$

15 Substitution of Eq. (13.948) into Eq. (11.79) is

$$c' = 1.09352a_0 = 5.78666 \times 10^{-11} \text{ m} \quad (13.949)$$

The internuclear distance given by multiplying Eq. (13.949) by two is

286

$$2c' = 2.18704a_0 = 1.15733 \times 10^{-10} \text{ m} \quad (13.950)$$

The experimental bond distance is [28]

$$2c' = 1.15077 \times 10^{-10} \text{ m} \quad (13.951)$$

Substitution of Eqs. (13.948-13.949) into Eq. (11.80) is

$$b = c = 2.12693a_0 = 1.12552 \times 10^{-10} \text{ m} \quad (13.952)$$

Substitution of Eqs. (13.948-13.949) into Eq. (11.67) is

$$e = 0.45724 \quad (13.953)$$

Using the electron configuration of NO (Eq. (13.916)), the radii of the $N1s = 0.14605a_0$ (Eq. (10.51)), $N2s = 0.69385a_0$ (Eq. (10.62)), $N2p = 0.74841a_0$ (Eq. (13.925)), $O1s = 0.12739a_0$ (Eq. (10.51)), $O2s = 0.59020a_0$ (Eq. (10.62)), and $O2p = 0.70460a_0$ (Eq. (13.935)) shells and the parameters of the σ MO of NO given by Eqs. (13.3-13.4), (13.948-13.950), and (13.952-13.953), the dimensional diagram and charge-density of the NO MO are shown in Figures 31 and 32, respectively.

15 SUM OF THE ENERGIES OF THE σ MO AND THE AOs OF THE NITRIC OXIDE RADICAL

The energies of the NO σ MO are given by the substitution of the semiprincipal axes (Eqs. (13.948-13.949) and (13.952)) into the energy equations (Eqs. (11.207-11.212)) of H_2 except that the terms based on charge are multiplied by four and the kinetic energy term is multiplied by two due to the σ -MO double bond with two pairs of paired electrons:

$$V_e = 2^2 \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -98.30623 \text{ eV} \quad (13.954)$$

$$V_p = 2^2 \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 49.76880 \text{ eV} \quad (13.955)$$

$$T = 2 \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 10.27631 \text{ eV} \quad (13.956)$$

$$V_m = 2^2 \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -10.27631 \text{ eV} \quad (13.957)$$

$$E_T = V_e + T + V_m + V_p \quad (13.958)$$

Substitution of Eqs. (11.79) and (13.954-13.957) into Eq. (13.958) gives

$$E_T(NO, \sigma) = \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(8 \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 4 \right) = -48.53743 \text{ eV} \quad (13.959)$$

where $E_T(NO, \sigma)$ is the total energy of the σ MO of NO . The total energy of NO , $E_T(NO)$, is given by the sum of $E(\text{ionization}; N^+)$, the energy of the second electron of nitrogen (Eq. (13.936)) donated to the double bond, $E(\text{ionization}; O^+)$, the energy of the second electron of oxygen (Eq. (13.937)) donated to the double bond, $E_T(N, 2p)$, the $N2p$ AO contribution due to the decrease in radius with bond formation (Eq. (13.939)), $E_T(O, 2p)$, the $O2p$ AO contribution due to the decrease in radius with bond formation (Eq. (13.941)), and $E_T(NO, \sigma)$, the σ MO contribution given by Eq. (13.959):

$$\begin{aligned} E_T(NO) &= \left(E(\text{ionization}; N^+) + E(\text{ionization}; O^+) \right) \\ &\quad + E_T(N, 2p) + E_T(O, 2p) + E_T(NO, \sigma) \\ &= \left(29.6013 \text{ eV} + 35.11730 \text{ eV} \right. \\ &\quad \left. - 10.68853 \text{ eV} - 39.92918 \text{ eV} - 48.53743 \text{ eV} \right) \\ &= -34.43653 \text{ eV} \end{aligned} \quad (13.960)$$

10

VIBRATION OF NO

The vibrational energy levels of NO may be solved by determining the Morse potential curve from the energy relationships for the transition from a N atom and O atom whose parameters are given by Eqs. (10.134-10.143) and (10.154-10.163), respectively, to a N atom whose parameter r_s is given by Eq. (13.925), an O atom whose parameter r_o is given by Eq. (13.935), and the σ MO whose parameters are given by Eqs. (13.948-13.950) and (13.952-13.953). As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter (K&P) [15] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods.

20

THE DOPPLER ENERGY TERMS OF THE NITRIC OXIDE RADICAL

The equations of the radiation reaction force of nitric oxide are the same as those of H_2 with the substitution of the NO parameters except that there is a factor of four increase in the central force in Eq. (11.231) due to the double bond. Using Eqs. (11.231-11.233) and 5 (13.901), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{4e^2}{4\pi\epsilon_0 a^3}} = 2.23557 \times 10^{16} \text{ rad/s} \quad (13.961)$$

where a is given by Eq. (13.948). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.23557 \times 10^{16} \text{ rad/s} = 14.71493 \text{ eV} \quad (13.962)$$

10 In Eq. (11.181), substitution of $E_T(NO)$ for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (13.962) for \bar{E}_K gives the Doppler energy of the electrons of the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -34.43653 \text{ eV} \sqrt{\frac{2e(14.71493 \text{ eV})}{m_e c^2}} = -0.26134 \text{ eV} \quad (13.963)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the 15 transition state at their corresponding frequency. The decrease in the energy of the NO MO due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (13.963) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy. Using the experimental NO ω_e of 1904.20 cm^{-1} (0.23609 eV) [28] for 20 \bar{E}_{Kvib} of the transition state, $\bar{E}'_{osc}(NO)$ per bond is

$$\bar{E}'_{osc}(NO) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (13.964)$$

$$\bar{E}'_{osc}(NO) = -0.26134 \text{ eV} + \frac{1}{2} (0.23609 \text{ eV}) = -0.14329 \text{ eV} \quad (13.965)$$

Since the σ MO bond is a double bond with twice as many electrons as a single bond, $\bar{E}'_{osc}(NO)$ is multiplied by two to give

$$25 \quad \bar{E}_{osc}(NO) = -0.28658 \text{ eV} \quad (13.966)$$

TOTAL AND BOND ENERGIES OF THE NITRIC OXIDE RADICAL

$E_{T+osc}(NO)$, the total energy of NO including the Doppler term, is given by the sum of $E_T(NO)$ (Eq. (13.960)) and $\bar{E}_{osc}(NO)$ given by Eq. (13.966):

$$\begin{aligned}
 E_{T+osc}(NO) &= \left(V_e + T + V_m + V_p + E(\text{ionization}; N^+) + E(\text{ionization}; O^+) \right. \\
 &\quad \left. + E_T(N, 2p) + E_T(O, 2p) + \bar{E}_{osc}(NO) \right) \\
 5 \quad &= \left(E_T(NO, \sigma) + E(\text{ionization}; N^+) + E(\text{ionization}; O^+) \right. \\
 &\quad \left. + E_T(N, 2p) + E_T(O, 2p) + \bar{E}_{osc}(NO) \right) \quad (13.967) \\
 &= E_T(NO) + \bar{E}_{osc}(NO)
 \end{aligned}$$

$$\begin{aligned}
 E_{T+osc}(NO) &= \left\{ \begin{aligned} &\frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(8\ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 4 \right) \\ &+ E(\text{ionization}; N^+) + E(\text{ionization}; O^+) \\ &- \sum_{N, n=4}^4 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_5} - \frac{1}{r_7} \right) - \sum_{O, n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_8} \right) \end{aligned} \right\} \quad (13.968) \\
 &\quad \left(1 + 2\sqrt{\frac{2\hbar\sqrt{\frac{4e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} + 2\left(\frac{1}{2}\hbar\sqrt{\frac{k}{\mu}}\right) \right) \\
 &= -34.43653 \text{ eV} - 2(0.26134 \text{ eV}) + 2\left(\frac{1}{2}\hbar\sqrt{\frac{k}{\mu}}\right)
 \end{aligned}$$

From Eqs. (13.966-13.968), the total energy of the NO MO is

$$\begin{aligned}
 E_{T+osc}(NO) &= -34.43653 \text{ eV} + \bar{E}_{osc}(NO) \\
 &= -34.43653 \text{ eV} + (-0.28658) \\
 &= -34.72312 \text{ eV} \quad (13.969)
 \end{aligned}$$

where the experimental ω_e was used for the $\hbar\sqrt{\frac{k}{\mu}}$ term.

10 The NO bond dissociation energy, $E_D(NO)$, is given by the difference between the sum of the energies of the N and O atoms and $E_{T+osc}(NO)$:

$$E_D(NO) = E(N) + E(O) - E_{T+osc}(NO) \quad (13.970)$$

where the energy of a nitrogen atom is [6]

$E(N) = -14.53414 \text{ eV}$ (13.871)

and the energy of an oxygen atom is [6]

$E(O) = -13.61806 \text{ eV}$ (13.972)

Thus, the NO bond dissociation energy, $E_D(NO)$, given by Eqs. (13.969-13.972) is

5
$$\begin{aligned} E_D(NO) &= -(14.53414 \text{ eV} + 13.61806 \text{ eV}) - E_{T+osc}(NO) \\ &= -28.15220 \text{ eV} - (-34.72312 \text{ eV}) \\ &= 6.57092 \text{ eV} \end{aligned}$$
 (13.973)

The experimental NO bond dissociation energy is [49]

$E_{D298}(NO) = 6.5353 \text{ eV}$ (13.974)

The results of the determination of bond parameters of NO are given in Table 13.1. The calculated results are based on first principles and given in closed-form, exact equations
10 containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

Table 13.1. The calculated and experimental bond parameters of H_3^+ , D_3^+ , OH, OD, H_2O , D_2O , NH, ND, NH_2 , ND_2 , NH_3 , ND_3 , CH, CD, CH_2 , CH_3 , CH_4 , N_2 , O_2 , F_2 , Cl_2 ,
15 CN, CO, and NO.

Parameter	Calculated	Experimental	Ref. for Exp.
H_3^+ Bond Energy	4.373 eV	4.373 eV	8
D_3^+ Bond Energy	4.374 eV		
OH Bond Energy	4.4104 eV	4.4117 eV	22
OD Bond Energy	4.4687 eV	4.454 eV	23
OH Bond Length	0.971651 Å	0.971 Å	13
OD Bond Length	0.971651 Å	0.971 Å	13
OH Vibrational Energy	0.4367 eV	0.4424 eV	16-17
OD Vibrational Energy	0.3219 eV	0.3263 eV	16-17
OH ω_e	3696.38 cm^{-1}	3735.21 cm^{-1}	14
OD ω_e	2689.51 cm^{-1}	2720.9 cm^{-1}	14
OH $\omega_e x_e$	87.18 cm^{-1}	82.81 cm^{-1}	14
OD $\omega_e x_e$	46.75 cm^{-1}	44.2 cm^{-1}	14
OH B_e	18.835 cm^{-1}	18.871 cm^{-1}	14

Parameter	Calculated	Experimental	Ref. for Exp.
$OD\ B_e$	9.971 cm^{-1}	10.01 cm^{-1}	14
H_2O Bond Energy	5.1059 eV	5.0991 eV	26
D_2O Bond Energy	5.178 eV	5.191 eV	31-32
H_2O O-H Bond Length	0.971574 Å	0.970 ± 0.005 Å	23
D_2O O-D Bond Length	0.971574 Å	0.970 ± 0.005 Å	23
H_2O H-H Distance	1.552 Å	1.55 ± 0.01 Å	13
D_2O D-D Distance	1.552 Å	1.55 ± 0.01 Å	13
H_2O Bond Angle	106°	106°	23
D_2O Bond Angle	106°	106°	23
NH Bond Energy	3.47530 eV	3.47 eV	30
ND Bond Energy	3.52556 eV	3.5134 eV	31
NH Bond Length	1.04262 Å	1.0362 Å	28
ND Bond Length	1.04262 Å	1.0361 Å	28
NH Vibrational Energy	0.38581 eV	0.38752 eV	28
ND Vibrational Energy	0.28583 eV	0.28690 eV	28
NH ω_e	3284.58 cm^{-1}	3282.3 cm^{-1}	28
ND ω_e	2398.72 cm^{-1}	2398 cm^{-1}	28
NH $\omega_e x_e$	86.37 cm^{-1}	78.4 cm^{-1}	28
ND $\omega_e x_e$	47.40 cm^{-1}	42 cm^{-1}	28
NH B_e	16.495 cm^{-1}	16.993 cm^{-1}	28
ND B_e	8.797 cm^{-1}	8.7913 cm^{-1}	28
NH_2 Bond Energy	3.9323 eV	3.9461 eV	35
ND_2 Bond Energy	3.9401 eV	3.9362 eV	33-35
NH_2 Bond Length	1.04262 Å	1.0240 Å	32
ND_2 Bond Length	1.04262 Å		
NH_2 Bond Angle	105.97	103.3°	32
ND_2 Bond Angle	105.97		
NH_3 Bond Energy	4.57913 eV	4.60155 eV	37
ND_3 Bond Energy	4.64499 eV	4.71252 eV	37

Parameter	Calculated	Experimental	Ref. for Exp.
NH_3 Bond Length	1.0368 Å	1.012 Å	32
ND_3 Bond Length	1.0368 Å		
NH_3 Bond Angle	106.67	106.67°	36
ND_3 Bond Angle	106.67	106.70	36
CH Bond Energy	3.47404 eV	3.47 eV	14
CD Bond Energy	3.51673 eV	3.52 eV	14
CH Bond Length	1.1183 Å	1.1198 Å	14
CD Bond Length	1.1183 Å	1.118 Å	14
CH Vibrational Energy	0.33879 eV	0.33885 eV	14
CD Vibrational Energy	0.25173 eV	0.25189 eV	14
CH ω_e	2865.86 cm^{-1}	2861.6 cm^{-1}	14
CD ω_e	2102.97 cm^{-1}	2101.0 cm^{-1}	14
CH $\omega_e x_e$	66.624 cm^{-1}	64.3 cm^{-1}	14
CD $\omega_e x_e$	36.335 cm^{-1}	34.7 cm^{-1}	14
CH B_e	14.498 cm^{-1}	14.457 cm^{-1}	14
CD B_e	7.807 cm^{-1}	7.808 cm^{-1}	14
CH_2 Bond Energy	4.36968 eV	4.33064 eV	39
CH_2 Bond Length	1.1067 Å	1.111 Å	38
CH_2 Bond Angle	100.22	102.4°	38
CH_3 Bond Energy	4.70075 eV	4.72444 eV	40
CH_3 Bond Length	1.1029 Å	1.079 Å	38
CH_3 Bond Angle	100.70°		
CH_4 Bond Energy	4.4900 eV	4.48464 eV	40
CH_4 Bond Length	1.1010 Å	1.087 Å	41
CH_4 Bond Angle	109.5°	109.5°	41
N_2 Bond Energy	9.71181 eV	9.756 eV	43
N_2 Bond Length	1.0955 Å	1.094 Å	43
O_2 Bond Energy	5.10711 eV	5.11665 eV	46
O_2 Bond Length	1.20862 Å	1.20752 Å	28

Parameter	293		
	Calculated	Experimental	Ref. for Exp.
F_2 Bond Energy	1.62168 eV	1.606 eV	48
F_2 Bond Length	1.41114 Å	1.41193 Å	28
Cl_2 Bond Energy	2.52236 eV	2.51412 eV	49
Cl_2 Bond Length	1.988 Å	1.988 Å	28
Cl_2 ω_e	538.52 cm^{-1}	559.7 cm^{-1}	28
Cl_2 $\omega_e x_e$	3.41 cm^{-1}	2.68 cm^{-1}	28
Cl_2 B_e	0.2420 cm^{-1}	0.2440 cm^{-1}	28
CN Bond Energy	7.77526 eV	7.7731 eV	50
CN Bond Length	1.17231 Å	1.17181 Å	28
CO Bond Energy	11.16652 eV	11.15696 eV	49
CO Bond Length	1.13290 Å	1.12823 Å	28
NO Bond Energy	6.57092 eV	6.5353 eV	49
NO Bond Length	1.15733 Å	1.15077 Å	28

References for Section II

1. G. C. Lie, D. Frye, "Vibrational analysis of a Hylleraas-configuration interaction potential for H_3^+ ", J. Chem. Phys., Vol. 96, No. 9, (1992), pp. 6784-6790.
- 5 2. G. R. Fowles, *Analytical Mechanics*, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 251-305.
3. N. V. Sidgwick, *The Chemical Elements and Their Compounds*, Volume I, Oxford, Clarendon Press, (1950), p.17.
4. D. A. McQuarrie, *Quantum Chemistry*, University Science Books, Mill Valley, CA, 10 (1983), p. 172.
5. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. 10-181.
6. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. 10-175.
- 15 7. P. W. Atkins, *Physical Chemistry*, 2nd Edition, W. H. Freeman, San Francisco, (1982), p. 589.
8. P. C. Cosby, H. Helm, "Experimental determination of the H_3^+ bond dissociation energy", Vol. 152, No. 1, (1988), pp. 71-74.

9. K. P. Huber, G. Herzberg, *Molecular Spectra and Molecular Structure, IV. Constants of Diatomic Molecules*, Van Nostrand Reinhold Company, New York, (1979).
10. NIST Atomic Spectra Database, www.physics.nist.gov/cgi-bin/AtData/display.ksh.
11. G. R. Fowles, *Analytical Mechanics*, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 154-156
12. R. C. Weast, *CRC Handbook of Chemistry and Physics*, 58th Edition, CRC Press, West Palm Beach, Florida, (1977), p. E-68.
13. P. C. Hariharan, J. A. Pople, Accuracy of AH_n equilibrium geometries by single determinant molecular orbital theory, *Mol. Phys.*, Vol. 27(1), (1974), pp. 209–214.
- 10 14. G. Herzberg, J. W. T. Spinks, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules*, 2nd Edition, Krieger Publishing Company, Malabar, FL, (1989), p. 560.
15. M. Karplus, R. N. Porter, *Atoms and Molecules an Introduction for Students of Physical Chemistry*, The Benjamin/Cummings Publishing, Menlo Park, CA, (1970), pp. 447-484.
- 15 16. K. P. Huber, G. Herzberg, *Molecular Spectra and Molecular Structure, IV. Constants of Diatomic Molecules*, Van Nostrand Reinhold Company, New York, (1979).
17. J. Crovisier, Molecular Database—Constants for molecules of astrophysical interest in the gas phase: photodissociation, microwave and infrared spectra, Ver. 4.2, Observatoire de Paris, Section de Meudon, Meudon, France, May 2002, pp. 34–37, available at
20 <http://www.usr.obspm.fr/~crovisie/>.
18. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. 10-175.
19. NIST Atomic Spectra Database, www.physics.nist.gov/cgi-bin/AtData/display.ksh.
20. J. A. Joens, The dissociation energy of $OH(X^2\Pi_{3/2})$ and the enthalpy of formation of $OH(X^2\Pi_{3/2})$, $ClOH$, and $BrOH$ from thermochemical cycles, *J. Phys. Chem.*, Vol. 105,
25 (2001), pp. 11041–11044.
21. Y.-M. Chen, D.E. Clemmer, P. B. Armentrout, Kinetic and electronic energy dependence of the reactions of Sc^+ and Ti^+ with D_2O , *J. Phys. Chem.*, Vol. 98, (1994), pp. 11490–11498.
- 30 22. L. V. Gurvich, I. V. Veyts, C. B. Alcock, *Thermodynamic Properties of Individual Substances*, 4th ed., Hemisphere: New York, 1989, Vol. 1, Part 2.

23. K. Ichikawa, Y. Kameda, T. Yamaguchi, H. Wakita and M. Misawa, Neutron diffraction investigation of the intramolecular structure of a water molecule in the liquid phase at high temperatures, *Mol. Phys.* Vol. 73, (1991), pp. 79-86.
24. H. A. Haus, J. R. Melcher, *Electromagnetic Fields and Energy*, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, (1985), Sec. 5.4.
25. R. Lemus, Vibrational excitations in H₂O in the framework of a local model, *J. Mol. Spectrosc.*, Vol. 225, (2004), pp. 73-92.
26. B. Ruscic, A. F. Wagner, L. B. Harding, R. L. Asher, D. Feller, D. A. Dixon, K. A. Peterson, Y. Song, X. Qian, C.-Y. Ng, J. Liu, W. Chen, D. W. Schwenke, On the enthalpy of formation of hydroxyl radical and gas-phase bond dissociation energies of water and hydroxyl, *J. Phys. Chem. A*, Vol. 106, (2002), pp. 2727-2747.
27. X.-M. Qian, Y. Song, K.-C. Lau, C. Y. Ng, J. Liu, W. Chen, G. Z. He, A pulsed field ionization photoelectron-photoion coincidence study of the dissociative photoionization process $D_2O + h\nu \rightarrow OD^+ + D + e^-$, *Chem. Phys. Letts.*, Vol. 353(1-2), (2002), pp. 19-26.
28. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), pp. 9-80 to 9-85.
29. J. B. Marquette, C. Rebrion, B. R. Rowe, "The reactions of $N^+ (^3P)$ ions with normal, para, and deuterated hydrogens at low temperatures", *J. Chem. Phys.*, Vol. 89, (4), (1988), pp. 2041-2047.
30. W. R. Graham, H. Lew, "Spectra of the $d^1\Sigma^+ - c^1\Pi$ and $d^1\Sigma^+ - b^1\Pi^+$ systems and dissociation energy of NH and ND ", *Can. J. Phys.*, Vol. 56, (1978), pp. 85-99.
31. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. 9-54.
32. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. 9-20.
33. T. Amano, P. F. Bernath, R. W. McKellar, "Direct observation of the ν_1 and ν_3 fundamental bands of NH_2 by difference frequency laser spectroscopy", *J. Mol. Spectrosc.*, Vol. 94, (1982), pp. 100-113.
34. A. P. Altshuller, "Heat of formation of $NH(g)$ and the bond dissociation energy of $D(NH-H)$ ", *J. Chem. Phys.*, Vol. 22, No. 11, (1954), pp. 1947-1948.

35. J. Berkowitz, G. B. Ellison, D. Gutman, "Three methods to measure RH bond energies", J. Phys. Chem., Vol. 98, (1994), pp. 2744-2765.
36. W. S. Benedict, E. K. Plyler, "Vibration-rotation bands of ammonia", Can. J. Phys., Vol. 35, (1957), pp. 1235-1241.
- 5 37. D. H. Mordaunt, R. N. Dixon, M. N. R. Ashfold, "Photodissociation dynamics of the \bar{A} state ammonia molecules. II. The isotopic dependence for partially and fully deuterated isotopomers", J. Chem. Phys., Vol. 104, (17), (1996), pp. 6472-6481.
38. D. J. DeFrees, B. A. Levi, S. K. Pollack, W. J. Hehre, J. S. Binkley, J. A. Pople, "Effect of correlation on theoretical equilibrium geometries", J. Am. Chem. Soc., 101:15, (1979),
10 pp. 4085-4089.
39. A. G. Csaszar, M. L. Leininger, V. Szalay, "The standard enthalpy of formation of CH_2 ", J. Chem. Phys., Vol. 118, No. 23, (2003), pp. 10631-10642.
40. B. Ruscic, M. Litorja, R. L. Asher, "Ionization energy of methylene revisited: Improved values for the enthalpy of formation of CH_2 and the bond dissociation energy of CH_3 via
15 simultaneous solution of the local thermodynamic network", J. Phys. Chem. A, Vol. 103, (1999), pp. 8625-8633.
41. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. 9-34.
42. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca
20 Raton, Florida, (1998-9), pp. 4-130 to 4-135.
43. G. Herzberg, J. W. T. Spinks, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules*, 2nd Edition, Krieger Publishing Company, Malabar, FL, (1989), pp. 551-553.
44. 24. J. Itatani, J. Levesque, D. Zeidler, H. Niikura, H. Pepin, J. C. Kieffer, P. B. Corkum,
25 D. M. Villeneuve, Tomographic imaging of molecular orbitals, Nature, Vol. 432, (2004), pp. 867-871.
45. R. H. Christian, R. E. Duff, F. L. Yarger, "Equation of state of gases by shock wave measurements. II. The dissociation energy of nitrogen", J. Chem. Phys., Vol. 23, No. 11, (1955), pp. 2045-2049.
- 30 46. K. M. Ervin, I. Anusiewicz, P. Skurski, J. Simons, W. C. Lineberger, The only stable state of O_2^- is the $X^2\Pi_g$ ground state and it (still!) has an adiabatic electron detachment energy of 0.45 eV, J. Phys. Chem. A, Vol. 107, (2003), pp. 8521-8529.

47. P. C. Cosby, D. L. Huestis, On the dissociation energy of O_2 and the energy of the $O_2^+ b^4\Sigma_g^-$ state, *J. Chem. Phys.*, Vol. 97, No. 9, (1992), pp. 6108-6112.
48. J. Yang, Y. Hao, J. Li, C. Zhou, Y. Mo, A combined zero electronic kinetic energy spectroscopy and ion-pair dissociation imaging study of the $F_2^+(X^2\Pi_g)$ structure, *J. Chem. Phys.*, Vol. 122, No. 13, (2005), 134308-1–134308-7.
49. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), pp. 9-51 to 9-57.
50. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 86th Edition, Taylor & Francis, Boca Raton, Florida, (2005-6), pp. 9-54 to 9-62.

Section III

MORE POLYATOMIC MOLECULES AND HYDROCARBONS

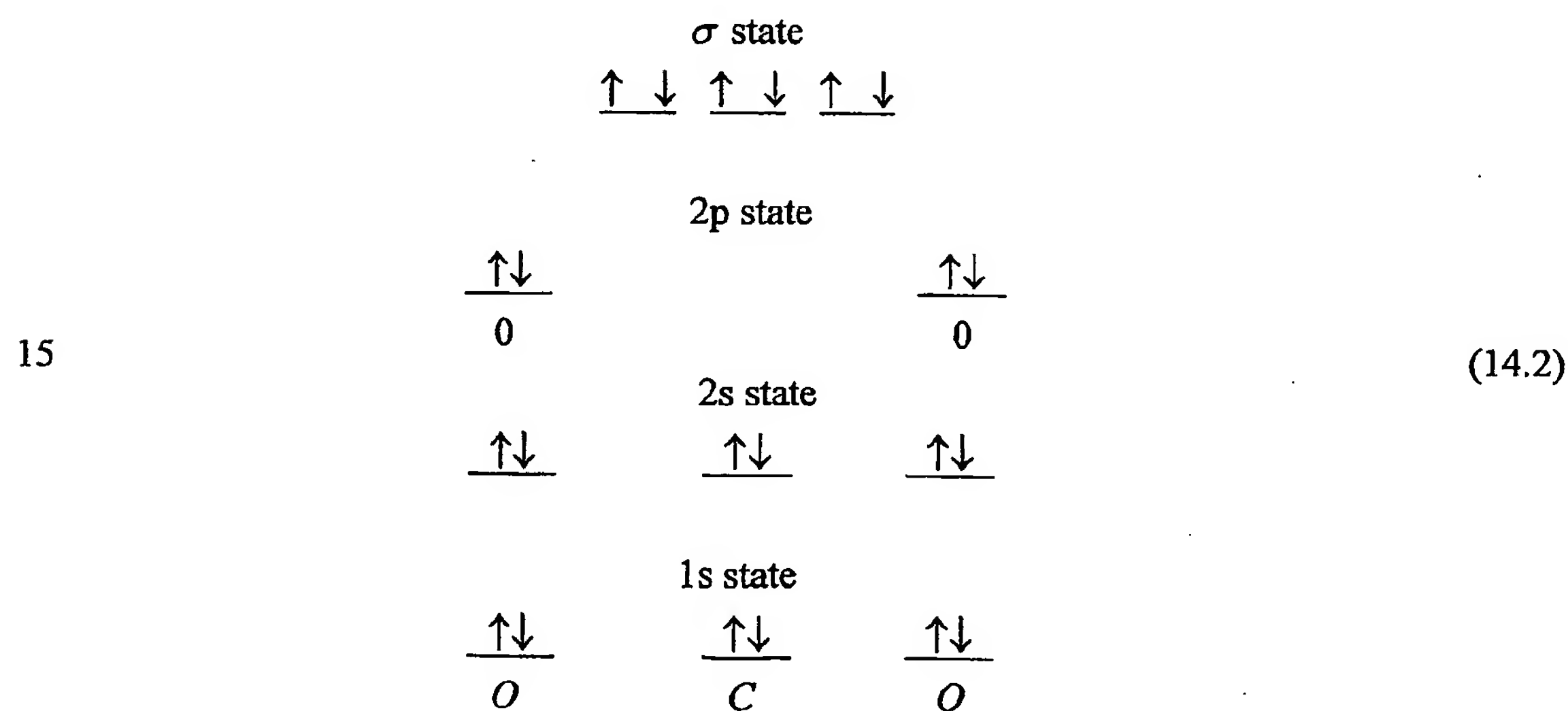
Additional polyatomic molecules can be solved using the same principles as those used to
5 solve hydrogen molecular ions and molecules wherein the hydrogen molecular orbitals (MOs)
and hydrogen atomic orbitals serve as basis functions for the MOs. The MO must (1) be a
solution of Laplace's equation to give a equipotential energy surface, (2) correspond to an
orbital solution of the Newtonian equation of motion in an inverse-radius-squared central
field having a constant total energy, (3) be stable to radiation, and (4) conserve the electron
10 angular momentum of \hbar . Energy of the MO must be matched to that of the outermost atomic
orbital of a bonding heteroatom in the case where a minimum energy is achieved with a direct
bond to the AO. Alternatively, the MO is continuous with the AO containing paired electrons
that do not participate in the bond. Rather, they only provide a means for the energy matched
MO to form a continuous equipotential energy surface. In the case that an independent MO is
15 formed, the AO force balance causes the remaining electrons to be at lower energy and a
smaller radius. In another case, the atomic orbital may hybridize in order to achieve a bond at
an energy minimum, and the sharing of electrons between two or more such orbitals to form a
MO permits the participating hybridized orbitals to decrease in energy through a decrease in
the radius of one or more of the participating orbitals. Representative cases were solved.
20 Specifically, the results of the determination of bond parameters of carbon dioxide (CO_2),
nitrogen dioxide (NO_2), ethane (CH_3CH_3), ethylene (CH_2CH_2), acetylene ($CHCH$),
benzene (C_6H_6), propane (C_3H_8), butane (C_4H_{10}), pentane (C_5H_{12}), hexane (C_6H_{14}),
heptane (C_7H_{16}), octane (C_8H_{18}), nonane (C_9H_{20}), decane ($C_{10}H_{22}$), undecane ($C_{11}H_{24}$),
dodecane ($C_{12}H_{26}$), and octadecane ($C_{18}H_{38}$) are given in Table 14.1. The calculated results
25 are based on first principles and given in closed-form, exact equations containing
fundamental constants only. The agreement between the experimental and calculated results
is excellent.

CARBON DIOXIDE MOLECULE

30 The carbon dioxide molecule can be formed by the reaction of carbon monoxide and an
oxygen atom:



Each equivalent bond in the carbon dioxide molecule comprises a double bond that is energy-matched to the filled $C2s$ orbital. Each such bond comprises 75% of a H_2 -type MO with four paired electrons as a basis set such that three electrons can be assigned to each $C=O$ bond. Thus, the two $C2p$ electrons combine with the four $O2p$ electrons, two from each O , as a linear combination to form the two $C=O$ bonds of CO_2 . The force balance equation and radius r_s of the $2p$ shell of O is derived in the Eight-Electron Atoms section. With the formation of the H_2 -type MOs by the contribution of two $2p$ electrons from each of the two O atoms, a factor of two increase of the central field on the remaining $O2p$ electrons arises. The resulting increased Coulombic as well as magnetic central forces decrease the radii of the $O2p$ shells such that the energy minimum is achieved that is lower than that of the reactant atoms. The resulting electron configuration of CO_2 is $Cl s^2 O_1 1s^2 O_2 1s^2 C 2s^2 O_1 2s^2 O_2 2s^2 O_1 2p^2 O_2 2p^2 \sigma_{O_2, C, O_1}^6$ where the subscripts designate the O atom, 1 or 2, σ designates the H_2 -type MO, and the orbital arrangement is



Carbon dioxide is predicted to be diamagnetic in agreement with observations [1].

FORCE BALANCE OF THE $2p$ SHELL OF THE OXYGEN ATOM OF THE CARBON DIOXIDE MOLECULE

For each O atom, force balance for the outermost $2p$ electron of CO_2 (electron 6) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due

to interactions between electron 6 and the other $2p$ electrons as well as the $2s$ -shell electrons due to spin and orbital angular momentum. The forces used are derived in the Eight-Electron Atoms section. The central Coulomb force on the outer-most $2p$ shell electron of CO (electron 6) due to the nucleus and the inner five electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$\mathbf{F}_{ele} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} \mathbf{i}_r \quad (14.3)$$

for $r > r_5$. The $2p$ shell possess a +2 external electric field given by Eq. (10.92) for $r > r_6$. The energy is minimized with conservation of angular momentum. This condition is met when the diamagnetic force, $\mathbf{F}_{diamagnetic}$, of Eq. (10.82) due to the p -orbital contribution is given by:

$$\mathbf{F}_{diamagnetic} = -\left(\frac{2}{3}\right) \frac{\hbar^2}{4m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (14.4)$$

where $s=1/2$. And, $\mathbf{F}_{mag 2}$ corresponding to the conserved spin and orbital angular momentum given by Eq. (10.157) is

$$\mathbf{F}_{mag 2} = \frac{1}{Z} \frac{2\hbar^2}{m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (14.5)$$

The electric field external to the $2p$ shell given by Eq. (10.92) for $r > r_6$ gives rise to a second diamagnetic force, $\mathbf{F}_{diamagnetic 2}$, given by Eq. (10.93). $\mathbf{F}_{diamagnetic 2}$ due to the binding of the p -orbital electron having an electric field of +2 outside of its radius is :

$$\mathbf{F}_{diamagnetic 2} = -\left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_6^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (14.6)$$

The radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (14.3)) and diamagnetic (Eqs. (14.4) and (14.6)), and paramagnetic (Eq. (14.5)) forces as follows:

$$\frac{m_e v_6^2}{r_6} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{s(s+1)} + \frac{2\hbar^2}{Zm_e r_6^2 r_3} \sqrt{s(s+1)} - \left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{s(s+1)} \quad (14.7)$$

Substitution of $v_6 = \frac{\hbar}{m_e r_6}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (14.7) gives:

$$\frac{\hbar^2}{m_e r_6^3} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} + \frac{2\hbar^2}{Zm_e r_6^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{\frac{3}{4}} \quad (14.8)$$

The quadratic equation corresponding to Eq. (14.8) is

$$r_6^2 - \frac{\frac{\hbar^2}{m_e}}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} r_6 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} = 0 \quad (14.9)$$

5 The solution of Eq. (14.9) using the quadratic formula is:

$$r_6 = \frac{\frac{a_0}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \right)^2 + \frac{20\sqrt{3} \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)}}}{2} \quad (14.10)$$

r_3 in units of a_0

The positive root of Eq. (14.10) must be taken in order that $r_6 > 0$. Substitution of

$\frac{r_3}{a_0} = 0.59020$ (Eq. (10.62) with $Z = 8$) into Eq. (14.10) gives

$$r_6 = 0.74776a_0 \quad (14.11)$$

10

ENERGIES OF THE $2s$ AND $2p$ SHELLS OF THE CARBON ATOM AND THE $2p$ SHELL OF THE OXYGEN ATOMS OF THE CARBON DIOXIDE MOLECULE

Consider the determination of the total energy of CO_2 from the reaction of a carbon atom
 15 with two oxygen atoms. With the formation of the H_2 -type MO by the contribution of two
 $2p$ electrons from the C atom, the remaining outer-shell atomic electrons comprise the $2s$
 electrons which are unchanged by bonding with two oxygen atoms. However, the total
 energy of the CO_2 molecule, which is subtracted from the sum of the energies of the oxygen
 atom and carbon monoxide molecule to determine the $O-CO$ bond energy, is increased by

the ionization energies of C , C^+ , O , and $2O^+$ given by Eqs. (14.12-14.15), respectively. Experimentally, the energies are [2]

$$E(\text{ionization}; C) = 11.26030 \text{ eV} \quad (14.12)$$

$$E(\text{ionization}; C^+) = 24.38332 \text{ eV} \quad (14.13)$$

$$5 \quad E(\text{ionization}; O) = 13.61806 \text{ eV} \quad (14.14)$$

$$E(\text{ionization}; O^+) = 35.11730 \text{ eV} \quad (14.15)$$

In addition, the central forces on the $2p$ shell of the O atom are increased with the formation of the σ MO which reduces the shell's radius and increases its total energy. The Coulombic energy terms of the total energy of each O atom at the new radius are calculated
10 and added to the ionization energies of C , C^+ , O , and $2O^+$, and the energy of the σ MO to give the total energy of CO_2 . Then, the bond energy is determined from the total CO_2 energy.

The radius r_8 of each oxygen atom before bonding is given by Eq. (10.162):

$$r_8 = a_0 \quad (14.16)$$

15 Using the initial radius r_8 of each O atom and the final radius r_6 of the $O2p$ shell (Eq. (14.11)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(O, 2p)$ of the Coulombic energy change of the $O2p$ electrons of each O atom is determined using Eq. (10.102):

$$\begin{aligned} E_T(O, 2p) &= -\sum_{n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_8} \right) \\ &= -(13.60580 \text{ eV})(0.33733)(3+4) \\ &= -32.12759 \text{ eV} \end{aligned} \quad (14.17)$$

20

FORCE BALANCE OF THE σ MO OF THE CARBON DIOXIDE MOLECULE

As in the case of H_2 , the σ MO is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into the C atom for distances shorter than the radius of the $C2s$
25 shell; nor, can it extend into the O atom for distances shorter than the radius of the $O2p$ shell. Thus, the MO surface of each $C=O$ bond comprises a prolate spheroid that bridges and is continuous with the $2s$ and $2p$ shells of the O and C atoms whose nuclei serve as the foci. The energy of each prolate spheroid is matched to that of the $C2s$ and $O2p$ shells.

As in the case of previous examples of energy-matched MOs such as OH and NH , the $C=O$ -bond MO must comprise 75% of a H_2 -type ellipsoidal MO in order to match potential, kinetic, and orbital energy relationships. However, the paired electrons of the $C2s$ and $O2p$ shells are not involved in bonding. Rather, the AOs permit a continuous surface
 5 comprising the two $C=O$ -bond MOs having six paired electrons, two from each of the C and the two O atoms:



10 The force balance of the CO_2 MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (14.18) and the energy matching condition between the carbon and oxygen components of the MO.

Similar to the OH and H_2O cases given by Eqs. (13.57) and (13.162), the H_2 -type ellipsoidal MO comprises 75% of the CO_2 MO; so, the electron charge density in Eq. (11.65)
 15 is given by $-0.75e$. Thus, k' of the each H_2 -type-ellipsoidal-MO component of the CO_2 MO is given by Eq. (13.59). The distance from the origin of each $C=O$ -bond MO to each focus c' is given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C=O$ -bond MO $b=c$ given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). Then, the solution of the semimajor
 20 axis a allows for the solution of the other axes of the prolate spheroidal and eccentricity of the CO_2 MO.

The energy components of V_e , V_p , T , V_m , and E_T of the CO_2 σ MO are the same as those of OH given by Eqs. (13.67-13.73), except that the terms based on charge are multiplied by four and the kinetic energy term is multiplied by two due to each σ -MO double
 25 bond:

$$E_T(C=O, \sigma) = -\frac{4e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \quad (14.19)$$

where $E_T(C=O, \sigma)$ is the total energy of each $C=O$ σ MO of CO_2 . The total energy of a H_2 -type ellipsoidal MO is given by Eqs. (11.212) and (13.75). A minimum energy is obtained when each double bond of the σ MO of CO_2 comprises the energy equivalent of
 30 four H_2 -type ellipsoidal MOs. For each $C=O$ bond to match the energy of the $C2s$ orbital,

the ionization energy of C and C^+ (Eqs. (14.12-14.13)) must be added for each bond of the double bond. Thus, the total energy of each $C=O$ -bond MOs is

$$\begin{aligned} E_T(C=O, \sigma) &= 2 \left(2 \left(-\frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] \right) \right. \\ &\quad \left. + E(\text{ionization}; C) + E(\text{ionization}; C^+) \right) \\ &= 2 \left(2(-31.63536831 \text{ eV}) + 11.26030 \text{ eV} + 24.38332 \text{ eV} \right) \\ &= -55.25423 \text{ eV} \end{aligned} \quad (14.20)$$

$E_T(C=O, \sigma)$ given by Eq. (14.19) is set equal to Eq. (14.20):

$$5 \quad \frac{4e^2}{8\pi\epsilon_0 c'} \left[\left(\frac{3}{2} \right) \ln \frac{a+c'}{a-c'} - 1 \right] = -55.25423 \text{ eV} \quad (14.21)$$

From the energy relationship given by Eq. (14.21) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CO_2 MO can be solved.

Substitution of Eq. (13.60) into Eq. (14.21) gives

$$E_T(C=O, \sigma) = \frac{4e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[\left(\frac{3}{2} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = -55.25423 \text{ eV} \quad (14.22)$$

10 The most convenient way to solve Eq. (14.22) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.80703a_0 = 9.56239 \times 10^{-11} \text{ m} \quad (14.23)$$

Substitution of Eq. (14.23) into Eq. (13.60) is

$$c' = 1.09758a_0 = 5.80815 \times 10^{-11} \text{ m} \quad (14.24)$$

15 The internuclear distance given by multiplying Eq. (14.24) by two is

$$2c' = 2.19516a_0 = 1.16163 \times 10^{-10} \text{ m} \quad (14.25)$$

The experimental bond distance is [3]

$$2c' = 1.1600 \times 10^{-10} \text{ m} \quad (14.26)$$

Substitution of Eqs. (14.23-14.24) into Eq. (13.62) is

$$20 \quad b = c = 1.43550a_0 = 7.59636 \times 10^{-11} \text{ m} \quad (14.27)$$

Substitution of Eqs. (14.23-14.24) into Eq. (13.63) is

$$e = 0.60740 \quad (14.28)$$

The C and O nuclei comprise the foci of each H_2 -type ellipsoidal MO defined as $O=C=O$. Consider the left-hand $C=O$ bond of the two equivalent bonds in the absence

of the right-hand bond. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C2s$ AO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_a = 0.84317a_0$ is the radius of the $C2s$ shell. Substitution of Eqs. (14.23-14.24) into Eq. (13.261) gives

$$5 \quad \theta' = 54.53^\circ \quad (14.29)$$

Then, the angle θ_{C2sAO} the radial vector of the $C2s$ AO makes with the internuclear axis is

$$\theta_{C2sAO} = 180^\circ - 54.53^\circ = 125.47^\circ \quad (14.30)$$

as shown in Figure 33. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{H_2MO}$

10 between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C2s$ radial vector obeys the following relationship:

$$r_a \sin \theta_{C2sAO} = 0.84317a_0 \sin \theta_{C2sAO} = b \sin \theta_{H_2MO} \quad (14.31)$$

such that

$$\theta_{H_2MO} = \sin^{-1} \frac{0.84317a_0 \sin \theta_{C2sAO}}{b} = \sin^{-1} \frac{0.84317a_0 \sin 125.47^\circ}{b} \quad (14.32)$$

15 with the use of Eq. (14.30). Substitution of Eq. (14.27) into Eq. (14.32) gives

$$\theta_{H_2MO} = 28.58^\circ \quad (14.33)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{H_2MO} = a \cos \theta_{H_2MO} \quad (14.34)$$

20 Substitution of Eqs. (14.23) and (14.33) into Eq. (14.34) gives

$$d_{H_2MO} = 1.58687a_0 = 8.39737 \times 10^{-11} \text{ m} \quad (14.35)$$

The distance d_{C2sAO} along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$d_{C2sAO} = d_{H_2MO} - c' \quad (14.36)$$

25 Substitution of Eqs. (14.24) and (14.35) into Eq. (14.36) gives

$$d_{C2sAO} = 0.48929a_0 = 2.58922 \times 10^{-11} \text{ m} \quad (14.37)$$

The C and O nuclei comprise the foci of each H_2 -type ellipsoidal MO defined as $O=C=O$. Consider the right-hand $C=O$ bond of the two equivalent bonds. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $O2p$ AO are

given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_6 = 0.74776a_0$ is the radius of the $O2p$ shell. Substitution of Eqs. (14.23-14.24) into Eq. (13.261) gives

$$\theta' = 30.18^\circ \quad (14.38)$$

5 Then, the angle θ_{O2pAO} the radial vector of the $O2p$ AO makes with the internuclear axis is

$$\theta_{O2pAO} = 180^\circ - 30.18^\circ = 149.82^\circ \quad (14.39)$$

as shown in Figure 33. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{H_2MO}$

between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO

10 with the $O2p$ radial vector obeys the following relationship:

$$r_6 \sin \theta_{O2pAO} = 0.74776a_0 \sin \theta_{O2pAO} = b \sin \theta_{H_2MO} \quad (14.40)$$

such that

$$\theta_{H_2MO} = \sin^{-1} \frac{0.74776a_0 \sin \theta_{O2pAO}}{b} = \sin^{-1} \frac{0.74776a_0 \sin 149.82^\circ}{b} \quad (14.41)$$

with the use of Eq. (14.39). Substitution of Eq. (14.27) into Eq. (14.41) gives

$$15 \quad \theta_{H_2MO} = 15.18^\circ \quad (14.42)$$

Then, the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{H_2MO} = a \cos \theta_{H_2MO} \quad (14.43)$$

Substitution of Eqs. (14.23) and (14.42) into Eq. (14.43) gives

$$20 \quad d_{H_2MO} = 1.74396a_0 = 9.22862 \times 10^{-11} \text{ m} \quad (14.44)$$

The distance d_{O2pAO} along the internuclear axis from the origin of each O atom to the point of intersection of the orbitals is given by

$$d_{O2pAO} = d_{H_2MO} - c' \quad (14.45)$$

Substitution of Eqs. (14.24) and (14.44) into Eq. (14.45) gives

$$25 \quad d_{O2pAO} = 0.64637a_0 = 3.42047 \times 10^{-11} \text{ m} \quad (14.46)$$

As shown in Eq. (14.18), each $C = O$ bond comprises a factor of 0.75 of the charge-density of double that of the H_2 -type ellipsoidal MO. Using the electron configuration of CO_2 (Eq. (14.2)), the radii of the $C1s = 0.17113a_0$ (Eq. (10.51)), $C2s = 0.84317a_0$ (Eq. (10.62)), $O1s = 0.12739a_0$ (Eq. (10.51)), $O2s = 0.59020a_0$ (Eq. (10.62)), and

$O2p = 0.74776a_0$ (Eq. (14.11)) shells and the parameters of the σ MO of CO_2 given by Eqs. (13.3-13.4), (14.23-14.25), and (14.27-14.28), the dimensional diagram and charge-density of the CO_2 MO are shown in Figures 33 and 34, respectively.

5 SUM OF THE ENERGIES OF THE σ MO AND THE AOs OF THE CARBON DIOXIDE MOLECULE

The energies of the CO_2 σ MO are given by the substitution of the semiprincipal axes (Eqs. (14.23-14.24) and (14.27)) into the energy equations of OH (Eqs. (13.67-13.73)), except that the terms based on charge are multiplied by four and the kinetic energy term is multiplied by
10 two due to each σ -MO double bond:

$$V_e = 2^2 \left(\frac{3}{4} \right) \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -104.83940 \text{ eV} \quad (14.47)$$

$$V_p = 2^2 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} = 49.58464 \text{ eV} \quad (14.48)$$

$$T = 2 \left(\frac{3}{4} \right) \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 14.50438 \text{ eV} \quad (14.49)$$

$$V_m = 2^2 \left(\frac{3}{4} \right) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -14.50438 \text{ eV} \quad (14.50)$$

$$15 \quad E_T = V_e + T + V_m + V_p \quad (14.51)$$

Substitution of Eqs. (13.60) and (14.47-14.50) into Eq. (14.51) gives

$$E_T(C=O, \sigma) = V_e + T + V_m + V_p = \frac{-4e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[\left(\frac{3}{2} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = -55.25476 \text{ eV} \quad (14.52)$$

where $E_T(C=O, \sigma)$ is the total energy of each $C=O$ σ MO of CO_2 given by Eq. (14.19)
20 which is reiteratively matched to Eq. (14.20) within five-significant-figure round off error.

The total energy of CO_2 , $E_T(CO_2)$, is given by the sum of $E(\text{ionization}; C)$ and $E(\text{ionization}; C^+)$, the sum of the energies of the first and second electrons of carbon (Eqs. (14.12-14.13)) donated to each double bond, the sum of $E(\text{ionization}; O)$ and two times $E(\text{ionization}; O^+)$, the energies of the first and second electrons of oxygen (Eqs. (14.14-

14.15)) donated to the double bonds, two times $E_T(O, 2p)$, the $O2p$ AO contribution due to the decrease in radius with the formation of each bond (Eq. (14.17)), and two times $E_T(C=O, \sigma)$, the σ MO contribution given by Eq. (14.22):

$$\begin{aligned}
 E_T(CO_2) &= \left(E(\text{ionization}; C) + E(\text{ionization}; C^+) + E(\text{ionization}; O) \right) \\
 &\quad + 2E(\text{ionization}; O^+) + 2E_T(O, 2p) + 2E_T(C=O, \sigma) \\
 &= \left(11.26030 \text{ eV} + 24.38332 \text{ eV} + 13.61806 \text{ eV} \right) \\
 &\quad + 2(35.11730 \text{ eV}) + 2(-32.12759 \text{ eV}) \\
 &\quad - 2 \left(\frac{4e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left(\left(\frac{3}{2} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right) \right) \\
 &= \left(11.26030 \text{ eV} + 24.38332 \text{ eV} + 13.61806 \text{ eV} \right) \\
 &\quad + 2(35.11730 \text{ eV}) + 2(-32.12759 \text{ eV}) - 2(55.25423 \text{ eV}) \\
 &= -55.26841 \text{ eV}
 \end{aligned} \tag{14.53}$$

5

VIBRATION OF CO_2

The vibrational energy levels of CO_2 may be solved by determining the Morse potential curve from the energy relationships for the transition from a C atom and two O atoms whose parameters are given by Eqs. (10.115-10.123) and (10.154-10.163), respectively, to a C atom whose parameter r_c is given by Eq. (10.61), two O atoms whose parameter r_o is given by Eq. (14.11), and the σ CO_2 MO whose parameters are given by Eqs. (14.23-14.25) and (14.27-14.28). As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter (K&P) [4] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by perturbation methods.

THE DOPPLER ENERGY TERMS OF THE CARBON DIOXIDE MOLECULE

The equations of the radiation reaction force of carbon dioxide are the same as those of OH with the substitution of the CO_2 parameters except that there is a factor of four increase in the

central force in Eq. (13.140) due to the double bond. Using Eqs. (13.140-13.142), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{4(0.75)e^2}{4\pi\epsilon_0 b^3}} = 4.16331 \times 10^{16} \text{ rad/s} \quad (14.54)$$

where b is given by Eq. (14.27). The kinetic energy, E_K , is given by Planck's equation (Eq. 5 (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 4.16331 \times 10^{16} \text{ rad/s} = 27.40365 \text{ eV} \quad (14.55)$$

In Eq. (11.181), substitution of $E_T(\text{CO}_2)/2$ for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.55) for \bar{E}_K gives the Doppler energy of the electrons of the reentrant orbit:

$$10 \quad \bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -27.63421 \text{ eV} \sqrt{\frac{2e(27.40365 \text{ eV})}{m_e c^2}} = -0.28619 \text{ eV} \quad (14.56)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The transition state comprises $O-\text{CO}$, oxygen binding to CO . Vibration of the linear XYZ-molecular transition state corresponds to ν_3 [5] with the maximum kinetic energy localized to the nascent $C-O$ bond. In this case, 15 the kinetic energy of the nuclei is the maximum for this bond. Thus, \bar{E}_{Kvib} is the vibrational energy. The decrease in the energy of the CO_2 MO due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.56) and \bar{E}_{Kvib} , the vibrational energy. Using the experimental CO_2 $E_{vib}(\nu_3)$ of 2349 cm^{-1} (0.29124 eV) [6] for \bar{E}_{Kvib} of 20 the transition state, $\bar{E}_{osc}(\text{CO}_2)$ is

$$\bar{E}_{osc}(\text{CO}_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + E_{vib} \quad (14.57)$$

$$\bar{E}_{osc}(\text{CO}_2) = -0.28619 \text{ eV} + 0.29124 \text{ eV} = 0.00505 \text{ eV} \quad (14.58)$$

TOTAL AND BOND ENERGIES OF THE CARBON DIOXIDE MOLECULE

$E_{T+osc}(CO_2)$, the total energy of CO_2 including the Doppler term, is given by the sum of $E_T(CO_2)$ (Eq. (14.53)) and $\bar{E}_{osc}(CO_2)$ given by Eq. (14.58):

$$\begin{aligned}
 E_{T+osc}(CO_2) &= \left(\begin{aligned} &2(V_e + T + V_m + V_p) + E(\text{ionization}; C) \\ &+ E(\text{ionization}; C^+) + E(\text{ionization}; O) + 2E(\text{ionization}; O^+) \\ &+ 2E_T(O, 2p) + \bar{E}_{osc}(CO_2) \end{aligned} \right) \\
 &= \left(\begin{aligned} &2E_T(C=O, \sigma) + E(\text{ionization}; C) + E(\text{ionization}; C^+) \\ &+ E(\text{ionization}; O) + 2E(\text{ionization}; O^+) \\ &+ 2E_T(O, 2p) + \bar{E}_{osc}(CO_2) \end{aligned} \right) \quad (14.59) \\
 &= E_T(CO_2) + \bar{E}_{osc}(CO_2)
 \end{aligned}$$

$$\begin{aligned}
 E_{T+osc}(CO_2) &= \left(\begin{aligned} &2 \left(\frac{-4e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left(\left(\frac{3}{2} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right) \right) + E(\text{ionization}; C) \\ &+ E(\text{ionization}; C^+) + E(\text{ionization}; O) \\ &+ 2E(\text{ionization}; O^+) - 2 \sum_{O, n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_8} \right) \\ &\left(1 + \frac{1}{2} \sqrt{\frac{2\hbar \sqrt{\frac{4(0.75)e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} \right) + E_{vib} \end{aligned} \right) \\
 &= -55.26841 \text{ eV} - 0.28619 \text{ eV} + E_{vib} \\
 &\quad (14.60)
 \end{aligned}$$

From Eqs. (14.57-14.60), the total energy of the CO_2 MO is

$$\begin{aligned}
 E_{T+osc}(CO_2) &= -55.25476 \text{ eV} + \bar{E}_{osc}(CO_2) \\
 &= -55.25476 \text{ eV} + 0.00505 \text{ eV} \\
 &= -55.26336 \text{ eV} \quad (14.61)
 \end{aligned}$$

where the experimental E_{vib} was used.

10 As in the case of the dissociation of the bond of the hydroxyl radical, an oxygen atom is formed with dissociation of CO_2 . O has two unpaired electrons as shown in Eq. (13.55) which interact to stabilize the atom as shown by Eq. (10.161-10.162). The lowering of the energy of the reactants decreases the bond energy. Thus, the total energy of oxygen is

reduced by the energy in the field of the two magnetic dipoles given by Eq. (7.46) and Eq. (13.101):

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 a_0^3} = \frac{8\pi\mu_0 \mu_B^2}{a_0^3} = 0.11441 \text{ eV} \quad (14.62)$$

The CO_2 bond dissociation energy, $E_D(\text{CO}_2)$, is given by the sum of the energies of the CO and the O atom minus the sum of $E_{T+\text{osc}}(\text{CO}_2)$ and $E(\text{magnetic})$:

$$E_D(\text{CO}_2) = E(\text{CO}) + E(\text{O}) - (E(\text{magnetic}) + E_{T+\text{osc}}(\text{CO}_2)) \quad (14.63)$$

The energy of an oxygen atom is given by Eq. (14.14) and $E_r(\text{CO})$ is given by the sum of the experimental energies of C (Eq. (14.12)), O (Eq. (14.14)), and the negative of the bond energy of CO (Eq. (13.914)):

$$E(\text{CO}) = -11.26030 \text{ eV} - 13.618060 \text{ eV} - 11.15696 \text{ eV} = -36.03532 \text{ eV} \quad (14.64)$$

The energy of O is given by the negative of the corresponding ionization energy given in Eq. (4.14). Thus, the CO_2 bond dissociation energy, $E_D(\text{CO}_2)$, given by the Eqs. (4.14) and (14.61-14.64) is

$$\begin{aligned} E_D(\text{CO}_2) &= -(36.03532 \text{ eV} + 13.618060 \text{ eV}) - (E(\text{magnetic}) + E_{T+\text{osc}}(\text{CO}_2)) \\ &= -49.65338 \text{ eV} - (0.11441 \text{ eV} - 55.26336 \text{ eV}) \\ &= 5.49557 \text{ eV} \end{aligned} \quad (14.65)$$

The experimental CO_2 bond dissociation energy is [7]

$$E_{D298}(\text{CO}_2) = 5.516 \text{ eV} \quad (14.66)$$

The results of the determination of bond parameters of CO_2 are given in Table 14.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

NITROGEN DIOXIDE MOLECULE

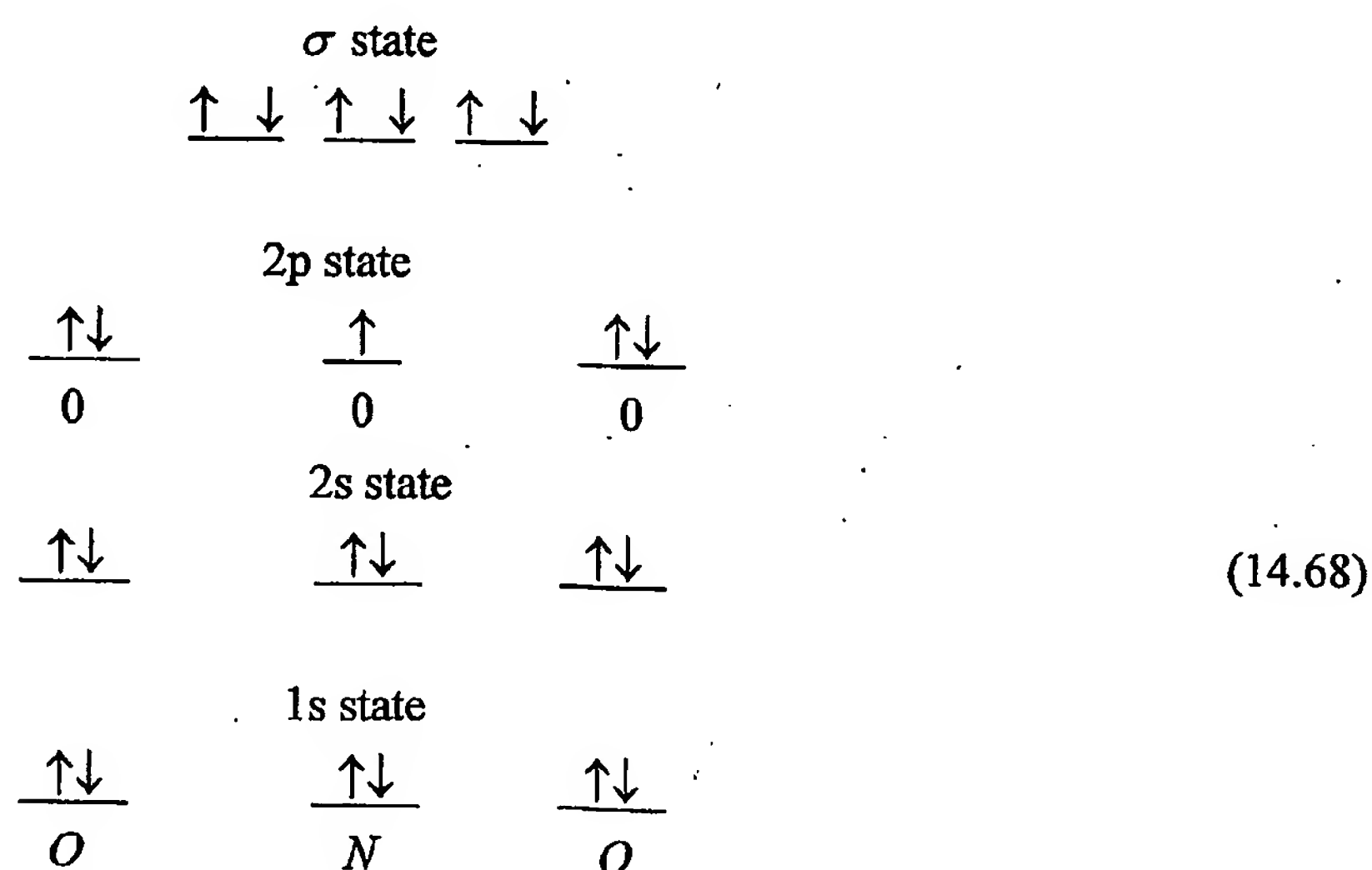
The nitrogen dioxide molecule can be formed by the reaction of nitric oxide and an oxygen atom:



The bonding in the nitrogen dioxide molecule comprises two double bonds, each a H_2 -type MO with four paired electrons wherein the central N atom is shared by both bonds such that six electrons can be assigned to the two $\text{N}=\text{O}$ bonds. Thus, two $\text{N}2p$ electrons combine

with the four $O2p$ electrons, two from each O , as a linear combination to form the two overlapping $N=O$ bonds of NO_2 . The force balance equation and radius r_7 of the $2p$ shell of N is derived in the Seven-Electron Atoms section. The force balance equation and radius r_8 of the $2p$ shell of O is derived in the Eight-Electron Atoms section. With the formation of each of the two H_2 -type MOs by the contribution of two $2p$ electrons each from the N and O atoms, a diamagnetic force arises between the remaining outer shell atomic electrons, the $2s$ and $2p$ electrons of N and O , and the H_2 -type MO. This force from N and O causes the H_2 -type MO to move to greater principal axes than would result with the Coulombic force alone. But, the factor of two increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining N and O electrons decrease the radii of the corresponding shells such that the energy minimum is achieved that is lower than that of the reactant atoms. The resulting electron configuration of NO_2 is $N1s^2O_11s^2O_21s^2N2s^2O_12s^2O_22s^2N2p^1O_12p^2O_22p^2\sigma_{O_1,N,O_1}^6$ where the subscripts designate the O atom, 1 or 2, σ designates the H_2 -type MO, and the orbital arrangement is

15



Nitrogen dioxide is predicted to be weakly paramagnetic in agreement with observations [1].

FORCE BALANCE OF THE 2p SHELL OF THE NITROGEN ATOM OF NITROGEN DIOXIDE

For the N atom, force balance for the outermost 2p electron of NO_2 (electron 5) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due to 5 interactions between electron 5 and the 2s-shell electrons due to spin and orbital angular momentum. The forces used are derived in the Seven-Electron Atoms section. The central Coulomb force on the outer-most 2p shell electron of NO (electron 5) due to the nucleus and the inner four electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$\mathbf{F}_{ele} = \frac{(Z-4)e^2}{4\pi\epsilon_0 r_5^2} \mathbf{i}_r \quad (14.69)$$

10 for $r > r_4$. The 2p shell possess a +2 external electric field given by Eq. (10.92) for $r > r_5$. The energy is minimized with conservation of angular momentum. This condition is met when the magnetic forces of N in NO_2 are the same as those of N in NO . They are also the same as those of N in the nitrogen molecule with r_5 replacing r_6 and with an increase of the central field by an integer. The diamagnetic force, $\mathbf{F}_{diamagnetic}$, of Eq. (10.82) due to the p-
15 orbital contribution is given by Eq. (13.918):

$$\mathbf{F}_{diamagnetic} = -\left(\frac{1}{3}\right) \frac{\hbar^2}{4m_e r_5^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{\hbar^2}{12m_e r_5^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (14.70)$$

And, $\mathbf{F}_{mag\ 2}$ corresponding to the conserved orbital angular momentum of the three orbitals is also the same as that of NO_2 given by Eq. (13.919):

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{3\hbar^2}{m_e r_5^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (14.71)$$

20 The electric field external to the 2p shell given by Eq. (10.92) for $r > r_5$ gives rise to a second diamagnetic force, $\mathbf{F}_{diamagnetic\ 2}$, given by Eq. (10.93). $\mathbf{F}_{diamagnetic\ 2}$ due to the binding of the p-orbital electron having an electric field of +2 outside of its radius is given by Eq. (13.920):

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-5}{Z-4}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_5^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (14.72)$$

25 The radius of the 2p shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (14.69)) and diamagnetic (Eqs. (14.70) and (14.72)), and paramagnetic (Eq. (14.71)) forces as follows:

314

$$\frac{m_e v_s^2}{r_s} = \left(\frac{(Z-4)e^2}{4\pi\epsilon_0 r_s^2} - \frac{\hbar^2}{12m_e r_s^2 r_3} \sqrt{s(s+1)} + \frac{3\hbar^2}{Zm_e r_s^2 r_3} \sqrt{s(s+1)} - \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_s^4 m_e} 10 \sqrt{s(s+1)} \right) \quad (14.73)$$

Substitution of $v_s = \frac{\hbar}{m_e r_s}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (14.73) gives:

$$\frac{\hbar^2}{m_e r_s^3} = \frac{(Z-4)e^2}{4\pi\epsilon_0 r_s^2} - \frac{\hbar^2}{12m_e r_s^2 r_3} \sqrt{\frac{3}{4}} + \frac{3\hbar^2}{Zm_e r_s^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_s^4 m_e} 10 \sqrt{\frac{3}{4}} \quad (14.74)$$

The quadratic equation corresponding to Eq. (14.74) is

$$5 \quad r_s^2 - \frac{\frac{\hbar^2}{m_e}}{\left(\frac{(Z-4)e^2}{4\pi\epsilon_0} - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} r_s - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-4)e^2}{4\pi\epsilon_0} - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} = 0 \quad (14.75)$$

The solution of Eq. (14.75) using the quadratic formula is:

$$r_s = \frac{\frac{a_0}{\left((Z-4) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \pm a_0 \sqrt{\left(\frac{1}{\left((Z-4) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \right)^2 + \frac{20\sqrt{3} \left[\frac{Z-5}{Z-4} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{\left((Z-4) - \left(\frac{1}{12} - \frac{3}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)}}}{2} \quad (14.76)$$

r_3 in units of a_0

The positive root of Eq. (14.76) must be taken in order that $r_s > 0$. Substitution of

10 $\frac{r_3}{a_0} = 0.69385$ (Eq. (10.62) with $Z = 7$) into Eq. (14.76) gives

$$r_s = 0.74841 a_0 \quad (14.77)$$

FORCE BALANCE OF THE $2p$ SHELL OF EACH OXYGEN ATOM OF NITROGEN DIOXIDE

15 For each O atom, force balance for the outermost $2p$ electron of NO_2 (electron 6) is achieved between the centrifugal force and the Coulombic and magnetic forces that arise due

to interactions between electron 6 and the other 2p electron as well as the 2s-shell electrons due to spin and orbital angular momentum. The forces used are derived in the Eight-Electron Atoms section. The central Coulomb force on the outer-most 2p shell electron of NO₂ (electron 6) due to the nucleus and the inner five electrons is given by Eq. (10.70) with the appropriate charge and radius:

$$\mathbf{F}_{ele} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} \mathbf{i}_r \quad (14.78)$$

for $r > r_5$. The 2p shell possess an external electric field of +2 given by Eq. (10.92) for $r > r_6$. The energy is minimized with conservation of angular momentum. This condition is met when the magnetic forces of O in NO₂ are the same as those of O in NO. The diamagnetic force, $\mathbf{F}_{diamagnetic}$, of Eq. (10.82) due to the p-orbital contribution given by Eq. (13.927) is

$$\mathbf{F}_{diamagnetic} = -\left(\frac{2}{3}\right) \frac{\hbar^2}{4m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r = -\frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r \quad (14.79)$$

And, $\mathbf{F}_{mag\ 2}$ corresponding to the conserved spin and orbital angular momentum given by Eq. (13.928) is

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{2\hbar^2}{m_e r_6^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r \quad (14.80)$$

The electric field external to the 2p shell given by Eq. (10.92) for $r > r_6$ gives rise to a second diamagnetic force, $\mathbf{F}_{diamagnetic\ 2}$, given by Eq. (10.93). $\mathbf{F}_{diamagnetic\ 2}$ due to the binding of the p-orbital electron having an electric field of +2 outside of its radius given by Eq. (13.929) is

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-6}{Z-5}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_6^4} 10 \sqrt{s(s+1)} \mathbf{i}_r \quad (14.81)$$

In addition, the contribution of two 2p electrons in the formation of the σ MO gives rise to a paramagnetic force on the remaining paired 2p electrons. The force $\mathbf{F}_{mag\ 3}$ is given by Eq. (13.930) is

$$\mathbf{F}_{mag\ 3} = \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \mathbf{i}_r \quad (14.82)$$

The radius of the 2p shell is calculated by equating the outward centrifugal force to the sum of the electric (Eq. (14.78)) and diamagnetic (Eqs. (14.79) and (14.81)), and

paramagnetic (Eqs. (14.80) and (14.82)) forces as follows:

$$\frac{m_e v_6^2}{r_6} = \left(\frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{s(s+1)} + \frac{2\hbar^2}{Zm_e r_6^2 r_3} \sqrt{s(s+1)} \right) - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{s(s+1)} + \frac{\hbar^2}{4m_e r_6^3} \sqrt{s(s+1)} \right) \quad (14.83)$$

Substitution of $v_6 = \frac{\hbar}{m_e r_6}$ (Eq. (1.56)) and $s = \frac{1}{2}$ into Eq. (14.83) gives:

$$\frac{\hbar^2}{m_e r_6^3} - \frac{\hbar^2}{4m_e r_6^3} \sqrt{\frac{3}{4}} = \frac{(Z-5)e^2}{4\pi\epsilon_0 r_6^2} - \frac{2\hbar^2}{12m_e r_6^2 r_3} \sqrt{\frac{3}{4}} + \frac{2\hbar^2}{Zm_e r_6^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_6^4 m_e} 10 \sqrt{\frac{3}{4}} \quad (14.84)$$

5

The quadratic equation corresponding to Eq. (14.84) is

$$r_6^2 - \frac{\frac{\hbar^2}{m_e} \left(1 - \frac{\sqrt{3}}{8} \right)}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} r_6 - \frac{\frac{\hbar^2}{m_e} \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 10 \sqrt{\frac{3}{4}}}{\left(\frac{(Z-5)e^2}{4\pi\epsilon_0} - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\hbar^2}{m_e r_3} \sqrt{\frac{3}{4}} \right)} = 0 \quad (14.85)$$

The solution of Eq. (14.85) using the quadratic formula is:

$$r_6 = \frac{a_0 \left(1 - \frac{\sqrt{3}}{8} \right)}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \pm a_0 \sqrt{\frac{\left(\frac{1 - \frac{\sqrt{3}}{8}}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \right)^2}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} + \frac{20\sqrt{3} \left[\frac{Z-6}{Z-5} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{\left((Z-5) - \left(\frac{2}{12} - \frac{2}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)}} \quad (14.86)$$

r_3 in units of a_0

The positive root of Eq. (14.86) must be taken in order that $r_6 > 0$. Substitution of

$\frac{r_3}{a_0} = 0.59020$ (Eq. (10.62) with $Z = 8$) into Eq. (14.86) gives

$$r_6 = 0.70460 a_0 \quad (14.87)$$

15 ENERGIES OF THE 2p SHELLS OF THE NITROGEN ATOM AND

OXYGEN ATOMS OF NITROGEN DIOXIDE

Consider the determination of the total energy of NO_2 from the reaction of a nitrogen atom with two oxygen atoms. With the formation of each H_2 -type MO by the contribution of two $2p$ electrons from each of the N and the two O atoms, the total energy of the NO_2 molecule, which is subtracted from the sum of the energies of the nitrogen and oxygen atoms to determine the bond energy, is increased by the ionization energies of N , N^+ , O , and $2O^+$ given by Eqs. (14.88-14.91), respectively. Experimentally, the energies are [2]

$$E(\text{ionization}; N) = 14.53414 \text{ eV} \quad (14.88)$$

$$E(\text{ionization}; N^+) = 29.6013 \text{ eV} \quad (14.89)$$

$$E(\text{ionization}; O) = 13.61806 \text{ eV} \quad (14.90)$$

$$E(\text{ionization}; O^+) = 35.11730 \text{ eV} \quad (14.91)$$

In addition, the central forces on the $2p$ shells of the N and O atoms are increased with the formation of the σ MOs which reduces each shell's radius and increases its total energy. The change per bond is the same as that of NO since the final radii given by Eq. (14.77) and (14.87) are the same for NO and NO_2 . The Coulombic energy terms of the total energy of the N and O atoms at the new radii are calculated and added to the ionization energies of N , N^+ , O , and $2O^+$, and the energy of the σ MOs to give the total energy of NO_2 . Then, the bond energy is determined from the total NO_2 energy.

The radius r_7 of the nitrogen atom before bonding is given by Eq. (10.142):

$$r_7 = 0.93084a_0 \quad (14.92)$$

Using the initial radius r_7 of the N atom and the final radius r_5 of the $N2p$ shell (Eq. (14.77)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(N, 2p)$ of the Coulombic energy change of the $N2p$ electrons of the N atom is determined using Eq. (10.102):

$$\begin{aligned} E_T(N, 2p) &= -\sum_{n=4}^4 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_5} - \frac{1}{r_7} \right) \\ &= -(13.60580 \text{ eV})(0.26186)(3) \\ &= -10.68853 \text{ eV} \end{aligned} \quad (14.93)$$

The radius r_8 of the oxygen atom before bonding is given by Eq. (10.162):

$$r_8 = a_0 \quad (14.94)$$

Using the initial radius r_8 of the O atom and the final radius r_6 of the $O2p$ shell (Eq. (14.87)) and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell, the sum $E_T(O, 2p)$ of the Coulombic energy change of the $O2p$ electrons of the O atom is determined using Eq. (10.102):

$$\begin{aligned}
 E_T(O, 2p) &= \sum_{n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_8} \right) \\
 &= (13.60580 \text{ eV}) (0.41925 a_0^{-1}) (3+4) \\
 &= -39.92918 \text{ eV}
 \end{aligned}
 \tag{14.95}$$

FORCE BALANCE OF THE σ MO OF NITROGEN DIOXIDE

The force balance can be considered due to a second pair of two electrons binding to a molecular ion having $+2e$ at each focus and a first bound pair. Then, the forces are the same as those of a molecule ion having $+e$ at each focus. The diamagnetic force $F_{\text{diamagneticMO1}}$ for each σ -MO of the NO_2 molecule due to the two paired electrons in the $O2p$ shell is given by Eq. (13.633) with $n_e = 2$:

$$F_{\text{diamagneticMO1}} = \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi \tag{14.96}$$

This is also the corresponding force of NO given by Eq. (13.942). $F_{\text{diamagneticMO2}}$ of the nitrogen dioxide molecule comprising nitrogen with charge $Z_1 = 7$ and $|L_1| = \hbar$ and $|L_2| = \sqrt{\frac{3}{4}}\hbar$ and the two oxygen atoms, each with $Z_2 = 8$ and $|L_3| = \hbar$ is given by the corresponding sum of the contributions. Using Eq. (13.835), $F_{\text{diamagneticMO2}}$ for NO_2 is

$$F_{\text{diamagneticMO2}} = \left(\frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{2}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi \tag{14.97}$$

This is also the corresponding force of NO given by Eq. (13.943) except the term due to oxygen is twice that of NO due to the two oxygen atoms of NO_2 . The general force balance equation for the σ -MO of the nitrogen dioxide molecule given by Eqs. (11.200), and (14.97-14.98) is also the same as that of CN (Eq. (14.836)) except for the doubling of the $\frac{2}{Z_2}$ term due to the two oxygen atoms:

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{2}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (14.98)$$

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D - \left(\frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{2}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \quad (14.99)$$

$$\left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{2}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D \quad (14.100)$$

$$a = \left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{2}{Z_2} \right) a_0 \quad (14.101)$$

5 Substitution of $Z_1 = 7$ and $Z_2 = 8$ into Eq. (14.101) gives

$$a = 2.51658a_0 = 1.33171 \times 10^{-10} \text{ m} \quad (14.102)$$

Substitution of Eq. (14.102) into Eq. (11.79) is

$$c' = 1.12173a_0 = 5.93596 \times 10^{-11} \text{ m} \quad (14.103)$$

The internuclear distance given by multiplying Eq. (14.103) by two is

$$10 \quad 2c' = 2.24347a_0 = 1.18719 \times 10^{-10} \text{ m} \quad (14.104)$$

The experimental bond distance is [3]

$$2c' = 1.193 \times 10^{-10} \text{ m} \quad (14.105)$$

Substitution of Eqs. (14.102-14.103) into Eq. (11.80) is

$$b = c = 2.25275a_0 = 1.19210 \times 10^{-10} \text{ m} \quad (14.106)$$

15 Substitution of Eqs. (14.102-14.103) into Eq. (11.67) is

$$e = 0.44574 \quad (14.107)$$

The bonding in the nitrogen dioxide molecule comprises two double bonds, each a H_2 -type MO with four paired electrons wherein the central N atom is shared by both bonds such that six electrons can be assigned to the two $N=O$ bonds. Thus, two $N2p$ electrons
20 combine with the four $O2p$ electrons, two from each O , as a linear combination to form the two overlapping $N=O$ bonds of NO_2 . Using the electron configuration of NO_2 (Eq.

(14.68)), the radii of the $N1s = 0.14605a_0$ (Eq. (10.51)), $N2s = 0.69385a_0$ (Eq. (10.62)), $N2p = 0.74841a_0$ (Eq. (14.77)), $O1s = 0.12739a_0$ (Eq. (10.51)), $O2s = 0.59020a_0$ (Eq. (10.62)), and $O2p = 0.70460a_0$ (Eq. (14.87)) shells and the parameters of the σ MOs of NO_2 given by Eqs. (13.3-13.4), (14.102-14.104), and (14.106-14.107), the dimensional
5 diagram and charge-density of the NO_2 MO are shown in Figures 35 and 36, respectively.

SUM OF THE ENERGIES OF THE σ MOs AND THE AOs OF NITROGEN DIOXIDE

The energies of each NO_2 σ MO are the same as those of NO (Eqs. (13.954-13.958)). They
10 are given by the substitution of the semiprincipal axes (Eqs. (14.102-14.103) and (14.106)) into the energy equations (Eqs. (11.207-11.212)) of H_2 except that the terms based on charge are multiplied by four and the kinetic energy term is multiplied by two due to the σ -MO double bond with two pairs of paired electrons:

$$V_e = 2^2 \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -93.03032 \text{ eV} \quad (14.108)$$

$$15 \quad V_p = 2^2 \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 48.51704 \text{ eV} \quad (14.109)$$

$$T = 2 \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 9.24176 \text{ eV} \quad (14.110)$$

$$V_m = 2^2 \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -9.24176 \text{ eV} \quad (14.111)$$

$$E_T = V_e + T + V_m + V_p \quad (14.112)$$

Substitution of Eqs. (11.79) and (14.108-14.111) into Eq. (14.112) gives

$$20 \quad E_T(N=O, \sigma) = \frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(8 \ln \frac{a+\sqrt{\frac{aa_0}{2}}}{a-\sqrt{\frac{aa_0}{2}}} - 4 \right) = -44.51329 \text{ eV} \quad (14.113)$$

where $E_T(N=O, \sigma)$ is the total energy of each σ MO of NO_2 . The total energy of NO_2 , $E_T(NO_2)$, is given by the sum of $E(\text{ionization}; N)$ and $E(\text{ionization}; N^+)$, the sum of the energies of the first and second electrons of nitrogen (Eqs. (14.88-14.89)) donated to each

double bond, the sum of $E(\text{ionization}; O)$ and two times $E(\text{ionization}; O^+)$, the energies of the first and second electrons of oxygen (Eqs. (14.90-14.91)) donated to the double bonds, $E_T(N, 2p)$, the $N2p$ AO contribution due to the decrease in radius with the formation of each bond (Eq. (14.93)), two times $E_T(O, 2p)$, the $O2p$ AO contribution due to the decrease in radius with the formation of each bond (Eq. (14.95)), and two times $E_T(N=O, \sigma)$, the σ MO contribution given by Eq. (14.113):

$$\begin{aligned}
 E_T(NO_2) &= \left(\begin{aligned} &E(\text{ionization}; N) + E(\text{ionization}; N^+) \\ &+ E(\text{ionization}; O) + 2E(\text{ionization}; O^+) + E_T(N, 2p) \\ &+ 2E_T(O, 2p) + 2E_T(N=O, \sigma) \end{aligned} \right) \\
 &= \left(\begin{aligned} &14.53414 \text{ eV} + 29.6013 \text{ eV} + 13.61806 \text{ eV} \\ &+ 2(35.11730 \text{ eV}) + (-10.68853) + 2(-39.92918 \text{ eV}) \\ &+ 2 \left(\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left(8 \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 4 \right) \right) \end{aligned} \right) \quad (14.114) \\
 &= \left(\begin{aligned} &14.53414 \text{ eV} + 29.6013 \text{ eV} + 13.61806 \text{ eV} \\ &+ 2(35.11730 \text{ eV}) + (-10.68853) \\ &+ 2(-39.92918 \text{ eV}) + 2(-44.51329 \text{ eV}) \end{aligned} \right) \\
 &= -51.58536 \text{ eV}
 \end{aligned}$$

VIBRATION OF NO_2

- 10 The vibrational energy levels of NO_2 may be solved by determining the Morse potential curve from the energy relationships for the transition from a N atom and two O atoms whose parameters are given by Eqs. (10.134-10.143) and (10.154-10.163), respectively, to a N atom whose parameter r_s is given by Eq. (14.77), two O atoms whose parameter r_o is given by Eq. (14.87), and the σ MOs whose parameters are given by Eqs. (14.102-14.104) and (14.106.-14.107). As shown in the Vibration of Hydrogen-type Molecular Ions section, the harmonic oscillator potential energy function can be expanded about the internuclear distance and expressed as a Maclaurin series corresponding to a Morse potential after Karplus and Porter (K&P) [4] and after Eq. (11.134). Treating the Maclaurin series terms as anharmonic perturbation terms of the harmonic states, the energy corrections can be found by
- 20 perturbation methods.

THE DOPPLER ENERGY TERMS OF NITROGEN DIOXIDE

The equations of the radiation reaction force of nitrogen dioxide are the same as those of NO with the substitution of the NO_2 parameters. Using Eq. (13.961), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{4e^2}{4\pi\epsilon_0 a^3}} = 2.07110 \times 10^{16} \text{ rad/s} \quad (14.115)$$

where a is given by Eq. (14.102). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.07110 \times 10^{16} \text{ rad/s} = 13.63231 \text{ eV} \quad (14.116)$$

In Eq. (11.181), substitution of $E_T(NO_2)/2$ for E_{mv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.116) for \bar{E}_K gives the Doppler energy of the electrons of the reentrant orbit:

$$\bar{E}_D \cong E_{mv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -25.79268 \text{ eV} \sqrt{\frac{2e(13.63231 \text{ eV})}{m_e c^2}} = -0.18840 \text{ eV} \quad (14.117)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The transition state comprises $O--NO$, oxygen binding to NO . As in the case of CO_2 bond formation, vibration in the transition state corresponds to ν_3 [5] with the maximum kinetic energy localized to the nascent $N-O$ bond. In this case, the kinetic energy of the nuclei is the maximum for this bond. Thus, \bar{E}_{Kvib} is the vibrational energy. The decrease in the energy of the NO_2 MO due to the reentrant orbit in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.117) and \bar{E}_{Kvib} , the vibrational energy. Using the experimental NO_2 $E_{vib}(\nu_3)$ of 1618 cm^{-1} (0.20061 eV) [6] for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(NO_2)$ is

$$\bar{E}_{osc}(NO_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + E_{vib} \quad (14.118)$$

$$\bar{E}_{osc}(NO_2) = -0.18840 \text{ eV} + 0.20061 \text{ eV} = 0.01221 \text{ eV} \quad (14.119)$$

TOTAL AND BOND ENERGIES OF NITROGEN DIOXIDE

$E_{T+osc}(NO_2)$; the total energy of NO_2 including the Doppler term, is given by the sum of $E_T(NO_2)$ (Eq. (14.114)) and $\bar{E}_{osc}(NO_2)$ given by Eq. (14.119):

$$\begin{aligned}
 E_{T+osc}(NO_2) &= \left(\begin{aligned} &2(V_e + T + V_m + V_p) + E(\text{ionization}; N) + E(\text{ionization}; N^+) \\ &+ E(\text{ionization}; O) + 2E(\text{ionization}; O^+) \\ &+ E_T(N, 2p) + 2E_T(O, 2p) + \bar{E}_{osc}(NO_2) \end{aligned} \right) \\
 &= \left(\begin{aligned} &2E_T(N=O, \sigma) + E(\text{ionization}; N) + E(\text{ionization}; N^+) \\ &+ E(\text{ionization}; O) + 2E(\text{ionization}; O^+) \\ &+ E_T(N, 2p) + 2E_T(O, 2p) + \bar{E}_{osc}(NO_2) \end{aligned} \right) \quad (14.120) \\
 &= E_T(NO_2) + \bar{E}_{osc}(NO_2)
 \end{aligned}$$

$$\begin{aligned}
 E_{T+osc}(NO_2) &= \left(\begin{aligned} &2 \left(\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left(8 \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 4 \right) \right) \\ &E(\text{ionization}; N) + E(\text{ionization}; N^+) \\ &+ E(\text{ionization}; O) + 2E(\text{ionization}; O^+) \\ &- \sum_{N, n=4}^4 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_5} - \frac{1}{r_7} \right) - 2 \sum_{O, n=4}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_6} - \frac{1}{r_8} \right) \\ &\left(1 + \frac{1}{2} \sqrt{\frac{2\hbar \sqrt{\frac{4e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right) + E_{vib} \end{aligned} \right) \quad (14.121) \\
 &= -51.58536 \text{ eV} - 0.18840 \text{ eV} + E_{vib}
 \end{aligned}$$

From Eqs. (14.119-14.121), the total energy of the NO_2 MO is

$$\begin{aligned}
 E_{T+osc}(NO_2) &= -51.58536 \text{ eV} + \bar{E}_{osc}(NO_2) \\
 &= -51.58536 \text{ eV} + 0.01221 \text{ eV} \\
 &= -51.57315 \text{ eV} \quad (14.122)
 \end{aligned}$$

where the experimental E_{vib} was used.

As in the case of the dissociation of the bond of the hydroxyl radical, an oxygen atom is formed with dissociation of NO_2 . O has two unpaired electrons as shown in Eq. (13.55) which interact to stabilize the atom as shown by Eq. (10.161-10.162). The lowering of the energy of the reactants decreases the bond energy. Thus, the total energy of oxygen is

reduced by the energy in the field of the two magnetic dipoles given by Eq. (7.46) and Eq. (13.101):

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 a_0^3} = \frac{8\pi\mu_0 \mu_B^2}{a_0^3} = 0.11441 \text{ eV} \quad (14.123)$$

The NO_2 bond dissociation energy, $E_D(\text{NO}_2)$, is given by the sum of the energies of the NO and the O atom minus the sum of $E_{T+\text{osc}}(\text{NO}_2)$ and $E(\text{magnetic})$:

$$E_D(\text{NO}_2) = E(\text{NO}) + E(\text{O}) - (E(\text{magnetic}) + E_{T+\text{osc}}(\text{NO}_2)) \quad (14.124)$$

The energy of an oxygen atom is given by the negative of Eq. (14.90), and $E_T(\text{NO})$ is given by the sum of the experimental energies of N (negative of Eq. (14.88)), O , and the negative of the bond energy of NO (Eq. (13.974)):

$$E(\text{NO}) = -14.53414 \text{ eV} - 13.618060 \text{ eV} - 6.53529 \text{ eV} = -34.68749 \text{ eV} \quad (14.125)$$

Thus, the NO_2 bond dissociation energy, $E_D(\text{NO}_2)$, given by Eqs. (4.90) and (14.112-14.125) is

$$\begin{aligned} E_D(\text{NO}_2) &= -(34.68749 \text{ eV} + 13.618060 \text{ eV}) - (E(\text{magnetic}) + E_{T+\text{osc}}(\text{NO}_2)) \\ &= -48.30555 \text{ eV} - (0.11441 \text{ eV} - 51.57315 \text{ eV}) \\ &= 3.15319 \text{ eV} \end{aligned} \quad (14.126)$$

The experimental NO_2 bond dissociation energy is [7]

$$E_{D298}(\text{NO}_2) = 3.161 \text{ eV} \quad (14.127)$$

BOND ANGLE OF NO_2

The NO_2 MO comprises a linear combination of two $\text{N}=\text{O}$ -bond MOs. A bond is also possible between the two O atoms of the $\text{N}=\text{O}$ bonds. Such $\text{O}=\text{O}$ bonding would decrease the $\text{N}=\text{O}$ bond strength since electron density would be shifted from the $\text{N}=\text{O}$ bonds to the $\text{O}=\text{O}$ bond. Thus, the bond angle between the two $\text{N}=\text{O}$ bonds is determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal O atoms of the $\text{N}=\text{O}$ bonds is zero. From Eqs. (11.79) and (13.228), the distance from the origin to each focus of the $\text{O}=\text{O}$ ellipsoidal MO is

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{aa_0}{2}} \quad (14.128)$$

The internuclear distance from Eq. (13.229) is

$$2c' = 2\sqrt{\frac{aa_0}{2}} \quad (14.129)$$

The length of the semiminor axis of the prolate spheroidal $O=O$ MO $b=c$ is given by Eq. (13.167).

The component energies and the total energy E_T of the $O=O$ bond are given by the 5 energy equations (Eqs. (11.207-11.212), (11.213-11.217), and (11.239)) of H_2 except that the terms based on charge are multiplied by four and the kinetic energy term is multiplied by two due to the $O=O$ double bond with two pairs of paired electrons. Substitution of Eq. (14.128) into Eqs. (11.207-11.212) gives

$$0 = \left[\frac{-e^2}{8\pi\epsilon_0\sqrt{\frac{aa_0}{2}}} \left(8\ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 4 \right) \left[1 + \sqrt{\frac{2\hbar\sqrt{\frac{4e^2}{4\pi\epsilon_0 a^3}}}}{m_e c^2} \right] \right. \\ \left. + \frac{1}{2}\hbar\sqrt{\frac{\frac{4e^2}{8\pi\epsilon_0 a^3} - \frac{4e^2}{8\pi\epsilon_0 (a+c')^3}}{8m_p}} \right] \quad (14.130)$$

10 From the energy relationship given by Eq. (14.130) and the relationship between the axes given by Eqs. (14.128-14.129) and (13.167-14.168), the dimensions of the $O=O$ MO can be solved.

The most convenient way to solve Eq. (14.130) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$15 \quad a = 8.3360a_0 = 4.4112 \times 10^{-10} \text{ m} \quad (14.131)$$

Substitution of Eq. (14.131) into Eq. (14.128) gives

$$c' = 2.0416a_0 = 1.0804 \times 10^{-10} \text{ m} \quad (14.132)$$

The internuclear distance given by multiplying Eq. (14.132) by two is

$$2c' = 4.0831a_0 = 2.1607 \times 10^{-10} \text{ m} \quad (14.133)$$

20 Substitution of Eqs. (14.131-14.132) into Eq. (14.167) gives

$$b = c = 8.0821a_0 = 4.2769 \times 10^{-10} \text{ m} \quad (14.134)$$

Substitution of Eqs. (14.131-14.132) into Eq. (14.168) gives

$$e = 0.2449 \quad (14.135)$$

From, $2c'_{c=c}$ (Eq. (14.133)), the distance between the two O atoms when the total 25 energy of the corresponding MO is zero (Eq. (14.130)), and $2c'_{N=O}$ (Eq. (14.104)), the

internuclear distance of each $N=O$ bond, the corresponding bond angle can be determined from the law of cosines. Using, Eqs. (13.240-13.242), the bond angle θ between the $N=O$ bonds is

$$\begin{aligned}\theta &= \cos^{-1} \left(\frac{2(2.24347)^2 - (4.0831)^2}{2(2.24347)^2} \right) \\ &= \cos^{-1}(-0.6562) \\ &= 131.012^\circ\end{aligned}\tag{14.136}$$

5 The experimental angle between the $N=O$ bonds is [3]

$$\theta = 134.1^\circ\tag{14.137}$$

The results of the determination of bond parameters of NO_2 are given in Table 14.1.

The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and
10 calculated results is excellent.

ETHANE MOLECULE (CH_3CH_3)

The ethane molecule CH_3CH_3 is formed by the reaction of two methyl radicals:



15 CH_3CH_3 can be solved using the same principles as those used to solve CH_3 , wherein the $2s$ and $2p$ shells of each C hybridize to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ hybridized orbitals (HOs) to form a molecular orbital (MO) permits each participating hybridized orbital to decrease in radius and energy. First, two sets of three H atomic orbitals (AOs) combine with two sets of three
20 carbon $2sp^3$ HOs to form two methyl groups comprising a linear combination of six diatomic H_2 -type MOs developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section. Then, the two CH_3 groups bond by forming a H_2 -type MO between the remaining $C2sp^3$ HO on each carbon.

25 FORCE BALANCE OF THE $C-C$ -BOND MO OF ETHANE

CH_3CH_3 comprises a chemical bond between two CH_3 radicals wherein each methyl radical comprises three chemical bonds between carbon and hydrogen atoms. The solution of the parameters of CH_3 is given in the Methyl Radical (CH_3) section. Each $C-H$ bond of

CH_3 having two spin-paired electrons, one from an initially unpaired electron of the carbon atom and the other from the hydrogen atom, comprises the linear combination of 75% H_2 -type ellipsoidal MO and 25% $C2sp^3$ HO. The proton of the H atom and the nucleus of the C atom are along each internuclear axis and serve as the foci. As in the case of H_2 , each of the three $C-H$ -bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell since it is energetically unfavorable. Thus, each MO surface comprises a prolate spheroid at the H proton that is continuous with the $C2sp^3$ shell at the C atom whose nucleus serves as the other focus. The electron configuration and the energy, $E(C, 2sp^3)$, of the $C2sp^3$ shell is given by Eqs. (13.422) and (13.428); respectively. The central paramagnetic force due to spin of each $C-H$ bond is provided by the spin-pairing force of the CH_3 MO that has the symmetry of an s orbital that superimposes with the $C2sp^3$ orbitals such that the corresponding angular momenta are unchanged.

Two CH_3 radicals bond to form CH_3CH_3 by forming a MO between the two remaining $C2sp^3$ -HO electrons of the two carbon atoms. However, in this case, the sharing of electrons between two $C2sp^3$ HOs to form a molecular orbital (MO) comprising two spin-paired electrons permits each $C2sp^3$ HO to decrease in radius and energy.

As in the case of the $C-H$ bonds, the $C-C$ -bond MO is a prolate-spheroidal-MO surface that cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell of each atom. Thus, the MO surface comprises a partial prolate spheroid in between the carbon nuclei and is continuous with the $C2sp^3$ shell at each C atom. The energy of the H_2 -type ellipsoidal MO is matched to that of the $C2sp^3$ shell. As in the case of previous examples of energy-matched MOs such as those of OH , NH , CH , and the $C=O$ -bond MO of CO_2 , the $C-C$ -bond MO of ethane must comprise 75% of a H_2 -type ellipsoidal MO in order to match potential, kinetic, and orbital energy relationships. Thus, the $C-C$ -bond MO must comprise two $C2sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the two $C2sp^3$ HOs:



The linear combination of the H_2 -type ellipsoidal MO with each $C2sp^3$ HO further comprises an excess 25% charge-density contribution from each $C2sp^3$ HO to the $C-C$ -bond MO to achieve an energy minimum. The force balance of the $C-C$ -bond MO is
 5 determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (14.139) and the energy matching condition between the $C2sp^3$ -HO components of the MO.

Similarly, the energies of each CH_3 MO involve each $C2sp^3$ and each $H1s$ electron with the formation of each $C-H$ bond. The sum of the energies of the H_2 -type ellipsoidal
 10 MOs is matched to that of the $C2sp^3$ shell. This energy is determined by the considering the effect of the donation of 25% electron density from the two $C2sp^3$ HOs to the $C-C$ -bond MO. The $2sp^3$ hybridized orbital arrangement given by Eq. (13.422) is

$$\begin{array}{cccc} & \text{2sp}^3 \text{ state} & & \\ \uparrow & \uparrow & \uparrow & \uparrow \\ \hline 0,0 & 1,-1 & 1,0 & 1,1 \end{array} \quad (14.140)$$

15

where the quantum numbers (ℓ, m_ℓ) are below each electron. The total energy of the state is given by the sum over the four electrons. The sum $E_T(C, 2sp^3)$ of calculated energies of C , C^+ , C^{2+} , and C^{3+} from Eqs. (10.123), (10.113-10.114), (10.68), and (10.48), respectively, is

$$\begin{aligned} E_T(C, 2sp^3) &= 64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + 11.27671 \text{ eV} \\ &= 148.25751 \text{ eV} \end{aligned} \quad (14.141)$$

20 which agrees well with the sum of 148.02532 eV from the experimental [2] values. Consider the case of the $C2sp^3$ HO of each methyl radical. The orbital-angular-momentum interactions cancel such that the energy of the $E_T(C, 2sp^3)$ is purely Coulombic. By considering that the central field decreases by an integer for each successive electron of the shell, the radius r_{2sp^3} of the $C2sp^3$ shell may be calculated from the Coulombic energy using

25 Eq. (10.102):

$$r_{2sp^3} = \sum_{n=2}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0 (e148.25751 \text{ eV})} = \frac{10e^2}{8\pi\epsilon_0 (e148.25751 \text{ eV})} = 0.91771a_0 \quad (14.142)$$

where $Z=6$ for carbon. Using Eqs. (10.102) and (14.142), the Coulombic energy $E_{Coulomb}(C, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$E_{Coulomb}(C, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.91771a_0} = -14.82575 \text{ eV} \quad (14.143)$$

During hybridization, one of the spin-paired $2s$ electrons is promoted to $C2sp^3$ shell as an unpaired electron. The energy for the promotion is the magnetic energy given by Eq. (13.152) at the initial radius of the $2s$ electrons. From Eq. (10.62) with $Z=6$, the radius r_3 of $C2s$ shell is

$$r_3 = 0.84317a_0 \quad (14.144)$$

Using Eqs. (13.152) and (14.144), the unpairing energy is

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} = \frac{8\pi\mu_0 \mu_B^2}{(0.84317a_0)^3} = 0.19086 \text{ eV} \quad (14.145)$$

Using Eqs. (14.143) and (14.145), the energy $E(C, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$\begin{aligned} E(C, 2sp^3) &= \frac{-e^2}{8\pi\epsilon_0 r_{2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} \\ &= -14.82575 \text{ eV} + 0.19086 \text{ eV} \\ &= -14.63489 \text{ eV} \end{aligned} \quad (14.146)$$

Next, consider the formation of the $C-C$ -bond MO of ethane from two methyl radicals, each having a $C2sp^3$ electron with an energy given by Eq. (14.146). The total energy of the state is given by the sum over the four electrons. The sum $E_T(C_{ethane}, 2sp^3)$ of calculated energies of $C2sp^3$, C^+ , C^{2+} , and C^{3+} from Eqs. (10.123), (10.113-10.114), (10.68), and (10.48), respectively, is

$$\begin{aligned} E_T(C_{ethane}, 2sp^3) &= -(64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + E(C, 2sp^3)) \\ &= -(64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + 14.63489 \text{ eV}) \\ &= -151.61569 \text{ eV} \end{aligned} \quad (14.147)$$

where $E(C, 2sp^3)$ is the sum of the energy of C , -11.27671 eV , and the hybridization energy. The orbital-angular-momentum interactions also cancel such that the energy of the $E_T(C_{ethane}, 2sp^3)$ is purely Coulombic.

The sharing of electrons between two $C2sp^3$ HOs to form a $C-C$ -bond MO permits

each participating hybridized orbital to decrease in radius and energy. In order to further satisfy the potential, kinetic, and orbital energy relationships, each $C2sp^3$ HO donates an excess of 25% of its electron density to the C–C-bond MO to form an energy minimum. By considering this electron redistribution in the ethane molecule as well as the fact that the central field decreases by an integer for each successive electron of the shell, the radius $r_{ethane2sp^3}$ of the $C2sp^3$ shell of ethane may be calculated from the Coulombic energy using Eq. (10.102):

$$\begin{aligned} r_{ethane2sp^3} &= \left(\sum_{n=2}^5 (Z-n) - 0.25 \right) \frac{e^2}{8\pi\epsilon_0 (e151.61569 \text{ eV})} \\ &= \frac{9.75e^2}{8\pi\epsilon_0 (e151.61569 \text{ eV})} \\ &= 0.87495a_0 \end{aligned} \quad (14.148)$$

Using Eqs. (10.102) and (14.148), the Coulombic energy $E_{Coulomb}(C_{ethane}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$\begin{aligned} E_{Coulomb}(C_{ethane}, 2sp^3) &= \frac{-e^2}{8\pi\epsilon_0 r_{ethane2sp^3}} \\ &= \frac{-e^2}{8\pi\epsilon_0 0.87495a_0} \\ &= -15.55033 \text{ eV} \end{aligned} \quad (14.149)$$

During hybridization, one of the spin-paired $2s$ electrons is promoted to $C2sp^3$ shell as an unpaired electron. The energy for the promotion is the magnetic energy given by Eq. (13.152). Using Eqs. (14.145) and (14.149), the energy $E(C_{ethane}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$\begin{aligned} E(C_{ethane}, 2sp^3) &= \frac{-e^2}{8\pi\epsilon_0 r_{ethane2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} \\ &= -15.55033 \text{ eV} + 0.19086 \text{ eV} \\ &= -15.35946 \text{ eV} \end{aligned} \quad (14.150)$$

Thus, $E_T(C-C, 2sp^3)$, the energy change of each $C2sp^3$ shell with the formation of the C–C-bond MO is given by the difference between Eq. (14.146) and Eq. (14.150):

$$\begin{aligned} E_T(C-C, 2sp^3) &= E(C_{ethane}, 2sp^3) - E(C, 2sp^3) \\ &= -15.35946 \text{ eV} - (-14.63489 \text{ eV}) \\ &= -0.72457 \text{ eV} \end{aligned} \quad (14.151)$$

The H_2 -type ellipsoidal MO comprises 75% of the $C-C$ -bond MO shared between two $C2sp^3$ HOs corresponding to the electron charge density in Eq. (11.65) of $\frac{-0.75e}{2}$. But, the additional 25% charge-density contribution to the $C-C$ -bond MO causes the electron charge density in Eq. (11.65) to be is given by $\frac{-e}{2} = -0.5e$. Thus, the force constant k' to
 5 determine the ellipsoidal parameter c' in terms of the central force of the foci given by Eq. (11.65) is

$$k' = \frac{(0.5)2e^2}{4\pi\epsilon_0} \quad (14.152)$$

The distance from the origin to each focus c' is given by substitution of Eq. (14.152) into Eq. (13.60). Thus, the distance from the origin of the $C-C$ -bond MO to each focus c' is given
 10 by

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 a}} = \sqrt{aa_0} \quad (14.153)$$

The internuclear distance from Eq. (14.153) is

$$2c' = 2\sqrt{aa_0} \quad (14.154)$$

The length of the semiminor axis of the prolate spheroidal $C-C$ -bond MO $b = c$ is given by
 15 Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of the $C-C$ -bond MO. Since the $C-C$ -bond MO comprises a H_2 -type-ellipsoidal MO that transitions to the $C_{ethane}2sp^3$ HO of each carbon, the energy $E(C_{ethane}, 2sp^3)$ in Eq. (14.150) adds to that of the H_2 -type ellipsoidal MO to give the total energy of the $C-C$ -bond MO.
 20 From the energy equation and the relationship between the axes, the dimensions of the $C-C$ -bond MO are solved. Similarly, $E(C_{ethane}, 2sp^3)$ is added to the energy of the H_2 -type ellipsoidal MO of each $C-H$ bond of the methyl groups to give its total energy. From the energy equation and the relationship between the axes, the dimensions of the equivalent $C-H$ -bond MOs of the methyl groups in ethane are solved.

25 The general equations for the energy components of V_e , V_p , T , V_m , and E_T of the $C-C$ -bond MO are the same as those of the CH MO as well as each $C-H$ -bond MO of the methyl groups except that energy of the $C_{ethane}2sp^3$ HO is used. Since the prolate

spheroidal H_2 -type MO transitions to the $C_{ethane} 2sp^3$ HO of each carbon and the energy of the $C_{ethane} 2sp^3$ shell must remain constant and equal to the $E(C_{ethane}, 2sp^3)$ given by Eq. (14.150), the total energy $E_T(C-C, \sigma)$ of the σ component of the $C-C$ -bond MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the $C_{ethane} 2sp^3$ HO and the H_2 -type ellipsoidal MO that forms the σ component of the $C-C$ -bond MO as given by Eq. (14.139) with the electron charge redistribution. Using Eqs. (13.431) and (14.150), $E_T(C-C, \sigma)$ is given by

$$E_T(C-C, \sigma) = E_T + E(C_{ethane}, 2sp^3) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.35946 \text{ eV} \quad (14.155)$$

To match the boundary condition that the total energy of the entire the H_2 -type ellipsoidal MO is given by Eqs. (11.212) and (13.75), $E_T(C-C, \sigma)$ given by Eq. (14.155) is set equal to Eq. (13.75):

$$E_T(C-C, \sigma) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.35946 \text{ eV} = -31.63536831 \text{ eV} \quad (14.156)$$

From the energy relationship given by Eq. (14.156) and the relationship between the axes given by Eqs. (14.153-14.154) and (13.62-13.63), the dimensions of the $C-C$ -bond MO can be solved.

Substitution of Eq. (14.153) into Eq. (14.156) gives

$$\frac{e^2}{8\pi\epsilon_0 \sqrt{aa_0}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+\sqrt{aa_0}}{a-\sqrt{aa_0}} - 1 \right] = e16.27589 \quad (14.157)$$

The most convenient way to solve Eq. (14.157) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 2.10725a_0 = 1.11511 \times 10^{-10} \text{ m} \quad (14.158)$$

Substitution of Eq. (14.158) into Eq. (14.153) gives

$$c' = 1.45164a_0 = 7.68173 \times 10^{-11} \text{ m} \quad (14.159)$$

The internuclear distance given by multiplying Eq. (14.159) by two is

$$2c' = 2.90327a_0 = 1.53635 \times 10^{-10} \text{ m} \quad (14.160)$$

The experimental bond distance is [3]

$$2c' = 1.5351 \times 10^{-10} \text{ m} \quad (14.161)$$

Substitution of Eqs. (14.158-14.159) into Eq. (13.62) gives

$$b = c = 1.52750a_0 = 8.08317 \times 10^{-11} \text{ m} \quad (14.162)$$

Substitution of Eqs. (14.158-14.159) into Eq. (13.63) gives

$$e = 0.68888 \quad (14.163)$$

The nucleus of the C atoms comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{ethane} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{ethane 2sp^3} = 0.87495a_0$ is the radius of the $C_{ethane} 2sp^3$ shell.

Substitution of Eqs. (14.158-14.159) into Eq. (13.261) gives

$$\theta' = 67.33^\circ \quad (14.164)$$

Then, the angle $\theta_{C-C_{ethane} 2sp^3 HO}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear axis is

$$\theta_{C-C_{ethane} 2sp^3 HO} = 180^\circ - 67.33^\circ = 112.67^\circ \quad (14.165)$$

as shown in Figure 37.

Consider the right-hand intersection point. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C-C_{ethane}, H_2 MO}$ between the internuclear axis and the point of intersection of the H_2 -

type ellipsoidal MO with the $C_{ethane} 2sp^3$ radial vector obeys the following relationship:

$$r_{ethane 2sp^3} \sin \theta_{C-C_{ethane} 2sp^3 HO} = 0.87495a_0 \sin \theta_{C-C_{ethane} 2sp^3 HO} = b \sin \theta_{C-C_{ethane}, H_2 MO} \quad (14.166)$$

such that

$$\theta_{C-C_{ethane}, H_2 MO} = \sin^{-1} \frac{0.87495a_0 \sin \theta_{C-C_{ethane} 2sp^3 HO}}{b} = \sin^{-1} \frac{0.87495a_0 \sin 112.67^\circ}{b} \quad (14.167)$$

with the use of Eq. (14.166). Substitution of Eq. (14.162) into Eq. (14.167) gives

$$\theta_{C-C_{ethane}, H_2 MO} = 31.91^\circ \quad (14.168)$$

Then, the distance $d_{C-C_{ethane}, H_2 MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C-C_{ethane}, H_2 MO} = a \cos \theta_{C-C_{ethane}, H_2 MO} \quad (14.169)$$

Substitution of Eqs. (14.158) and (14.168) into Eq. (14.169) gives

$$d_{C-C_{ethane},H_2MO} = 1.78885a_0 = 9.46617 \times 10^{-11} m \quad (14.170)$$

The distance $d_{C-C_{ethane},2sp^3HO}$ along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$d_{C-C_{ethane},2sp^3HO} = d_{C-C_{ethane},H_2MO} - c' \quad (14.171)$$

5 Substitution of Eqs. (14.159) and (14.170) into Eq. (14.171) gives

$$d_{C-C_{ethane},2sp^3HO} = 0.33721a_0 = 1.78444 \times 10^{-11} m \quad (14.172)$$

FORCE BALANCE OF THE CH_3 MOs OF ETHANE

Each of the two equivalent CH_3 MOs must comprise three $C-H$ bonds with each
10 comprising 75% of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO as given by Eq. (13.540):

$$3[1 C2sp^3 + 0.75 H_2 MO] \rightarrow CH_3 MO \quad (14.173)$$

The force balance of the CH_3 MO is determined by the boundary conditions that arise from
15 the linear combination of orbitals according to Eq. (13.540) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-ellipsoidal-MO component of the CH_3 MO in terms of the central force of the foci is given by Eq. (13.59). The distance from the origin of each $C-H$ -bond MO to each focus c' is
20 given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of each $C-H$ -bond MO. Since each of the three prolate spheroidal $C-H$ -bond MOs comprises an H_2 -type-
25 ellipsoidal MO that transitions to the $C_{ethane}2sp^3$ HO of ethane, the energy $E(C_{ethane},2sp^3)$ of Eq. (14.150) adds to that of the three corresponding H_2 -type ellipsoidal MOs to give the total energy of the CH_3 MO. From the energy equation and the relationship between the axes, the dimensions of the CH_3 MO are solved.

The energy components of V_e , V_p , T , and V_m are the same as those of methyl radical, three times those of CH corresponding to the three $C-H$ bonds except that energy of the $C_{ethane} 2sp^3$ HO is used. Since the each prolate spheroidal H_2 -type MO transitions to the $C_{ethane} 2sp^3$ HO and the energy of the $C_{ethane} 2sp^3$ shell must remain constant and equal to the $E(C_{ethane}, 2sp^3)$ given by Eq. (14.150), the total energy $E_{T_{ethane}}(CH_3)$ of the CH_3 MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the $C_{ethane} 2sp^3$ HO and the three H_2 -type ellipsoidal MOs that forms the CH_3 MO as given by Eq. (13.540). Using Eq. (13.431), $E_{T_{ethane}}(CH_3)$ is given by

$$E_{T_{ethane}}(CH_3) = E_T + E(C_{ethane}, 2sp^3) = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.35946 \text{ eV} \quad (14.174)$$

$E_{T_{ethane}}(CH_3)$ given by Eq. (14.174) is set equal to three times the energy of the H_2 -type ellipsoidal MO minus two times the Coulombic energy of H given by Eq. (13.542):

$$E_T(CH_3) = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.35946 \text{ eV} = -67.69450 \text{ eV} \quad (14.175)$$

From the energy relationship given by Eq. (14.175) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH_3 MO can be solved.

Substitution of Eq. (13.60) into Eq. (14.175) gives

$$\frac{3e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e52.33505 \quad (14.176)$$

The most convenient way to solve Eq. (14.176) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.64469a_0 = 8.70331 \times 10^{-11} \text{ m} \quad (14.177)$$

Substitution of Eq. (14.177) into Eq. (14.60) gives

$$c' = 1.04712a_0 = 5.54111 \times 10^{-11} \text{ m} \quad (14.178)$$

The internuclear distance given by multiplying Eq. (14.178) by two is

$$2c' = 2.09424a_0 = 1.10822 \times 10^{-10} \text{ m} \quad (14.179)$$

The experimental bond distance is [3]

$$2c' = 1.0940 \times 10^{-10} \text{ m} \quad (14.180)$$

Substitution of Eqs. (14.177-14.178) into Eq. (14.62) gives

$$b = c = 1.26828a_0 = 6.71145 \times 10^{-11} \text{ m} \quad (14.181)$$

Substitution of Eqs. (14.177-14.178) into Eq. (14.63) gives

$$5 \quad e = 0.63667 \quad (14.182)$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{\text{ethane}} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{\text{ethane} 2sp^3} = 0.87495a_0$ is

10 the radius of the $C_{\text{ethane}} 2sp^3$ shell. Substitution of Eqs. (14.177-14.178) into Eq. (13.261) gives

$$\theta' = 79.34^\circ \quad (14.183)$$

Then, the angle $\theta_{C-H_{\text{ethane}} 2sp^3 HO}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear axis is

$$15 \quad \theta_{C-H_{\text{ethane}} 2sp^3 HO} = 180^\circ - 79.34^\circ = 100.66^\circ \quad (14.184)$$

as shown in Figure 38.

The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C-H_{\text{ethane}} H_2 MO}$ between the
20 internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C_{\text{ethane}} 2sp^3$ radial vector obeys the following relationship:

$$r_{\text{ethane} 2sp^3} \sin \theta_{C-H_{\text{ethane}} 2sp^3 HO} = 0.87495a_0 \sin \theta_{C-H_{\text{ethane}} 2sp^3 HO} = b \sin \theta_{C-H_{\text{ethane}} H_2 MO} \quad (14.185)$$

such that

$$\theta_{C-H_{\text{ethane}} H_2 MO} = \sin^{-1} \frac{0.87495a_0 \sin \theta_{C-H_{\text{ethane}} 2sp^3 HO}}{b} = \sin^{-1} \frac{0.87495a_0 \sin 100.66^\circ}{b} \quad (14.186)$$

25 with the use of Eq. (14.184). Substitution of Eq. (14.181) into Eq. (14.186) gives

$$\theta_{C-H_{\text{ethane}} H_2 MO} = 42.68^\circ \quad (14.187)$$

Then, the distance $d_{C-H_{\text{ethane}} H_2 MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C-H_{\text{ethane}} H_2 MO} = a \cos \theta_{C-H_{\text{ethane}} H_2 MO} \quad (14.188)$$

Substitution of Eqs. (14.177) and (14.187) into Eq. (14.188) gives

$$d_{C-H_{ethane}, H_2MO} = 1.20901a_0 = 6.39780 \times 10^{-11} m \quad (14.189)$$

The distance $d_{C-H_{ethane} 2sp^3 HO}$ along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$5 \quad d_{C-H_{ethane} 2sp^3 HO} = d_{C-H_{ethane}, H_2MO} - c' \quad (14.190)$$

Substitution of Eqs. (14.178) and (14.189) into Eq. (14.190) gives

$$d_{C-H_{ethane} 2sp^3 HO} = 0.16189a_0 = 8.56687 \times 10^{-12} m \quad (14.191)$$

BOND ANGLE OF THE CH_3 GROUPS

10 Each CH_3 MO comprises a linear combination of three $C-H$ -bond MOs. Each $C-H$ -bond MO comprises the superposition of a H_2 -type ellipsoidal MO and the $C_{ethane} 2sp^3$ HO. A bond is also possible between the two H atoms of the $C-H$ bonds. Such $H-H$ bonding would decrease the $C-H$ bond strength since electron density would be shifted from the $C-H$ bonds to the $H-H$ bond. Thus, the bond angle between the two $C-H$ bonds is
15 determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal H atoms of the $C-H$ bonds is zero. From Eqs. (11.79) and (13.228), the distance from the origin to each focus of the $H-H$ ellipsoidal MO is

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{aa_0}{2}} \quad (14.192)$$

The internuclear distance from Eq. (13.229) is

$$20 \quad 2c' = 2\sqrt{\frac{aa_0}{2}} \quad (14.193)$$

The length of the semiminor axis of the prolate spheroidal $H-H$ MO $b = c$ is given by Eq. (14.62).

The bond angle of the CH_3 groups of ethane is derived by using the orbital composition and an energy matching factor as in the case with the CH_3 radical. Since the
25 two H_2 -type ellipsoidal MOs initially comprise 75% of the H electron density of H_2 and the energy of each H_2 -type ellipsoidal MO is matched to that of the $C_{ethane} 2sp^3$ HO, the component energies and the total energy E_T of the $H-H$ bond are given by Eqs. (13.67-13.73) except that V_e , T , and V_m are corrected for the hybridization-energy-matching factor

of 0.87495. Hybridization with 25% electron donation to the $C-C$ -bond gives rise to the $C_{ethane} 2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C_{ethane}, 2sp^3)$ given by Eq. (14.149). The corresponding normalization factor for determining the zero of the total $H-H$ bond energy is given by the ratio of 15.55033 eV, the magnitude of $E_{Coulomb}(C_{ethane}, 2sp^3)$ given by Eq. (14.149), and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). The hybridization energy factor $C_{ethaneC2sp^3HO}$ is

$$C_{ethaneC2sp^3HO} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{ethane2sp^3}}} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 0.87495 a_0}} = \frac{13.605804 \text{ eV}}{15.55033 \text{ eV}} = 0.87495 \quad (14.194)$$

Substitution of Eq. (14.152) into Eq. (13.233) with the hybridization factor of 0.87495 gives

$$0 = \left[\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left[(0.87495)^{-1} \left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \left[1 + \sqrt{\frac{\frac{0.75e^2}{4\pi\epsilon_0 a^3}}{m_e c^2}} \right] \right. \\ \left. + \hbar \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 (a+c')^3}}{0.5m_p}} \right] \quad (14.195)$$

From the energy relationship given by Eq. (14.195) and the relationship between the axes given by Eqs. (14.192-14.193) and (14.62-14.63), the dimensions of the $H-H$ MO can be solved.

The most convenient way to solve Eq. (14.195) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 5.7000a_0 = 3.0163 \times 10^{-10} \text{ m} \quad (14.196)$$

Substitution of Eq. (14.196) into Eq. (14.192) gives

$$c' = 1.6882a_0 = 8.9335 \times 10^{-11} \text{ m} \quad (14.197)$$

The internuclear distance given by multiplying Eq. (14.197) by two is

$$2c' = 3.3764a_0 = 1.7867 \times 10^{-10} \text{ m} \quad (14.198)$$

Substitution of Eqs. (14.196-14.197) into Eq. (14.62) gives

$$b = c = 5.4443a_0 = 2.8810 \times 10^{-10} \text{ m} \quad (14.199)$$

Substitution of Eqs. (14.196-14.197) into Eq. (14.63) gives

$$e = 0.2962 \quad (14.200)$$

From, $2c'_{H-H}$ (Eq. (14.198)), the distance between the two H atoms when the total energy of the corresponding MO is zero (Eq. (14.195)), and $2c'_{C-H}$ (Eq. (14.179)), the internuclear distance of each $C-H$ bond, the corresponding bond angle can be determined from the law of cosines. Using, Eq. (13.242), the bond angle θ between the $C-H$ bonds is

$$\theta = \cos^{-1} \left(\frac{2(2.09424)^2 - (3.3764)^2}{2(2.09424)^2} \right) = \cos^{-1}(-0.29964) = 107.44^\circ \quad (14.201)$$

The experimental angle between the $C-H$ bonds is [8]

$$\theta = 107.4^\circ \quad (14.202)$$

10 The CH_3 radical has a pyramidal structure with the carbon atom along the z -axis at the apex and the hydrogen atoms at the base in the xy -plane. The distance $d_{origin-H}$ from the origin to the nucleus of a hydrogen atom given by Eqs. (14.198) and (13.412) is

$$d_{origin-H} = 1.94936a_0 \quad (14.203)$$

The height along the z -axis of the pyramid from the origin to C nucleus d_{height} given by Eqs. (13.414), (14.179), and (14.203) is

$$d_{height} = 0.76540a_0 \quad (14.204)$$

The angle θ_v of each $C-H$ bond from the z -axis given by Eqs. (13.416), (14.203), and (14.204) is

$$\theta_v = 68.563^\circ \quad (14.205)$$

20 The $C-C$ bond is along the z -axis. Thus, the bond angle θ_{C-C-H} between the internuclear axis of the $C-C$ bond and a H atom of the methyl groups is given by

$$\theta_{C-C-H} = 180 - \theta_v \quad (14.206)$$

Substitution of Eq. (14.205) into Eq. (14.206) gives

$$\theta_{C-C-H} = 111.44^\circ \quad (14.207)$$

25 The experimental angle between the $C-C-H$ bonds is [3]

$$\theta_{C-C-H} = 111.17^\circ \quad (14.208)$$

The CH_3CH_3 MO shown in Figure 39 was rendered using these parameters. A minimum energy is obtained with a staggered configuration consistent with observations [3].

The charge-density in the $C-C$ -bond MO is increased by a factor of 0.25 with the formation of the $C_{ethane} 2sp^3$ HOs each having a smaller radius. Using the orbital composition of the CH_3 groups (Eq. (14.173)) and the $C-C$ -bond MO (Eq. (14.139), the radii of $C1s = 0.17113a_0$ (Eq. (10.51)) and $C_{ethane} 2sp^3 = 0.87495a_0$ (Eq. (14.148)) shells, and the
 5 parameters of the $C-C$ -bond (Eqs. (13.3-13.4), (14.158-14.160), and (14.162-14.172)), the parameters of the $C-H$ -bond MOs (Eqs. (13.3-13.4), (14.177-14.179), and (14.181-14.191)), and the bond-angle parameters (Eqs. (14.195-14.208)), the charge-density of the CH_3CH_3 MO comprising the linear combination of two sets of three $C-H$ -bond MOs and a
 10 $C-C$ -bond MO bridging the two methyl groups is shown in Figure 39. Each $C-H$ -bond MO comprises a H_2 -type ellipsoidal MO and a $C_{ethane} 2sp^3$ HO having the dimensional diagram shown in Figure 38. The $C-C$ -bond MO comprises a H_2 -type ellipsoidal MO bridging two $C_{ethane} 2sp^3$ HOs having the dimensional diagram shown in Figure 37.

ENERGIES OF THE CH_3 GROUPS

The energies of each CH_3 group of ethane are given by the substitution of the semiprincipal
 15 axes (Eqs. (14.177-14.178) and (14.181)) into the energy equations of the methyl radical (Eqs. (13.556-13.560)), with the exception that $E(C_{ethane}, 2sp^3)$ replaces $E(C, 2sp^3)$ in Eq. (13.560):

$$V_e = 3(0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -107.68424 \text{ eV} \quad (14.209)$$

$$V_p = \frac{3e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 38.98068 \text{ eV} \quad (14.210)$$

$$20 \quad T = 3(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 32.73700 \text{ eV} \quad (14.211)$$

$$V_m = 3(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -16.36850 \text{ eV} \quad (14.212)$$

$$E_{T_{ethane}}(CH_3) = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.35946 \text{ eV} = -67.69451 \text{ eV} \quad (14.213)$$

where $E_{T_{ethane}}(CH_3)$ is given by Eq. (14.174) which is reiteratively matched to Eq. (13.542)
 25 within five-significant-figure round off error.

VIBRATION OF THE $^{12}\text{CH}_3$ GROUPS

The vibrational energy levels of CH_3 in ethane may be solved as three equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERMS OF THE $^{12}\text{CH}_3$ GROUPS

10 The equations of the radiation reaction force of the methyl groups in ethane are the same as those of the methyl radical with the substitution of the methyl-group parameters. Using Eq. (13.561), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 2.50664 \times 10^{16} \text{ rad/s} \quad (14.214)$$

where b is given by Eq. (14.181). The kinetic energy, E_K , is given by Planck's equation (Eq.

15 (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.50664 \times 10^{16} \text{ rad/s} = 16.49915 \text{ eV} \quad (14.215)$$

In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.215) for \bar{E}_K gives the Doppler energy of the electrons of each of the

20 three bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(16.49915 \text{ eV})}{m_e c^2}} = -0.25422 \text{ eV} \quad (14.216)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH_3 due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.216) and \bar{E}_{Kvib} , the average kinetic energy of vibration which

is $1/2$ of the vibrational energy of each $C-H$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition state having three independent bonds, $\bar{E}'_{ethane\ osc}({}^{12}CH_3)$ per bond is

$$\bar{E}'_{ethane\ osc}({}^{12}CH_3) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}} \quad (14.217)$$

$$\bar{E}'_{ethane\ osc}({}^{12}CH_3) = -0.25422\ eV + \frac{1}{2}(0.35532\ eV) = -0.07656\ eV \quad (14.218)$$

5 Given that the vibration and reentrant oscillation is for three $C-H$ bonds, $\bar{E}_{ethane\ osc}({}^{12}CH_3)$, is:

$$\begin{aligned} \bar{E}_{ethane\ osc}({}^{12}CH_3) &= 3\left(\bar{E}_D + \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}}\right) \\ &= 3\left(-0.25422\ eV + \frac{1}{2}(0.35532\ eV)\right) \\ &= -0.22967\ eV \end{aligned} \quad (14.219)$$

TOTAL AND DIFFERENCE ENERGIES OF THE ${}^{12}CH_3$ GROUPS

10 $E_{ethaneT+osc}({}^{12}CH_3)$, the total energy of each ${}^{12}CH_3$ group including the Doppler term, is given by the sum of $E_{T_{ethane}}(CH_3)$ (Eq. (14.213)) and $\bar{E}_{ethane\ osc}({}^{12}CH_3)$ given by Eq. (14.219):

$$\begin{aligned} E_{ethaneT+osc}(CH_3) &= V_e + T + V_m + V_p + E(C_{ethane}, 2sp^3) + \bar{E}_{ethane\ osc}({}^{12}CH_3) \\ &= E_{T_{ethane}}(CH_3) + \bar{E}_{ethane\ osc}({}^{12}CH_3) \end{aligned} \quad (14.220)$$

$$\begin{aligned} E_{ethaneT+osc}({}^{12}CH_3) &= \left\{ \left(\frac{-3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.359469\ eV \right) \right. \\ &\quad \left. - 3 \left((31.63536831\ eV) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2} - \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}}} \right) \right\} \\ &= -67.69450\ eV - 3 \left(0.25422\ eV - \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}} \right) \end{aligned} \quad (14.221)$$

From Eqs. (14.217-14.221), the total energy of each ${}^{12}CH_3$ is

343

$$\begin{aligned}
 E_{ethaneT+osc}({}^{12}CH_3) &= -67.69450 \text{ eV} + \bar{E}_{ethane \text{ osc}}({}^{12}CH_3) \\
 &= -67.69450 \text{ eV} - 3 \left(0.25422 \text{ eV} - \frac{1}{2} (0.35532 \text{ eV}) \right) \\
 &= -67.92417 \text{ eV}
 \end{aligned} \tag{14.222}$$

where ω_e given by Eq. (13.458) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

The total energy for each methyl radical given by Eq. (13.569) is

$$\begin{aligned}
 E_{radicalT+osc}({}^{12}CH_3) &= -67.69450 \text{ eV} + \bar{E}_{radical \text{ osc}}({}^{12}CH_3) \\
 &= -67.69450 \text{ eV} - 3 \left(0.25670 \text{ eV} - \frac{1}{2} (0.35532 \text{ eV}) \right) \\
 &= -67.93160 \text{ eV}
 \end{aligned} \tag{14.223}$$

- 5 The difference in energy between the methyl groups and the methyl radical $\Delta E_{T+osc}({}^{12}CH_3)$ is given by two times the difference between Eqs. (14.222) and (14.223):

$$\begin{aligned}
 \Delta E_{T+osc}({}^{12}CH_3) &= 2 \left(E_{ethaneT+osc}({}^{12}CH_3) - E_{radicalT+osc}({}^{12}CH_3) \right) \\
 &= 2 \left(-67.92417 \text{ eV} - (-67.93160 \text{ eV}) \right) \\
 &= 0.01487 \text{ eV}
 \end{aligned} \tag{14.224}$$

SUM OF THE ENERGIES OF THE C-C σ MO AND THE HOs OF 10 ETHANE

The energy components of V_e , V_p , T , V_m , and E_T of the C-C-bond MO are the same as those of the CH MO as well as each C-H-bond MO of the methyl groups except that energy of the $C_{ethane} 2sp^3$ HO is used. The energies of each C-C-bond MO are given by the substitution of the semiprincipal axes (Eqs. (14.158-14.159) and (14.162)) into the energy
15 equations of the CH MO (Eqs. (13.449-13.453)), with the exception that $E(C_{ethane}, 2sp^3)$ replaces $E(C, 2sp^3)$ in Eq. (13.453):

$$V_e = (0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -29.101124 \text{ eV} \tag{14.225}$$

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 9.37273 \text{ eV} \tag{14.226}$$

$$T = (0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 6.90500 \text{ eV} \tag{14.227}$$

$$V_m = (0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -3.45250 \text{ eV} \quad (14.228)$$

$$E_T(C-C, \sigma) = -\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.35946 \text{ eV} = -31.63535 \text{ eV} \quad (14.229)$$

where $E_T(C-C, \sigma)$ is the total energy of the $C-C$ σ MO given by Eq. (14.155) which is reiteratively matched to Eq. (13.75) within five-significant-figure round off error.

The total energy of the $C-C$ -bond MO, $E_T(C-C)$, is given by the sum of two times $E_T(C-C, 2sp^3)$, the energy change of each $C2sp^3$ shell due to the decrease in radius with the formation of the $C-C$ -bond MO (Eq. (14.151)), and $E_T(C-C, \sigma)$, the σ MO contribution given by Eq. (14.156):

$$\begin{aligned} E_T(C-C) &= 2E_T(C-C, 2sp^3) + E_T(C-C, \sigma) \\ &= \left(2(-0.72457 \text{ eV}) + \left(-\frac{e^2}{8\pi\epsilon_0 \sqrt{aa_0}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{aa_0}}{a - \sqrt{aa_0}} - 1 \right] - 15.35946 \text{ eV} \right) \right) \quad (14.230) \\ &= 2(-0.72457 \text{ eV}) + (-31.63537 \text{ eV}) \\ &= -33.08452 \text{ eV} \end{aligned}$$

VIBRATION OF ETHANE

The vibrational energy levels of CH_3CH_3 may be solved as two sets of three equivalent coupled harmonic oscillators with a bridging harmonic oscillator by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERMS OF THE $C-C$ -BOND MO OF ETHANE

The equations of the radiation reaction force of the symmetrical $C-C$ -bond MO are given by Eqs. (11.231-11.233), except the force-constant factor is 0.5 based on the force constant k' of Eq. (14.152), and the $C-C$ -bond MO parameters are used. The angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.5e^2}{4\pi\epsilon_0 a^3}} = 9.55643 \times 10^{15} \text{ rad/s} \quad (14.231)$$

where a is given by Eq. (14.158). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 9.55643 \times 10^{15} \text{ rad/s} = 6.29021 \text{ eV} \quad (14.232)$$

5 In Eq. (11.181), substitution of $E_T(C-C)$ (Eq. (14.230)) for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.232) for \bar{E}_K gives the Doppler energy of the electrons of each of the three bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -33.08450 \text{ eV} \sqrt{\frac{2e(6.29021 \text{ eV})}{m_e c^2}} = -0.16416 \text{ eV} \quad (14.233)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the
10 transition state at their corresponding frequency. The decrease in the energy of the $C-C$ -bond MO due to the reentrant orbit of the bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.233) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of the $C-C$ bond. Using the experimental
15 $C-C$ $E_{vib}(\nu_3)$ of 993 cm^{-1} (0.12312 eV) [10] for \bar{E}_{Kvib} of the transition state, $\bar{E}_{osc}(C-C, \sigma)$ is

$$\bar{E}_{osc}(C-C, \sigma) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.234)$$

$$\bar{E}_{osc}(C-C, \sigma) = -0.16416 \text{ eV} + \frac{1}{2} (0.12312 \text{ eV}) = -0.10260 \text{ eV} \quad (14.235)$$

20 TOTAL ENERGIES OF THE $C-C$ -BOND MO OF ETHANE

$E_{T+osc}(C-C)$, the total energy of the $C-C$ -bond MO including the Doppler term, is given by the sum of $E_T(C-C)$ (Eq. (14.230)) and $\bar{E}_{osc}(C-C, \sigma)$ given by Eq. (14.235):

$$\begin{aligned}
 E_{T+osc}(C-C) &= V_e + T + V_m + V_p + E(C_{ethane}, 2sp^3) + 2E_T(C-C, 2sp^3) + \bar{E}_{osc}(C-C, \sigma) \\
 &= E_T(C-C, \sigma) + 2E_T(C-C, 2sp^3) + \bar{E}_{osc}(C-C, \sigma) \\
 &= E_T(C-C) + \bar{E}_{osc}(C-C, \sigma)
 \end{aligned}
 \tag{14.236}$$

$$\begin{aligned}
 E_{T+osc}(C-C) &= \left\{ \left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.35946 \text{ eV} + 2E_T(C-C, 2sp^3) \right) \right. \\
 &\quad \left. \left(1 + \sqrt{\frac{2\hbar \sqrt{\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\} \\
 &= -33.08452 \text{ eV} - 0.16416 \text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}
 \end{aligned}
 \tag{14.237}$$

5 From Eqs. (14.234-14.237), the total energy of the C-C-bond MO is

$$\begin{aligned}
 E_{T+osc}(C-C) &= -31.63537 \text{ eV} + 2E_T(C-C, 2sp^3) + \bar{E}_{osc}(C-C, \sigma) \\
 &= -31.63537 \text{ eV} + 2(-0.72457 \text{ eV}) - 0.16416 \text{ eV} + \frac{1}{2}(0.12312 \text{ eV}) \\
 &= -33.18712 \text{ eV}
 \end{aligned}
 \tag{14.238}$$

where the experimental E_{vib} was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

BOND ENERGY OF THE C-C BOND OF ETHANE

- 10 The dissociation energy of the C-C bond of CH_3CH_3 , $E_D(H_3C-CH_3)$, is given by two times $E(C, 2sp^3)$ (Eq. (14.146)), the initial energy of the $C2sp^3$ HO of each CH_3 radical that bond with a single C-C bond, minus the sum of $\Delta E_{T+osc}(^{12}CH_3)$ (Eq. (14.224)), the energy change going from the methyl radicals to the methyl groups of ethane, and $E_{T+osc}(C-C)$ (Eq. (14.238)). Thus, the dissociation energy of the C-C bond of CH_3CH_3 ,
- 15 is

$$\begin{aligned}
 E_D(H_3C-CH_3) &= 2(E(C, 2sp^3)) - (\Delta E_{T+osc}(^{12}CH_3) + E_{T+osc}(C-C)) \\
 &= 2(-14.63489 \text{ eV}) - (0.01487 \text{ eV} - 33.18712 \text{ eV}) \\
 &= 2(-14.63489 \text{ eV}) - (33.17225 \text{ eV}) \\
 &= 3.90247 \text{ eV}
 \end{aligned}
 \tag{14.239}$$

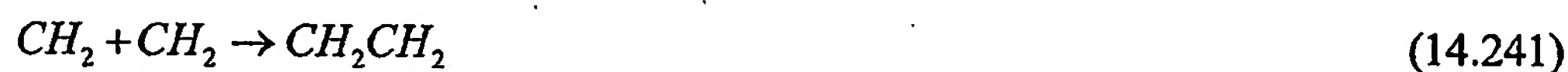
The experimental dissociation energy of the $C - C$ bond of CH_3CH_3 is [6]

$$E_D(H_3C - CH_3) = 3.89690 \text{ eV} \quad (14.240)$$

The results of the determination of bond parameters of CH_3CH_3 are given in Table 14.1. The calculated results are based on first principles and given in closed-form, exact 5 equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

ETHYLENE MOLECULE (CH_2CH_2)

The ethylene molecule CH_2CH_2 is formed by the reaction of two dihydrogen carbide 10 radicals:



CH_2CH_2 can be solved using the same principles as those used to solve the methane series $CH_{n=1,2,3,4}$, wherein the $2s$ and $2p$ shells of each C hybridize to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ hybridized orbitals 15 (HOs) to form a molecular orbital (MO) permits each participating hybridized orbital to decrease in radius and energy. First, two sets of two H atomic orbitals (AOs) combine with two sets of two carbon $2sp^3$ HOs to form two dihydrogen carbide groups comprising a linear combination of four diatomic H_2 -type MOs developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section. Then, the two CH_2 groups bond by 20 forming a H_2 -type MO between the remaining two $C2sp^3$ HOs on each carbon atom.

FORCE BALANCE OF THE $C = C$ -BOND MO OF ETHYLENE

CH_2CH_2 comprises a chemical bond between two CH_2 radicals wherein each radical comprises two chemical bonds between carbon and hydrogen atoms. The solution of the 25 parameters of CH_2 is given in the Dihydrogen Carbide (CH_2) section. Each $C - H$ bond of CH_2 having two spin-paired electrons, one from an initially unpaired electron of the carbon atom and the other from the hydrogen atom, comprises the linear combination of 75% H_2 -type ellipsoidal MO and 25% $C2sp^3$ HO. The proton of the H atom and the nucleus of the C atom are along each internuclear axis and serve as the foci. As in the case of H_2 , each of

the two $C-H$ -bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell since it is energetically unfavorable. Thus, each MO surface comprises a prolate spheroid at the H proton that is continuous with the $C2sp^3$ shell at the C atom whose nucleus serves as
 5 the other focus. The electron configuration and the energy, $E(C, 2sp^3)$, of the $C2sp^3$ shell is given by Eqs. (13.422) and (13.428), respectively. The central paramagnetic force due to spin of each $C-H$ bond is provided by the spin-pairing force of the CH_2 MO that has the symmetry of an s orbital that superimposes with the $C2sp^3$ orbitals such that the corresponding angular momenta are unchanged.

10 Two CH_2 radicals bond to form CH_2CH_2 by forming a MO between the two pairs of remaining $C2sp^3$ -HO electrons of the two carbon atoms. However, in this case, the sharing of electrons between four $C2sp^3$ HOs to form a molecular orbital (MO) comprising four spin-paired electrons permits each $C2sp^3$ HO to decrease in radius and energy.

As in the case of the $C-H$ bonds, the $C=C$ -bond MO is a prolate-spheroidal-MO
 15 surface that cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell of each atom. Thus, the MO surface comprises a partial prolate spheroid in between the carbon nuclei and is continuous with the $C2sp^3$ shell at each C atom. The energy of the H_2 -type ellipsoidal MO is matched to that of the $C2sp^3$ shell. As in the case of previous examples of energy-matched MOs such as those of OH , NH , CH , the $C=O$ -bond MO of
 20 CO_2 , and the $C-C$ -bond MO of CH_3CH_3 , the $C=C$ -bond MO of ethylene must comprise 75% of a H_2 -type ellipsoidal MO in order to match potential, kinetic, and orbital energy relationships. Thus, the $C=C$ -bond MO must comprise a linear combination of two MOs wherein each comprises two $C2sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the $C2sp^3$ HOs:

25



The linear combination of each H_2 -type ellipsoidal MO with each $C2sp^3$ HO further comprises an excess 25% charge-density contribution from each $C2sp^3$ HO to the $C=C$ -

bond MO to achieve an energy minimum. The force balance of the $C=C$ -bond MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (14.242) and the energy matching condition between the $C2sp^3$ -HO components of the MO.

5 Similarly, the energies of each CH_2 MO involve each $C2sp^3$ and each $H1s$ electron with the formation of each $C-H$ bond. The sum of the energies of the H_2 -type ellipsoidal MOs is matched to that of the $C2sp^3$ shell. This energy is determined by the considering the effect of the donation of 25% electron density from the two pairs of $C2sp^3$ HOs to the $C=C$ -bond MO with the formation of the $C_{ethylene}2sp^3$ HOs each having a smaller radius.

10 The $2sp^3$ hybridized orbital arrangement is given by Eq. (14.140). The sum $E_T(C, 2sp^3)$ of calculated energies of C , C^+ , C^{2+} , and C^{3+} is given by Eq. (14.141). The radius r_{2sp^3} of the $C2sp^3$ shell is given by Eq. (14.142). The Coulombic energy $E_{Coulomb}(C, 2sp^3)$ and the energy $E(C, 2sp^3)$ of the outer electron of the $C2sp^3$ shell are given by Eqs. (14.143) and (14.146), respectively.

15 Next, consider the formation of the $C=C$ -bond MO of ethylene from two CH_2 radicals, each having a $C2sp^3$ electron with an energy given by Eq. (14.146). The total energy of the state is given by the sum over the four electrons. The sum $E_T(C_{ethylene}, 2sp^3)$ of calculated energies of $C2sp^3$, C^+ , C^{2+} , and C^{3+} from Eqs. (10.123), (10.113-10.114), (10.68), and (10.48), respectively, is

$$\begin{aligned} E_T(C_{ethylene}, 2sp^3) &= -(64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + E(C, 2sp^3)) \\ &= -(64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + 14.63489 \text{ eV}) \\ &= -151.61569 \text{ eV} \end{aligned}$$

(14.243)

where $E(C, 2sp^3)$ (Eq. (14.146)) is the sum of the energy of C , -11.27671 eV , and the hybridization energy. The orbital-angular-momentum interactions also cancel such that the energy of the $E_T(C_{ethylene}, 2sp^3)$ is purely Coulombic.

25 The sharing of electrons between two pairs of $C2sp^3$ HOs to form a $C=C$ -bond MO permits each participating hybridized orbital to decrease in radius and energy. In order to further satisfy the potential, kinetic, and orbital energy relationships, each participating

$C2sp^3$ HO donates an excess of 25% per bond of its electron density to the $C = C$ -bond MO to form an energy minimum. By considering this electron redistribution in the ethylene molecule as well as the fact that the central field decreases by an integer for each successive electron of the shell, the radius $r_{ethylene2sp^3}$ of the $C2sp^3$ shell of ethylene may be calculated from the Coulombic energy using Eq. (10.102):

$$r_{ethylene2sp^3} = \left(\sum_{n=2}^5 (Z - n) - 0.5 \right) \frac{e^2}{8\pi\epsilon_0 (e151.61569 \text{ eV})} = \frac{9.5e^2}{8\pi\epsilon_0 (e151.61569 \text{ eV})} = 0.85252a_0 \quad (14.244)$$

where $Z = 6$ for carbon. Using Eqs. (10.102) and (14.244), the Coulombic energy $E_{Coulomb}(C_{ethylene}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$10 \quad E_{Coulomb}(C_{ethylene}, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{ethylene2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.85252a_0} = -15.95955 \text{ eV} \quad (14.245)$$

During hybridization, one of the spin-paired $2s$ electrons is promoted to $C2sp^3$ shell as an unpaired electron. The energy for the promotion is the magnetic energy given by Eq. (13.152). Using Eqs. (14.145) and (14.245), the energy $E(C_{ethylene}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$\begin{aligned} E(C_{ethylene}, 2sp^3) &= \frac{-e^2}{8\pi\epsilon_0 r_{ethylene2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} \\ 15 \quad &= -15.95955 \text{ eV} + 0.19086 \text{ eV} \\ &= -15.76868 \text{ eV} \end{aligned} \quad (14.246)$$

Thus, $E_T(C = C, 2sp^3)$, the energy change of each $C2sp^3$ shell with the formation of the $C = C$ -bond MO is given by the difference between Eq. (14.146) and Eq. (14.246):

$$\begin{aligned} E_T(C = C, 2sp^3) &= E(C_{ethylene}, 2sp^3) - E(C, 2sp^3) \\ &= -15.76868 \text{ eV} - (-14.63489 \text{ eV}) \\ &= -1.13380 \text{ eV} \end{aligned} \quad (14.247)$$

As in the case of Cl_2 , each H_2 -type ellipsoidal MO comprises 75% of the $C = C$ -bond MO shared between two $C2sp^3$ HOs corresponding to the electron charge density in Eq. (11.65) of $\frac{-0.75e}{2}$. But, the additional 25% charge-density contribution to each bond of the $C = C$ -

bond MO causes the electron charge density in Eq. (11.65) to be is given by $\frac{-e}{2} = -0.5e$. The

corresponding force constant k' is given by Eq. (14.152). In addition, the energy matching at both $C2sp^3$ HOs further requires that k' be corrected by the hybridization factor given by Eq. (13.430). Thus, the force constant k' to determine the ellipsoidal parameter c' in terms of the central force of the foci (Eq. (11.65)) is given by

$$5 \quad k' = C_{C2sp^3HO} \frac{(0.5)2e^2}{4\pi\epsilon_0} = 0.91771 \frac{(0.5)2e^2}{4\pi\epsilon_0} \quad (14.248)$$

The distance from the origin to each focus c' is given by substitution of Eq. (14.248) into Eq. (13.60). Thus, the distance from the origin of the component of the double $C=C$ -bond MO to each focus c' is given by

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{(0.91771)m_e e^2 a}} = \sqrt{\frac{aa_0}{0.91771}} \quad (14.249)$$

10 The internuclear distance from Eq. (14.249) is

$$2c' = 2\sqrt{\frac{aa_0}{0.91771}} \quad (14.250)$$

The length of the semiminor axis of the prolate spheroidal $C=C$ -bond MO $b=c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of
15 the $C=C$ -bond MO. From the energy equation and the relationship between the axes, the dimensions of the $C=C$ -bond MO are solved.

The general equations for the energy components of V_e , V_p , T , V_m , and E_T of the $C=C$ -bond MO are the same as those of the CH MO except that energy of the $C_{ethylene}2sp^3$ HO is used and the double-bond nature is considered. In the case of a single bond, the prolate
20 spheroidal H_2 -type MO transitions to the $C_{ethylene}2sp^3$ HO of each carbon, and the energy of the $C_{ethylene}2sp^3$ shell must remain constant and equal to the $E(C_{ethylene}, 2sp^3)$ given by Eq. (14.246). Thus, the energy $E(C_{ethylene}, 2sp^3)$ in Eq. (14.246) adds to that of the energies of the corresponding H_2 -type ellipsoidal MO. The second bond of the double $C=C$ -bond MO also transitions to the $C_{ethylene}2sp^3$ HO of each C . The energy of a second H_2 -type
25 ellipsoidal MO adds to the first energy component, and the two bonds achieve an energy minimum as a linear combination of the two H_2 -type ellipsoidal MOs each having the carbon nuclei as the foci. Each $C-C$ -bond MO comprises the same $C_{ethylene}2sp^3$ HO shells of

constant energy given by Eq. (14.246). As in the case of the water, NH_2 , and ammonia molecules given by Eqs. (13.180), (13.320), and (13.372), respectively, the energy of the redundant shell is subtracted from the total energy of the linear combination of the σ MO.

Thus, the total energy $E_T(C=C, \sigma)$ of the σ component of the $C=C$ -bond MO is given by

5 the sum of the energies of the two bonds each comprising the linear combination of the $C_{ethylene} 2sp^3$ HO and the H_2 -type ellipsoidal MO as given by Eq. (14.242) wherein the E_T terms add positively, the $E(C_{ethylene}, 2sp^3)$ terms cancel, and the energy matching condition between the components is provided by Eq. (14.248). Using Eqs. (13.431) and (14.246), $E_T(C=C, \sigma)$ is given by

$$10 \quad \begin{aligned} E_T(C=C, \sigma) &= E_T + E(C_{ethylene}, 2sp^3) - E(C_{ethylene}, 2sp^3) \\ &= -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \end{aligned} \quad (14.251)$$

The total energy term of the double $C=C$ -bond MO is given by the sum of the two H_2 -type ellipsoidal MOs given by Eq. (11.212). To match this boundary condition, $E_T(C=C, \sigma)$ given by Eq. (14.251) is set equal to two times Eq. (13.75):

$$E_T(C=C, \sigma) = -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] = -63.27074 \text{ eV} \quad (14.252)$$

15 From the energy relationship given by Eq. (14.252) and the relationship between the axes given by Eqs. (14.249-14.250) and (13.62-13.63), the dimensions of the $C=C$ -bond MO can be solved.

Substitution of Eq. (14.249) into Eq. (14.252) gives

$$\frac{2e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{0.91771}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{0.91771}}}{a - \sqrt{\frac{aa_0}{0.91771}}} - 1 \right] = e63.27074 \quad (14.253)$$

20 The most convenient way to solve Eq. (14.253) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.47228a_0 = 7.79098 \times 10^{-11} \text{ m} \quad (14.254)$$

Substitution of Eq. (14.254) into Eq. (14.249) gives

$$c' = 1.26661a_0 = 6.70259 \times 10^{-11} \text{ m} \quad (14.255)$$

25 The internuclear distance given by multiplying Eq. (14.255) by two is

$$2c' = 2.53321a_0 = 1.34052 \times 10^{-10} \text{ m} \quad (14.256)$$

The experimental bond distance is [3]

$$2c' = 1.339 \times 10^{-10} \text{ m} \quad (14.257)$$

Substitution of Eqs. (14.254-14.255) into Eq. (13.62) gives

$$5 \quad b = c = 0.75055a_0 = 3.97173 \times 10^{-11} \text{ m} \quad (14.258)$$

Substitution of Eqs. (14.252-14.255) into Eq. (13.63) gives

$$e = 0.86030 \quad (14.259)$$

The nucleus of the C atoms comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{ethylene} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{ethylene 2sp^3} = 0.85252a_0$ is the radius of the $C_{ethylene} 2sp^3$ shell.

Substitution of Eqs. (14.254-14.255) into Eq. (13.261) gives

$$\theta' = 129.84^\circ \quad (14.260)$$

Then, the angle $\theta_{C=C_{ethylene} 2sp^3 HO}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear axis is

$$\theta_{C=C_{ethylene} 2sp^3 HO} = 180^\circ - 129.84^\circ = 50.16^\circ \quad (14.261)$$

as shown in Figure 40.

Consider the right-hand intersection point. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C=C_{ethylene}, H_2 MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with each $C_{ethylene} 2sp^3$ radial vector obeys the following relationship:

$$r_{ethylene 2sp^3} \sin \theta_{C=C_{ethylene} 2sp^3 HO} = 0.85252a_0 \sin \theta_{C=C_{ethylene} 2sp^3 HO} = b \sin \theta_{C=C_{ethylene}, H_2 MO} \quad (14.262)$$

such that

$$25 \quad \theta_{C=C_{ethylene}, H_2 MO} = \sin^{-1} \frac{0.85252a_0 \sin \theta_{C=C_{ethylene} 2sp^3 HO}}{b} = \sin^{-1} \frac{0.85252a_0 \sin 50.16^\circ}{b} \quad (14.263)$$

with the use of Eq. (14.261). Substitution of Eq. (14.258) into Eq. (14.263) gives

$$\theta_{C=C_{ethylene}, H_2 MO} = 60.70^\circ \quad (14.264)$$

Then, the distance $d_{C=C_{ethylene}, H_2 MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C=C_{ethylene}, H_2MO} = a \cos \theta_{C=C_{ethylene}, H_2MO} \quad (14.265)$$

Substitution of Eqs. (14.254) and (14.264) into Eq. (14.265) gives

$$d_{C=C_{ethylene}, H_2MO} = 0.72040a_0 = 3.81221 \times 10^{-11} \text{ m} \quad (14.266)$$

The distance $d_{C=C_{ethylene} 2sp^3HO}$ along the internuclear axis from the origin of the C atom to the
5 point of intersection of the orbitals is given by

$$d_{C=C_{ethylene} 2sp^3HO} = c' - d_{C=C_{ethylene}, H_2MO} \quad (14.267)$$

Substitution of Eqs. (14.255) and (14.266) into Eq. (14.267) gives

$$d_{C=C_{ethylene} 2sp^3HO} = 0.54620a_0 = 2.89038 \times 10^{-11} \text{ m} \quad (14.268)$$

10 FORCE BALANCE OF THE CH_2 MOs OF ETHYLENE

Each of the two equivalent CH_2 MOs must comprise two $C-H$ bonds with each comprising 75% of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO as given by Eq. (13.494):

$$2[1 C2sp^3 + 0.75 H_2 MO] \rightarrow CH_2 MO \quad (14.269)$$

15

The force balance of the CH_2 MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (13.494) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-
20 ellipsoidal-MO component of the CH_2 MO in terms of the central force of the foci is given by Eq. (13.59). The distance from the origin of each $C-H$ -bond MO to each focus c' is given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for
25 the solution of the other axes of each prolate spheroid and eccentricity of each $C-H$ -bond MO. From the energy equation and the relationship between the axes, the dimensions of the CH_2 MO are solved.

Consider the formation of the double $C=C$ -bond MO of ethylene from two CH_2 radicals, each having a $C2sp^3$ shell with an energy given by Eq. (14.146). The energy

components of V_e , V_p , T , V_m , and E_T are the same as those of the dihydrogen carbide radical, two times those of CH corresponding to the two $C-H$ bonds, except that two times $E_T(C=C, 2sp^3)$ is subtracted from $E_T(CH_2)$ of Eq. (13.495). The subtraction of the energy change of the $C2sp^3$ shells with the formation of the $C=C$ -bond MO matches the energy of the $C-H$ -bond MOs to the decrease in the energy of the $C2sp^3$ HOs. Using Eqs. (13.495) and (14.247), $E_{T_{ethylene}}(CH_2)$ is given by

$$E_{T_{ethylene}}(CH_2) = E_T + E(C, 2sp^3) - 2E_T(C=C, 2sp^3) \\ = \left[-\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right] \\ -14.63489 \text{ eV} - (-2.26758 \text{ eV}) \quad (14.270)$$

$E_{T_{ethylene}}(CH_2)$ given by Eq. (14.270) is set equal to two times the energy of the H_2 -type ellipsoidal MO minus the Coulombic energy of H given by Eq. (13.496):

$$10 \quad E_{T_{ethylene}}(CH_2) = \left[-\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right] = -49.66493 \text{ eV} \\ (14.271)$$

From the energy relationship given by Eq. (14.271) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH_2 MO can be solved.

Substitution of Eq. (13.60) into Eq. (14.271) gives

$$15 \quad \frac{2e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e37.29762 \quad (14.272)$$

The most convenient way to solve Eq. (14.272) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.56946a_0 = 8.30521 \times 10^{-11} \text{ m} \quad (14.273)$$

Substitution of Eq. (14.273) into Eq. (13.60) gives

$$20 \quad c' = 1.02289a_0 = 5.41290 \times 10^{-11} \text{ m} \quad (14.274)$$

The internuclear distance given by multiplying Eq. (14.274) by two is

$$2c' = 2.04578a_0 = 1.08258 \times 10^{-10} \text{ m} \quad (14.275)$$

The experimental bond distance is [3]

$$2c' = 1.087 \times 10^{-10} \text{ m} \quad (14.276)$$

Substitution of Eqs. (14.273-14.274) into Eq. (14.62) gives

$$b = c = 1.19033a_0 = 6.29897 \times 10^{-11} \text{ m} \quad (14.277)$$

Substitution of Eqs. (14.273-14.274) into Eq. (14.63) gives

$$5 \quad e = 0.65175 \quad (14.278)$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{ethylene} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270).

The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{ethylene 2sp^3} = 0.85252a_0$ is

10 the radius of the $C_{ethylene} 2sp^3$ shell. Substitution of Eqs. (14.273-14.274) into Eq. (13.261) gives

$$\theta' = 84.81^\circ \quad (14.279)$$

Then, the angle $\theta_{C-H_{ethylene} 2sp^3 HO}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear axis is

$$15 \quad \theta_{C-H_{ethylene} 2sp^3 HO} = 180^\circ - 84.81^\circ = 95.19^\circ \quad (14.280)$$

as shown in Figure 41.

The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C-H_{ethylene} H_2 MO}$ between the

20 internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C_{ethylene} 2sp^3$ radial vector obeys the following relationship:

$$r_{ethylene 2sp^3} \sin \theta_{C-H_{ethylene} 2sp^3 HO} = 0.85252a_0 \sin \theta_{C-H_{ethylene} 2sp^3 HO} = b \sin \theta_{C-H, H_2 MO} \quad (14.281)$$

such that

$$\theta_{C-H_{ethylene} H_2 MO} = \sin^{-1} \frac{0.85252a_0 \sin \theta_{C-H_{ethylene} 2sp^3 HO}}{b} = \sin^{-1} \frac{0.85252a_0 \sin 95.19^\circ}{b} \quad (14.282)$$

25 with the use of Eq. (14.280). Substitution of Eq. (14.277) into Eq. (14.282) gives

$$\theta_{C-H_{ethylene} H_2 MO} = 45.50^\circ \quad (14.283)$$

Then, the distance $d_{C-H_{ethylene} H_2 MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C-H_{ethylene}, H_2MO} = a \cos \theta_{C-H_{ethylene}, H_2MO} \quad (14.284)$$

Substitution of Eqs. (14.273) and (14.283) into Eq. (14.284) gives

$$d_{C-H_{ethylene}, H_2MO} = 1.10002a_0 = 5.82107 \times 10^{-11} \text{ m} \quad (14.285)$$

The distance $d_{C-H_{ethylene} 2sp^3HO}$ along the internuclear axis from the origin of the C atom to the
5 point of intersection of the orbitals is given by

$$d_{C-H_{ethylene} 2sp^3HO} = d_{C-H_{ethylene}, H_2MO} - c' \quad (14.286)$$

Substitution of Eqs. (14.274) and (14.285) into Eq. (14.286) gives

$$d_{C-H_{ethylene} 2sp^3HO} = 0.07713a_0 = 4.08171 \times 10^{-12} \text{ m} \quad (14.287)$$

10 BOND ANGLE OF THE CH_2 GROUPS

Each CH_2 MO comprises a linear combination of two $C-H$ -bond MOs. Each $C-H$ -bond MO comprises the superposition of a H_2 -type ellipsoidal MO and the $C_{ethylene} 2sp^3$ HO. A bond is also possible between the two H atoms of the $C-H$ bonds. Such $H-H$ bonding would decrease the $C-H$ bond strength since electron density would be shifted from the
15 $C-H$ bonds to the $H-H$ bond. Thus, the bond angle between the two $C-H$ bonds is determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal H atoms of the $C-H$ bonds is zero. From Eqs. (11.79) and (13.228), the distance from the origin to each focus of the $H-H$ ellipsoidal MO is

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{aa_0}{2}} \quad (14.288)$$

20 The internuclear distance from Eq. (13.229) is

$$2c' = 2\sqrt{\frac{aa_0}{2}} \quad (14.289)$$

The length of the semiminor axis of the prolate spheroidal $H-H$ MO $b = c$ is given by Eq. (14.62).

The bond angle of the CH_2 groups of ethane is derived by using the orbital
25 composition and an energy matching factor as in the case with the dihydrogen carbide radical and the CH_3 groups of ethane. Since the two H_2 -type ellipsoidal MOs initially comprise 75% of the H electron density of H_2 and the energy of each H_2 -type ellipsoidal MO is matched to that of the $C_{ethylene} 2sp^3$ HO, the component energies and the total energy E_T of the

$H-H$ bond are given by Eqs. (13.67-13.73) except that V_e , T , and V_m are corrected for the hybridization-energy-matching factor of 0.85252. Hybridization with 25% electron donation to the $C=C$ -bond gives rise to the $C_{ethylene}2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C_{ethylene}, 2sp^3)$ given by Eq. (14.245). The corresponding normalization factor for determining the zero of the total $H-H$ bond energy is given by the ratio of 15.95955 eV, the magnitude of $E_{Coulomb}(C_{ethylene}, 2sp^3)$ given by Eq. (14.245), and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). The hybridization energy factor $C_{ethyleneC2sp^3HO}$ is

$$C_{ethyleneC2sp^3HO} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{ethylene2sp^3}}} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 0.85252 a_0}} = \frac{13.605804 \text{ eV}}{15.95955 \text{ eV}} = 0.85252 \quad (14.290)$$

10 Substitution of Eq. (14.290) into Eq. (13.233) or Eq. (14.195) with the hybridization factor of 0.85252 gives

$$0 = \left[\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left[(0.85252)^{-1} \left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 a^3}}}}{m_e c^2} \right] \right. \\ \left. + \hbar \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 a^3}}{0.5m_p}} \right] \quad (14.291)$$

15 From the energy relationship given by Eq. (14.291) and the relationship between the axes given by Eqs. (14.192-14.193) and (14.62-14.63), the dimensions of the $H-H$ MO can be solved.

The most convenient way to solve Eq. (14.291) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 6.0400a_0 = 3.1962 \times 10^{-10} \text{ m} \quad (14.292)$$

20 Substitution of Eq. (14.292) into Eq. (14.288) gives

$$c' = 1.7378a_0 = 9.1961 \times 10^{-11} \text{ m} \quad (14.293)$$

The internuclear distance given by multiplying Eq. (14.293) by two is

$$2c' = 3.4756a_0 = 1.8392 \times 10^{-10} \text{ m} \quad (14.294)$$

Substitution of Eqs. (14.292-14.293) into Eq. (14.62) gives

$$b = c = 5.7846a_0 = 3.0611 \times 10^{-10} \text{ m} \quad (14.295)$$

Substitution of Eqs. (14.292-14.293) into Eq. (14.63) gives

$$e = 0.2877 \quad (14.296)$$

From, $2c'_{H-H}$ (Eq. (14.294)), the distance between the two H atoms when the total energy of the corresponding MO is zero (Eq. (14.291)), and $2c'_{C-H}$ (Eq. (14.275)), the internuclear distance of each $C-H$ bond, the corresponding bond angle can be determined from the law of cosines. Using, Eq. (13.242), the bond angle θ_{HCH} between the $C-H$ bonds is

$$\theta_{HCH} = \cos^{-1} \left(\frac{2(2.04578)^2 - (3.4756)^2}{2(2.04578)^2} \right) = \cos^{-1}(-0.44318) = 116.31^\circ \quad (14.297)$$

The experimental angle between the $C-H$ bonds is [11]

$$\theta_{HCH} = 116.6^\circ \quad (14.298)$$

The $C=C$ bond is along the z -axis. Thus, based on the symmetry of the equivalent bonds, the bond angle $\theta_{C=C-H}$ between the internuclear axis of the $C=C$ bond and a H atom of the CH_2 groups is given by

$$\theta_{C=C-H} = \frac{(360^\circ - \theta_{CHC})}{2} \quad (14.299)$$

Substitution of Eq. (14.298) into Eq. (14.299) gives

$$\theta_{C=C-H} = 121.85^\circ \quad (14.300)$$

The experimental angle between the $C=C-H$ bonds is [11]

$$\theta_{C=C-H} = 121.7^\circ \quad (14.301)$$

and [3]

$$\theta_{C=C-H} = 121.3^\circ \quad (14.302)$$

The $C=C$ bond and H atoms of ethylene lie in a plane, and rotation about the $C=C$ is not possible due to conservation of angular momentum in the two sets of spin-paired electrons of the double bond. The CH_2CH_2 MO shown in Figure 42 was rendered using these parameters.

The charge-density in the $C=C$ -bond MO is increased by a factor of 0.25 per bond with the formation of the $C_{ethylene} 2sp^3$ HOs each having a smaller radius. Using the orbital

composition of the CH_2 groups (Eq. (14.269)) and the $C=C$ -bond MO (Eq. (14.242), the radii of $C1s = 0.17113a_0$ (Eq. (10.51)) and $C_{ethylene} 2sp^3 = 0.85252a_0$ (Eq. (14.244)) shells, and the parameters of the $C=C$ -bond (Eqs. (13.3-13.4), (14.254-14.256), and (14.258-14.268)), the parameters of the $C-H$ -bond MOs (Eqs. (13.3-13.4), (14.273-14.275), and (14.277-14.287)), and the bond-angle parameters (Eqs. (14.297-14.302)), the charge-density of the CH_2CH_2 MO comprising the linear combination of two sets of two $C-H$ -bond MOs and a $C=C$ -bond MO bridging the two CH_2 groups is shown in Figure 42. Each $C-H$ -bond MO comprises a H_2 -type ellipsoidal MO and a $C_{ethylene} 2sp^3$ HO having the dimensional diagram shown in Figure 41. The $C=C$ -bond MO comprises a H_2 -type ellipsoidal MO
 10 bridging two $C_{ethylene} 2sp^3$ HOs having the dimensional diagram shown in Figure 40.

ENERGIES OF THE CH_2 GROUPS

The energies of each CH_2 group of ethylene are given by the substitution of the semiprincipal axes (Eqs. (14.273-14.274) and (14.277)) into the energy equations of dihydrogen carbide
 15 (Eqs. (13.510-13.514)), with the exception that two times $E_T(C=C, 2sp^3)$ (Eq. (14.247)) is subtracted from $E_T(CH_2)$ in Eq. (13.514):

$$V_e = 2(0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -76.00757 \text{ eV} \quad (14.303)$$

$$V_p = \frac{2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 26.60266 \text{ eV} \quad (14.304)$$

$$T = 2(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 24.21459 \text{ eV} \quad (14.305)$$

$$20 \quad V_m = 2(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -12.10730 \text{ eV} \quad (14.306)$$

$$E_{T_{ethylene}}(CH_2) = \left(\frac{-2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) - 14.63489 \text{ eV} - (-2.26758 \text{ eV})$$

(14.307)

where $E_{T_{ethylene}}(CH_2)$ is given by Eq. (14.270) which is reiteratively matched to Eq. (13.496) within five-significant-figure round off error.

VIBRATION OF THE $^{12}CH_2$ GROUPS

- 5 The vibrational energy levels of CH_2 in ethylene may be solved as two equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

10

THE DOPPLER ENERGY TERMS OF THE $^{12}CH_2$ GROUPS

The equations of the radiation reaction force of the CH_2 groups in ethylene are the same as those of the dihydrogen carbide radical with the substitution of the CH_2 -group parameters. Using Eq. (13.515), the angular frequency of the reentrant oscillation in the transition state is

$$15 \quad \omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 2.75685 \times 10^{16} \text{ rad/s} \quad (14.308)$$

where b is given by Eq. (14.277). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.75685 \times 10^{16} \text{ rad/s} = 18.14605 \text{ eV} \quad (14.309)$$

- In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total
20 energy of each H_2 -type MO, for E_{lv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.309) for \bar{E}_K gives the Doppler energy of the electrons of each of the two bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{lv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(18.14605 \text{ eV})}{m_e c^2}} = -0.26660 \text{ eV} \quad (14.310)$$

- In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the
25 transition state at their corresponding frequency. The decrease in the energy of CH_2 due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding

energies, \bar{E}_D given by Eq. (14.310) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of each $C-H$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition state having two independent bonds, $\bar{E}'_{ethylene\ osc} (^{12}CH_2)$ per bond is

$$\bar{E}'_{ethylene\ osc} (^{12}CH_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.311)$$

$$5 \quad \bar{E}'_{ethylene\ osc} (^{12}CH_2) = -0.26660 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) = -0.08894 \text{ eV} \quad (14.312)$$

Given that the vibration and reentrant oscillation is for two $C-H$ bonds, $\bar{E}_{ethylene\ osc} (^{12}CH_2)$, is:

$$\begin{aligned} \bar{E}_{ethylene\ osc} (^{12}CH_2) &= 2 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\ &= 2 \left(-0.26660 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -0.17788 \text{ eV} \end{aligned} \quad (14.313)$$

10 TOTAL AND DIFFERENCE ENERGIES OF THE $^{12}CH_2$ GROUPS

$E_{ethyleneT+osc} (^{12}CH_2)$, the total energy of each $^{12}CH_2$ group including the Doppler term, is given by the sum of $E_{T_{ethylene}} (CH_2)$ (Eq. (14.307)) and $\bar{E}_{ethylene\ osc} (^{12}CH_2)$ given by Eq. (14.313):

$$\begin{aligned} E_{ethyleneT+osc} (CH_2) &= \left(V_e + T + V_m + V_p + E(C, 2sp^3) \right. \\ &\quad \left. - 2E_T(C=C, 2sp^3) + \bar{E}_{ethylene\ osc} (^{12}CH_2) \right) \\ &= E_{T_{ethylene}} (CH_2) + \bar{E}_{ethylene\ osc} (^{12}CH_2) \end{aligned} \quad (14.314)$$

$$\begin{aligned} 15 \quad E_{ethyleneT+osc} (^{12}CH_2) &= \left\{ \begin{aligned} &\left[\frac{-2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right. \\ &\quad \left. - 14.63489 \text{ eV} - (-2.26759 \text{ eV}) \right] \\ &- 2 \left((31.63536831 \text{ eV}) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}} \right) \end{aligned} \right\} \\ &= -49.66493 \text{ eV} - 2 \left(0.26660 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \end{aligned} \quad (14.315)$$

From Eqs. (14.313-14.315), the total energy of each $^{12}\text{CH}_2$ is

$$\begin{aligned} E_{\text{ethylene}T+\text{osc}}(^{12}\text{CH}_2) &= -49.66493 \text{ eV} + \bar{E}_{\text{ethylene osc}}(^{12}\text{CH}_2) \\ &= -49.66493 \text{ eV} - 2 \left(0.26660 \text{ eV} - \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -49.84282 \text{ eV} \end{aligned} \quad (14.316)$$

where ω_e given by Eq. (13.458) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

The total energy for each dihydrogen carbide radical given by Eq. (13.523) is

$$\begin{aligned} E_{\text{radical}T+\text{osc}}(^{12}\text{CH}_2) &= -49.66493 \text{ eV} + \bar{E}_{\text{radical osc}}(^{12}\text{CH}_2) \\ &= -49.66493 \text{ eV} - 2 \left(0.25493 \text{ eV} - \frac{1}{2} (0.35532 \text{ eV}) \right) \\ &= -49.81948 \text{ eV} \end{aligned} \quad (14.317)$$

The difference in energy between the CH_2 groups and the dihydrogen carbide radical $\Delta E_{T+\text{osc}}(^{12}\text{CH}_2)$ is given by two times the difference between Eqs. (14.316) and (14.317):

$$\begin{aligned} \Delta E_{T+\text{osc}}(^{12}\text{CH}_2) &= 2 \left(E_{\text{ethylene}T+\text{osc}}(^{12}\text{CH}_2) - E_{\text{radical}T+\text{osc}}(^{12}\text{CH}_2) \right) \\ &= 2 \left(-49.84282 \text{ eV} - (-49.81948 \text{ eV}) \right) \\ &= -0.04667 \text{ eV} \end{aligned} \quad (14.318)$$

10 SUM OF THE ENERGIES OF THE $\text{C}=\text{C}$ σ MO AND THE HOs OF ETHYLENE

The energy components of V_e , V_p , T , V_m , and E_T of the $\text{C}=\text{C}$ -bond MO are the same as those of the CH MO except that each term is multiplied by two corresponding to the double bond and the energy term corresponding to the $\text{C}_{\text{ethylene}} 2sp^3$ HOs in the equation for E_T is zero. The energies of each $\text{C}=\text{C}$ -bond MO are given by the substitution of the semiprincipal axes (Eqs. (14.254-14.255) and (14.258)) into two times the energy equations of the CH MO (Eqs. (13.449-13.453)), with the exception that zero replaces $E(\text{C}, 2sp^3)$ in Eq. (13.453):

$$V_e = 2(0.91771) \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -102.08992 \text{ eV} \quad (14.319)$$

$$V_p = 2 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} = 21.48386 \text{ eV} \quad (14.320)$$

$$T = 2(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 34.67062 \text{ eV} \quad (14.321)$$

$$V_m = 2(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -17.33531 \text{ eV} \quad (14.322)$$

$$E_T(C=C, \sigma) = -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] = -63.27075 \text{ eV} \quad (14.323)$$

where $E_T(C=C, \sigma)$ is the total energy of the $C=C$ σ MO given by Eq. (14.251) which is
 5 reiteratively matched to two times Eq. (13.75) within five-significant-figure round off error.

The total energy of the $C=C$ -bond MO, $E_T(C=C)$, is given by the sum of two times $E_T(C=C, 2sp^3)$, the energy change of each $C2sp^3$ shell due to the decrease in radius with the formation of the $C=C$ -bond MO (Eq. (14.247)), and $E_T(C=C, \sigma)$, the σ MO contribution given by Eq. (14.252):

$$\begin{aligned} E_T(C=C) &= 2E_T(C=C, 2sp^3) + E_T(C=C, \sigma) \\ &= \left(2(-1.13380 \text{ eV}) + \left(-\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \right) \\ &= 2(-1.13380 \text{ eV}) + (-63.27074 \text{ eV}) \\ &= -65.53833 \text{ eV} \end{aligned} \quad (14.324)$$

VIBRATION OF ETHYLENE

The vibrational energy levels of CH_2CH_2 may be solved as two sets of two equivalent coupled harmonic oscillators with a bridging harmonic oscillator by developing the
 15 Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERMS OF THE $C=C$ -BOND MO OF 20 ETHYLENE

The equations of the radiation reaction force of the $C=C$ -bond MO are given by Eq. (13.142), except the force-constant factor is $(0.93172)0.5$ based on the force constant k' of

Eq. (14.248), and the $C = C$ -bond MO parameters are used. The angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.91771 \frac{(0.5)e^2}{4\pi\epsilon_0 b^3}}{m_e}} = 4.30680 \times 10^{16} \text{ rad/s} \quad (14.325)$$

where b is given by Eq. (14.258). The kinetic energy, E_K , is given by Planck's equation (Eq. 5 (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 4.30680 \times 10^{16} \text{ rad/s} = 28.34813 \text{ eV} \quad (14.326)$$

In Eq. (11.181), substitution of $E_T(C = C)/2$ (Eq. (14.324)) for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.326) for \bar{E}_K gives the Doppler energy of the electrons of each of the two bonds for the reentrant orbit:

$$10 \quad \bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -32.76916 \text{ eV} \sqrt{\frac{2e(28.34813 \text{ eV})}{m_e c^2}} = -0.34517 \text{ eV} \quad (14.327)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of the $C = C$ -bond MO due to the reentrant orbit of the bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the
15 corresponding energies, \bar{E}_D given by Eq. (14.327) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of the $C = C$ bond. Using the experimental $C = C$ $E_{vib}(\nu_3)$ of 1443.5 cm^{-1} (0.17897 eV) [12] for \bar{E}_{Kvib} of the transition state having two bonds, $\bar{E}'_{osc}(C = C, \sigma)$ per bond is

$$\bar{E}'_{osc}(C = C, \sigma) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.328)$$

$$20 \quad \bar{E}'_{osc}(C = C, \sigma) = -0.34517 \text{ eV} + \frac{1}{2} (0.17897 \text{ eV}) = -0.25568 \text{ eV} \quad (14.329)$$

Given that the vibration and reentrant oscillation is for two $C - C$ bonds of the $C = C$ double bond, $\bar{E}_{ethylene\ osc}(C = C, \sigma)$, is:

$$\bar{E}_{ethylene\ osc}(C = C, \sigma) = 2 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) = 2 \left(-0.34517 \text{ eV} + \frac{1}{2} (0.17897 \text{ eV}) \right) = -0.51136 \text{ eV} \quad (14.330)$$

TOTAL ENERGIES OF THE $C=C$ -BOND MO OF ETHYLENE

$E_{T+osc}(C=C)$, the total energy of the $C=C$ -bond MO including the Doppler term, is given by the sum of $E_T(C=C)$ (Eq. (14.324)) and $\bar{E}_{ethylene\ osc}(C=C, \sigma)$ given by Eq. (14.330):

$$\begin{aligned} E_{T+osc}(C=C) &= V_e + T + V_m + V_p + 2E_T(C=C, 2sp^3) + \bar{E}_{ethylene\ osc}(C=C, \sigma) \\ &= E_T(C=C, \sigma) + 2E_T(C=C, 2sp^3) + \bar{E}_{ethylene\ osc}(C=C, \sigma) \\ &= E_T(C=C) + \bar{E}_{ethylene\ osc}(C=C, \sigma) \end{aligned} \quad (14.331)$$

$$\begin{aligned} E_{T+osc}(C=C) &= \left\{ \left(\frac{-2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] + 2E_T(C=C, 2sp^3) \right) \right. \\ &\quad \left. \left(1 + (2) \left(\frac{1}{2} \right) \sqrt{\frac{2\hbar \sqrt{(0.91771) \frac{1}{2} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} + 2 \left(\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right) \right\} \\ &= -65.53833 \text{ eV} - 2 \left(0.34517 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \end{aligned} \quad (14.332)$$

From Eqs. (14.330-14.332), the total energy of the $C=C$ -bond MO is

$$\begin{aligned} E_{T+osc}(C=C) &= -63.27074 \text{ eV} + 2E_T(C=C, 2sp^3) + \bar{E}_{ethylene\ osc}(C=C, \sigma) \\ &= -63.27074 \text{ eV} + 2(-1.13380 \text{ eV}) - 2 \left(0.34517 \text{ eV} - \frac{1}{2} (0.17897 \text{ eV}) \right) \\ &= -66.04969 \text{ eV} \end{aligned} \quad (14.333)$$

where the experimental E_{vib} was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

10

BOND ENERGY OF THE $C=C$ BOND OF ETHYLENE

The dissociation energy of the $C=C$ bond of CH_2CH_2 , $E_D(H_2C=CH_2)$, is given by four times $E(C, 2sp^3)$ (Eq. (14.146)), the initial energy of each $C2sp^3$ HO of each CH_2 radical that forms the double $C=C$ bond, minus the sum of $\Delta E_{T+osc}(^{12}CH_2)$ (Eq. (14.318)), the energy change going from the dihydrogen carbide radicals to the CH_2 groups of ethylene, and $E_{T+osc}(C=C)$ (Eq. (14.333)). Thus, the dissociation energy of the $C=C$ bond of CH_2CH_2 , is

15

$$\begin{aligned}
 E_D(H_2C=CH_2) &= 4 \left(E(C, 2sp^3) \right) - \left(\Delta E_{T+osc}({}^{12}CH_2) + E_{T+osc}(C=C) \right) \\
 &= 4(-14.63489 \text{ eV}) - (-0.04667 \text{ eV} - 66.04969 \text{ eV}) \\
 &= 4(-14.63489 \text{ eV}) - (-66.09636 \text{ eV}) \\
 &= 7.55681 \text{ eV}
 \end{aligned}
 \tag{14.334}$$

The experimental dissociation energy of the $C=C$ bond of CH_2CH_2 is [7]

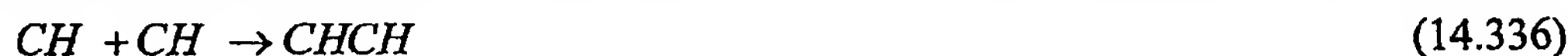
$$E_D(H_2C-CH_2) = 7.5969 \text{ eV} \tag{14.335}$$

The results of the determination of bond parameters of CH_2CH_2 are given in Table

- 5 14.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

ACETYLENE MOLECULE ($CHCH$)

- 10 The acetylene molecule $CHCH$ is formed by the reaction of two hydrogen carbide radicals:



$CHCH$ can be solved using the same principles as those used to solve the methane series $CH_{n=1,2,3,4}$ as well as ethane, wherein the $2s$ and $2p$ shells of each C hybridize to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$

- 15 hybridized orbitals (HOs) to form a molecular orbital (MO) permits each participating hybridized orbital to decrease in radius and energy. First, two sets of one H atomic orbital (AO) combine with two sets of one carbon $2sp^3$ HO to form two hydrogen carbide groups comprising a linear combination of two diatomic H_2 -type MOs developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section. Then, the two
20 CH groups bond by forming a H_2 -type MO between the remaining three $C2sp^3$ HOs on each carbon atom.

FORCE BALANCE OF THE $C \equiv C$ -BOND MO OF ACETYLENE

- $CHCH$ comprises a chemical bond between two CH radicals wherein each radical
25 comprises a chemical bond between a carbon and a hydrogen atom. The solution of the parameters of CH is given in the Hydrogen Carbide (CH) section. The $C-H$ bond of CH having two spin-paired electrons, one from an initially unpaired electron of the carbon atom and the other from the hydrogen atom, comprises the linear combination of 75% H_2 -type

ellipsoidal MO and 25% $C2sp^3$ HO. The proton of the H atom and the nucleus of the C atom are along each internuclear axis and serve as the foci. As in the case of H_2 , the $C-H$ -bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell since it is energetically unfavorable. Thus, the MO surface comprises a prolate spheroid at the H proton that is continuous with the $C2sp^3$ shell at the C atom whose nucleus serves as the other focus. The electron configuration and the energy, $E(C, 2sp^3)$, of the $C2sp^3$ shell is given by Eqs. (13.422) and (13.428), respectively. The central paramagnetic force due to spin of the $C-H$ bond is provided by the spin-pairing force of the CH MO that has the symmetry of an s orbital that superimposes with the $C2sp^3$ orbitals such that the corresponding angular momenta are unchanged.

Two CH radicals bond to form $CHCH$ by forming a MO between the two pairs of three remaining $C2sp^3$ -HO electrons of the two carbon atoms. However, in this case, the sharing of electrons between two $C2sp^3$ HOs to form a MO comprising six spin-paired electrons permits each $C2sp^3$ HO to decrease in radius and energy.

As in the case of the $C-H$ bonds, the $C \equiv C$ -bond MO is a prolate-spheroidal-MO surface that cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell of each atom. Thus, the MO surface comprises a partial prolate spheroid in between the carbon nuclei and is continuous with the $C2sp^3$ shell at each C atom. The energy of the H_2 -type ellipsoidal MO is matched to that of the $C2sp^3$ shell. As in the case of previous examples of energy-matched MOs such as those of OH , NH , CH , the $C=O$ -bond MO of CO_2 , the $C-C$ -bond MO of CH_3CH_3 , and the $C=C$ -bond MO of CH_2CH_2 , the $C \equiv C$ -bond MO of acetylene must comprise 75% of a H_2 -type ellipsoidal MO in order to match potential, kinetic, and orbital energy relationships. Thus, the $C \equiv C$ -bond MO must comprise a linear combination of three MOs wherein each comprises two $C2sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the $C2sp^3$ HOs:



The linear combination of each H_2 -type ellipsoidal MO with each $C2sp^3$ HO further comprises an excess 25% charge-density contribution from each $C2sp^3$ HO to the $C \equiv C$ -bond MO to achieve an energy minimum. The force balance of the $C \equiv C$ -bond MO is determined by the boundary conditions that arise from the linear combination of orbitals 5 according to Eq. (14.337) and the energy matching condition between the $C2sp^3$ -HO components of the MO.

Similarly, the energies of each CH MO involve each $C2sp^3$ and each $H1s$ electron with the formation of each $C-H$ bond. The sum of the energies of the H_2 -type ellipsoidal MOs is matched to that of the $C2sp^3$ shell. This energy is determined by the considering the 10 effect of the donation of 25% electron density from the three pairs of $C2sp^3$ HOs to the $C \equiv C$ -bond MO with the formation of the $C_{acetylene}2sp^3$ HOs each having a smaller radius. The $2sp^3$ hybridized orbital arrangement is given by Eq. (14.140). The sum $E_T(C, 2sp^3)$ of calculated energies of C , C^+ , C^{2+} , and C^{3+} is given by Eq. (14.141). The radius r_{2sp^3} of the $C2sp^3$ shell is given by Eq. (14.142). The Coulombic energy $E_{Coulomb}(C, 2sp^3)$ and the 15 energy $E(C, 2sp^3)$ of the outer electron of the $C2sp^3$ shell are given by Eqs. (14.143) and (14.146), respectively.

Next, consider the formation of the $C \equiv C$ -bond MO of acetylene from two CH radicals, each having a $C2sp^3$ electron with an energy given by Eq. (14.146). The total energy of the state is given by the sum over the four electrons. The sum $E_T(C_{acetylene}, 2sp^3)$ of 20 calculated energies of $C2sp^3$, C^+ , C^{2+} , and C^{3+} from Eqs. (10.123), (10.113-10.114), (10.68), and (10.48), respectively, is

$$\begin{aligned} E_T(C_{acetylene}, 2sp^3) &= -(64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + E(C, 2sp^3)) \\ &= -(64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + 14.63489 \text{ eV}) \\ &= -151.61569 \text{ eV} \end{aligned}$$

(14.338)

where $E(C, 2sp^3)$ (Eq. (14.146)) is the sum of the energy of C , -11.27671 eV , and the 25 hybridization energy. The orbital-angular-momentum interactions also cancel such that the energy of the $E_T(C_{acetylene}, 2sp^3)$ is purely Coulombic.

The sharing of electrons between three pairs of $C2sp^3$ HOs to form a $C \equiv C$ -bond MO permits each participating hybridized orbital to decrease in radius and energy. In order to further satisfy the potential, kinetic, and orbital energy relationships, each participating $C2sp^3$ HO donates an excess of 25% of its electron density to the $C \equiv C$ -bond MO to form an energy minimum. By considering this electron redistribution in the acetylene molecule as well as the fact that the central field decreases by an integer for each successive electron of the shell, the radius $r_{acetylene2sp^3}$ of the $C2sp^3$ shell of acetylene may be calculated from the Coulombic energy using Eq. (10.102):

$$\begin{aligned} r_{acetylene2sp^3} &= \left(\sum_{n=2}^5 (Z-n) - 0.75 \right) \frac{e^2}{8\pi\epsilon_0 (e151.61569 \text{ eV})} \\ &= \frac{9.25e^2}{8\pi\epsilon_0 (e151.61569 \text{ eV})} \\ &= 0.83008a_0 \end{aligned} \quad (14.339)$$

where $Z = 6$ for carbon. Using Eqs. (10.102) and (14.339), the Coulombic energy $E_{Coulomb}(C_{acetylene}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$E_{Coulomb}(C_{acetylene}, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{acetylene2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.83008a_0} = -16.39089 \text{ eV} \quad (14.340)$$

During hybridization, one of the spin-paired $2s$ electrons is promoted to $C2sp^3$ shell as an unpaired electron. The energy for the promotion is the magnetic energy given by Eq. (13.152). Using Eqs. (14.145) and (14.340), the energy $E(C_{acetylene}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$\begin{aligned} E(C_{acetylene}, 2sp^3) &= \frac{-e^2}{8\pi\epsilon_0 r_{acetylene2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} = -16.39089 \text{ eV} + 0.19086 \text{ eV} = -16.20002 \text{ eV} \\ & \quad (14.341) \end{aligned}$$

Thus, $E_T(C \equiv C, 2sp^3)$, the energy change of each $C2sp^3$ shell with the formation of the $C \equiv C$ -bond MO is given by the difference between Eq. (14.146) and Eq. (14.341):

$$\begin{aligned} E_T(C \equiv C, 2sp^3) &= E(C_{acetylene}, 2sp^3) - E(C, 2sp^3) \\ &= -16.20002 \text{ eV} - (-14.63489 \text{ eV}) \\ &= -1.56513 \text{ eV} \end{aligned} \quad (14.342)$$

As in the case of Cl_2 , each H_2 -type ellipsoidal MO comprises 75% of the $C \equiv C$ -bond MO shared between two $C2sp^3$ HOs corresponding to the electron charge density in Eq. (11.65) of $\frac{-0.75e}{2}$. But, the additional 25% charge-density contribution to each bond of the $C \equiv C$ -bond MO causes the electron charge density in Eq. (11.65) to be is given by $\frac{-e}{2} = -0.5e$. The

5 corresponding force constant k' to determine the ellipsoidal parameter c' in terms of the central force of the foci (Eq. (11.65)) is given by Eq. (14.152). The distance from the origin to each focus c' is given by Eq. (14.153). The internuclear distance is given by Eq. (14.154). The length of the semiminor axis of the prolate spheroidal $C \equiv C$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis

10 a then allows for the solution of the other axes of each prolate spheroid and eccentricity of the $C \equiv C$ -bond MO. From the energy equation and the relationship between the axes, the dimensions of the $C \equiv C$ -bond MO are solved.

The general equations for the energy components of V_e , V_p , T , V_m , and E_T of the $C \equiv C$ -bond MO are the same as those of the CH MO except that energy of the $C_{acetylene}2sp^3$

15 HO is used and the triple-bond nature is considered. In the case of a single bond, the prolate spheroidal H_2 -type MO transitions to the $C_{acetylene}2sp^3$ HO of each carbon, and the energy of the $C_{acetylene}2sp^3$ shell must remain constant and equal to the $E(C_{acetylene}, 2sp^3)$ given by Eq. (14.391). Thus, the energy $E(C_{acetylene}, 2sp^3)$ in Eq. (14.391) adds to that of the energies of the corresponding H_2 -type ellipsoidal MO. The second and third bonds of the triple $C \equiv C$ -

20 bond MO also transition to each $C_{acetylene}2sp^3$ HO of each C . The energy of a second and a third H_2 -type ellipsoidal MO adds to the first energy component, and the three bonds achieve an energy minimum as a linear combination of the three H_2 -type ellipsoidal MOs each having the carbon nuclei as the foci. Each $C-C$ -bond MO comprises the same $C_{acetylene}2sp^3$ HO shells of constant energy given by Eq. (14.391). As in the case of the water, NH_2 ,

25 ammonia, and ethylene molecules given by Eqs. (13.180), (13.320), (13.372), and (14.251), respectively, the energy of the redundant shell is subtracted from the total energy of the linear combination of the σ MO. Thus, the total energy $E_T(C \equiv C, \sigma)$ of the σ component of the $C \equiv C$ -bond MO is given by the sum of the energies of the three bonds each comprising the

linear combination of the $C_{acetylene} 2sp^3$ HO and the H_2 -type ellipsoidal MO as given by Eq. (14.337) wherein the E_T terms add positively and the $E(C_{acetylene}, 2sp^3)$ term is positive due to the sum over a negative and two positive terms. Using Eqs. (13.431) and (14.341), $E_T(C \equiv C, \sigma)$ is given by

$$\begin{aligned}
 E_T(C \equiv C, \sigma) &= E_T + E(C_{acetylene}, 2sp^3) - E(C_{acetylene}, 2sp^3) - E(C_{acetylene}, 2sp^3) \\
 5 \quad &= -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - E(C_{acetylene}, 2sp^3) \\
 &= -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] + 16.20002 \text{ eV} \\
 (14.343)
 \end{aligned}$$

The total energy term of the double $C \equiv C$ -bond MO is given by the sum of the three H_2 -type ellipsoidal MOs given by Eq. (11.212). To match this boundary condition, $E_T(C \equiv C, \sigma)$ given by Eq. (14.343) is set equal to three times Eq. (13.75):

$$10 \quad E_T(C \equiv C, \sigma) = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] + 16.20002 \text{ eV} = -94.90610 \text{ eV} \quad (14.344)$$

From the energy relationship given by Eq. (14.344) and the relationship between the axes given by Eqs. (14.153-14.154) and (13.62-13.63), the dimensions of the $C \equiv C$ -bond MO can be solved.

15 Substitution of Eq. (14.153) into Eq. (14.344) gives

$$\frac{3e^2}{8\pi\epsilon_0 \sqrt{aa_0}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+\sqrt{aa_0}}{a-\sqrt{aa_0}} - 1 \right] = e111.10613 \quad (14.345)$$

The most convenient way to solve Eq. (14.345) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.28714a_0 = 6.81122 \times 10^{-11} \text{ m} \quad (14.346)$$

20 Substitution of Eq. (14.346) into Eq. (14.153) gives

$$c' = 1.13452a_0 = 6.00362 \times 10^{-11} \text{ m} \quad (14.347)$$

The internuclear distance given by multiplying Eq. (14.347) by two is

$$2c' = 2.26904a_0 = 1.20072 \times 10^{-10} \text{ m} \quad (14.348)$$

The experimental bond distance is [3]

$$25 \quad 2c' = 1.203 \times 10^{-10} \text{ m} \quad (14.349)$$

Substitution of Eqs. (14.346-14.347) into Eq. (13.62) gives

$$b = c = 0.60793a_0 = 3.21704 \times 10^{-11} \text{ m} \quad (14.350)$$

Substitution of Eqs. (14.346-14.347) into Eq. (13.63) gives

$$e = 0.88143 \quad (14.351)$$

5 The nucleus of the C atoms comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{\text{acetylene}} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{\text{acetylene} 2sp^3} = 0.83008a_0$ is the radius of the $C_{\text{acetylene}} 2sp^3$ shell. Substitution of Eqs. (14.346-14.347) into Eq. (13.261) gives

$$10 \quad \theta' = 137.91^\circ \quad (14.352)$$

Then, the angle $\theta_{C \equiv C_{\text{acetylene}} 2sp^3 \text{ HO}}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear axis is

$$\theta_{C \equiv C_{\text{acetylene}} 2sp^3 \text{ HO}} = 180^\circ - 137.91^\circ = 42.09^\circ \quad (14.353)$$

as shown in Figure 43.

15

Consider the right-hand intersection point. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C \equiv C_{\text{acetylene}}, H_2 \text{ MO}}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with each $C_{\text{acetylene}} 2sp^3$ radial vector obeys the following

20 relationship:

$$r_{\text{acetylene} 2sp^3} \sin \theta_{C \equiv C_{\text{acetylene}} 2sp^3 \text{ HO}} = 0.83008a_0 \sin \theta_{C \equiv C_{\text{acetylene}} 2sp^3 \text{ HO}} = b \sin \theta_{C \equiv C_{\text{acetylene}}, H_2 \text{ MO}} \quad (14.354)$$

such that

$$\theta_{C \equiv C_{\text{acetylene}}, H_2 \text{ MO}} = \sin^{-1} \frac{0.83008a_0 \sin \theta_{C \equiv C_{\text{acetylene}} 2sp^3 \text{ HO}}}{b} = \sin^{-1} \frac{0.83008a_0 \sin 42.09^\circ}{b} \quad (14.355)$$

with the use of Eq. (14.353). Substitution of Eq. (14.350) into Eq. (14.355) gives

$$25 \quad \theta_{C \equiv C_{\text{acetylene}}, H_2 \text{ MO}} = 66.24^\circ \quad (14.356)$$

Then, the distance $d_{C \equiv C_{\text{acetylene}}, H_2 \text{ MO}}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C \equiv C_{\text{acetylene}}, H_2 \text{ MO}} = a \cos \theta_{C \equiv C_{\text{acetylene}}, H_2 \text{ MO}} \quad (14.357)$$

Substitution of Eqs. (14.346) and (14.356) into Eq. (14.357) gives

374

$$d_{C \equiv C_{acetylene}, H_2 MO} = 0.51853a_0 = 2.74396 \times 10^{-11} m \quad (14.358)$$

The distance $d_{C \equiv C_{acetylene}, 2sp^3 HO}$ along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

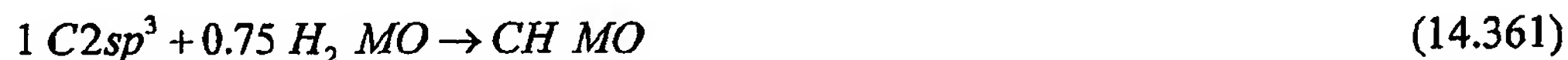
$$d_{C \equiv C_{acetylene}, 2sp^3 HO} = c' - d_{C \equiv C_{acetylene}, H_2 MO} \quad (14.359)$$

5 Substitution of Eqs. (14.347) and (14.358) into Eq. (14.359) gives

$$d_{C \equiv C_{acetylene}, 2sp^3 HO} = 0.61599a_0 = 3.25966 \times 10^{-11} m \quad (14.360)$$

FORCE BALANCE OF THE CH MOs OF ACETYLENE

The $C-H$ bond of each of the two equivalent CH MOs must comprise 75% of a H_2 -type
10 ellipsoidal MO and a $C2sp^3$ HO as given by Eq. (13.429):



The force balance of the CH MO is determined by the boundary conditions that arise from
15 the linear combination of orbitals according to Eq. (13.429) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-ellipsoidal-MO component of the CH MO in terms of the central force of the foci is given by Eq. (13.59). The distance from the origin of each $C-H$ -bond MO to each focus c' is given
20 by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b=c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of each $C-H$ -bond MO. From the energy equation and the relationship between the axes, the dimensions of the
25 CH MO are solved.

Consider the formation of the triple $C \equiv C$ -bond MO of acetylene from two CH radicals, each having a $C2sp^3$ shell with an energy given by Eq. (14.146). The energy components of V_e , V_p , T , V_m , and E_T are the same as those of the hydrogen carbide radical, except that two times $E_T(C \equiv C, 2sp^3)$ is subtracted from $E_T(CH)$ of Eq. (13.495). The
30 subtraction of the energy change of the $C2sp^3$ shells with the formation of the $C \equiv C$ -bond

MO matches the energy of the $C-H$ -bond MOs to the decrease in the energy of the $C2sp^3$ HOs. Using Eqs. (13.495) and (14.342), $E_{T_{acetylene}}(CH)$ is given by

$$\begin{aligned} E_{T_{acetylene}}(CH) &= E_T + E(C, 2sp^3) - 2E_T(C \equiv C, 2sp^3) \\ &= \left(-\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \\ &\quad \left(-14.63489 \text{ eV} - (-3.13026 \text{ eV}) \right) \end{aligned} \quad (14.362)$$

$E_{T_{acetylene}}(CH)$ given by Eq. (14.362) is set equal to the energy of the H_2 -type ellipsoidal MO 5 given by Eq. (13.75):

$$\begin{aligned} E_{T_{acetylene}}(CH) &= \left(-\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \\ &\quad \left(-14.63489 \text{ eV} - (-3.13026 \text{ eV}) \right) \\ &= -31.63537 \text{ eV} \end{aligned} \quad (14.363)$$

From the energy relationship given by Eq. (14.363) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH MO can be solved.

10 Substitution of Eq. (13.60) into Eq. (14.363) gives

$$\frac{e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e20.13074 \quad (14.364)$$

The most convenient way to solve Eq. (14.364) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.48719a_0 = 7.86987 \times 10^{-11} \text{ m} \quad (14.365)$$

15 Substitution of Eq. (14.365) into Eq. (14.60) gives

$$c' = 0.99572a_0 = 5.26913 \times 10^{-11} \text{ m} \quad (14.366)$$

The internuclear distance given by multiplying Eq. (14.366) by two is

$$2c' = 1.99144a_0 = 1.05383 \times 10^{-10} \text{ m} \quad (14.367)$$

The experimental bond distance is [3]

$$20 \quad 2c' = 1.060 \times 10^{-10} \text{ m} \quad (14.368)$$

Substitution of Eqs. (14.365-14.366) into Eq. (14.62) gives

$$b = c = 1.10466a_0 = 5.84561 \times 10^{-11} \text{ m} \quad (14.369)$$

Substitution of Eqs. (14.365-14.366) into Eq. (14.63) gives

$$e = 0.66953 \quad (14.370)$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{acetylene} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where

5 $r_n = r_{acetylene 2sp^3} = 0.83008a_0$ is the radius of the $C_{acetylene} 2sp^3$ shell. Substitution of Eqs. (14.365-14.366) into Eq. (13.261) gives

$$\theta' = 90.99^\circ \quad (14.371)$$

Then, the angle $\theta_{C-H_{acetylene} 2sp^3 HO}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear axis is

$$10 \quad \theta_{C-H_{acetylene} 2sp^3 HO} = 180^\circ - 90.99^\circ = 89.01^\circ \quad (14.372)$$

as shown in Figure 43. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C-H_{acetylene}, H_2 MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C_{acetylene} 2sp^3$ radial vector obeys the following relationship:

$$15 \quad r_{acetylene 2sp^3} \sin \theta_{C-H_{acetylene} 2sp^3 HO} = 0.83008a_0 \sin \theta_{C-H_{acetylene} 2sp^3 HO} = b \sin \theta_{C-H_{acetylene}, H_2 MO} \quad (14.373)$$

such that

$$\theta_{C-H_{acetylene}, H_2 MO} = \sin^{-1} \frac{0.83008a_0 \sin \theta_{C-H_{acetylene} 2sp^3 HO}}{b} = \sin^{-1} \frac{0.83008a_0 \sin 89.01^\circ}{b} \quad (14.374)$$

with the use of Eq. (14.372). Substitution of Eq. (14.369) into Eq. (14.374) gives

$$\theta_{C-H_{acetylene}, H_2 MO} = 48.71^\circ \quad (14.375)$$

20 Then, the distance $d_{C-H_{acetylene}, H_2 MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C-H_{acetylene}, H_2 MO} = a \cos \theta_{C-H_{acetylene}, H_2 MO} \quad (14.376)$$

Substitution of Eqs. (14.365) and (14.375) into Eq. (14.376) gives

$$d_{C-H_{acetylene}, H_2 MO} = 0.98145a_0 = 5.19359 \times 10^{-11} \text{ m} \quad (14.377)$$

25 The distance $d_{C-H_{acetylene} 2sp^3 HO}$ along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$d_{C-H_{acetylene} 2sp^3 HO} = c' - d_{C-H_{acetylene}, H_2 MO} \quad (14.378)$$

Substitution of Eqs. (14.366) and (14.377) into Eq. (14.378) gives

$$d_{C-H_{acetylene} 2sp^3 HO} = 0.01427a_0 = 7.55329 \times 10^{-13} m \quad (14.379)$$

With the $C \equiv C$ double bond along one axis, the minimum energy is obtained with the $C-H$ -bond MO at a maximum separation. Thus, the bond angle $\theta_{C \equiv C-H}$ between the internuclear axis of the $C \equiv C$ bond and the H atom of the CH groups is

$$5 \quad \theta_{C \equiv C-H} = 180^\circ \quad (14.380)$$

The experimental angle between the $C \equiv C-H$ bonds is [6]

$$\theta_{C \equiv C-H} = 180^\circ \quad (14.381)$$

The $CHCH$ MO shown in Figure 44 was rendered using these parameters.

The charge-density in the $C \equiv C$ -bond MO is increased by a factor of 0.25 per bond
 10 with the formation of the $C_{acetylene} 2sp^3$ HOs each having a smaller radius. Using the orbital composition of the CH groups (Eq. (14.361)) and the $C \equiv C$ -bond MO (Eq. (14.337), the radii of $C1s = 0.17113a_0$ (Eq. (10.51)) and $C_{acetylene} 2sp^3 = 0.83008a_0$ (Eq. (14.339)) shells, and the parameters of the $C \equiv C$ -bond (Eqs. (13.3-13.4), (14.346-14.348), and (14.350-14.360)), the parameters of the $C-H$ -bond MOs (Eqs. (13.3-13.4), (14.365-14.367), and
 15 (14.369-14.379)), and the bond-angle parameter (Eqs. (14.380-14.381)), the charge-density of the $CHCH$ MO comprising the linear combination of two $C-H$ -bond MOs and a $C \equiv C$ -bond MO bridging the two CH groups is shown in Figure 44. Each $C-H$ -bond MO comprises a H_2 -type ellipsoidal MO and a $C_{acetylene} 2sp^3$ HO having the dimensional diagram shown in Figure 43. The $C \equiv C$ -bond MO comprises a H_2 -type ellipsoidal MO bridging two
 20 $C_{acetylene} 2sp^3$ HOs having the dimensional diagram also shown in Figure 43.

ENERGIES OF THE CH GROUPS

The energies of each CH group of acetylene are given by the substitution of the semiprincipal axes (Eqs. (14.365-14.366) and (14.369)) into the energy equations of hydrogen
 25 carbide (Eqs. (13.510-13.514)), with the exception that two times $E_T(C \equiv C, 2sp^3)$ (Eq. (14.342)) is subtracted from $E_T(CH)$ in Eq. (13.514):

$$V_e = (0.91771) \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -40.62396 eV \quad (14.382)$$

$$V_p = \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} = 13.66428 eV \quad (14.383)$$

$$T = (0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 13.65796 \text{ eV} \quad (14.384)$$

$$V_m = (0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -6.82898 \text{ eV} \quad (14.385)$$

$$E_{T_{acetylene}}(CH) = \left(\frac{-\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right]}{-14.63489 \text{ eV} - (-3.13026 \text{ eV})} \right) = -31.63532 \text{ eV} \quad (14.386)$$

5 where $E_{T_{acetylene}}(CH)$ is given by Eq. (14.362) which is reiteratively matched to Eq. (13.75) within five-significant-figure round off error.

VIBRATION OF THE ^{12}CH GROUPS

The vibrational energy levels of CH in acetylene may be solved using the methods given in
10 the Vibration and Rotation of CH section.

THE DOPPLER ENERGY TERMS OF THE ^{12}CH GROUPS

The equations of the radiation reaction force of the CH groups in acetylene are the same as those of the hydrogen carbide radical with the substitution of the CH -group parameters.

15 Using Eq. (13.477), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{\frac{0.75e^2}{4\pi\epsilon_0 b^3}}{m_e}} = 3.08370 \times 10^{16} \text{ rad/s} \quad (14.387)$$

where b is given by Eq. (14.369). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 3.08370 \times 10^{16} \text{ rad/s} = 20.29747 \text{ eV} \quad (14.388)$$

20 In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_m , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.388) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit:

$$\bar{E}_D \cong E_m \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(20.29747 \text{ eV})}{m_e c^2}} = -0.28197 \text{ eV} \quad (14.389)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding 5 energies, \bar{E}_D given by Eq. (14.389) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of each $C-H$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition state, $\bar{E}_{acetylene\ osc} (^{12}CH)$ is

$$\bar{E}_{acetylene\ osc} (^{12}CH) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.390)$$

$$\bar{E}_{acetylene\ osc} (^{12}CH) = -0.28197\ eV + \frac{1}{2} (0.35532\ eV) = -0.10430\ eV \quad (14.391)$$

10

TOTAL AND DIFFERENCE ENERGIES OF THE ^{12}CH GROUPS

$E_{acetyleneT+osc} (^{12}CH)$, the total energy of each ^{12}CH group including the Doppler term, is given by the sum of $E_{T_{acetylene}} (CH)$ (Eq. (14.386)) and $\bar{E}_{acetylene\ osc} (^{12}CH)$ given by Eq. (14.391):

$$\begin{aligned} E_{acetyleneT+osc} (CH) &= \left(V_e + T + V_m + V_p + E(C, 2sp^3) \right. \\ &\quad \left. - 2E_T (C \equiv C, 2sp^3) + \bar{E}_{acetylene\ osc} (^{12}CH) \right) \\ &= E_{T_{acetylene}} (CH) + \bar{E}_{acetylene\ osc} (^{12}CH) \end{aligned} \quad (14.392)$$

15

$$\begin{aligned} E_{acetyleneT+osc} (^{12}CH) &= \left\{ \left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right. \right. \\ &\quad \left. \left. - 14.63489\ eV - (-3.13026\ eV) \right) \right. \\ &\quad \left. - \left((31.63536831\ eV) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}} \right) \right\} \\ &= -31.63537\ eV - \left(0.28197\ eV - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \end{aligned} \quad (14.393)$$

From Eqs. (14.391-14.393), the total energy of each ^{12}CH is

$$\begin{aligned}
 E_{\text{acetylene}T+\text{osc}}(^{12}\text{CH}) &= -31.63537 \text{ eV} + \bar{E}_{\text{acetylene osc}}(^{12}\text{CH}) \\
 &= -31.63537 \text{ eV} - \left(0.28197 \text{ eV} - \frac{1}{2}(0.35532 \text{ eV}) \right) \\
 &= -31.73967 \text{ eV}
 \end{aligned} \tag{14.394}$$

where ω_e given by Eq. (13.458) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

The total energy for each hydrogen carbide radical given by Eq. (13.485) is

$$\begin{aligned}
 E_{\text{radical}T+\text{osc}}(^{12}\text{CH}) &= -31.63537 \text{ eV} + \bar{E}_{\text{radical osc}}(^{12}\text{CH}) \\
 &= -31.63537 \text{ eV} - 0.24966 \text{ eV} + \frac{1}{2}(0.35532 \text{ eV}) \\
 &= -31.70737 \text{ eV}
 \end{aligned} \tag{14.395}$$

5 The difference in energy between the CH groups and the hydrogen carbide radical $\Delta E_{T+\text{osc}}(^{12}\text{CH})$ is given by two times the difference between Eqs. (14.394) and (14.395):

$$\begin{aligned}
 \Delta E_{T+\text{osc}}(^{12}\text{CH}) &= 2(E_{\text{acetylene}T+\text{osc}}(^{12}\text{CH}) - E_{\text{radical}T+\text{osc}}(^{12}\text{CH})) \\
 &= 2(-31.73967 \text{ eV} - (-31.70737 \text{ eV})) \\
 &= -0.06460 \text{ eV}
 \end{aligned} \tag{14.396}$$

SUM OF THE ENERGIES OF THE $\text{C} \equiv \text{C}$ σ MO AND THE HOs OF 10 ACETYLENE

The energy components of V_e , V_p , T , V_m , and E_T of the $\text{C} \equiv \text{C}$ -bond MO are the same as those of the CH MO except that each term is multiplied by three corresponding to the triple bond and the energy term corresponding to the $C_{\text{acetylene}} 2sp^3$ HOs in the equation for E_T is positive. The energies of each $\text{C} \equiv \text{C}$ -bond MO are given by the substitution of the
15 semiprincipal axes (Eqs. (14.346-14.347) and (14.350)) into three times the energy equations of the CH MO (Eqs. (13.449-13.453)), with the exception that $E(C, 2sp^3)$ in Eq. (13.453) is positive and given by Eq. (14.341):

$$V_e = 3(0.91771) \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -182.53826 \text{ eV} \tag{14.397}$$

$$V_p = 3 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} = 35.97770 \text{ eV} \tag{14.398}$$

$$T = 3(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 70.90876 \text{ eV} \quad (14.399)$$

$$V_m = 3(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -35.45438 \text{ eV} \quad (14.400)$$

$$E_T(C \equiv C, \sigma) = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + 16.20002 \text{ eV} = -94.90616 \text{ eV} \quad (14.401)$$

5 where $E_T(C \equiv C, \sigma)$ is the total energy of the $C \equiv C$ σ MO given by Eq. (14.343) which is reiteratively matched to three times Eq. (13.75) within five-significant-figure round off error.

The total energy of the $C \equiv C$ -bond MO, $E_T(C \equiv C)$, is given by the sum of two times $E_T(C \equiv C, 2sp^3)$, the energy change of each $C2sp^3$ shell due to the decrease in radius with the formation of the $C \equiv C$ -bond MO (Eq. (14.342)), and $E_T(C \equiv C, \sigma)$, the σ MO
10 contribution given by Eq. (14.344):

$$\begin{aligned} E_T(C \equiv C) &= 2E_T(C \equiv C, 2sp^3) + E_T(C \equiv C, \sigma) \\ &= \left(2(-1.56513 \text{ eV}) + \left(-\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + 16.20002 \text{ eV} \right) \right) \\ &= 2(-1.56513 \text{ eV}) + (-94.90610 \text{ eV}) \\ &= -98.03637 \text{ eV} \end{aligned} \quad (14.402)$$

VIBRATION OF ACETYLENE

15 The vibrational energy levels of $CHCH$ may be solved as two equivalent coupled harmonic oscillators with a bridging harmonic oscillator by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERMS OF THE $C \equiv C$ -BOND MO OF ACETYLENE

The equations of the radiation reaction force of the $C \equiv C$ -bond MO are given by Eq. (14.231), except that the $C \equiv C$ -bond MO parameters are used. The angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.5e^2}{4\pi\epsilon_0 a^3}} = 2.00186 \times 10^{16} \text{ rad/s} \quad (14.403)$$

where a is given by Eq. (14.346). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.00186 \times 10^{16} \text{ rad/s} = 13.17659 \text{ eV} \quad (14.404)$$

In Eq. (11.181), substitution of $E_T(C \equiv C)/3$ (Eq. (14.402)) for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.404) for \bar{E}_K gives the Doppler energy of the electrons of each of the three bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -32.67879 \text{ eV} \sqrt{\frac{2e(13.17659 \text{ eV})}{m_e c^2}} = -0.23468 \text{ eV} \quad (14.405)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of the $C \equiv C$ -bond MO due to the reentrant orbit of the bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.405) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of the $C \equiv C$ bond. Using the experimental $C \equiv C$ $E_{vib}(\nu_3)$ of 3374 cm^{-1} (0.41833 eV) [6] for \bar{E}_{Kvib} of the transition state having three bonds, $\bar{E}'_{osc}(C \equiv C, \sigma)$ per bond is

$$\bar{E}'_{osc}(C \equiv C, \sigma) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.406)$$

$$\bar{E}'_{osc}(C \equiv C, \sigma) = -0.23468 \text{ eV} + \frac{1}{2} (0.41833 \text{ eV}) = -0.02551 \text{ eV} \quad (14.407)$$

Given that the vibration and reentrant oscillation is for three $C-C$ bonds of the $C \equiv C$ triple bond, $\bar{E}_{acetylene\ osc}(C \equiv C, \sigma)$, is:

$$\begin{aligned}
 \bar{E}_{acetylene\ osc} (C \equiv C, \sigma) &= 3 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\
 &= 3 \left(-0.23468\ eV + \frac{1}{2} (0.41833\ eV) \right) \\
 &= -0.07654\ eV
 \end{aligned}
 \tag{14.408}$$

TOTAL ENERGIES OF THE $C \equiv C$ -BOND MO OF ACETYLENE

$E_{T+osc} (C \equiv C)$, the total energy of the $C \equiv C$ -bond MO including the Doppler term, is given

5 by the sum of $E_T (C \equiv C)$ (Eq. (14.402)) and $\bar{E}_{acetylene\ osc} (C \equiv C, \sigma)$ given by Eq. (14.408):

$$\begin{aligned}
 E_{T+osc} (C \equiv C) &= \left(V_e + T + V_m + V_p - E(C_{acetylene}, 2sp^3) \right. \\
 &\quad \left. + 2E_T (C \equiv C, 2sp^3) + \bar{E}_{acetylene\ osc} (C \equiv C, \sigma) \right) \\
 &= E_T (C \equiv C, \sigma) + 2E_T (C \equiv C, 2sp^3) + \bar{E}_{acetylene\ osc} (C \equiv C, \sigma) \\
 &= E_T (C \equiv C) + \bar{E}_{acetylene\ osc} (C \equiv C, \sigma)
 \end{aligned}
 \tag{14.409}$$

$$\begin{aligned}
 E_{T+osc} (C \equiv C) &= \left\{ \left(\frac{-3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \right. \\
 &\quad \left. - E(C_{acetylene}, 2sp^3) + 2E_T (C \equiv C, 2sp^3) \right\} \\
 &\quad \left\{ \left(1 + (3) \left(\frac{1}{3} \right) \sqrt{\frac{2\hbar \sqrt{\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 a^3}}}}{m_e c^2} \right) + 3 \left(\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\} \\
 &= -98.03637\ eV - 3 \left(0.23468\ eV - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)
 \end{aligned}
 \tag{14.410}$$

From Eqs. (14.408-14.410), the total energy of the $C \equiv C$ -bond MO is

$$\begin{aligned}
 E_{T+osc} (C \equiv C) &= -94.90610\ eV + 2E_T (C \equiv C, 2sp^3) + \bar{E}_{acetylene\ osc} (C \equiv C, \sigma) \\
 &= -94.90610\ eV + 2(-1.56513\ eV) - 3 \left(0.23468\ eV - \frac{1}{2} (0.41833\ eV) \right) \\
 &= -98.11291\ eV
 \end{aligned}
 \tag{14.411}$$

10

where the experimental E_{vib} was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

BOND ENERGY OF THE $C \equiv C$ BOND OF ACETYLENE

As in the case of $^{12}CH_2$ and ^{14}NH , the dissociation of the $C \equiv C$ bond forms three unpaired electrons per central atom wherein the magnetic moments cannot all cancel. The energy per atom $E(\text{magnetic})$ is given by Eq. (13.524). Thus, the dissociation energy of the $C \equiv C$ bond of $CHCH$, $E_D(HC \equiv CH)$, is given by six times $E(C, 2sp^3)$ (Eq. (14.146)), the initial energy of each $C2sp^3$ HO of each CH radical that forms the triple $C \equiv C$ bond, minus the sum of $\Delta E_{T+osc}(^{12}CH)$ (Eq. (14.396)), the energy change going from the hydrogen carbide radicals to the CH groups of acetylene, $E_{T+osc}(C \equiv C)$ (Eq. (14.411)), and two times $E(\text{magnetic})$ given by Eq. (13.524). Thus, the dissociation energy of the $C \equiv C$ bond of $CHCH$, is

$$\begin{aligned} E_D(HC \equiv CH) &= 6\left(E(C, 2sp^3)\right) - \left(\Delta E_{T+osc}(^{12}CH) + E_{T+osc}(C \equiv C) + 2E(\text{magnetic})\right) \\ &= 6(-14.63489 \text{ eV}) - (-0.06460 \text{ eV} - 98.11291 \text{ eV} + 0.29606 \text{ eV}) \quad (14.412) \\ &= 6(-14.63489 \text{ eV}) - (-97.88145 \text{ eV}) \\ &= 10.07212 \text{ eV} \end{aligned}$$

The experimental dissociation energy of the $C \equiv C$ bond of $CHCH$ is [7]

$$E_D(HC \equiv CH) = 10.0014 \text{ eV} \quad (14.413)$$

The results of the determination of bond parameters of $CHCH$ are given in Table 14.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

BENZENE MOLECULE (C_6H_6)

The benzene molecule C_6H_6 is formed by the reaction of three ethylene molecules:



C_6H_6 can be solved using the same principles as those used to solve ethylene wherein the $2s$ and $2p$ shells of each C hybridize to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ hybridized orbitals (HOs) to form a molecular orbital (MO) permits each participating hybridized orbital to decrease in radius and energy. Each $2sp^3$ HO of each carbon atom initially has four unpaired electrons. Thus, the $6H$ atomic orbitals (AOs) of benzene contribute six electrons and the six sp^3 -hybridized carbon

atoms contribute twenty-four electrons to form six $C-H$ bonds and six $C=C$ bonds. Each $C-H$ bond has two paired electrons with one donated from the H AO and the other from the $C2sp^3$ HO. Each $C=C$ bond comprises a linear combination of a factor of 0.75 of four paired electrons (three electrons) from two sets of two $C2sp^3$ HOs of the participating carbon
 5 atoms. Each $C-H$ and each $C=C$ bond comprises a linear combination of one and two diatomic H_2 -type MOs developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section, respectively.

FORCE BALANCE OF THE $C=C$ -BOND MO OF BENZENE

10 C_6H_6 can be considered a linear combination of three ethylene molecules wherein a $C-H$ bond of each CH_2 group of $H_2C=CH_2$ is replaced by a $C=C$ bond to form a six-member ring of carbon atoms. The solution of the ethylene molecule is given in the Ethylene Molecule (CH_2CH_2) section. Before forming ethylene groups, the $2sp^3$ hybridized orbital arrangement of each carbon atom is given by Eq. (14.140). The sum $E_T(C, 2sp^3)$ of
 15 calculated energies of C , C^+ , C^{2+} , and C^{3+} is given by Eq. (14.141). The radius r_{2sp^3} of the $C2sp^3$ shell is given by Eq. (14.142). The Coulombic energy $E_{Coulomb}(C, 2sp^3)$ and the energy $E(C, 2sp^3)$ of the outer electron of the $C2sp^3$ shell are given by Eqs. (14.143) and (14.146), respectively. Two CH_2 radicals bond to form CH_2CH_2 by forming a MO between the two pairs of remaining $C2sp^3$ -HO electrons of the two carbon atoms. However, in this
 20 case, the sharing of electrons between four $C2sp^3$ HOs to form a MO comprising four spin-paired electrons permits each $C2sp^3$ HO to decrease in radius and energy. The $C=C$ -bond MO is a prolate-spheroidal-MO surface that cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell of each atom. Thus, the MO surface comprises a partial prolate spheroid in between the carbon nuclei and is continuous with the $C2sp^3$ shell
 25 at each C atom. The energy of the H_2 -type ellipsoidal MO is matched to that of each $C2sp^3$ shell. As in the case of previous examples of energy-matched MOs such as those of OH , NH , CH , the $C=O$ -bond MO of CO_2 , and the $C-C$ -bond MO of CH_3CH_3 , the $C=C$ -bond MO of ethylene must comprise 75% of a H_2 -type ellipsoidal MO in order to match potential, kinetic, and orbital-energy relationships. Thus, the $C=C$ -bond MO must comprise

a linear combination of two MOs wherein each comprises two $C2sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the $C2sp^3$ HOs:

$$2(2 C2sp^3 + 0.75 H_2 MO) \rightarrow C = C - bond MO \quad (14.415)$$

5

The linear combination of each H_2 -type ellipsoidal MO with each $C2sp^3$ HO further comprises an excess 25% charge-density contribution from each $C2sp^3$ HO to the $C = C$ -bond MO to achieve an energy minimum. The force balance of the $C = C$ -bond MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (14.415) and the energy matching condition between the $C2sp^3$ -HO components of the MO.

The sharing of electrons between two pairs of $C2sp^3$ HOs to form a $C = C$ -bond MO permits each participating hybridized orbital to decrease in radius and energy. The sum $E_T(C_{ethylene}, 2sp^3)$ of calculated energies of $C2sp^3$, C^+ , C^{2+} , and C^{3+} is given by Eq. (14.243). In order to further satisfy the potential, kinetic, and orbital energy relationships, each participating $C2sp^3$ HO donates an excess of 25% of its electron density to the $C = C$ -bond MO to form an energy minimum. By considering this electron redistribution in the ethylene molecule as well as the fact that the central field decreases by an integer for each successive electron of the shell, the radius $r_{ethylene 2sp^3}$ of the $C2sp^3$ shell of ethylene calculated from the Coulombic energy is given by Eq. (14.244). The Coulombic energy $E_{Coulomb}(C_{ethylene}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is given by Eq. (14.245). The energy $E(C_{ethylene}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is given by Eq. (14.246). $E_T(C = C, 2sp^3)$ (Eq. (14.247), the energy change of each $C2sp^3$ shell with the formation of the $C = C$ -bond MO is given by the difference between $E(C_{ethylene}, 2sp^3)$ and $E(C, 2sp^3)$.

25

Consider the case where three sets of $C = C$ -bond MOs form bonds between the two carbon atoms of each molecule to form a six-member ring such that the six resulting bonds comprise eighteen paired electrons. Each bond comprises a linear combination of two MOs wherein each comprises two $C2sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the $C2sp^3$ HOs:

$$\left(\begin{array}{l} 3(2 C2sp^3 + 0.75 H_2 MO) \rightarrow 3(C \overset{4e}{=C}) - \text{ethylene-type-bond MO} \\ \rightarrow 6(C \overset{3e}{=C}) - \text{bond MO of benzene} \end{array} \right) \quad (14.416)$$

The linear combination of each H_2 -type ellipsoidal MO with each $C2sp^3$ HO further
 5 comprises an excess 25% charge-density contribution per bond from each $C2sp^3$ HO to the
 $C = C$ -bond MO to achieve an energy minimum. Thus, the dimensional parameters of each
 bond $C = C$ -bond are determined using the same equations as those used to determine the
 same parameters of the $C = C$ -bond MO of ethylene (Eqs. (14.242-14.268)) while matching
 the boundary conditions of the structure of benzene. The energies of each $C = C$ bond of
 10 benzene are also determined using the same equations as those of ethylene with the
 parameters of benzene. The result is that the energies are essentially given as 0.75 times the
 energies of the $C = C$ -bond MO of ethylene (Eqs. (14.251-14.253) and (14.319-14.333)).

The derivation of the dimensional parameters of benzene follows the same procedure
 as the determination of those of ethylene. As in the case of ethylene, each H_2 -type
 15 ellipsoidal MO comprises 75% of the $C = C$ -bond MO shared between two $C2sp^3$ HOs
 corresponding to the electron charge density in Eq. (11.65) of $\frac{-0.75e}{2}$. But, the additional
 25% charge-density contribution to each bond of the $C = C$ -bond MO causes the electron
 charge density in Eq. (11.65) to be given by $\frac{-e}{2} = -0.5e$. The corresponding force constant
 k' is given by Eq. (14.152). In addition, the energy matching at all six $C2sp^3$ HOs further
 20 requires that k' be corrected by a hybridization factor (Eq. (13.430)) as in the case of
 ethylene, expect that the constraint that the bonds connect a six-member ring of $C = C$ bonds
 of benzene rather two $C2sp^3$ HOs of ethylene decreases the hybridization factor of benzene
 compared to that of ethylene (Eq. (14.248)).

Since the energy of each H_2 -type ellipsoidal MO is matched to that of all the
 25 continuously connected $C_{benzene}2sp^3$ HOs, the hybridization-energy-matching factor is
 0.85252. Hybridization with 25% electron donation to each $C = C$ -bond gives rise to the
 $C_{benzene}2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C_{benzene}, 2sp^3)$ given by Eq. (14.245). The
 corresponding hybridization factor is given by the ratio of 15.95955 eV, the magnitude of

$E_{Coulomb}(C_{benzene}, 2sp^3)$ given by Eq. (14.245), and 13.605804 eV , the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). The hybridization energy factor $C_{benzeneC2sp^3HO}$ is

$$C_{benzeneC2sp^3HO} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{benzene2sp^3}}} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 0.85252 a_0}} = \frac{13.605804 \text{ eV}}{15.95955 \text{ eV}} = 0.85252 \quad (14.417)$$

5 Thus, the force constant k' to determine the ellipsoidal parameter c' in terms of the central force of the foci (Eq. (11.65)) is given by

$$k' = C_{benzeneC2sp^3HO} \frac{(0.5)2e^2}{4\pi\epsilon_0} = 0.85252 \frac{(0.5)2e^2}{4\pi\epsilon_0} \quad (14.418)$$

The distance from the origin to each focus c' is given by substitution of Eq. (14.418) into Eq. (13.60). Thus, the distance from the origin of the component of the double $C=C$ -bond MO

10 to each focus c' is given by

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{(0.85252)m_e e^2 a}} = \sqrt{\frac{a a_0}{0.85252}} \quad (14.419)$$

The internuclear distance from Eq. (14.419) is

$$2c' = 2\sqrt{\frac{a a_0}{0.85252}} \quad (14.420)$$

The length of the semiminor axis of the prolate spheroidal $C=C$ -bond MO $b=c$ is given by
15 Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of the $C=C$ -bond MO. From the energy equation and the relationship between the axes, the dimensions of the $C=C$ -bond MO are solved.

The general equations for the energy components of V_e , V_p , T , V_m , and E_T of the
20 $C=C$ -bond MO of benzene are the same as those of the CH_2CH_2 MO except that energy of the $C_{benzene}2sp^3$ HO is used and the hybridization factor is given by Eq. (14.417). Using Eqs. (14.251) and (14.417), $E_T(C=C, \sigma)$ is given by

$$\begin{aligned} E_T(C=C, \sigma) &= E_T + E(C_{benzene}, 2sp^3) - E(C_{benzene}, 2sp^3) \\ &= -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.85252) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \end{aligned} \quad (14.421)$$

The total energy term of the double $C = C$ -bond MO is given by the sum of the two H_2 -type ellipsoidal MOs given by Eq. (11.212). To match this boundary condition, $E_T(C = C, \sigma)$ given by Eq. (14.421) is set equal to two times Eq. (13.75):

$$E_T(C = C, \sigma) = -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.85252) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] = -63.27074 \text{ eV} \quad (14.422)$$

- 5 From the energy relationship given by Eq. (14.422) and the relationship between the axes given by Eqs. (14.419-14.420) and (13.62-13.63), the dimensions of the $C = C$ -bond MO can be solved.

Substitution of Eq. (14.419) into Eq. (14.422) gives

$$\frac{2e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{0.85252}}} \left[(0.85252) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{0.85252}}}{a - \sqrt{\frac{aa_0}{0.85252}}} - 1 \right] = -63.27074 \quad (14.423)$$

- 10 The most convenient way to solve Eq. (14.423) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.47348a_0 = 7.79733 \times 10^{-11} \text{ m} \quad (14.424)$$

Substitution of Eq. (14.424) into Eq. (14.4129) gives

$$c' = 1.31468a_0 = 6.95699 \times 10^{-11} \text{ m} \quad (14.425)$$

- 15 The internuclear distance given by multiplying Eq. (14.425) by two is

$$2c' = 2.62936a_0 = 1.39140 \times 10^{-10} \text{ m} \quad (14.426)$$

The experimental bond distance is [3]

$$2c' = 1.339 \times 10^{-10} \text{ m} \quad (14.427)$$

Substitution of Eqs. (14.424-14.425) into Eq. (13.62) gives

$$20 \quad b = c = 0.66540a_0 = 3.52116 \times 10^{-11} \text{ m} \quad (14.428)$$

Substitution of Eqs. (14.424-14.425) into Eq. (13.63) gives

$$e = 0.89223 \quad (14.429)$$

- The nucleus of the C atoms comprise the foci of the H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{\text{benzene}} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). Each benzene carbon atom contributes 25 $(0.75)(-1.13380 \text{ eV}) = -0.85035 \text{ eV}$ (Eqs. (14.483) and (14.493)) to each of the two $C = C$ -bond MOs and $(0.5)(-1.13380 \text{ eV}) = -0.56690 \text{ eV}$ (Eq. (14.467)) to the corresponding

$C-H$ -bond MO. The energy contribution due to the charge donation at each carbon superimposes linearly. The radius of $r_{\text{benzene } 2sp^3} = 0.79597a_0$ is calculated using Eq. (14.518) using the total energy donation to each bond with which it is participates in bonding. The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{\text{benzene } 2sp^3} = 0.79597a_0$ is the

5 radius of the $C_{\text{benzene}} 2sp^3$ shell. Substitution of Eqs. (14.424-14.425) into Eq. (13.261) gives

$$\theta' = 134.24^\circ \quad (14.430)$$

Then, the angle $\theta_{C=C_{\text{benzene}} 2sp^3 HO}$ the radial vector of the $C2sp^3$ HO makes with the internuclear axis is

$$\theta_{C=C_{\text{benzene}} 2sp^3 HO} = 180^\circ - 134.24^\circ = 45.76^\circ \quad (14.431)$$

10 as shown in Figure 45.

Thus, the $^{12}CH_4$ bond dissociation energy, $E_D(^{12}CH_4)$, given by Eqs. (13.154), and (13.614-13.616) is

$$\begin{aligned} E_D(^{12}CH_4) &= -(67.95529 \text{ eV} + 13.59844 \text{ eV}) - E_{T+osc}(^{12}CH_4) \\ &= -81.55373 \text{ eV} - (-86.04373 \text{ eV}) \\ &= 4.4900 \text{ eV} \end{aligned} \quad (13.617)$$

The experimental $^{12}CH_4$ bond dissociation energy is [40]

$$15 \quad E_D(^{12}CH_4) = 4.48464 \text{ eV} \quad (13.618)$$

Consider the right-hand intersection point. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C=C_{\text{benzene}}, H_2 MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with each $C_{\text{benzene}} 2sp^3$ radial vector obeys the following relationship:

$$20 \quad r_{\text{benzene } 2sp^3} \sin \theta_{C=C_{\text{benzene}} 2sp^3 HO} = 0.79597a_0 \sin \theta_{C=C_{\text{benzene}} 2sp^3 HO} = b \sin \theta_{C=C_{\text{benzene}}, H_2 MO} \quad (14.432)$$

such that

$$\theta_{C=C_{\text{benzene}}, H_2 MO} = \sin^{-1} \frac{0.79597a_0 \sin \theta_{C=C_{\text{benzene}} 2sp^3 HO}}{b} = \sin^{-1} \frac{0.79597a_0 \sin 45.76^\circ}{b} \quad (14.433)$$

with the use of Eq. (14.431). Substitution of Eq. (14.428) into Eq. (14.433) gives

$$\theta_{C=C_{\text{benzene}}, H_2 MO} = 58.98^\circ \quad (14.434)$$

25 Then, the distance $d_{C=C_{\text{benzene}}, H_2 MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C=C_{benzene}, H_2MO} = a \cos \theta_{C=C_{benzene}, H_2MO} \quad (14.435)$$

Substitution of Eqs. (14.424) and (14.434) into Eq. (14.435) gives

$$d_{C=C_{benzene}, H_2MO} = 0.75935a_0 = 4.01829 \times 10^{-11} \text{ m} \quad (14.436)$$

The distance $d_{C=C_{benzene} 2sp^3 HO}$ along the internuclear axis from the origin of the C atom to the
5 point of intersection of the orbitals is given by

$$d_{C=C_{benzene} 2sp^3 HO} = c' - d_{C=C_{benzene}, H_2MO} \quad (14.437)$$

Substitution of Eqs. (14.425) and (14.436) into Eq. (14.437) gives

$$d_{C=C_{benzene} 2sp^3 HO} = 0.55533a_0 = 2.93870 \times 10^{-11} \text{ m} \quad (14.438)$$

10 FORCE BALANCE OF THE CH MOs OF BENZENE

Benzene can also be considered as comprising chemical bonds between six CH radicals wherein each radical comprises a chemical bond between carbon and hydrogen atoms. The solution of the parameters of CH is given in the Hydrogen Carbide (CH) section. Each $C-H$ bond of CH having two spin-paired electrons, one from an initially unpaired electron
15 of the carbon atom and the other from the hydrogen atom, comprises the linear combination of 75% H_2 -type ellipsoidal MO and 25% $C2sp^3$ HO as given by Eq. (13.439):



20 The proton of the H atom and the nucleus of the C atom are along each internuclear axis and serve as the foci. As in the case of H_2 , the $C-H$ -bond MO is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell since it is energetically unfavorable. Thus, each MO surface comprises a prolate spheroid at the H proton that is continuous with the $C2sp^3$
25 shell at the C atom whose nucleus serves as the other focus.

The force balance of the CH MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (14.439) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO. The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-ellipsoidal-MO component
30 of the CH MO in terms of the central force of the foci is given by Eq. (13.59). The distance

from the origin of each $C-H$ -bond MO to each focus c' is given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b=c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other
 5 axes of each prolate spheroid and eccentricity of each $C-H$ -bond MO. From the energy equation and the relationship between the axes, the dimensions of the CH MO are solved.

Consider the formation of the double $C=C$ -bond MOs of benzene wherein ethylene formed from two CH_2 radicals, each having a $C2sp^3$ shell with an energy given by Eq. (14.146), serves as a basis element. The energy components of V_e , V_p , T , V_m , and E_T are
 10 the same as those of the hydrogen carbide radical, except that $E_T(C=C, 2sp^3)$ is subtracted from $E_T(CH)$ of Eq. (13.495). As in the case of the CH_2 groups of ethylene (Eq. (14.270)), the subtraction of the energy change of the $C2sp^3$ shell per H with the formation of the $C=C$ -bond MO matches the energy of each $C-H$ -bond MO to the decrease in the energy of the corresponding $C2sp^3$ HO. Using Eqs. (13.431) and (14.247), $E_{T_{benzene}}(CH)$ is given by

$$\begin{aligned}
 E_{T_{benzene}}(CH) &= E_T + E(C, 2sp^3) - E_T(C=C, 2sp^3) \\
 15 \quad &= \left(-\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \\
 &\quad -14.63489 \text{ eV} - (-1.13379 \text{ eV})
 \end{aligned} \tag{14.440}$$

$E_{T_{benzene}}(CH)$ given by Eq. (14.440) is set equal to the energy of the H_2 -type ellipsoidal MO given by Eq. (13.75):

$$\begin{aligned}
 E_{T_{benzene}}(CH) &= \left(-\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \\
 &\quad -14.63489 \text{ eV} - (-1.13379 \text{ eV}) \\
 (14.441)
 \end{aligned}$$

20 From the energy relationship given by Eq. (14.441) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH MO can be solved.

Substitution of Eq. (13.60) into Eq. (14.441) gives

$$\frac{e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e18.13427 \tag{14.442}$$

The most convenient way to solve Eq. (14.442) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.60061a_0 = 8.47006 \times 10^{-11} \text{ m} \quad (14.443)$$

Substitution of Eq. (14.443) into Eq. (14.60) gives

$$5 \quad c' = 1.03299a_0 = 5.46636 \times 10^{-11} \text{ m} \quad (14.444)$$

The internuclear distance given by multiplying Eq. (14.444) by two is

$$2c' = 2.06598a_0 = 1.09327 \times 10^{-10} \text{ m} \quad (14.445)$$

The experimental bond distance is [3]

$$2c' = 1.101 \times 10^{-10} \text{ m} \quad (14.446)$$

10 Substitution of Eqs. (14.443-14.444) into Eq. (14.62) gives

$$b = c = 1.22265a_0 = 6.47000 \times 10^{-11} \text{ m} \quad (14.447)$$

Substitution of Eqs. (14.443-14.444) into Eq. (14.63) gives

$$e = 0.64537 \quad (14.448)$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of each
15 H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{benzene} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{benzene 2sp^3} = 0.79597a_0$ is the radius of the $C_{benzene} 2sp^3$ shell. Substitution of Eqs. (14.443-14.444) into Eq. (13.261) gives

$$20 \quad \theta' = 74.42^\circ \quad (14.449)$$

Then, the angle $\theta_{C-H_{benzene} 2sp^3 HO}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear axis is

$$\theta_{C-H_{benzene} 2sp^3 HO} = 180^\circ - 74.42^\circ = 105.58^\circ \quad (14.450)$$

as shown in Figure 46.

25

The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C-H_{benzene} H_2 MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C_{benzene} 2sp^3$ radial vector obeys the following relationship:

$$30 \quad r_{benzene 2sp^3} \sin \theta_{C-H_{benzene} 2sp^3 HO} = 0.79597a_0 \sin \theta_{C-H_{benzene} 2sp^3 HO} = b \sin \theta_{C-H_{benzene} H_2 MO} \quad (14.451)$$

such that

$$\theta_{C-H_{benzene}, H_2MO} = \sin^{-1} \frac{0.79597a_0 \sin \theta_{C-H_{benzene} 2sp^3HO}}{b} = \sin^{-1} \frac{0.79597a_0 \sin 105.58^\circ}{b} \quad (14.452)$$

with the use of Eq. (14.450). Substitution of Eq. (14.447) into Eq. (14.452) gives

$$\theta_{C-H_{benzene}, H_2MO} = 38.84^\circ \quad (14.453)$$

- 5 Then, the distance $d_{C-H_{benzene}, H_2MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C-H_{benzene}, H_2MO} = a \cos \theta_{C-H_{benzene}, H_2MO} \quad (14.454)$$

Substitution of Eqs. (14.443) and (14.453) into Eq. (14.454) gives

$$d_{C-H_{benzene}, H_2MO} = 1.24678a_0 = 6.59767 \times 10^{-11} \text{ m} \quad (14.455)$$

- 10 The distance $d_{C-H_{benzene} 2sp^3HO}$ along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$d_{C-H_{benzene} 2sp^3HO} = d_{C-H_{benzene}, H_2MO} - c' \quad (14.456)$$

Substitution of Eqs. (14.444) and (14.455) into Eq. (14.456) gives

$$d_{C-H_{benzene} 2sp^3HO} = 0.21379a_0 = 1.13131 \times 10^{-11} \text{ m} \quad (14.457)$$

- 15 The basis set of benzene, the ethylene molecule, is planar with bond angles of approximately 120° (Eqs. (14.298-14.302)). To form a closed ring of equivalent planar bonds, the $C=C$ bonds of benzene form a planar hexagon. The bond angle $\theta_{C=C=C}$ between the internuclear axis of any two adjacent $C=C$ bonds is

$$\theta_{C=C=C} = 120^\circ \quad (14.458)$$

- 20 The bond angle $\theta_{C=C-H}$ between the internuclear axis of each $C=C$ bond and the corresponding H atom of each CH group is

$$\theta_{C=C-H} = 120^\circ \quad (14.459)$$

The experimental angle between the $C=C=C$ bonds is [13-15]

$$\theta_{C=C=C} = 120^\circ \quad (14.460)$$

- 25 The experimental angle between the $C=C-H$ bonds is [13-15]

$$\theta_{C=C-H} = 120^\circ \quad (14.461)$$

The C_6H_6 MO shown in Figure 47 was rendered using these parameters.

The charge-density in the $C=C$ -bond MO is increased by a factor of 0.25 per bond with the formation of the $C_{benzene} 2sp^3$ HOs each having a smaller radius. Using the orbital

composition of the CH groups (Eq. (14.439)) and the $C=C$ -bond MO (Eq. (14.416), the radii of $Cl_s = 0.17113a_0$ (Eq. (10.51)) and $C_{benzene}2sp^3 = 0.79597a_0$ (Eq. (14.520)) shells, and the parameters of the $C=C$ -bond (Eqs. (13.3-13.4), (14.424-14.426), and (14.428-14.438)), the parameters of the $C-H$ -bond MOs (Eqs. (13.3-13.4), (14.443-14.445), and 5 (14.447-14.457)), and the bond-angle parameters (Eqs. (14.458-14.459)), the charge-density of the C_6H_6 MO comprising the linear combination of six sets of $C-H$ -bond MOs with bridging $C=C$ -bond MOs is shown in Figure 47. Each $C-H$ -bond MO comprises a H_2 -type ellipsoidal MO and a $C_{benzene}2sp^3$ HO having the dimensional diagram shown in Figure 46. The $C=C$ -bond MO comprises a H_2 -type ellipsoidal MO bridging two sets of two 10 $C_{benzene}2sp^3$ HOs having the dimensional diagram shown in Figure 45.

ENERGIES OF THE CH GROUPS

The energies of each CH group of benzene are given by the substitution of the semiprincipal axes (Eqs. (14.443-14.444) and (14.447)) into the energy equations of hydrogen carbide (Eqs. 15 (13.449-13.453)), with the exception that $E_T(C=C, 2sp^3)$ (Eq. (14.247)) is subtracted from $E_T(CH)$ in Eq. (13.453):

$$V_e = (0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -37.10024 \text{ eV} \quad (14.462)$$

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 13.17125 \text{ eV} \quad (14.463)$$

$$T = (0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 11.58941 \text{ eV} \quad (14.464)$$

$$20 \quad V_m = (0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -5.79470 \text{ eV} \quad (14.465)$$

$$E_{T_{benzene}}(CH) = \left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \bigg/ \left(-14.63489 \text{ eV} - (-1.13379 \text{ eV}) \right) = -31.63539 \text{ eV}$$

(14.466)

where $E_{T_{benzene}}(CH)$ is given by Eq. (14.440) which is reiteratively matched to Eq. (13.75) within five-significant-figure round off error.

The total energy of the $C-H$ -bond MO, $E_{T_{benzene}}(C-H)$, is given by the sum of $0.5E_T(C=C, 2sp^3)$, the energy change of each $C2sp^3$ shell per single bond due to the decrease in radius with the formation of the corresponding $C=C$ -bond MO (Eq. (14.247)), and $E_{T_{benzene}}(CH)$, the σ MO contribution given by Eq. (14.441):

$$\begin{aligned}
 E_{T_{benzene}}(C-H) &= (0.5)E_T(C=C, 2sp^3) + E_{T_{benzene}}(CH) \\
 &= \left((0.5)(-1.13379 \text{ eV}) + \left[-\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right] \right) \\
 &= (0.5)(-1.13379 \text{ eV}) + (-31.63537 \text{ eV}) \\
 &= -32.20226 \text{ eV}
 \end{aligned} \tag{14.467}$$

VIBRATION OF THE ^{12}CH GROUPS

The vibrational energy levels of CH in benzene may be solved using the methods given in the Vibration and Rotation of CH section.

THE DOPPLER ENERGY TERMS OF THE ^{12}CH GROUPS

The equations of the radiation reaction force of the CH groups in benzene are the same as those of the hydrogen carbide radical with the substitution of the CH -group parameters. Using Eq. (13.477), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 2.64826 \times 10^{16} \text{ rad/s} \tag{14.468}$$

where b is given by Eq. (14.447). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.64826 \times 10^{16} \text{ rad/s} = 17.43132 \text{ eV} \tag{14.469}$$

In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.469) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(17.43132 \text{ eV})}{m_e c^2}} = -0.26130 \text{ eV} \quad (14.470)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.470) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of the $C-H$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition, $\bar{E}_{benzene\ osc} (^{12}CH)$ per bond is

$$\bar{E}_{benzene\ osc} (^{12}CH) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.471)$$

$$10 \quad \bar{E}_{benzene\ osc} (^{12}CH) = -0.26130 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) = -0.08364 \text{ eV} \quad (14.472)$$

TOTAL AND BOND ENERGIES OF THE ^{12}CH GROUPS

$E_{benzeneT+osc} (^{12}CH)$, the total energy of each ^{12}CH group including the Doppler term, is given by the sum of $E_{T_{benzene}} (C-H)$ (Eq. (14.467)) and $\bar{E}_{benzene\ osc} (^{12}CH)$ given by Eq. (14.472):

$$15 \quad E_{benzeneT+osc} (CH) = \left(\left(V_e + T + V_m + V_p + E(C, 2sp^3) - E_T (C=C, 2sp^3) \right) + 0.5 E_T (C=C, 2sp^3) + \bar{E}_{benzene\ osc} (^{12}CH) \right) \quad (14.473)$$

$$= E_{T_{benzene}} (C-H) + \bar{E}_{benzene\ osc} (^{12}CH)$$

$$E_{benzeneT+osc} (^{12}CH) = \left\{ \begin{aligned} & \left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 0.5(1.13379 \text{ eV}) \right. \\ & \left. - 14.63489 \text{ eV} - (-1.13379 \text{ eV}) \right) \\ & - \left((31.63536831 \text{ eV}) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \end{aligned} \right\}$$

$$= -32.20226 \text{ eV} - \left(0.26130 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (14.474)$$

From Eqs. (14.472-14.474), the total energy of each ^{12}CH is

398

$$\begin{aligned}
 E_{benzeneT+osc}({}^{12}CH) &= -32.20226 \text{ eV} + \bar{E}_{benzene\ osc}({}^{12}CH_2) \\
 &= -32.20226 \text{ eV} - \left(0.26130 \text{ eV} - \frac{1}{2}(0.35532 \text{ eV}) \right) \\
 &= -32.28590 \text{ eV}
 \end{aligned} \tag{14.475}$$

where ω_e given by Eq. (13.458) was used for the $\hbar\sqrt{\frac{k}{\mu}}$ term.

As in the case of ${}^{12}CH_2$, ${}^{14}NH$, and acetylene, the dissociation of the $C=C$ bonds forms three unpaired electrons per central atom wherein the magnetic moments cannot all cancel. The energy per atom $E(magnetic)$ is given by Eq. (13.524). Thus, the bond dissociation energy of each CH group of the linear combination to form benzene, $E_{D_{benzene}}({}^{12}CH)$, is given by the sum of the total energies of the $C2sp^3$ HO and the hydrogen atom minus the sum of $E_{benzeneT+osc}({}^{12}CH)$ and $E(magnetic)$ given by Eq. (13.524):

$$E_{D_{benzene}}({}^{12}CH) = E(C, 2sp^3) + E(H) - (E_{benzeneT+osc}({}^{12}CH) + E(magnetic)) \tag{14.476}$$

$E(C, 2sp^3)$ is given by Eq. (13.428), $E_D(H)$ is given by Eq. (13.154), and $E(magnetic)$ is given by Eq. (13.524). Thus, $E_{D_{benzene}}({}^{12}CH)$ given by Eqs. (13.154), (13.428), (13.524), (14.475), and (14.476) is

$$\begin{aligned}
 E_{D_{benzene}}({}^{12}CH) &= -(14.63489 \text{ eV} + 13.59844 \text{ eV}) - (E_{benzeneT+osc}(CH) + E(magnetic)) \\
 &= -28.23333 \text{ eV} - (-32.28590 \text{ eV} + 0.14803 \text{ eV}) \\
 &= 3.90454 \text{ eV}
 \end{aligned} \tag{14.477}$$

15 SUM OF THE ENERGIES OF THE $C=C$ σ MO ELEMENT AND THE HOs OF BENZENE

The energy components of V_e , V_p , T , V_m , and E_T of the $C=C$ -bond MO of benzene are the same as those of the CH_2CH_2 MO except that the hybridization factor is given by Eq. (14.417). The energies of each $C=C$ -bond MO are given by the substitution of the semiprincipal axes (Eqs. (14.424-14.425) and (14.428)) into energy equations of the CH_2CH_2 MO (Eqs. (14.319-14.323)), with the exception that the hybridization factor is 0.85252 (Eq. (14.417)):

$$V_e = 2(0.85252) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -101.12679 \text{ eV} \tag{14.478}$$

$$V_p = 2 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} = 20.69825 \text{ eV} \quad (14.479)$$

$$T = 2(0.85252) \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = 34.31559 \text{ eV} \quad (14.480)$$

$$V_m = 2(0.85252) \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} = -17.15779 \text{ eV} \quad (14.481)$$

$$E_T(C=C, \sigma) = -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.85252) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] = -63.27075 \text{ eV} \quad (14.482)$$

5 where $E_T(C=C, \sigma)$ is the total energy of the $C=C$ σ MO given by Eq. (14.421) which is reiteratively matched to two times Eq. (13.75) within five-significant-figure round off error.

The total energy of the $C=C$ -bond MO, $E_T(C=C)$, is given by the sum of two times $E_T(C=C, 2sp^3)$, the energy change of each $C2sp^3$ shell due to the decrease in radius with the formation of the $C=C$ -bond MO (Eq. (14.247)), and $E_T(C=C, \sigma)$, the σ MO

10 contribution given by Eq. (14.422):

$$\begin{aligned} E_T(C=C) &= 2E_T(C=C, 2sp^3) + E_T(C=C, \sigma) \\ &= \left(2(-1.13380 \text{ eV}) + \left(-\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.85252) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \right) \\ &= 2(-1.13380 \text{ eV}) + (-63.27074 \text{ eV}) = -65.53833 \text{ eV} \end{aligned} \quad (14.483)$$

which is the same $E_T(C=C, \sigma)$ of ethylene given by Eq. (14.324).

VIBRATION OF BENZENE

15 The $C=C$ vibrational energy levels of C_6H_6 may be solved as six sets of equivalent coupled harmonic oscillators where each C is a further coupled to the corresponding $C-H$ oscillator by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of

20 Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERMS OF THE $C=C$ -BOND MO ELEMENT

OF BENZENE

The equations of the radiation reaction force of the $C = C$ -bond MO of benzene are given by Eq. (13.142), except the force-constant factor is $(0.85252)0.5$ based on the force constant k' of Eq. (14.418), and the $C = C$ -bond MO parameters are used. The angular frequency of the
5 reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.85252 \frac{(0.5)e^2}{4\pi\epsilon_0 b^3}}{m_e}} = 4.97272 \times 10^{16} \text{ rad/s} \quad (14.484)$$

where b is given by Eq. (14.428). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 4.97272 \times 10^{16} \text{ rad/s} = 32.73133 \text{ eV} \quad (14.485)$$

10 In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.485) for \bar{E}_K gives the Doppler energy of the electrons for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63536831 \text{ eV} \sqrt{\frac{2e(32.73133 \text{ eV})}{m_e c^2}} = -0.35806 \text{ eV} \quad (14.486)$$

15 In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of the $C = C$ -bond MO due to the reentrant orbit of the bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.486) and \bar{E}_{Kvib} , the average kinetic energy of
20 vibration which is 1/2 of the vibrational energy of the $C = C$ bond. Using the experimental $C = C$ $E_{vib}(\nu_{16})$ of 1584.8 cm^{-1} (0.19649 eV) [16] for \bar{E}_{Kvib} of the transition state having two bonds, $\bar{E}'_{osc}(C = C, \sigma)$ per bond is

$$\bar{E}'_{osc}(C = C, \sigma) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.487)$$

$$\bar{E}'_{osc}(C = C, \sigma) = -0.35806 \text{ eV} + \frac{1}{2} (0.19649 \text{ eV}) = -0.25982 \text{ eV} \quad (14.488)$$

Given that the vibration and reentrant oscillation is for two $C-C$ bonds of each $C=C$ double bond, $\bar{E}_{benzene\ osc}(C=C, \sigma)$, is:

$$\bar{E}_{benzene\ osc}(C=C, \sigma) = 2 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) = 2 \left(-0.35806\ eV + \frac{1}{2} (0.19649\ eV) \right) = -0.51963\ eV \quad (14.489)$$

5

TOTAL ENERGIES OF THE $C=C$ -BOND MO ELEMENT OF BENZENE

$E_{T+osc}(C=C)$, the total energy of the $C=C$ -bond MO of benzene including the Doppler term, is given by the sum of $E_T(C=C)$ (Eq. (14.483)) and $\bar{E}_{benzene\ osc}(C=C, \sigma)$ given by Eq. (14.489):

$$\begin{aligned} E_{T+osc}(C=C) &= V_e + T + V_m + V_p + 2E_T(C=C, 2sp^3) + \bar{E}_{benzene\ osc}(C=C, \sigma) \\ 10 \quad &= E_T(C=C, \sigma) + 2E_T(C=C, 2sp^3) + \bar{E}_{benzene\ osc}(C=C, \sigma) \\ &= E_T(C=C) + \bar{E}_{benzene\ osc}(C=C, \sigma) \end{aligned} \quad (14.490)$$

$$E_{T+osc}(C=C) = \left\{ \left(\frac{-2e^2}{8\pi\epsilon_0 c'} \left[(0.85252) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] + 2E_T(C=C, 2sp^3) \right) \right. \\ \left. - 2 \left((31.63536831\ eV) \sqrt{\frac{2\hbar \sqrt{(0.85252) \frac{1}{2} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\} \quad (14.491)$$

$$= -65.53833\ eV - 2 \left(0.35806\ eV - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)$$

From Eqs. (14.489-14.491), the total energy of the $C=C$ -bond MO is

$$\begin{aligned} E_{T+osc}(C=C) &= -63.27074\ eV + 2E_T(C=C, 2sp^3) + \bar{E}_{benzene\ osc}(C=C, \sigma) \\ &= -63.27074\ eV + 2(-1.13380\ eV) - 2 \left(0.35806\ eV - \frac{1}{2} (0.19649\ eV) \right) \\ &= -66.05796\ eV \end{aligned} \quad (14.492)$$

15

where the experimental E_{vib} was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

TOTAL BOND DISSOCIATION ENERGY OF BENZENE

Ethylene serves as a basis element for the $C=C$ bonding of benzene wherein each of the six

$C=C$ bonds of benzene comprises $(0.75)(4)=3$ electrons according to Eq. (14.416). The

total energy of the bonds of the eighteen electrons of the $C=C$ bonds of benzene,

5 $E_T(C_6H_6, C=C)$, is given by $(6)(0.75)$ times $E_{T+osc}(C=C)$ (Eq. (14.492)), the total energy of the $C=C$ -bond MO of benzene including the Doppler term, minus eighteen times $E(C, 2sp^3)$ (Eq. (14.146)), the initial energy of each $C2sp^3$ HO of each C that forms the double $C=C$ bonds. Thus, the total energy of the six $C=C$ bonds of benzene is

$$\begin{aligned} E_T(C_6H_6, C=C) &= (6)(0.75)E_{T+osc}(C=C) - 18E(C, 2sp^3) \\ &= (6)(0.75)(-66.05796 \text{ eV}) - 18(-14.63489 \text{ eV}) \\ &= -297.26081 \text{ eV} - (-263.42798 \text{ eV}) \\ &= -33.83284 \text{ eV} \end{aligned} \quad (14.493)$$

10 Each of the $C-H$ bonds of benzene comprises two electrons according to Eq. (14.439).

From the energy of each $C-H$ bond, $-E_{D_{benzene}}(^{12}CH)$ (Eq. (14.477)), the total energy of the twelve electrons of the six $C-H$ bonds of benzene, $E_T(C_6H_6, C-H)$, is given by

$$E_T(C_6H_6, C-H) = (6)(-E_{D_{benzene}}(^{12}CH)) = 6(-3.90454 \text{ eV}) = -23.42724 \text{ eV} \quad (14.494)$$

The total bond dissociation energy of benzene, $E_D(C_6H_6)$, is given by the negative sum of

15 $E_T(C_6H_6, C=C)$ (Eq. (14.493)) and $E_T(C_6H_6, C-H)$ (Eq. (14.494)):

$$\begin{aligned} E_D(C_6H_6) &= -\left(E_T(C_6H_6, C=C) + E_T(C_6H_6, C-H)\right) \\ &= -((-33.83284 \text{ eV}) + (-23.42724 \text{ eV})) \\ &= 57.2601 \text{ eV} \end{aligned} \quad (14.495)$$

The experimental total bond dissociation energy of benzene, $E_T(C_6H_6)$, is given by the negative difference between the enthalpy of its formation ($\Delta H_f(\text{benzene}(\text{gas}))$) and the sum of the enthalpy of the formation of the gaseous carbons ($\Delta H_f(C(\text{gas}))$) and hydrogen

20 ($\Delta H_f(H(\text{gas}))$) atoms. The heats of formation are [17-18]

$$\Delta H_f(\text{benzene}(\text{gas})) = 82.9 \text{ kJ/mole} \quad (0.8592 \text{ eV/molecule}) \quad (14.496)$$

403

$$\Delta H_f(C(gas)) = 716.68 \text{ kJ/mole } (7.42774 \text{ eV/molecule}) \quad (14.497)$$

$$\Delta H_f(H(gas)) = 217.998 \text{ kJ/mole } (2.259353 \text{ eV/molecule}) \quad (14.498)$$

Thus, the total bond dissociation energy of benzene, $E_D(C_6H_6)$, is

$$\begin{aligned} E_D(C_6H_6) - E_T(C_6H_6) &= -(\Delta H_f(\text{benzene}(gas)) - (6\Delta H_f(C(gas)) + 6\Delta H_f(H(gas)))) \\ &= -(0.8592 \text{ eV} - 6(7.42774 \text{ eV} + 2.259353 \text{ eV})) \\ &= 57.26 \text{ eV} \end{aligned}$$

5 (14.499)

where $E_T(C_6H_6)$ is the total energy of the bonds. The results of the determination of bond parameters of C_6H_6 are given in Table 14.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

10

CONTINUOUS-CHAIN ALKANES (C_nH_{2n+2} , $n = 3, 4, 5 \dots \infty$)

The continuous chain alkanes, C_nH_{2n+2} , are the homologous series comprising terminal methyl groups at each end of the chain with $n-2$ methylene (CH_2) groups in between:



- 15 C_nH_{2n+2} can be solved using the same principles as those used to solve ethane and ethylene wherein the $2s$ and $2p$ shells of each C hybridize to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ hybridized orbitals (HOs) to form a molecular orbital (MO) permits each participating hybridized orbital to decrease in radius and energy. Three H AOs combine with three carbon $2sp^3$ HOs and two H AOs combine
- 20 with two carbon $2sp^3$ HOs to form each methyl and methylene group, respectively, where each bond comprises a H_2 -type MO developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section. The CH_3 and CH_2 groups bond by forming H_2 -type MOs between the remaining $C2sp^3$ HOs on the carbons such that each carbon forms four bonds involving its four $C2sp^3$ HOs.

25

FORCE BALANCE OF THE C-C-BOND MOs OF CONTINUOUS-CHAIN ALKANES

C_nH_{2n+2} comprises a chemical bond between two terminal CH_3 radicals and $n-2$ CH_2 radicals wherein each methyl and methylene radical comprises three and two chemical bonds, respectively, between carbon and hydrogen atoms. The solution of the parameters of CH_3 is given in the Methyl Radical (CH_3) section. The solution of the parameters of CH_2 is given in the Dihydrogen Carbide Radical (CH_2) section and follows the same procedure. Each C-H bond having two spin-paired electrons, one from an initially unpaired electron of the carbon atom and the other from the hydrogen atom, comprises the linear combination of 75% H_2 -type ellipsoidal MO and 25% $C2sp^3$ HO as given by Eq. (13.429):



The proton of the H atom and the nucleus of the C atom are along each internuclear axis and serve as the foci. As in the case of H_2 , each of the C-H-bond MOs is a prolate spheroid with the exception that the ellipsoidal MO surface cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell since it is energetically unfavorable. Thus, each MO surface comprises a prolate spheroid at the H proton that is continuous with the $C2sp^3$ shell at the C atom whose nucleus serves as the other focus. The electron configuration and the energy, $E(C, 2sp^3)$, of the $C2sp^3$ shell is given by Eqs. (13.422) and (13.428), respectively. The central paramagnetic force due to spin of each C-H bond is provided by the spin-pairing force of the CH_3 or CH_2 MO that has the symmetry of an s orbital that superimposes with the $C2sp^3$ orbitals such that the corresponding angular momenta are unchanged. The energies of each CH_3 and CH_2 MO involve each $C2sp^3$ and each $H1s$ electron with the formation of each C-H bond. The sum of the energies of the H_2 -type ellipsoidal MOs is matched to that of the $C2sp^3$ shell. The force balance of the C-H-bond MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (14.139) and the energy matching condition between the $C2sp^3$ -HO components of the MO.

The CH_3 and CH_2 groups form $C-C$ bonds comprising H_2 -type MOs between the remaining $C2sp^3$ HOs on the carbons such that each carbon forms four bonds involving its four $C2sp^3$ HOs. The sharing of electrons between any two $C2sp^3$ HOs to form a molecular orbital (MO) comprising two spin-paired electrons permits each $C2sp^3$ HO to
 5 decrease in radius and energy. As in the case of the $C-H$ bonds, each $C-C$ -bond MO is a prolate-spheroidal-MO surface that cannot extend into $C2sp^3$ HO for distances shorter than the radius of the $C2sp^3$ shell of each atom. Thus, the MO surface comprises a partial prolate spheroid in between the carbon nuclei and is continuous with the $C2sp^3$ shell at each C atom. The energy of the H_2 -type ellipsoidal MO is matched to that of the $C2sp^3$ shell. As
 10 in the case of previous examples of energy-matched MOs such as the $C-C$ -bond MO of ethane, each $C-C$ -bond MO of C_nH_{2n+2} must comprise 75% of a H_2 -type ellipsoidal MO in order to match potential, kinetic, and orbital energy relationships. Thus, the $C-C$ -bond MO must comprise two $C2sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the two $C2sp^3$ HOs:

15



The linear combination of the H_2 -type ellipsoidal MO with each $C2sp^3$ HO further comprises an excess 25% charge-density contribution from each $C2sp^3$ HO to the $C-C$ -
 20 bond MO to achieve an energy minimum. The force balance of the $C-C$ -bond MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (14.502) and the energy matching condition between the $C2sp^3$ -HO components of the MO.

Before bonding, the $2sp^3$ hybridized orbital arrangement of each carbon atom is given by Eq. (14.140). The sum $E_T(C, 2sp^3)$ of calculated energies of C , C^+ , C^{2+} , and C^{3+} is given by
 25 Eq. (14.141). The radius r_{2sp^3} of the $C2sp^3$ shell is given by Eq. (14.142). The Coulombic energy $E_{Coulomb}(C, 2sp^3)$ and the energy $E(C, 2sp^3)$ of the outer electron of the $C2sp^3$ shell are given by Eqs. (14.143) and (14.146), respectively.

The formation of each $C-C$ bond of C_nH_{2n+2} further requires that the energy of all

H_2 -type prolate spheroidal MOs (σ MOs) be matched at all $C2sp^3$ HOs since they are continuous throughout the molecule. Thus, the energy of each $C2sp^3$ HO must be a linear combination of that of the CH_3 and CH_2 groups that serve as basis elements. Each CH_3 forms one $C-C$ bond, and each CH_2 group forms two. Thus, the energy of each $C2sp^3$ HO of each CH_3 and CH_2 group alone is given by that in ethane and ethylene, respectively. The parameters of ethane and ethylene are given by Eqs. (14.147-14.151) and (14.244-14.247), respectively. The alkane parameters can be determined by first reviewing those of ethane and ethylene.

With the formation of the $C-C$ -bond MO of ethane from two methyl radicals, each having a $C2sp^3$ electron with an energy given by Eq. (14.146), the total energy of the state is given by the sum over the four electrons. The sum $E_T(C_{ethane}, 2sp^3)$ of calculated energies of $C2sp^3$, C^+ , C^{2+} , and C^{3+} given by Eq. (14.147), is

$$\begin{aligned} E_T(C_{ethane}, 2sp^3) &= -(64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + E(C, 2sp^3)) \\ &= -(64.3921 \text{ eV} + 48.3125 \text{ eV} + 24.2762 \text{ eV} + 14.63489 \text{ eV}) \quad (14.503) \\ &= -151.61569 \text{ eV} \end{aligned}$$

where $E(C, 2sp^3)$ is the sum of the energy of C , -11.27671 eV , and the hybridization energy. The orbital-angular-momentum interactions also cancel such that the energy of the $E_T(C_{ethane}, 2sp^3)$ is purely Coulombic.

The sharing of electrons between two $C2sp^3$ HOs to form a $C-C$ -bond MO permits each participating hybridized orbital to decrease in radius and energy. In order to further satisfy the potential, kinetic, and orbital energy relationships, each $C2sp^3$ HO donates an excess of 25% of its electron density to the $C-C$ -bond MO to form an energy minimum. By considering this electron redistribution in the ethane molecule as well as the fact that the central field decreases by an integer for each successive electron of the shell, the radius $r_{ethane 2sp^3}$ of the $C2sp^3$ shell of ethane may be calculated from the Coulombic energy using Eq.

(10.102):

$$r_{ethane 2sp^3} = \left(\sum_{n=2}^5 (Z-n) - 0.25 \right) \frac{e^2}{8\pi\epsilon_0 (e151.61569 \text{ eV})} = \frac{9.75e^2}{8\pi\epsilon_0 (e151.61569 \text{ eV})} = 0.87495a_0 \quad (14.504)$$

Using Eqs. (10.102) and (14.504), the Coulombic energy $E_{Coulomb}(C_{ethane}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$E_{Coulomb}(C_{ethane}, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{ethane 2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.87495a_0} = -15.55033 \text{ eV} \quad (14.505)$$

During hybridization, one of the spin-paired $2s$ electrons is promoted to $C2sp^3$ shell as an unpaired electron. The energy for the promotion is the magnetic energy given by Eq. (14.145). Using Eqs. (14.145) and (14.505), the energy $E(C_{ethane}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$E(C_{ethane}, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{ethane 2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} = -15.55033 \text{ eV} + 0.19086 \text{ eV} = -15.35946 \text{ eV} \quad (14.506)$$

10 Thus, $E_T(C-C, 2sp^3)$, the energy change of each $C2sp^3$ shell with the formation of the $C-C$ -bond MO is given by the difference between Eq. (14.146) and Eq. (14.506):

$$E_T(C-C, 2sp^3) = E(C_{ethane}, 2sp^3) - E(C, 2sp^3) = -15.35946 \text{ eV} - (-14.63489 \text{ eV}) = -0.72457 \text{ eV} \quad (14.507)$$

Next, consider the formation of the $C=C$ -bond MO of ethylene from two CH_2 radicals, each having a $C2sp^3$ electron with an energy given by Eq. (14.146). The sum $E_T(C_{ethylene}, 2sp^3)$ of calculated energies of $C2sp^3$, C^+ , C^{2+} , and C^{3+} is given by Eq. (14.147). The sharing of electrons between two pairs of $C2sp^3$ HOs to form a $C=C$ -bond MO permits each participating HO to decrease in radius and energy. In order to further satisfy the potential, kinetic, and orbital energy relationships, each participating $C2sp^3$ HO donates an excess of 25% of its electron density to the $C=C$ -bond MO to form an energy minimum. By considering this electron redistribution in the ethylene molecule as well as the fact that the central field decreases by an integer for each successive electron of the shell, the radius $r_{ethylene 2sp^3}$ of the $C2sp^3$ shell of ethylene may be calculated from the Coulombic energy using Eqs. (10.102) and (14.147):

$$\begin{aligned}
 r_{ethylene 2sp^3} &= \left(\sum_{n=2}^5 (Z-n) - 0.5 \right) \frac{408 e^2}{8\pi\epsilon_0 (e151.61569 eV)} \\
 &= \frac{9.5e^2}{8\pi\epsilon_0 (e151.61569 eV)} \\
 &= 0.85252a_0
 \end{aligned} \tag{14.508}$$

where $Z = 6$ for carbon. Using Eqs. (10.102) and (14.508), the Coulombic energy $E_{Coulomb}(C_{ethylene}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$E_{Coulomb}(C_{ethylene}, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{ethylene 2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.85252a_0} = -15.95955 eV \tag{14.509}$$

5 During hybridization, one of the spin-paired $2s$ electrons is promoted to $C2sp^3$ shell as an unpaired electron. The energy for the promotion is the magnetic energy given by Eq. (14.145). Using Eqs. (14.145) and (14.509), the energy $E(C_{ethylene}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$E(C_{ethylene}, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{ethylene 2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} = -15.95955 eV + 0.19086 eV = -15.76868 eV \tag{14.510}$$

10 Thus, $E_T(C=C, 2sp^3)$, the energy change of each $C2sp^3$ shell with the formation of the $C=C$ -bond MO is given by the difference between Eq. (14.146) and Eq. (14.510):

$$\begin{aligned}
 E_T(C=C, 2sp^3) &= E(C_{ethylene}, 2sp^3) - E(C, 2sp^3) \\
 &= -15.76868 eV - (-14.63489 eV) \\
 &= -1.13380 eV
 \end{aligned} \tag{14.511}$$

To meet the energy matching condition for all σ MOs at all $C2sp^3$ HOs, the energy
15 $E(C_{alkane}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell of each alkane carbon atom must be the average of $E(C_{ethane}, 2sp^3)$ (Eq. (14.506)) and $E(C_{ethylene}, 2sp^3)$ (Eq. (14.510)):

$$\begin{aligned}
 E(C_{alkane}, 2sp^3) &= \frac{E(C_{ethane}, 2sp^3) + E(C_{ethylene}, 2sp^3)}{2} \\
 &= \frac{(-15.35946 eV) + (-15.76868 eV)}{2} \\
 &= -15.56407 eV
 \end{aligned} \tag{14.512}$$

And, $E_{T_{alkane}}(C-C, 2sp^3)$, the energy change of each $C2sp^3$ shell with the formation of each $C-C$ -bond MO, must be the average of $E_T(C-C, 2sp^3)$ (Eq. (14.507)) and $E_T(C=C, 2sp^3)$ (Eq. (14.511)):

$$\begin{aligned} E_{T_{alkane}}(C-C, 2sp^3) &= \frac{E_T(C-C, 2sp^3) + E_T(C=C, 2sp^3)}{2} \\ &= \frac{(-0.72457 \text{ eV}) + (-1.13379 \text{ eV})}{2} \\ &= -0.92918 \text{ eV} \end{aligned} \quad (14.513)$$

- 5 Using Eq. (10.102), the radius $r_{alkane 2sp^3}$ of the $C2sp^3$ shell of each carbon atom of C_nH_{2n+2} may be calculated from the Coulombic energy using the initial energy $E_{Coulomb}(C, 2sp^3) = -14.82575 \text{ eV}$ (Eq. (14.143)) and $E_{T_{alkane}}(C-C, 2sp^3)$ Eq. (14.513)), the energy change of each $C2sp^3$ shell with the formation of each $C-C$ -bond MO. Consider the case of a methyl carbon which donates $E_{T_{alkane}}(C-C, 2sp^3)$ Eq. (14.513)) to a single
- 10 $C-C$ bond:

$$\begin{aligned} r_{alkane 2sp^3} &= \frac{-e^2}{8\pi\epsilon_0 (E_{Coulomb}(C, 2sp^3) + E_{T_{alkane}}(C-C, 2sp^3))} \\ &= \frac{e^2}{8\pi\epsilon_0 (e14.825751 \text{ eV} + e0.92918 \text{ eV})} \\ &= 0.86359a_0 \end{aligned} \quad (14.514)$$

Using Eqs. (10.102) and (14.514), the Coulombic energy $E_{Coulomb}(C_{alkane}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$15 \quad E_{Coulomb}(C_{alkane}, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{alkane 2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.86359a_0} = -15.75493 \text{ eV} \quad (14.515)$$

During hybridization, one of the spin-paired $2s$ electrons is promoted to $C2sp^3$ shell as an unpaired electron. The energy for the promotion is the magnetic energy given by Eq. (14.145). Using Eqs. (14.145) and (14.515), the energy $E(C_{alkane}, 2sp^3)$ of the outer electron of the $C2sp^3$ shell is

$$20 \quad E(C_{alkane}, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{alkane 2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} = -15.75493 \text{ eV} + 0.19086 \text{ eV} = -15.56407 \text{ eV}$$

(14.516)

Thus, $E_{T_{alkane}}(C-C, 2sp^3)$, the energy change of each $C2sp^3$ shell with the formation of each $C-C$ bond MO is given by the difference between Eq. (14.146) and Eq. (14.516):

$$E_{T_{alkane}}(C-C, 2sp^3) = E(C_{alkane}, 2sp^3) - E(C, 2sp^3) = -15.56407 \text{ eV} - (-14.63489 \text{ eV}) = -0.92918 \text{ eV} \quad (14.517)$$

which agrees with Eq. (14.513).

The energy contribution due to the charge donation at each carbon superimposes linearly. In general, the radius $r_{mol 2sp^3}$ of the $C2sp^3$ HO of a carbon atom of a group of a given molecule is calculated using Eq. (14.514) by considering $\sum E_{T_{mol}}(MO, 2sp^3)$, the total energy donation to each bond with which it participates in bonding. The general equation for the radius is given by

$$\begin{aligned} r_{mol 2sp^3} &= \frac{-e^2}{8\pi\epsilon_0 \left(E_{Coulomb}(C, 2sp^3) + \sum E_{T_{mol}}(MO, 2sp^3) \right)} \\ &= \frac{e^2}{8\pi\epsilon_0 \left(e14.825751 \text{ eV} + \sum |E_{T_{mol}}(MO, 2sp^3)| \right)} \end{aligned} \quad (14.518)$$

The $C2sp^3$ HO of each methyl group of an alkane contributes -0.92918 eV to the corresponding single $C-C$ bond; thus, the corresponding $C2sp^3$ HO radius is given by Eq. (14.514). The $C2sp^3$ HO of each methylene group of C_nH_{2n+2} contributes -0.92918 eV to each of the two corresponding $C-C$ bond MOs. Thus, the radius of each methylene group of an alkane is given by

$$\begin{aligned} r_{alkane C_{methylene} 2sp^3} &= \frac{-e^2}{8\pi\epsilon_0 \left(E_{Coulomb}(C, 2sp^3) + \sum E_{T_{alkane}}(methylene C-C, 2sp^3) \right)} \\ &= \frac{e^2}{8\pi\epsilon_0 (e14.825751 \text{ eV} + e0.92918 \text{ eV} + e0.92918 \text{ eV})} \\ &= 0.81549a_0 \end{aligned} \quad (14.519)$$

As in the case with ethane, the H_2 -type ellipsoidal MO comprises 75% of the $C-C$ bond MO shared between two $C2sp^3$ HOs corresponding to the electron charge density in Eq. (11.65) of $\frac{-0.75e}{2}$. But, the additional 25% charge-density contribution to the $C-C$

bond MO causes the electron charge density in Eq. (11.65) to be given by $\frac{-e}{2} = -0.5e$.

Thus, the force constant k' to determine the ellipsoidal parameter c' in terms of the central force of the foci is given by Eq. (14.152). The distance from the origin of the $C-C$ -bond MO to each focus c' is given by Eq. (14.153). The internuclear distance from is given by Eq. (14.154). The length of the semiminor axis of the prolate spheroidal $C-C$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of the $C-C$ -bond MO. Since the $C-C$ -bond MO comprises a H_2 -type-ellipsoidal MO that transitions to the $C_{alkane}2sp^3$ HO of each carbon, the energy $E(C_{alkane}, 2sp^3)$ in Eq. (14.512) adds to that of the H_2 -type ellipsoidal MO to give the total energy of the $C-C$ -bond MO. From the energy equation and the relationship between the axes, the dimensions of the $C-C$ -bond MO are solved. Similarly, $E(C_{alkane}, 2sp^3)$ is added to the energy of the H_2 -type ellipsoidal MO of each $C-H$ bond of the methyl and methylene groups to give their total energy. From the energy equation and the relationship between the axes, the dimensions of the equivalent $C-H$ -bond MOs of the methyl and methylene groups in the alkane are solved:

The general equations for the energy components of V_e , V_p , T , V_m , and E_T of each $C-C$ -bond MO are the same as those of the CH MO except that energy of the $C_{alkane}2sp^3$ HO is used. The energy components at each carbon atom superimpose linearly and may be treated independently. Since each prolate spheroidal H_2 -type MO transitions to the $C_{alkane}2sp^3$ HO of each corresponding carbon of the bond and the energy of the $C_{alkane}2sp^3$ shell treated independently must remain constant and equal to the $E(C_{alkane}, 2sp^3)$ given by Eq. (14.512), the total energy $E_{T_{alkane}}(C-C, \sigma)$ of the σ component of each $C-C$ -bond MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the $C_{alkane}2sp^3$ HO and the H_2 -type ellipsoidal MO that forms the σ component of the $C-C$ -bond MO as given by Eq. (14.502) with the electron charge redistribution. The total number of $C-C$ bonds in C_nH_{2n+2} is $n-1$. Using Eqs. (13.431) and (14.512), $E_{T_{alkane}}(C-C, \sigma)$ of the $n-1$ bonds is given by

$$E_{T_{alkane}}(C-C, \sigma) = (n-1) \left(E_T + E(C_{alkane}, 2sp^3) \right) \\ = (n-1) \left(-\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} \right) \quad (14.520)$$

To match the boundary condition that the total energy of each H_2 -type ellipsoidal MO is given by Eqs. (11.212) and (13.75), $E_{T_{alkane}}(C-C, \sigma)$ given by Eq. (14.520) is set equal to $(n-1)$ times Eq. (13.75):

$$E_{T_{alkane}}(C-C, \sigma) = \\ 5 \quad (n-1) \left(-\frac{e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} \right) = (n-1)(-31.63536831 \text{ eV}) \quad (14.521)$$

From the energy relationship given by Eq. (14.521) and the relationship between the axes given by Eqs. (14.153-14.154) and (13.62-13.63), the dimensions of the $C-C$ -bond MO can be solved.

10 Substitution of Eq. (14.153) into Eq. (14.521) gives

$$\frac{e^2}{8\pi\epsilon_0 \sqrt{aa_0}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+\sqrt{aa_0}}{a-\sqrt{aa_0}} - 1 \right] = e16.07130 \quad (14.522)$$

The most convenient way to solve Eq. (14.522) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 2.12499a_0 = 1.12450 \times 10^{-10} \text{ m} \quad (14.523)$$

15 Substitution of Eq. (14.523) into Eq. (14.155) gives

$$c' = 1.45774a_0 = 7.71400 \times 10^{-11} \text{ m} \quad (14.524)$$

The internuclear distance given by multiplying Eq. (14.524) by two is

$$2c' = 2.91547a_0 = 1.54280 \times 10^{-10} \text{ m} \quad (14.525)$$

The experimental $C-C$ bond distance of propane is [3]

$$20 \quad 2c' = 1.532 \times 10^{-10} \text{ m} \quad (14.526)$$

The experimental $C-C$ bond distance of butane is [3]

$$2c' = 1.531 \times 10^{-10} \text{ m} \quad (14.527)$$

Substitution of Eqs. (14.523-14.524) into Eq. (13.62) gives

$$b = c = 1.54616a_0 = 8.18192 \times 10^{-11} \text{ m} \quad (14.528)$$

25 Substitution of Eqs. (14.523-14.524) into Eq. (13.63) gives

$$e = 0.68600 \quad (14.529)$$

The nucleus of the C atoms comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{alkane}2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270). The polar intersection angle θ' is given by Eq. (13.261) where for methylene bonds $r_n = r_{alkane2sp^3} = r_{methylene2sp^3} = 0.81549a_0$ is the radius of the $C_{alkane}2sp^3$ shell given by Eq. (14.519). Substitution of Eqs. (14.523-14.524) into Eq. (13.261) gives

$$\theta' = 56.41^\circ \quad (14.530)$$

Then, the angle $\theta_{C-C_{alkane}2sp^3HO}$ the radial vector of the $C2sp^3$ HO makes with the internuclear axis is

$$\theta_{C-C_{alkane}2sp^3HO} = 180^\circ - 56.41^\circ = 123.59^\circ \quad (14.531)$$

as shown in Figure 48.

15 Consider the right-hand intersection point. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C-C_{alkane}H_2MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C_{alkane}2sp^3$ radial vector obeys the following relationship:

$$r_{alkane2sp^3} \sin \theta_{C-C_{alkane}2sp^3HO} = 0.81549a_0 \sin \theta_{C-C_{alkane}2sp^3HO} = b \sin \theta_{C-C_{alkane}H_2MO} \quad (14.532)$$

20 such that

$$\theta_{C-C_{alkane}H_2MO} = \sin^{-1} \frac{0.81549a_0 \sin \theta_{C-C_{alkane}2sp^3HO}}{b} = \sin^{-1} \frac{0.81549a_0 \sin 123.59^\circ}{b} \quad (14.533)$$

with the use of Eq. (14.531). Substitution of Eq. (14.528) into Eq. (14.533) gives

$$\theta_{C-C_{alkane}H_2MO} = 26.06^\circ \quad (14.534)$$

Then, the distance $d_{C-C_{alkane}H_2MO}$ along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C-C_{alkane}H_2MO} = a \cos \theta_{C-C_{alkane}H_2MO} \quad (14.535)$$

Substitution of Eqs. (14.523) and (14.534) into Eq. (14.535) gives

$$d_{C-C_{alkane}H_2MO} = 1.90890a_0 = 1.01015 \times 10^{-10} \text{ m} \quad (14.536)$$

The distance $d_{C-C_{alkane} 2sp^3 HO}$ along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$d_{C-C_{alkane} 2sp^3 HO} = d_{C-C_{alkane}, H_2 MO} - c' \quad (14.537)$$

Substitution of Eqs. (14.524) and (14.536) into Eq. (14.537) gives

$$5 \quad d_{C-C_{alkane} 2sp^3 HO} = 0.45117a_0 = 2.38748 \times 10^{-11} \text{ m} \quad (14.538)$$

FORCE BALANCE OF THE CH_3 MOs OF CONTINUOUS-CHAIN ALKANES

Each of the two CH_3 MOs must comprise three equivalent C-H bonds with each
10 comprising 75% of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO as given by Eq. (13.540):



The force balance of the CH_3 MO is determined by the boundary conditions that arise from
15 the linear combination of orbitals according to Eq. (14.539) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-ellipsoidal-MO component of the CH_3 MO in terms of the central force of the foci is given by Eq. (13.59). The distance from the origin of each C-H-bond MO to each focus c' is
20 given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal C-H-bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of each C-H-bond MO. Since each of the three prolate spheroidal C-H-bond MOs comprises an H_2 -type-
25 ellipsoidal MO that transitions to the $C_{alkane} 2sp^3$ HO of $C_n H_{2n+2}$, the energy $E(C_{alkane}, 2sp^3)$ of Eq. (14.512) adds to that of the three corresponding H_2 -type ellipsoidal MOs to give the total energy of the CH_3 MO. From the energy equation and the relationship between the axes, the dimensions of the CH_3 MO are solved.

The energy components of V_e , V_p , T , and V_m are the same as those of methyl radical, three times those of CH corresponding to the three $C-H$ bonds except that energy of the $C_{alkane} 2sp^3$ HO is used. Since the each prolate spheroidal H_2 -type MO transitions to the $C_{alkane} 2sp^3$ HO and the energy of the $C_{alkane} 2sp^3$ shell must remain constant and equal to the $E(C_{alkane}, 2sp^3)$ given by Eq. (14.512), the total energy $E_{T_{alkane}}(CH_3)$ of the CH_3 MO is given by the sum of the energies of the orbitals corresponding to the composition of the linear combination of the $C_{alkane} 2sp^3$ HO and the three H_2 -type ellipsoidal MOs that forms the CH_3 MO as given by Eq. (14.539). Using Eq. (13.431) or Eq. (13.541), $E_{T_{alkane}}(CH_3)$ is given by

$$E_{T_{alkane}}(CH_3) = E_T + E(C_{alkane}, 2sp^3) \\ = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} \quad (14.540)$$

$E_{T_{alkane}}(CH_3)$ given by Eq. (14.540) is set equal to three times the energy of the H_2 -type ellipsoidal MO minus two times the Coulombic energy of H given by Eq. (13.542):

$$E_T(CH_3) = -\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} = -67.69450 \text{ eV} \quad (14.541)$$

From the energy relationship given by Eq. (14.541) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH_3 MO can be solved.

Substitution of Eq. (13.60) into Eq. (14.541) gives

$$\frac{3e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e52.13044 \quad (14.542)$$

The most convenient way to solve Eq. (14.542) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.64920a_0 = 8.72720 \times 10^{-11} \text{ m} \quad (14.543)$$

Substitution of Eq. (14.543) into Eq. (14.60) gives

$$c' = 1.04856a_0 = 5.54872 \times 10^{-11} \text{ m} \quad (14.544)$$

The internuclear distance given by multiplying Eq. (14.544) by two is

$$2c' = 2.09711a_0 = 1.10974 \times 10^{-10} \text{ m} \quad (14.545)$$

The experimental $C-H$ bond distance of propane is [3]

$$2c' = 1.107 \times 10^{-10} \text{ m} \quad (14.546)$$

Substitution of Eqs. (14.543-14.544) into Eq. (14.62) gives

$$b = c = 1.27295a_0 = 6.73616 \times 10^{-11} \text{ m} \quad (14.547)$$

5 Substitution of Eqs. (14.543-14.544) into Eq. (14.63) gives

$$e = 0.63580 \quad (14.548)$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{alkane} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270).

10 The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{alkane 2sp^3} = 0.86359a_0$ is the radius of the $C_{alkane} 2sp^3$ shell. Substitution of Eqs. (14.543-14.544) into Eq. (13.261) gives

$$\theta' = 77.49^\circ \quad (14.549)$$

Then, the angle $\theta_{C-H_{alkane} 2sp^3 HO}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear

15 axis is

$$\theta_{C-H_{alkane} 2sp^3 HO} = 180^\circ - 77.49^\circ = 102.51^\circ \quad (14.550)$$

as shown in Figure 49.

20 The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C-H_{alkane} H_2 MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C_{alkane} 2sp^3$ radial vector obeys the following relationship:

$$r_{alkane 2sp^3} \sin \theta_{C-H_{alkane} 2sp^3 HO} = 0.86359a_0 \sin \theta_{C-H_{alkane} 2sp^3 HO} = b \sin \theta_{C-H_{alkane} H_2 MO} \quad (14.551)$$

25 such that

$$\theta_{C-H_{alkane} H_2 MO} = \sin^{-1} \frac{0.86359a_0 \sin \theta_{C-H_{alkane} 2sp^3 HO}}{b} = \sin^{-1} \frac{0.86359a_0 \sin 102.51^\circ}{b} \quad (14.552)$$

with the use of Eq. (14.550). Substitution of Eq. (14.547) into Eq. (14.552) gives

$$\theta_{C-H_{alkane} H_2 MO} = 41.48^\circ \quad (14.553)$$

Then, the distance d_{C-H_{alkane}, H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C-H_{alkane}, H_2MO} = a \cos \theta_{C-H_{alkane}, H_2MO} \quad (14.554)$$

Substitution of Eqs. (14.543) and (14.553) into Eq. (14.554) gives

$$5 \quad d_{C-H_{alkane}, H_2MO} = 1.23564a_0 = 6.53871 \times 10^{-11} \text{ m} \quad (14.555)$$

The distance $d_{C-H_{alkane} 2sp^3HO}$ along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$d_{C-H_{alkane} 2sp^3HO} = d_{C-H_{alkane}, H_2MO} - c' \quad (14.556)$$

Substitution of Eqs. (14.544) and (14.555) into Eq. (14.556) gives

$$10 \quad d_{C-H_{alkane} 2sp^3HO} = 0.18708a_0 = 9.89999 \times 10^{-12} \text{ m} \quad (14.557)$$

BOND ANGLE OF THE CH_3 AND CH_2 GROUPS

Each CH_3 MO comprises a linear combination of three $C-H$ -bond MOs. Each $C-H$ -bond MO comprises the superposition of a H_2 -type ellipsoidal MO and the $C_{alkane} 2sp^3$ HO.

- 15 A bond is also possible between the two H atoms of the $C-H$ bonds. Such $H-H$ bonding would decrease the $C-H$ bond strength since electron density would be shifted from the $C-H$ bonds to the $H-H$ bond. Thus, the bond angle between the two $C-H$ bonds is determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal H atoms of the $C-H$ bonds is zero. From Eqs. (11.79) and (13.228), the distance
- 20 from the origin to each focus of the $H-H$ ellipsoidal MO is

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{aa_0}{2}} \quad (14.558)$$

The internuclear distance from Eq. (14.558) is

$$2c' = 2\sqrt{\frac{aa_0}{2}} \quad (14.559)$$

- The length of the semiminor axis of the prolate spheroidal $H-H$ MO $b=c$ is given by Eq.
- 25 (14.62).

The bond angle of the CH_3 groups of C_nH_{2n+2} is derived by using the orbital composition and an energy matching factor as in the case with the CH_3 radical. Since each pair of H_2 -type ellipsoidal MOs initially comprise 75% of the H electron density of H_2 and

the energy of each H_2 -type ellipsoidal MO is matched to that of the $C_{alkane} 2sp^3$ HO, the component energies and the total energy E_T of the $H-H$ bond are given by Eqs. (13.67-13.73) except that V_e , T , and V_m are corrected for the hybridization-energy-matching factor of 0.86359. Hybridization with 25% electron donation to the $C-C$ -bond gives rise to the

5 $C_{alkane} 2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C_{alkane}, 2sp^3)$ given by Eq. (14.515). The corresponding normalization factor for determining the zero of the total $H-H$ bond energy is given by the ratio of 15.75493 eV, the magnitude of $E_{Coulomb}(C_{alkane}, 2sp^3)$ given by Eq. (14.515), and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). The hybridization energy factor $C_{alkaneC2sp^3HO}$ is

$$10 \quad C_{alkaneC2sp^3HO} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{alkane2sp^3}}} = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 0.86359 a_0}} = \frac{13.605804 \text{ eV}}{15.75493 \text{ eV}} = 0.86359 \quad (14.560)$$

Substitution of Eq. (14.558) into Eq. (13.233) with the hybridization factor of 0.86359 gives

$$0 = \left[\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2}}} \left[(0.86359)^{-1} \left(\frac{3}{2} - \frac{3}{8} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2}}}{a - \sqrt{\frac{aa_0}{2}}} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 a^3}}}}{m_e c^2} \right] \right. \\ \left. + \hbar \sqrt{\frac{\frac{0.75e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 (a+c')^3}}{0.5m_p}} \right] \quad (14.561)$$

15 From the energy relationship given by Eq. (14.561) and the relationship between the axes given by Eqs. (14.558-14.559) and (14.62-14.63), the dimensions of the $H-H$ MO can be solved.

The most convenient way to solve Eq. (14.561) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$20 \quad a = 5.8660a_0 = 3.1042 \times 10^{-10} \text{ m} \quad (14.562)$$

Substitution of Eq. (14.562) into Eq. (14.558) gives

$$c' = 1.7126a_0 = 9.0627 \times 10^{-11} \text{ m} \quad (14.563)$$

The internuclear distance given by multiplying Eq. (14.563) by two is

$$2c' = 3.4252a_0 = 1.8125 \times 10^{-10} \text{ m} \quad (14.564)$$

Substitution of Eqs. (14.562-14.563) into Eq. (14.62) gives

$$b = c = 5.6104a_0 = 2.9689 \times 10^{-10} \text{ m} \quad (14.565)$$

5 Substitution of Eqs. (14.562-14.563) into Eq. (14.63) gives

$$e = 0.2920 \quad (14.566)$$

Using $2c'_{H-H}$ (Eq. (14.564)), the distance between the two H atoms when the total energy of the corresponding MO is zero (Eq. (14.561)), and $2c'_{C-H}$, the internuclear distance of each $C-H$ bond, the corresponding bond angle can be determined from the law of
10 cosines. Since the internuclear distance of each $C-H$ bond of CH_3 (Eq. (14.545)) and CH_2 (Eq. (14.597)) are sufficiently equivalent, the bond angle determined with either is within experimental error of being the same. Using, Eqs. (13.242), (14.545), and (14.564), the bond angle θ between the $C-H$ bonds is

$$\theta = \cos^{-1} \left(\frac{2(2.09711)^2 - (3.4252)^2}{2(2.09711)^2} \right) = \cos^{-1}(-0.33383) = 109.50^\circ \quad (14.567)$$

15 The experimental angle between the $C-H$ bonds is [19]

$$\theta = 109.3^\circ \quad (14.568)$$

The CH_3 radical has a pyramidal structure with the carbon atom along the z-axis at the apex and the hydrogen atoms at the base in the xy-plane. The distance $d_{origin-H}$ from the origin to the nucleus of a hydrogen atom given by Eqs. (14.564) and (13.412) is

$$20 \quad d_{origin-H} = 1.97754a_0 \quad (14.569)$$

The height along the z-axis of the pyramid from the origin to C nucleus d_{height} given by Eqs. (13.414), (14.545), and (14.569) is

$$d_{height} = 0.69800a_0 \quad (14.570)$$

The angle θ_v of each $C-H$ bond from the z-axis given by Eqs. (13.416), (14.569), and

25 (14.570) is

$$\theta_v = 70.56^\circ \quad (14.571)$$

The $C-C$ bond is along the z-axis. Thus, the bond angle θ_{C-C-H} between the internuclear axis of the $C-C$ bond and a H atom of the methyl groups is given by

$$\theta_{C-C-H} = 180 - \theta_v \quad (14.572)$$

Substitution of Eq. (14.571) into Eq. (14.572) gives

$$\theta_{C-C-H} = 109.44^\circ \quad (14.573)$$

The experimental angle between the $C-C-H$ bonds is [19]

$$\theta_{C-C-H} = 109.3^\circ \quad (14.574)$$

- 5 The C_nH_{2n+2} MOs shown in Figures 50-60 were rendered using these parameters. A minimum energy is obtained with a staggered configuration consistent with observations [3].

ENERGIES OF THE CH_3 GROUPS

The energies of each CH_3 group of C_nH_{2n+2} are given by the substitution of the
10 semiprincipal axes (Eqs. (14.543-14.544) and (14.547)) into the energy equations of methyl radical (Eqs. (13.556-13.560)), with the exception that $E(C_{alkane}, 2sp^3)$ (Eq. (14.514)) replaces $E(C, 2sp^3)$ in Eq. (13.560):

$$V_e = 3(0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -107.32728 \text{ eV} \quad (14.575)$$

$$V_p = \frac{3e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 38.92728 \text{ eV} \quad (14.576)$$

$$15 \quad T = 3(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 32.53914 \text{ eV} \quad (14.577)$$

$$V_m = 3(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -16.26957 \text{ eV} \quad (14.578)$$

$$E_{T_{alkane}}(CH_3) = \left(-\frac{3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) = -67.69451 \text{ eV} \quad (14.579)$$

where $E_{T_{alkane}}(CH_3)$ is given by Eq. (14.540) which is reiteratively matched to Eq. (13.542) within five-significant-figure round off error.

20

VIBRATION OF THE $^{12}CH_3$ GROUPS

The vibrational energy levels of the $C-H$ bonds of CH_3 in C_nH_{2n+2} may be solved as three equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived

from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERMS OF THE $^{12}\text{CH}_3$ GROUPS

- 5 The equations of the radiation reaction force of the methyl groups in $\text{C}_n\text{H}_{2n+2}$ are the same as those of the methyl radical with the substitution of the methyl-group parameters. Using Eq. (13.561), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 2.49286 \times 10^{16} \text{ rad/s} \quad (14.580)$$

where b is given by Eq. (14.547). The kinetic energy, E_K , is given by Planck's equation
10 (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.49286 \times 10^{16} \text{ rad/s} = 16.40846 \text{ eV} \quad (14.581)$$

In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.581) for \bar{E}_K gives the Doppler energy of the electrons of each of the
15 three bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(16.40846 \text{ eV})}{m_e c^2}} = -0.25352 \text{ eV} \quad (14.582)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH_3 due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic
20 oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.582) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of each $\text{C}-\text{H}$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition state having three independent bonds, $\bar{E}'_{alkane\ osc} (^{12}\text{CH}_3)$ per bond is

$$\bar{E}'_{alkane\ osc} (^{12}\text{CH}_3) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.583)$$

$$25 \quad \bar{E}'_{alkane\ osc} (^{12}\text{CH}_3) = -0.25352 \text{ eV} + \frac{1}{2} (0.35532 \text{ eV}) = -0.07586 \text{ eV} \quad (14.584)$$

Given that the vibration and reentrant oscillation is for three $C-H$ bonds, $\bar{E}_{alkane\ osc} (^{12}CH_3)$, is:

$$\begin{aligned}\bar{E}_{alkane\ osc} (^{12}CH_3) &= 3 \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\ &= 3 \left(-0.25352\ eV + \frac{1}{2} (0.35532\ eV) \right) \\ &= -0.22757\ eV\end{aligned}\quad (14.585)$$

5 TOTAL BOND ENERGIES OF THE $^{12}CH_3$ GROUPS

$E_{alkaneT+osc} (^{12}CH_3)$, the total energy of each $^{12}CH_3$ group including the Doppler term, is given by the sum of $E_{T_{alkane}} (CH_3)$ (Eq. (14.579)) and $\bar{E}_{alkane\ osc} (^{12}CH_3)$ given by Eq. (14.585):

$$\begin{aligned}E_{alkaneT+osc} (CH_3) &= V_e + T + V_m + V_p + E(C_{alkane}, 2sp^3) + \bar{E}_{alkane\ osc} (^{12}CH_3) \\ &= E_{T_{alkane}} (CH_3) + \bar{E}_{alkane\ osc} (^{12}CH_3)\end{aligned}\quad (14.586)$$

$$\begin{aligned}E_{alkaneT+osc} (^{12}CH_3) &= \left\{ \left(\frac{-3e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407\ eV \right) \right. \\ &\quad \left. - 3 \left((31.63536831\ eV) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}} \right) \right\} \\ &= -67.69450\ eV - 3 \left(0.25352\ eV - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)\end{aligned}\quad (14.587)$$

From Eqs. (14.585-14.587), the total energy of each $^{12}CH_3$ is

$$\begin{aligned}E_{alkaneT+osc} (^{12}CH_3) &= -67.69450\ eV + \bar{E}_{alkane\ osc} (^{12}CH_3) \\ &= -67.69450\ eV - 3 \left(0.25352\ eV - \frac{1}{2} (0.35532\ eV) \right) \\ &= -67.92207\ eV\end{aligned}\quad (14.588)$$

where ω_e given by Eq. (13.458) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

The total CH_3 bond dissociation energy, $E_{D_{alkane}}(^{12}CH_3)$ is given by the sum of the initial $C2sp^3$ HO energy, $E(C, 2sp^3)$ (Eq. (14.146)), and three times the energy of the hydrogen atom, $E_D(H)$ (Eq. (13.154)), minus $E_{alkaneT+osc}(^{12}CH_3)$ (Eq. (14.588)):

$$E_{D_{alkane}}(^{12}CH_3) = E(C, 2sp^3) + 3E(H) - E_{alkaneT+osc}(^{12}CH_3) \quad (14.589)$$

5 Thus, the total $^{12}CH_3$ bond dissociation energy, $E_{D_{alkane}}(^{12}CH_3)$ is

$$\begin{aligned} E_{D_{alkane}}(^{12}CH_3) &= -(14.63489 \text{ eV} + 3(13.59844 \text{ eV})) - (E_{alkaneT+osc}(^{12}CH_2)) \\ &= -55.43021 \text{ eV} - (-67.92207 \text{ eV}) \\ &= 12.49186 \text{ eV} \end{aligned} \quad (14.590)$$

FORCE BALANCE OF THE CH_2 MOs OF CONTINUOUS-CHAIN ALKANES

10 Each of the CH_2 MOs must comprise two equivalent $C-H$ bonds with each comprising 75% of a H_2 -type ellipsoidal MO and a $C2sp^3$ HO as given by Eq. (13.494):

$$2[1 C2sp^3 + 0.75 H_2 MO] \rightarrow CH_2 MO \quad (14.591)$$

15 The force balance of each CH_2 MO is determined by the boundary conditions that arise from the linear combination of orbitals according to Eq. (14.591) and the energy matching condition between the hydrogen and $C2sp^3$ HO components of the MO.

The force constant k' to determine the ellipsoidal parameter c' of the each H_2 -type-ellipsoidal-MO component of the CH_2 MO in terms of the central force of the foci is given
20 by Eq. (13.59). The distance from the origin of each $C-H$ -bond MO to each focus c' is given by Eq. (13.60). The internuclear distance is given by Eq. (13.61). The length of the semiminor axis of the prolate spheroidal $C-H$ -bond MO $b = c$ is given by Eq. (13.62). The eccentricity, e , is given by Eq. (13.63). The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of each $C-H$ -bond
25 MO. Since each of the two prolate spheroidal $C-H$ -bond MOs comprises an H_2 -type-ellipsoidal MO that transitions to the $C_{alkane}2sp^3$ HO of C_nH_{2n+2} , the energy $E(C_{alkane}, 2sp^3)$

of Eq. (14.512) adds to that of the two corresponding H_2 -type ellipsoidal MOs to give the total energy of the CH_2 MO. From the energy equation and the relationship between the axes, the dimensions of the CH_2 MO are solved.

The energy components of V_e , V_p , T , and V_m are the same as those of dihydrogen
5 carbide radical, two times those of CH corresponding to the two $C-H$ bonds except that energy of the $C_{alkane} 2sp^3$ HO is used. Since the each prolate spheroidal H_2 -type MO transitions to the $C_{alkane} 2sp^3$ HO and the energy of the $C_{alkane} 2sp^3$ shell treated independently must remain constant and equal to the $E(C_{alkane}, 2sp^3)$ given by Eq. (14.512), the total energy $E_{T_{alkane}}(CH_2)$ of the CH_2 MO is given by the sum of the energies of the orbitals
10 corresponding to the composition of the linear combination of the $C_{alkane} 2sp^3$ HO and the two H_2 -type ellipsoidal MOs that forms the CH_2 MO as given by Eq. (14.591). Using Eq. (13.431) or Eq. (13.495), $E_{T_{alkane}}(CH_2)$ is given by

$$\begin{aligned} E_{T_{alkane}}(CH_2) &= E_T + E(C_{alkane}, 2sp^3) \\ &= -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} \end{aligned} \quad (14.592)$$

$E_{T_{alkane}}(CH_2)$ given by Eq. (14.592) is set equal to two times the energy of the H_2 -type
15 ellipsoidal MO minus the Coulombic energy of H given by Eq. (13.496):

$$E_T(CH_2) = -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} = -49.66493 \text{ eV} \quad (14.593)$$

From the energy relationship given by Eq. (14.593) and the relationship between the axes given by Eqs. (13.60-13.63), the dimensions of the CH_2 MO can be solved.

20 Substitution of Eq. (13.60) into Eq. (14.593) gives

$$\frac{2e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right] = e34.10086 \quad (14.594)$$

The most convenient way to solve Eq. (14.594) is by the reiterative technique using a computer. The result to within the round-off error with five-significant figures is

$$a = 1.67122a_0 = 8.84370 \times 10^{-11} \text{ m} \quad (14.595)$$

Substitution of Eq. (14.595) into Eq. (14.60) gives

$$c' = 1.05553a_0 = 5.58563 \times 10^{-11} \text{ m} \quad (14.596)$$

The internuclear distance given by multiplying Eq. (14.596) by two is

$$2c' = 2.11106a_0 = 1.11713 \times 10^{-10} \text{ m} \quad (14.597)$$

5 The experimental $C-H$ bond distance of butane is [3]

$$2c' = 1.117 \times 10^{-10} \text{ m} \quad (14.598)$$

Substitution of Eqs. (14.595-14.596) into Eq. (14.62) gives

$$b = c = 1.29569a_0 = 6.85652 \times 10^{-11} \text{ m} \quad (14.599)$$

Substitution of Eqs. (14.595-14.596) into Eq. (14.63) gives

$$10 \quad e = 0.63159 \quad (14.600)$$

The nucleus of the H atom and the nucleus of the C atom comprise the foci of each H_2 -type ellipsoidal MO. The parameters of the point of intersection of the H_2 -type ellipsoidal MO and the $C_{alkane} 2sp^3$ HO are given by Eqs. (13.84-13.95) and (13.261-13.270).

The polar intersection angle θ' is given by Eq. (13.261) where $r_n = r_{methylene 2sp^3} = 0.81549a_0$ is

15 the radius of the $C_{methylene} 2sp^3$ shell (Eq. (14.521)). Substitution of Eqs. (14.595-14.596) into Eq. (13.261) gives

$$\theta' = 68.47^\circ \quad (14.601)$$

Then, the angle $\theta_{C-H_{alkane} 2sp^3 HO}$ the radial vector of the $C 2sp^3$ HO makes with the internuclear axis is

$$20 \quad \theta_{C-H_{alkane} 2sp^3 HO} = 180^\circ - 68.47^\circ = 111.53^\circ \quad (14.602)$$

as shown in Figure 49. The distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals. Thus, the angle $\omega t = \theta_{C-H_{alkane}, H_2 MO}$ between the internuclear axis and the point of intersection of the H_2 -type ellipsoidal MO with the $C_{alkane} 2sp^3$ radial vector obeys the following relationship:

$$25 \quad r_{alkane 2sp^3} \sin \theta_{C-H_{alkane} 2sp^3 HO} = 0.81549a_0 \sin \theta_{C-H_{alkane} 2sp^3 HO} = b \sin \theta_{C-H_{alkane}, H_2 MO} \quad (14.603)$$

such that

$$\theta_{C-H_{alkane}, H_2 MO} = \sin^{-1} \frac{0.81549a_0 \sin \theta_{C-H_{alkane} 2sp^3 HO}}{b} = \sin^{-1} \frac{0.81549a_0 \sin 111.53^\circ}{b} \quad (14.604)$$

with the use of Eq. (14.602). Substitution of Eq. (14.599) into Eq. (14.604) gives

$$\theta_{C-H_{alkane}, H_2 MO} = 35.84^\circ \quad (14.605)$$

Then, the distance d_{C-H_{alkane}, H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{C-H_{alkane}, H_2MO} = a \cos \theta_{C-H_{alkane}, H_2MO} \quad (14.606)$$

Substitution of Eqs. (14.595) and (14.605) into Eq. (14.606) gives

$$5 \quad d_{C-H_{alkane}, H_2MO} = 1.35486a_0 = 7.16963 \times 10^{-11} \text{ m} \quad (14.607)$$

The distance $d_{C-H_{alkane} 2sp^3HO}$ along the internuclear axis from the origin of the C atom to the point of intersection of the orbitals is given by

$$d_{C-H_{alkane} 2sp^3HO} = d_{C-H_{alkane}, H_2MO} - c' \quad (14.608)$$

Substitution of Eqs. (14.596) and (14.605) into Eq. (14.608) gives

$$10 \quad d_{C-H_{alkane} 2sp^3HO} = 0.29933a_0 = 1.58400 \times 10^{-11} \text{ m} \quad (14.609)$$

The charge-density in each $C-C$ -bond MO is increased by a factor of 0.25 with the formation of the $C_{alkane} 2sp^3$ HOs each having a smaller radius. Using the orbital composition of the $C-C$ -bond MOs (Eq. (14.504), CH_3 groups (Eq. (14.539)), and the CH_2 groups (Eq. (14.591)), the radii of $C1s = 0.17113a_0$ (Eq. (10.51)), $C_{alkane} 2sp^3 = 0.86359a_0$ (Eq. (14.514)),
 15 and $C_{alkane} 2sp^3 = C_{methylene} 2sp^3 = 0.81549a_0$ (Eq. (14.521)) shells, the parameters of the $C-C$ -bonds (Eqs. (13.3-13.4), (14.523-14.525), and (14.528-14.538)), the parameters of the $C-H$ -bond MOs of the CH_3 groups (Eqs. (13.3-13.4), (14.544-14.545), and (14.547-14.557)), the parameters of the $C-H$ -bond MOs of the CH_2 groups (Eqs. (13.3-13.4), (14.595-14.597), and (14.599-14.609)), and the bond-angle parameters (Eqs. (14.562-
 20 14.574)), the charge-density of the C_nH_{2n+2} MO comprising the linear combination $2n+2$ $C-H$ -bond MOs and $n-1$ $C-C$ -bond MOs, each bridging one or more methyl or methylene groups is shown for representative cases where data was available [17-18]. Propane, butane, pentane, hexane, heptane, octane, nonane, decane, undecane, dodecane, and octadecane are shown in Figures 50-60, respectively. Each $C-H$ -bond MO comprises a
 25 H_2 -type ellipsoidal MO and a $C_{alkane} 2sp^3$ HO having the dimensional diagram shown in Figure 48. Each $C-C$ -bond MO comprises a H_2 -type ellipsoidal MO bridging two $C_{alkane} 2sp^3$ HOs having the dimensional diagram shown in Figure 49.

ENERGIES OF THE CH_2 GROUPS

The energies of each CH_2 group of C_nH_{2n+2} are given by the substitution of the semiprincipal axes (Eqs. (14.595-14.596) and (14.599)) into the energy equations of dihydrogen carbide radical (Eqs. (13.510-13.514)), with the exception that $E(C_{alkane}, 2sp^3)$

5 (Eq. (14.512)) replaces $E(C, 2sp^3)$ in Eq. (13.514):

$$V_s = 2(0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -70.41425 \text{ eV} \quad (14.610)$$

$$V_p = \frac{2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 25.78002 \text{ eV} \quad (14.611)$$

$$T = 2(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 21.06675 \text{ eV} \quad (14.612)$$

$$V_m = 2(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -10.53337 \text{ eV} \quad (14.613)$$

$$10 \quad E_{T_{alkane}}(CH_2) = -\frac{2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} = -49.66493 \text{ eV} \quad (14.614)$$

where $E_{T_{alkane}}(CH_2)$ is given by Eq. (14.592) which is reiteratively matched to Eq. (13.496) within five-significant-figure round off error.

15 VIBRATION OF THE $^{12}CH_2$ GROUPS

The vibrational energy levels of the $C-H$ bonds of CH_2 in C_nH_{2n+2} may be solved as two equivalent coupled harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section 20 and the Vibration of Hydrogen-Type Molecules section.

THE DOPPLER ENERGY TERMS OF THE $^{12}CH_2$ GROUPS

The equations of the radiation reaction force of the methylene groups in C_nH_{2n+2} are the same as those of the dihydrogen carbide radical with the substitution of the methylene-group

parameters. Using Eq. (13.515), the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.75e^2}{4\pi\epsilon_0 b^3}} = 2.42751 \times 10^{16} \text{ rad/s} \quad (14.615)$$

where b is given by Eq. (14.599). The kinetic energy, E_K , is given by Planck's equation 5 (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 2.42751 \times 10^{16} \text{ rad/s} = 15.97831 \text{ eV} \quad (14.616)$$

In Eq. (11.181), substitution of $E_T(H_2)$ (Eqs. (11.212) and (13.75)), the maximum total energy of each H_2 -type MO, for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.616) for \bar{E}_K gives the Doppler energy of the electrons of each of the 10 three bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -31.63537 \text{ eV} \sqrt{\frac{2e(15.97831 \text{ eV})}{m_e c^2}} = -0.25017 \text{ eV} \quad (14.617)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of CH_2 due to the reentrant orbit of each bond in the transition state corresponding to simple harmonic 15 oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.617) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of each $C-H$ bond. Using ω_e given by Eq. (13.458) for \bar{E}_{Kvib} of the transition state having two independent bonds, $\bar{E}'_{alkane\ osc}({}^{12}CH_2)$ per bond is

$$\bar{E}'_{alkane\ osc}({}^{12}CH_2) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}} \quad (14.618)$$

$$20 \quad \bar{E}'_{alkane\ osc}({}^{12}CH_2) = -0.25017 \text{ eV} + \frac{1}{2}(0.35532 \text{ eV}) = -0.07251 \text{ eV} \quad (14.619)$$

Given that the vibration and reentrant oscillation is for two $C-H$ bonds, $\bar{E}_{alkane\ osc}({}^{12}CH_2)$, is:

$$\bar{E}_{alkane\ osc}({}^{12}CH_2) = 2\left(\bar{E}_D + \frac{1}{2}\hbar\sqrt{\frac{k}{\mu}}\right) = 2\left(-0.25017 \text{ eV} + \frac{1}{2}(0.35532 \text{ eV})\right) = -0.14502 \text{ eV}$$

TOTAL BOND ENERGIES OF THE $^{12}\text{CH}_2$ GROUPS

$E_{\text{alkane}T+\text{osc}}(^{12}\text{CH}_2)$, the total energy of each $^{12}\text{CH}_2$ group including the Doppler term, is given
 5 by the sum of $E_{T_{\text{alkane}}}(^1\text{CH}_2)$ (Eq. (14.614)) and $\bar{E}_{\text{alkane osc}}(^{12}\text{CH}_2)$ given by Eq. (14.620):

$$\begin{aligned}
 E_{\text{alkane}T+\text{osc}}(^1\text{CH}_2) &= V_e + T + V_m + V_p + E(C_{\text{alkane}}, 2sp^3) + \bar{E}_{\text{alkane osc}}(^{12}\text{CH}_2) \\
 &= E_{T_{\text{alkane}}} (^1\text{CH}_2) + \bar{E}_{\text{alkane osc}} (^{12}\text{CH}_2) \\
 E_{\text{alkane}T+\text{osc}} (^{12}\text{CH}_2) &= \left\{ \left(\frac{-2e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} \right) \right. \\
 &\quad \left. - 2 \left((31.63536831 \text{ eV}) \sqrt{\frac{2\hbar \sqrt{\frac{3}{4} \frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \right\} \\
 &= -49.66493 \text{ eV} - 2 \left(0.25017 \text{ eV} - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)
 \end{aligned}
 \tag{14.621}$$

(14.622)

From Eqs. (14.620-14.622), the total energy of each $^{12}\text{CH}_2$ is

$$\begin{aligned}
 E_{\text{alkane}T+\text{osc}} (^{12}\text{CH}_2) &= -49.66493 \text{ eV} + \bar{E}_{\text{alkane osc}} (^{12}\text{CH}_2) \\
 &= -49.66493 \text{ eV} - 2 \left(0.25017 \text{ eV} - \frac{1}{2} (0.35532 \text{ eV}) \right) \\
 &= -49.80996 \text{ eV}
 \end{aligned}
 \tag{14.623}$$

where ω_e given by Eq. (13.458) was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

The derivation of the total CH_2 bond dissociation energy, $E_{D_{\text{alkane}}} (^{12}\text{CH}_2)$ follows from that of the bond dissociation energy of dihydrogen carbide radical, $E_D (^{12}\text{CH}_2)$, given by Eqs. (13.524-13.527). $E_{D_{\text{alkane}}} (^{12}\text{CH}_2)$ is given by the sum of the initial $\text{C}2sp^3$ HO energy,
 15 $E(C, 2sp^3)$ (Eq. (14.146)), and two times the energy of the hydrogen atom, $E(H)$ (Eq. (13.154)), minus the sum of $E_{\text{alkane}T+\text{osc}} (^{12}\text{CH}_2)$ (Eq. (14.623)) and $E(\text{magnetic})$ (Eq. (13.524)):

430

$$E_{D_{alkane}}(^{12}CH_2) = E(C, 2sp^3) + 2E(H) - E_{alkaneT+osc}(^{12}CH_2) - E(magnetic) \quad (14.624)$$

Thus, the total $^{12}CH_2$ bond dissociation energy, $E_{D_{alkane}}(^{12}CH_2)$ is

$$\begin{aligned} E_{D_{alkane}}(^{12}CH_2) &= -(14.63489 \text{ eV} + 2(13.59844 \text{ eV})) - (E_{alkaneT+osc}(^{12}CH_2) + E(magnetic)) \\ &= -41.83177 \text{ eV} - (-49.80996 \text{ eV} + 0.14803 \text{ eV}) \\ &= 7.83016 \text{ eV} \end{aligned} \quad (14.625)$$

5

SUM OF THE ENERGIES OF THE C-C σ MOs AND THE HOs OF CONTINUOUS-CHAIN ALKANES

The energy components of V_e , V_p , T , V_m , and E_T of the C-C-bond MOs are the same as those of the CH MO except that energy of the $C_{alkane} 2sp^3$ HO is used. The energies of each C-C-bond MO are given by the substitution of the semiprincipal axes (Eqs. (14.523-14.524) and (14.528)) into the energy equations of the CH MO (Eqs. (13.449-13.453)), with the exception that $E(C_{alkane}, 2sp^3)$ (Eq. (14.512)) replaces $E(C, 2sp^3)$ in Eq. (13.453). The total number of C-C bonds of C_nH_{2n+2} is $n-1$. Thus, the energies of the $n-1$ bonds is given by

$$15 \quad V_e = (n-1)(0.91771) \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -(n-1)28.79214 \text{ eV} \quad (14.626)$$

$$V_p = \frac{(n-1)e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = (n-1)9.33352 \text{ eV} \quad (14.627)$$

$$T = (n-1)(0.91771) \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = (n-1)6.77464 \text{ eV} \quad (14.628)$$

$$V_m = (n-1)(0.91771) \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -(n-1)3.38732 \text{ eV} \quad (14.629)$$

$$\begin{aligned} E_{T_{alkane}}(C-C, \sigma) &= -\frac{(n-1)e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] - 15.56407 \text{ eV} \\ &= -(n-1)31.63537 \text{ eV} \end{aligned} \quad (14.630)$$

20

where $E_{T_{alkane}}(C-C, \sigma)$ is the total energy of the C-C σ MOs given by Eq. (14.520) which is reiteratively matched to Eq. (13.75) within five-significant-figure round off error.

Since there are two carbon atoms per bond, the number of $C-C$ bonds is $n-1$, and the energy change of each $C2sp^3$ shell due to the decrease in radius with the formation of each $C-C$ -bond MO is $E_{T_{alkane}}(C-C, 2sp^3)$ (Eq. (14.517)), the total energy of the $C-C$ -bond MOs, $E_{T_{alkane}}(C-C)$, is given by the sum of $2(n-1)E_{T_{alkane}}(C-C, 2sp^3)$ and

5 $E_{T_{alkane}}(C-C, \sigma)$, the σ MO contribution given by Eq. (14.630):

$$\begin{aligned}
 E_{T_{alkane}}(C-C) &= 2(n-1)E_{T_{alkane}}(C-C, 2sp^3) + E_{T_{alkane}}(C-C, \sigma) \\
 &= (n-1) \left[\begin{aligned} &2(-0.92918 \text{ eV}) \\ &+ \left[-\frac{e^2}{8\pi\epsilon_0\sqrt{aa_0}} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{aa_0}}{a - \sqrt{aa_0}} - 1 \right] \right] \\ &-15.56407 \text{ eV} \end{aligned} \right] \quad (14.631) \\
 &= (n-1)(2(-0.92918 \text{ eV}) + (-31.63537 \text{ eV})) \\
 &= -(n-1)33.49373 \text{ eV}
 \end{aligned}$$

VIBRATION OF CONTINUOUS-CHAIN ALKANES

The vibrational energy levels of the $C-C$ bonds of C_nH_{2n+2} may be solved as $n-1$ sets of

10 coupled carbon harmonic oscillators wherein each carbon is further coupled to two or three equivalent H harmonic oscillators by developing the Lagrangian, the differential equation of motion, and the eigenvalue solutions [9] wherein the spring constants are derived from the central forces as given in the Vibration of Hydrogen-Type Molecular Ions section and the Vibration of Hydrogen-Type Molecules section.

15

THE DOPPLER ENERGY TERMS OF THE $C-C$ -BOND MOs OF CONTINUOUS-CHAIN ALKANES

The equations of the radiation reaction force of each symmetrical $C-C$ -bond MO are given by Eqs. (11.231-11.233), except the force-constant factor is 0.5 based on the force constant

20 k' of Eq. (14.152), and the $C-C$ -bond MO parameters are used. The angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{0.5e^2}{4\pi\epsilon_0 a^3}} = 9.43699 \times 10^{15} \text{ rad/s} \quad (14.632)$$

where a is given by Eq. (14.523). The kinetic energy, E_K , is given by Planck's equation (Eq. (11.127)):

$$\bar{E}_K = \hbar\omega = \hbar 9.43699 \times 10^{15} \text{ rad/s} = 6.21159 \text{ eV} \quad (14.633)$$

In Eq. (11.181), substitution of $E_{T_{alkane}}(C-C)$ (Eq. (14.631)) with $n=2$ for E_{hv} , the mass of the electron, m_e , for M , and the kinetic energy given by Eq. (14.633) for \bar{E}_K gives the Doppler energy of the electrons of each of the bonds for the reentrant orbit:

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{Mc^2}} = -33.49373 \text{ eV} \sqrt{\frac{2e(6.21159 \text{ eV})}{m_e c^2}} = -0.16515 \text{ eV} \quad (14.634)$$

In addition to the electrons, the nuclei also undergo simple harmonic oscillation in the transition state at their corresponding frequency. The decrease in the energy of each $C-C$ bond MO due to the reentrant orbit of the bond in the transition state corresponding to simple harmonic oscillation of the electrons and nuclei, \bar{E}_{osc} , is given by the sum of the corresponding energies, \bar{E}_D given by Eq. (14.634) and \bar{E}_{Kvib} , the average kinetic energy of vibration which is 1/2 of the vibrational energy of each $C-C$ bond. Using the ethane experimental $C-C$ $E_{vib}(\nu_3)$ of 993 cm^{-1} (0.12312 eV) [10] for \bar{E}_{Kvib} of the transition state having $n-1$ independent bonds, $\bar{E}'_{alkane\ osc}(C-C, \sigma)$ per bond is

$$\bar{E}'_{alkane\ osc}(C-C, \sigma) = \bar{E}_D + \bar{E}_{Kvib} = \bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \quad (14.635)$$

$$\bar{E}'_{alkane\ osc}(C-C, \sigma) = -0.16515 \text{ eV} + \frac{1}{2} (0.12312 \text{ eV}) = -0.10359 \text{ eV} \quad (14.636)$$

Given that the vibration and reentrant oscillation is for $n-1$ $C-C$ bonds, $\bar{E}_{alkane\ osc}(C-C, \sigma)$, is:

$$\begin{aligned} \bar{E}_{alkane\ osc}(C-C, \sigma) &= (n-1) \left(\bar{E}_D + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\ &= (n-1) \left(-0.16515 \text{ eV} + \frac{1}{2} (0.12312 \text{ eV}) \right) \\ &= -(n-1) 0.10359 \text{ eV} \end{aligned} \quad (14.637)$$

TOTAL ENERGIES OF THE C-C-BOND MOs OF CONTINUOUS-CHAIN ALKANES

$E_{alkaneT+osc}(C-C)$, the total energy of the $n-1$ bonds of the C-C-bond MOs including the Doppler term, is given by the sum of $E_{T_{alkane}}(C-C)$ (Eq. (14.631)) and $\bar{E}_{alkane\ osc}(C-C, \sigma)$ given by Eq. (14.637):

$$\begin{aligned}
 E_{alkaneT+osc}(C-C) &= \left((n-1) \left(V_e + T + V_m + V_p + E(C_{alkane}, 2sp^3) \right) + \bar{E}_{alkane\ osc}(C-C, \sigma) \right) \\
 &= E_{T_{alkane}}(C-C, \sigma) + 2(n-1)E_{T_{alkane}}(C-C, 2sp^3) + \bar{E}_{alkane\ osc}(C-C, \sigma) \\
 &= E_{T_{alkane}}(C-C) + \bar{E}_{alkane\ osc}(C-C, \sigma)
 \end{aligned}
 \tag{14.638}$$

$$\begin{aligned}
 E_{alkaneT+osc}(C-C) &= (n-1) \left\{ \left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[(0.91771) \left(2 - \frac{1}{2} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \right) \right. \\
 &\quad \left. -15.56407\text{ eV} + 2E_{T_{alkane}}(C-C, 2sp^3) \right\} \\
 &\quad \left(1 + \sqrt{\frac{2\hbar \sqrt{\frac{1}{2} \frac{e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \\
 &= (n-1) \left(-33.49373\text{ eV} - 0.16515\text{ eV} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)
 \end{aligned}
 \tag{14.639}$$

From Eqs. (14.637-14.639), the total energy of the $n-1$ bonds of the C-C-bond MOs is

$$\begin{aligned}
 E_{alkaneT+osc}(C-C) &= (n-1) \left(-31.63537\text{ eV} + 2E_{T_{alkane}}(C-C, 2sp^3) \right) + \bar{E}_{alkane\ osc}(C-C, \sigma) \\
 &= (n-1) \left(-31.63537\text{ eV} + 2(-0.92918\text{ eV}) \right) \\
 &\quad \left(-0.16515\text{ eV} + \frac{1}{2} (0.12312\text{ eV}) \right) \\
 &= -(n-1)33.59732\text{ eV}
 \end{aligned}
 \tag{14.640}$$

where the experimental E_{vib} was used for the $\hbar \sqrt{\frac{k}{\mu}}$ term.

TOTAL BOND ENERGY OF THE C-C BONDS OF CONTINUOUS-CHAIN ALKANES

Since there are two carbon atoms per bond and the number of C-C bonds is $n-1$, the total bond energy of the C-C bonds of C_nH_{2n+2} , $E_D(C-C)_{n-1}$, is given by $2(n-1)E(C, 2sp^3)$ minus $E_{alkaneT+osc}(C-C)$ (Eq. (14.640)) where $E(C, 2sp^3)$ (Eq. (14.146)) is the initial energy of each $C2sp^3$ HO of the CH_3 and CH_2 groups that bond to the C-C bonds. Thus, the total dissociation energy of the C-C bonds of C_nH_{2n+2} , is

$$\begin{aligned} E_D(C-C)_{n-1} &= 2(n-1)(E(C, 2sp^3)) - (E_{alkaneT+osc}(C-C)) \\ &= 2(n-1)(-14.63489 \text{ eV}) - (n-1)(-33.59732 \text{ eV}) \\ &= (n-1)(2(-14.63489 \text{ eV}) - (-33.59732 \text{ eV})) \\ &= (n-1)(4.32754 \text{ eV}) \end{aligned} \quad (14.641)$$

10 TOTAL ENERGY OF CONTINUOUS-CHAIN ALKANES

$E_D(C_nH_{2n+2})$, the total bond dissociation energy of C_nH_{2n+2} , is given as the sum of the energy components due to the two methyl groups, $n-2$ methylene groups, and $n-1$ C-C bonds where each energy component is given by Eqs. (14.590), (14.625), and (14.641), respectively. Thus, the total bond dissociation energy of C_nH_{2n+2} is

$$\begin{aligned} E_D(C_nH_{2n+2}) &= E_D(C-C)_{n-1} + 2E_{D_{alkane}}(^{12}CH_3) + (n-2)E_{D_{alkane}}(^{12}CH_2) \\ &= (n-1)(4.32754 \text{ eV}) + 2(12.49186 \text{ eV}) + (n-2)(7.83016 \text{ eV}) \end{aligned} \quad (14.642)$$

The experimental total bond dissociation energy of C_nH_{2n+2} , $E_{D_{exp}}(C_nH_{2n+2})$, is given by the negative difference between the enthalpy of its formation ($\Delta H_f(C_nH_{2n+2}(gas))$) and the sum of the enthalpy of the formation of the reactant gaseous carbons ($\Delta H_f(C(gas))$) and hydrogen ($\Delta H_f(H(gas))$) atoms:

$$\begin{aligned} E_{D_{exp}}(C_nH_{2n+2}) &= -\{\Delta H_f(C_nH_{2n+2}(gas)) - [n\Delta H_f(C(gas)) + (2n+2)\Delta H_f(H(gas))]\} \\ &= -\{\Delta H_f(C_nH_{2n+2}(gas)) - [n7.42774 \text{ eV} + (2n+2)2.259353 \text{ eV}]\} \\ &\quad (14.643) \end{aligned}$$

where the heats of formation atomic carbon and hydrogen gas are given by [17-18]

$$\Delta H_f(C(gas)) = 716.68 \text{ kJ/mole } (7.42774 \text{ eV/molecule}) \quad (14.644)$$

$$\Delta H_f(H(gas)) = 217.998 \text{ kJ/mole } (2.259353 \text{ eV/molecule}) \quad (14.645)$$

Using the corresponding experimental $\Delta H_f(C_nH_{2n+2}(gas))$ [18], $E_D(C_nH_{2n+2})$ was determined from propane, butane, pentane, hexane, heptane, octane, nonane, decane, undecane, dodecane, and octadecane in the corresponding sections, and the results of the determination of the total energies are given in Table 14.1. The calculated results are based on first principles and given in closed-form, exact equations containing fundamental constants only. The agreement between the experimental and calculated results is excellent.

Using the results for C_nH_{2n+2} and the functional groups as basis sets that are linearly combined, the exact solution for the dimensional parameters, charge density functions, and energies of all molecules can be obtained. For example, one or more of the hydrogen atoms of the solution for C_nH_{2n+2} can be substituted with one or more of the previously solved functional groups or derivative functional groups to give a desired molecule. The solution is given by energy matching each group to C_nH_{2n+2} . Substitution of one or more H 's of C_nH_{2n+2} with functional groups from the list of CH_3 , other C_nH_{2n+2} groups, $H_2C=CH_2$, $HC\equiv CH$, F , Cl , O , OH , NH , NH_2 , CN , NO , NO_2 , CO , CO_2 , and C_6H_6 give the solutions of branched alkanes, alkenes, and alkynes, alkyl halides, ethers, alcohols, amides, amines, nitriles, alkyl nitrosos, alkyl nitrates, aldehydes, ketones, carboxylic acids, esters, and substituted aromatics.

PROPANE (C_3H_8)

Using Eq. (14.642) with $n = 3$, the total bond dissociation energy of C_3H_8 is

$$\begin{aligned} E_D(C_3H_8) &= E_D(C-C)_2 + 2E_{D_{alkane}}(^{12}CH_3) + E_{D_{alkane}}(^{12}CH_2) \\ &= (2)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (1)(7.83016 \text{ eV}) \\ &= 41.46896 \text{ eV} \end{aligned} \quad (14.646)$$

Using Eq. (14.643), the experimental total bond dissociation energy of C_3H_8 , $E_{D_{exp}}(C_3H_8)$, given by the negative difference between the enthalpy of its formation ($\Delta H_f(C_3H_8(gas)) = -1.0758 \text{ eV}$) [18] and the sum of the enthalpy of the formation of the gaseous carbons ($\Delta H_f(C(gas))$) and hydrogen ($\Delta H_f(H(gas))$) atoms is

$$\begin{aligned} E_{D_{exp}}(C_3H_8) &= -\left\{ \Delta H_f(C_3H_8(gas)) - [3\Delta H_f(C(gas)) + 8\Delta H_f(H(gas))] \right\} \\ &= -\left\{ -1.0758 \text{ eV} - [(3)7.42774 \text{ eV} + (8)2.259353 \text{ eV}] \right\} \\ &= 41.434 \text{ eV} \end{aligned} \quad (14.647)$$

The charge-density of the C_3H_8 molecular orbital (MO) comprising a linear combination of two methyl groups and one methylene group is shown in Figure 50.

BUTANE (C_4H_{10})

5 Using Eq. (14.642) with $n = 4$, the total bond dissociation energy of C_4H_{10} is

$$\begin{aligned} E_D(C_4H_{10}) &= E_D(C-C)_3 + 2E_{D_{alkane}}(^{12}CH_3) + 2E_{D_{alkane}}(^{12}CH_2) \\ &= (3)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (2)(7.83016 \text{ eV}) \quad (14.648) \\ &= 53.62666 \text{ eV} \end{aligned}$$

Using Eq. (14.643), the experimental total bond dissociation energy of C_4H_{10} , $E_{D_{exp}}(C_4H_{10})$, given by the negative difference between the enthalpy of its formation ($\Delta H_f(C_4H_{10}(gas)) = -1.3028 \text{ eV}$) [18] and the sum of the enthalpy of the formation of the

10 gaseous carbons ($\Delta H_f(C(gas))$) and hydrogen ($\Delta H_f(H(gas))$) atoms is

$$\begin{aligned} E_{D_{exp}}(C_4H_{10}) &= -\left\{ \Delta H_f(C_4H_{10}(gas)) - [4\Delta H_f(C(gas)) + 10\Delta H_f(H(gas))] \right\} \\ &= -\left\{ -1.3028 \text{ eV} - [(4)7.42774 \text{ eV} + (10)2.259353 \text{ eV}] \right\} \\ &= 53.61 \text{ eV} \end{aligned}$$

(14.649)

The charge-density of the C_4H_{10} molecular orbital (MO) comprising a linear combination of two methyl and two methylene groups is shown in Figure 51.

15

PENTANE (C_5H_{12})

Using Eq. (14.642) with $n = 5$, the total bond dissociation energy of C_5H_{12} is

$$\begin{aligned} E_D(C_5H_{12}) &= E_D(C-C)_4 + 2E_{D_{alkane}}(^{12}CH_3) + 3E_{D_{alkane}}(^{12}CH_2) \\ &= (4)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (3)(7.83016 \text{ eV}) \quad (14.650) \\ &= 65.78436 \text{ eV} \end{aligned}$$

Using Eq. (14.643), the experimental total bond dissociation energy of C_5H_{12} , $E_{D_{exp}}(C_5H_{12})$,

20 given by the negative difference between the enthalpy of its formation ($\Delta H_f(C_5H_{12}(gas)) = -1.5225 \text{ eV}$) [18] and the sum of the enthalpy of the formation of the gaseous carbons ($\Delta H_f(C(gas))$) and hydrogen ($\Delta H_f(H(gas))$) atoms is

437

$$\begin{aligned}
 E_{D_{\text{exp}}}(C_5H_{12}) &= -\left\{\Delta H_f(C_5H_{12}(\text{gas})) - [5\Delta H_f(C(\text{gas})) + 12\Delta H_f(H(\text{gas}))]\right\} \\
 &= -\left\{-1.5225 \text{ eV} - [(5)7.42774 \text{ eV} + (12)2.259353 \text{ eV}]\right\} \quad (14.651) \\
 &= 65.77 \text{ eV}
 \end{aligned}$$

The charge-density of the C_5H_{12} molecular orbital (MO) comprising a linear combination of two methyl and three methylene groups is shown in Figure 52.

5 HEXANE (C_6H_{14})

Using Eq. (14.642) with $n = 6$, the total bond dissociation energy of C_6H_{14} is

$$\begin{aligned}
 E_D(C_6H_{14}) &= E_D(C-C)_5 + 2E_{D_{\text{alkane}}}(^{12}\text{CH}_3) + 4E_{D_{\text{alkane}}}(^{12}\text{CH}_2) \\
 &= (5)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (4)(7.83016 \text{ eV}) \quad (14.652) \\
 &= 77.94206 \text{ eV}
 \end{aligned}$$

Using Eq. (14.643), the experimental total bond dissociation energy of C_6H_{14} , $E_{D_{\text{exp}}}(C_6H_{14})$, given by the negative difference between the enthalpy of its formation
 10 ($\Delta H_f(C_6H_{14}(\text{gas})) = -1.7298 \text{ eV}$) [18] and the sum of the enthalpy of the formation of the gaseous carbons ($\Delta H_f(C(\text{gas}))$) and hydrogen ($\Delta H_f(H(\text{gas}))$) atoms is

$$\begin{aligned}
 E_{D_{\text{exp}}}(C_6H_{14}) &= -\left\{\Delta H_f(C_6H_{14}(\text{gas})) - [6\Delta H_f(C(\text{gas})) + 14\Delta H_f(H(\text{gas}))]\right\} \\
 &= -\left\{-1.7298 \text{ eV} - [(6)7.42774 \text{ eV} + (14)2.259353 \text{ eV}]\right\} \\
 &= 77.93 \text{ eV} \\
 &(14.653)
 \end{aligned}$$

The charge-density of the C_6H_{14} molecular orbital (MO) comprising a linear combination of
 15 two methyl and four methylene groups is shown in Figure 53.

HEPTANE (C_7H_{16})

Using Eq. (14.642) with $n = 7$, the total bond dissociation energy of C_7H_{16} is

$$\begin{aligned}
 E_D(C_7H_{16}) &= E_D(C-C)_6 + 2E_{D_{\text{alkane}}}(^{12}\text{CH}_3) + 5E_{D_{\text{alkane}}}(^{12}\text{CH}_2) \\
 &= (6)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (5)(7.83016 \text{ eV}) \quad (14.654) \\
 &= 90.09976 \text{ eV}
 \end{aligned}$$

20 Using Eq. (14.643), the experimental total bond dissociation energy of C_7H_{16} , $E_{D_{\text{exp}}}(C_7H_{16})$, given by the negative difference between the enthalpy of its formation

$(\Delta H_f(C_7H_{16}(gas)) = -1.9443 \text{ eV})$ [18] and the sum of the enthalpy of the formation of the gaseous carbons $(\Delta H_f(C(gas)))$ and hydrogen $(\Delta H_f(H(gas)))$ atoms is

$$\begin{aligned} E_{D_{exp}}(C_7H_{16}) &= -\left\{\Delta H_f(C_7H_{16}(gas)) - [7\Delta H_f(C(gas)) + 16\Delta H_f(H(gas))]\right\} \\ &= -\left\{-1.9443 \text{ eV} - [(7)7.42774 \text{ eV} + (16)2.259353 \text{ eV}]\right\} \\ &= 90.09 \text{ eV} \end{aligned}$$

(14.655)

- 5 The charge-density of the C_7H_{16} MO comprising a linear combination of two methyl and five methylene groups is shown in Figure 54.

OCTANE (C_8H_{18})

Using Eq. (14.642) with $n = 8$, the total bond dissociation energy of C_8H_{18} is

$$\begin{aligned} E_D(C_8H_{18}) &= E_D(C-C)_7 + 2E_{D_{alkane}}(^{12}CH_3) + 6E_{D_{alkane}}(^{12}CH_2) \\ 10 \quad &= (7)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (6)(7.83016 \text{ eV}) \quad (14.656) \\ &= 102.25746 \text{ eV} \end{aligned}$$

Using Eq. (14.643), the experimental total bond dissociation energy of C_8H_{18} , $E_{D_{exp}}(C_8H_{18})$, given by the negative difference between the enthalpy of its formation $(\Delta H_f(C_8H_{18}(gas)) = -2.1609 \text{ eV})$ [18] and the sum of the enthalpy of the formation of the gaseous carbons $(\Delta H_f(C(gas)))$ and hydrogen $(\Delta H_f(H(gas)))$ atoms is

$$\begin{aligned} E_{D_{exp}}(C_8H_{18}) &= -\left\{\Delta H_f(C_8H_{18}(gas)) - [8\Delta H_f(C(gas)) + 18\Delta H_f(H(gas))]\right\} \\ 15 \quad &= -\left\{-2.1609 \text{ eV} - [(8)7.42774 \text{ eV} + (18)2.259353 \text{ eV}]\right\} \\ &= 102.25 \text{ eV} \end{aligned}$$

(14.657)

The charge-density of the C_8H_{18} MO comprising a linear combination of two methyl and six methylene groups is shown in Figure 55.

20 NONANE (C_9H_{20})

Using Eq. (14.642) with $n = 9$, the total bond dissociation energy of C_9H_{20} is

439

$$\begin{aligned}
 E_D(C_9H_{20}) &= E_D(C-C)_8 + 2E_{D_{alkane}}(^{12}CH_3) + 7E_{D_{alkane}}(^{12}CH_2) \\
 &= (8)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (7)(7.83016 \text{ eV}) \quad (14.658) \\
 &= 114.41516 \text{ eV}
 \end{aligned}$$

Using Eq. (14.643), the experimental total bond dissociation energy of C_9H_{20} , $E_{D_{exp}}(C_9H_{20})$, given by the negative difference between the enthalpy of its formation ($\Delta H_f(C_9H_{20}(gas)) = -2.3651 \text{ eV}$) [18] and the sum of the enthalpy of the formation of the 5 gaseous carbons ($\Delta H_f(C(gas))$) and hydrogen ($\Delta H_f(H(gas))$) atoms is

$$\begin{aligned}
 E_{D_{exp}}(C_9H_{20}) &= -\left\{ \Delta H_f(C_9H_{20}(gas)) - [9\Delta H_f(C(gas)) + 20\Delta H_f(H(gas))] \right\} \\
 &= -\left\{ -2.3651 \text{ eV} - [(9)7.42774 \text{ eV} + (20)2.259353 \text{ eV}] \right\} \\
 &= 114.40 \text{ eV} \\
 &\quad (14.659)
 \end{aligned}$$

The charge-density of the C_9H_{20} MO comprising a linear combination of two methyl and seven methylene groups is shown in Figure 56.

10

DECANE ($C_{10}H_{22}$)

Using Eq. (14.642) with $n=10$, the total bond dissociation energy of $C_{10}H_{22}$ is

$$\begin{aligned}
 E_D(C_{10}H_{22}) &= E_D(C-C)_9 + 2E_{D_{alkane}}(^{12}CH_3) + 8E_{D_{alkane}}(^{12}CH_2) \\
 &= (9)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (8)(7.83016 \text{ eV}) \quad (14.660) \\
 &= 126.57286 \text{ eV}
 \end{aligned}$$

Using Eq. (14.643), the experimental total bond dissociation energy of $C_{10}H_{22}$, $E_{D_{exp}}(C_{10}H_{22})$, given by the negative difference between the enthalpy of its formation ($\Delta H_f(C_{10}H_{22}(gas)) = -2.5858 \text{ eV}$) [18] and the sum of the enthalpy of the formation of the 15 gaseous carbons ($\Delta H_f(C(gas))$) and hydrogen ($\Delta H_f(H(gas))$) atoms is

$$\begin{aligned}
 E_{D_{exp}}(C_{10}H_{22}) &= -\left\{ \Delta H_f(C_{10}H_{22}(gas)) - [10\Delta H_f(C(gas)) + 22\Delta H_f(H(gas))] \right\} \\
 &= -\left\{ -2.5858 \text{ eV} - [(10)7.42774 \text{ eV} + (22)2.259353 \text{ eV}] \right\} \\
 &= 126.57 \text{ eV} \\
 &\quad (14.661)
 \end{aligned}$$

20 The charge-density of the $C_{10}H_{22}$ molecular orbital (MO) comprising a linear combination of two methyl and eight methylene groups is shown in Figure 57.

UNDECANE ($C_{11}H_{24}$)

Using Eq. (14.642) with $n = 11$, the total bond dissociation energy of $C_{11}H_{24}$ is

$$\begin{aligned} E_D(C_{11}H_{24}) &= E_D(C-C)_{10} + 2E_{D_{alkane}}(^{12}CH_3) + 9E_{D_{alkane}}(^{12}CH_2) \\ &= (10)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (9)(7.83016 \text{ eV}) \quad (14.662) \\ &= 138.73056 \text{ eV} \end{aligned}$$

- 5 Using Eq. (14.643), the experimental total bond dissociation energy of $C_{11}H_{24}$, $E_{D_{exp}}(C_{11}H_{24})$, given by the negative difference between the enthalpy of its formation ($\Delta H_f(C_{11}H_{24}(gas) = -2.8066 \text{ eV})$) [18] and the sum of the enthalpy of the formation of the gaseous carbons ($\Delta H_f(C(gas))$) and hydrogen ($\Delta H_f(H(gas))$) atoms is

$$\begin{aligned} E_{D_{exp}}(C_{11}H_{24}) &= -\left\{\Delta H_f(C_{11}H_{24}(gas)) - [11\Delta H_f(C(gas)) + 24\Delta H_f(H(gas))]\right\} \\ &= -\left\{-2.8066 \text{ eV} - [(11)7.42774 \text{ eV} + (24)2.259353 \text{ eV}]\right\} \\ &= 138.736 \text{ eV} \end{aligned}$$

10 (14.663)

The charge-density of the $C_{11}H_{24}$ MO comprising a linear combination of two methyl and nine methylene groups is shown in Figure 58.

15 DODECANE ($C_{12}H_{26}$)

Using Eq. (14.642) with $n = 12$, the total bond dissociation energy of $C_{12}H_{26}$ is

$$\begin{aligned} E_D(C_{12}H_{26}) &= E_D(C-C)_{11} + 2E_{D_{alkane}}(^{12}CH_3) + 10E_{D_{alkane}}(^{12}CH_2) \\ &= (11)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (10)(7.83016 \text{ eV}) \quad (14.664) \\ &= 150.88826 \text{ eV} \end{aligned}$$

- Using Eq. (14.643), the experimental total bond dissociation energy of $C_{12}H_{26}$, $E_{D_{exp}}(C_{12}H_{26})$, given by the negative difference between the enthalpy of its formation ($\Delta H_f(C_{12}H_{26}(gas) = -2.9994 \text{ eV})$) [18] and the sum of the enthalpy of the formation of the gaseous carbons ($\Delta H_f(C(gas))$) and hydrogen ($\Delta H_f(H(gas))$) atoms is
- 20

$$\begin{aligned}
 E_{D_{\text{exp}}}(C_{12}H_{26}) &= -\left\{\Delta H_f(C_{12}H_{26}(\text{gas})) - [12\Delta H_f(C(\text{gas})) + 26\Delta H_f(H(\text{gas}))]\right\} \\
 &= -\left\{-2.9994 \text{ eV} - [(12)7.42774 \text{ eV} + (26)2.259353 \text{ eV}]\right\} \quad (14.665) \\
 &= 150.88 \text{ eV}
 \end{aligned}$$

The charge-density of the $C_{12}H_{26}$ MO comprising a linear combination of two methyl and ten methylene groups is shown in Figure 59.

5 OCTADECANE ($C_{18}H_{38}$)

Using Eq. (14.642) with $n = 18$, the total bond dissociation energy of $C_{18}H_{38}$ is

$$\begin{aligned}
 E_D(C_{18}H_{38}) &= E_D(C-C)_{17} + 2E_{D_{\text{alkane}}}(^{12}\text{CH}_3) + 16E_{D_{\text{alkane}}}(^{12}\text{CH}_2) \\
 &= (17)(4.32754 \text{ eV}) + (2)(12.49186 \text{ eV}) + (16)(7.83016 \text{ eV}) \quad (14.666) \\
 &= 223.83446 \text{ eV}
 \end{aligned}$$

Using Eq. (14.643), the experimental total bond dissociation energy of $C_{18}H_{38}$, $E_{D_{\text{exp}}}(C_{18}H_{38})$, given by the negative difference between the enthalpy of its formation

10 ($\Delta H_f(C_{18}H_{38}(\text{gas})) = -4.2970 \text{ eV}$) [18] and the sum of the enthalpy of the formation of the gaseous carbons ($\Delta H_f(C(\text{gas}))$) and hydrogen ($\Delta H_f(H(\text{gas}))$) atoms is

$$\begin{aligned}
 E_{D_{\text{exp}}}(C_{18}H_{38}) &= -\left\{\Delta H_f(C_{18}H_{38}(\text{gas})) - [18\Delta H_f(C(\text{gas})) + 38\Delta H_f(H(\text{gas}))]\right\} \\
 &= -\left\{-4.2970 \text{ eV} - [(18)7.42774 \text{ eV} + (38)2.259353 \text{ eV}]\right\} \\
 &= 223.85 \text{ eV}
 \end{aligned}$$

(14.667)

The charge-density of the $C_{18}H_{38}$ molecular orbital (MO) comprising a linear combination of
15 two methyl and sixteen methylene groups is shown in Figure 60.

Table 14.1. The calculated and experimental bond parameters of CO_2 , NO_2 , CH_3CH_3 , CH_2CH_2 , $CHCH$, benzene, propane, butane, pentane, hexane, heptane, octane, nonane, decane, undecane, dodecane, and octadecane.

Parameter	Calculated	Experimental	Ref. for Exp.
CO_2 Bond Energy	5.49553 eV	5.51577 eV	7
CO_2 Bond Length	1.1616 Å	1.1600 Å	3

Parameter	442		
	Calculated	Experimental	Ref. for Exp.
NO_2 Bond Energy	3.1532 eV	3.161 eV	7
NO_2 Bond Length	1.1872 Å	1.193 Å	3
NO_2 Bond Angle	131.012°	134.1°	3
H_3C-CH_3 Bond Energy	3.90245 eV	3.8969 eV	7
H_3C-CH_3 Bond Length	1.53635 Å	1.5351 Å	3
$H-CH_2CH_3$ Bond Length	1.10822 Å	1.0940 Å	3
Ethane $H-C-H$ Bond Angle	107.44°	107.4°	8
Ethane $C-C-H$ Bond Angle	111.44°	111.17°	3
$H_2C=CH_2$ Bond Energy	7.55681 eV	7.597 eV	7
$H_2C=CH_2$ Bond Length	1.3405 Å	1.339 Å	3
$H-CHCH_2$ Bond Length	1.0826 Å	1.087 Å	3
Ethylene $H-C-H$ Bond Angle	116.31°	116.6°	11
Ethylene $C=C-H$ Bond Angle	121.85°	121.7°	11
$HC\equiv CH$ Bond Energy	10.07212 eV	10.0014 eV	7
$HC\equiv CH$ Bond Length	1.2007 Å	1.203 Å	3
$H-CCH$ Bond Length	1.0538 Å	1.060 Å	3
Acetylene $C\equiv C-H$ Bond Angle	180°	180°	6
C_6H_6 Total Bond Energy	57.2601 eV	57.26 eV	17-18
Benzene $C=C$ Bond Length	1.3914 Å	1.399 Å	3
$H-C_6H_5$ Bond Length	1.0933 Å	1.101 Å	3
C_6H_6 $C=C=C$ Bond Angle	120°	120°	13-15
C_6H_6 $C=C-H$ Bond Angle	120°	120°	13-15
C_3H_8 Total Bond Energy	41.46896 eV	41.434 eV	17-18
Propane $C-C$ Bond Length	1.5428 Å	1.532 Å	3
Propane $C-H$ Bond Length	1.1097 Å	1.107 Å	3
Alkane $H-C-H$ Bond Angle	109.50°	109.3°	19
Alkane $C-C-H$ Bond Angle	109.44°	109.3°	19
C_4H_{10} Total Bond Energy	53.62666 eV	53.61 eV	17-18
Butane $C-C$ Bond Length	1.5428 Å	1.531 Å	3
Butane $C-H$ Bond Length	1.11713 Å	1.117 Å	3

443			
Parameter	Calculated	Experimental	Ref. for Exp.
C_5H_{12} Total Bond Energy	65.78436 eV	65.77 eV	17-18
C_6H_{14} Total Bond Energy	77.94206 eV	77.93 eV	17-18
C_7H_{16} Total Bond Energy	90.09976 eV	90.09 eV	17-18
C_8H_{18} Total Bond Energy	102.25746 eV	102.25 eV	17-18
C_9H_{20} Total Bond Energy	114.41516 eV	114.40 eV	17-18
$C_{10}H_{22}$ Total Bond Energy	126.57286 eV	126.57 eV	17-18
$C_{11}H_{24}$ Total Bond Energy	138.73056 eV	138.736 eV	17-18
$C_{12}H_{26}$ Total Bond Energy	150.88826 eV	150.88 eV	17-18
$C_{18}H_{38}$ Total Bond Energy	223.83446 eV	223.85 eV	17-18

References for Section III

1. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), pp. 4-130 to 4-135.
- 5 2. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), p. 10-175.
3. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), pp. 9-15 to 9-41.
4. M. Karplus, R. N. Porter, *Atoms and Molecules an Introduction for Students of Physical*
10 *Chemistry*, The Benjamin/Cummings Publishing Company, Menlo Park, California, (1970), pp. 447-484.
5. G. Herzberg, *Molecular Spectra and Molecular Structure II. Infrared and Raman Spectra of Polyatomic Molecules*, Krieger Publishing Company, Malabar, FL, (1945), p. 174.
6. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca
15 Raton, Florida, (1998-9), pp. 9-76 to 9-79.
7. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), pp. 9-63 to 9-69.
8. R. L. DeKock, H. B. Gray, *Chemical Structure and Bonding*, The Benjamin/Cummings Publishing Company, Menlo Park, CA, (1980), p. 162.
- 20 9. G. R. Fowles, *Analytical Mechanics*, Third Edition, Holt, Rinehart, and Winston, New York, (1977), pp. 251-305.

10. G. Herzberg, *Molecular Spectra and Molecular Structure II. Infrared and Raman Spectra of Polyatomic Molecules*, Van Nostrand Reinhold Company, New York, New York, (1945), p. 344.
11. R. L. DeKock, H. B. Gray, *Chemical Structure and Bonding*, The Benjamin/Cummings
5 Publishing Company, Menlo Park, CA, (1980), p. 179.
12. G. Herzberg, *Molecular Spectra and Molecular Structure II. Infrared and Raman Spectra of Polyatomic Molecules*, Van Nostrand Reinhold Company, New York, New York, (1945), p. 326.
13. W. I. F. David, R. M. Ibberson, G. A. Jeffrey, J. R. Ruble, "The structure analysis of
10 deuterated benzene and deuterated nitromethane by pulsed-neutron powder diffraction: a comparison with single crystal neutron analysis", *Physica B* (1992), 180 & 181, pp. 597-600..
14. G. A. Jeffrey, J. R. Ruble, R. K. McMullan, J. A. Pople, "The crystal structure of
15 deuterated benzene," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 414, No. 1846, (Nov. 9, 1987), pp. 47-57.
15. H. B. Burgi, S. C. Capelli, "Getting more out of crystal-structure analyses," *Helvetica Chimica Acta*, Vol. 86, (2003), pp. 1625-1640.
16. G. Herzberg, *Molecular Spectra and Molecular Structure II. Infrared and Raman Spectra of Polyatomic Molecules*, Van Nostrand Reinhold Company, New York, New York,
20 (1945), pp. 362-369.
17. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), pp. 9-63.
18. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca Raton, Florida, (1998-9), pp. 5-1 to 5-60.
- 25 19. R. J. Fessenden, J. S. Fessenden, *Organic Chemistry*, Willard Grant Press. Boston, Massachusetts, (1979), pp. 46-48.

I CLAIM:

1. A system of computing and rendering the nature of at least one specie selected from a group of diatomic molecules having at least one atom that is other than hydrogen, polyatomic
5 molecules, molecular ions, polyatomic molecular ions, or molecular radicals, or any functional group therein, comprising physical, Maxwellian solutions of charge, mass, and current density functions of said specie, said system comprising:
processing means for processing physical, Maxwellian equations representing charge, mass, and current density functions of said specie; and
10 an output device in communication with the processing means for displaying said physical, Maxwellian solutions of charge, mass, and current density functions of said specie.
2. The system of claim 1 wherein the output device is a display that displays at least one of visual or graphical media.
15
3. The system of claim 2 wherein the display is at least one of static or dynamic.
4. The system of claim 3 wherein at least one of vibration and rotation is be displayed.
- 20 5. The system of claim 4 wherein displayed information is used to model reactivity and physical properties.
6. The system of claim 5, wherein the output device is a monitor, video projector, printer, or three-dimensional rendering device.
25
7. The system of claim 6 wherein displayed information is used to model other species and provides utility to anticipate their reactivity and physical properties.
8. The system of claim 7 wherein the processing means is a general purpose computer.
30
9. The system of claim 8 wherein the general purpose computer comprises a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage device, an input means.

10. The system of claim 9, wherein the input means comprises a serial port, usb port, microphone input, camera input, keyboard or mouse.
- 5 11. The system of claim 10 wherein the processing means comprises a special purpose computer or other hardware system.
12. The system of claim 11 further comprising computer program products.
- 10 13. The system of claim 12 further comprising computer readable medium having embodied therein program code means.
14. The system of claim 13 wherein the computer readable media is any available media which can be accessed by a general purpose or special purpose computer.
- 15 15. The system of claim 14 wherein the computer readable media comprises at least one of RAM, ROM, EPROM, CD ROM, DVD or other optical disk storage, magnetic disk storage or other magnetic storage devices, or any other medium which can embody the desired program code means and which can be accessed by a general purpose or special purpose
- 20 computer.
16. The system of claim 15 wherein the program code means comprises executable instructions and data which cause a general purpose computer or special purpose computer to perform a certain function of a group of functions.
- 25 17. The system of claim 16 wherein the program code is Mathematica programmed with an algorithm based on the physical solutions, and the computer is a PC.
18. The system of claim 17 wherein the algorithm is `ParametricPlot3D[{2*Sqrt[1-`
- 30 `z*z]*Cos[u],Sqrt[(1-z*z)]*Sin[u],z},{u,0,2* Pi},{z,-1,.9999}]`, and the rendering is viewed from different perspectives.

19. The system of claim 18 wherein the algorithms for viewing from different perspectives comprises Show[Out[1], ViewPoint->{x,y,z}] where x, y, and z are Cartesian coordinates.

20. The system of claim 19 wherein the physical, Maxwellian solutions of the charge, mass,
5 and current density functions of said specie comprises a solution of the classical wave

equation $\left[\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] \rho(r, \theta, \phi, t) = 0$ which is the equation of motion of the charge.

21. The system of claim 20 wherein the boundary constraint of the wave equation solution is nonradiation according to Maxwell's equations.

10

22. The system of claim 21 wherein the boundary condition is met for an ellipsoidal-time harmonic function when

$$\omega_n = \frac{\pi \hbar}{m_e A} = \frac{\hbar}{m_e ab}$$

where the area of an ellipse is

15

$$A = \pi ab$$

where $2b$ is the length of the semiminor axis and $2a$ is the length of the semimajor axis.

23. The system of claim 22 wherein the specie charge and current density functions, bond distance, and energies are solved from the Laplacian in ellipsoidal coordinates:

$$20 \quad (\eta - \zeta) R_\xi \frac{\partial}{\partial \xi} \left(R_\xi \frac{\partial \phi}{\partial \xi} \right) + (\zeta - \xi) R_\eta \frac{\partial}{\partial \eta} \left(R_\eta \frac{\partial \phi}{\partial \eta} \right) + (\xi - \eta) R_\zeta \frac{\partial}{\partial \zeta} \left(R_\zeta \frac{\partial \phi}{\partial \zeta} \right) = 0.$$

with the constraint of nonradiation.

24. The system of claim 23 wherein each bond of the said specie defined as a molecular orbital (MO) has the ellipsoidal charge-density function given by

$$25 \quad \sigma = \frac{q}{4\pi abc} \frac{1}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}}$$

25. The system of claim 24 wherein the bonds of the said specie defined as a molecular orbital (MO) has the charge-density function comprising a linear combination of ellipsoids wherein the charge density of one said ellipsoid is given by

$$\sigma = \frac{q}{4\pi abc} \frac{1}{\sqrt{\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}}}$$

26. The system of claim 25 wherein the equation of motion has the parametric form

$$r(t) = ia \cos \omega t + jb \sin \omega t$$

5

27. The system of claim 26 wherein the force balance of the hydrogen-type molecular ion ellipsoidal MO is given by

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{pe^2}{8\pi\epsilon_0} D$$

$$a = 2a_0$$

10

28. The system of claim 27 where the force constant k of a H_2^+ -type ellipsoidal MO due to the equivalent of two point charges of at the foci is given by:

$$k = \frac{2e^2}{4\pi\epsilon_0}$$

15 29. The system of claim 28 wherein the distance from the origin of the H_2 -type-ellipsoidal-MO to each focus c' is given by :

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{aa_0}{2}}$$

30. The system of claim 29 wherein the internuclear distance $2c'$ is given by:

$$20 \quad 2c' = 2\sqrt{\frac{aa_0}{2}}$$

31. The system of claim 30 wherein the length of the semiminor axis of the prolate spheroidal H_2 -type MO $b = c$ is given by:

$$b = \sqrt{a^2 - c'^2}$$

25

32. The system of claim 31 wherein the length of the semiminor axis of the prolate spheroidal H_2 -type MO $b = c$ is given by:

$$e = \frac{c'}{a}$$

5 33. The system of claim 32 wherein the internuclear distance, $2c'$, which is the distance between the foci is

$$2c' = 2a_0;$$

the semiminor axis is

$$b = \sqrt{3}a_0, \text{ and}$$

10 the eccentricity, e , is

$$e = \frac{1}{2}.$$

34. The system of claim 33 wherein the potential energy of the electron in the central field of the protons at the foci is

$$\begin{aligned} V_e &= \frac{-4e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} \\ 15 \quad &= \frac{-4e^2}{8\pi\epsilon_0 c'} \ln \frac{a+c'}{a-c'} ; \\ &= \frac{-4e^2}{8\pi\epsilon_0 a_H} \ln 3 = -59.7575 \text{ eV} \end{aligned}$$

The potential energy of the two protons is

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = \frac{e^2}{8\pi\epsilon_0 a_H} = 13.5984 \text{ eV};$$

The kinetic energy of the electron is

$$T = \frac{2\hbar^2}{m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = \frac{2e^2}{8\pi\epsilon_0 a_H} \ln 3 = 29.8787 \text{ eV}, \text{ and}$$

20 The total energy, E_T , is given by the sum of the energy terms:

$$E_T = V_e + T + V_p$$

$$\begin{aligned}
 E_T &= \frac{-e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \left[\left(4 - \frac{4}{a}\right) \ln \frac{a + \sqrt{a^2-b^2}}{a - \sqrt{a^2-b^2}} - 1 \right] \\
 &= \frac{-e^2}{8\pi\epsilon_0 a_H} (4 \ln 3 - 1 - 2 \ln 3) \\
 &= -16.2803 \text{ eV}
 \end{aligned}$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

5

35. The system of claim 34 wherein during bond formation, the electron undergoes a reentrant oscillatory orbit with vibration of the protons, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

10 The total energy is

$$E_T = V_e + T + V_p + \bar{E}_{osc}$$

$$\begin{aligned}
 E_T &= - \left\{ \frac{e^2}{8\pi\epsilon_0 a_H} (4 \ln 3 - 1 - 2 \ln 3) \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{2e^2}{4\pi\epsilon_0 (2a_H)^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} \\
 &= -16.2803 \text{ eV} - 0.118811 \text{ eV} + \frac{1}{2} (0.29282 \text{ eV}) \\
 &= -16.2527 \text{ eV}
 \end{aligned}$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

15

36. The system of claim 35 wherein the bond dissociation energy, E_D , is the difference between the total energy of the hydrogen atom and E_T :

$$E_D = E(H) - E_T = 2.654 \text{ eV}$$

20 wherein the total energy of a hydrogen atom is

$E(H) = -13.59844 \text{ eV}$, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

37. The system of claim 36 wherein the H_2^+ -type ellipsoidal MO and the hydrogen atomic orbital (AO) serve as basis functions for the MOs of specie.

5

38. The system of claim 37 wherein the MO must (1) be a solution of Laplace's equation to give a equipotential energy surface, (2) correspond to an orbital solution of the Newtonian equation of motion in an inverse-radius-squared central field having a constant total energy,
10 (3) be stable to radiation, and (4) conserve the electron angular momentum of \hbar .

39. The system of claim 38 wherein the potential energy of electron in the central field of the protons at the foci is

$$V_e = \frac{-4e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = \frac{-4e^2}{8\pi\epsilon_0 c'} \ln \frac{a+c'}{a-c'};$$

15 The potential energy of the two protons is

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = \frac{e^2}{8\pi\epsilon_0 c'};$$

The kinetic energy of the electron is

$$T = \frac{2\hbar^2}{m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = \frac{2\hbar^2}{m_e a c'} \ln \frac{a+c'}{a-c'}, \text{ and}$$

The total energy, E_T , is given by the sum of the energy terms:

20

$$E_T = V_e + T + V_p$$

$$E_T = \frac{-e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \left[\left(4 - \frac{4}{a} \right) \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} - 1 \right] = \frac{-e^2}{8\pi\epsilon_0 c'} \left[\left(4 - \frac{4}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right].$$

40. The system of claim 39 wherein during bond formation, the electron undergoes a reentrant oscillatory orbit with vibration of the protons, and the corresponding energy \bar{E}_{osc} is
25 the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

The total energy is

$$E_T = V_e + T + V_p + \bar{E}_{osc}$$

$$E_T = - \left\{ \frac{-e^2}{8\pi\epsilon_0 c'} \left[\left(4 - \frac{4}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{2e^2}{4\pi\epsilon_0 (a)^3}}}{m_e}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\}$$

- 5 41. The system of claim 40 wherein the force balance equation derived of a H_2 -type ellipsoidal MO is given by

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{p e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D$$

$$a = \frac{a_0}{p}$$

42. The system of claim 41 where the force constant k of a H_2 -type ellipsoidal MO due to
10 the equivalent of two point charges of at the foci is given by:

$$k = \frac{2e^2}{4\pi\epsilon_0}$$

43. The system of claim 42 wherein the distance from the origin of the H_2 -type-ellipsoidal-MO to each focus c' is given by :

15
$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2a}} = \sqrt{\frac{a a_0}{2}}$$

44. The system of claim 43 wherein the internuclear distance $2c'$ is given by:

$$2c' = 2\sqrt{\frac{a a_0}{2}}$$

20

45. The system of claim 44 wherein the length of the semiminor axis of the prolate spheroidal H_2 -type MO $b = c$ is given by:

$$b = \sqrt{a^2 - c'^2}$$

46. The system of claim 45 wherein the length of the semiminor axis of the prolate spheroidal H_2 -type MO $b = c$ is given by:

$$e = \frac{c'}{a}$$

5

47. The system of claim 46 wherein the internuclear distance, $2c'$, which is the distance between the foci is

$$2c' = \sqrt{2}a_0;$$

the semiminor axis is

$$10 \quad b = \frac{1}{\sqrt{2}} a_0, \text{ and}$$

the eccentricity, e , is

$$e = \frac{1}{\sqrt{2}}.$$

48. The system of claim 47 wherein the potential energy of the two electrons in the central
15 field of the protons at the foci is

$$V_e = \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -67.8358 \text{ eV};$$

The potential energy of the two protons is

$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} = 19.2415 \text{ eV};$$

The kinetic energy of the electrons is

$$20 \quad T = \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = 33.9179 \text{ eV}, \text{ and}$$

The energy, V_m , of the magnetic force between the electrons is

$$V_m = \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} = -16.9589 \text{ eV}, \text{ and}$$

The total energy, E_T , is given by the sum of the energy terms:

$$E_T = V_e + T + V_m + V_p$$

$$E_T = -13.60 \text{ eV} \left[\left(2p^2\sqrt{2} - p^2\sqrt{2} + \frac{p^2\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - p^2\sqrt{2} \right] = -p^2 31.63$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

- 5 49. The system of claim 48 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the protons, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

The total energy is

$$10 \quad E_T = V_e + T + V_m + V_p + \bar{E}_{osc}$$

$$E_T = - \left\{ \frac{e^2}{8\pi\epsilon_0 a_0} \left[\left(2\sqrt{2} - \sqrt{2} + \frac{\sqrt{2}}{2} \right) \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} - \sqrt{2} \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{e^2}{4\pi\epsilon_0 a_0^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\}$$

$$= -31.635 \text{ eV} - 0.326469 \text{ eV} + \frac{1}{2} (0.56764 \text{ eV})$$

$$= -31.6776 \text{ eV}$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

- 15 50. The system of claim 49 wherein the bond dissociation energy, E_D , is the difference between the total energy of the hydrogen atoms and E_T :

$$E_D = E(2H[a_H]) - E_T = 4.478 \text{ eV}$$

wherein the total energy of two hydrogen atoms is

$$E(2H[a_H]) = -27.21 \text{ eV}, \text{ wherein the calculated and measured values and constants recited}$$

- 20 in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

51. The system of claim 50 wherein the H_2 -type ellipsoidal MO and the hydrogen atomic orbital (AO) serve as basis functions for the MOs of the specie.

52. The system of claim 51 wherein the MO must (1) be a solution of Laplace's equation to give a equipotential energy surface, (2) correspond to an orbital solution of the Newtonian equation of motion in an inverse-radius-squared central field having a constant total energy, (3) be stable to radiation, and (4) conserve the electron angular momentum of \hbar .

5

53. The system of claim 52 wherein the potential energy of the two electrons in the central field of the nuclei at the foci is

$$V_e = \frac{-2e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}};$$

The potential energy of the two nuclei is

10
$$V_p = \frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}};$$

The kinetic energy of the electrons is

$$T = \frac{\hbar^2}{2m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}, \text{ and}$$

The energy, V_m , of the magnetic force between the electrons is

$$V_m = \frac{-\hbar^2}{4m_e a \sqrt{a^2-b^2}} \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}}, \text{ and}$$

15 The total energy, E_T , is given by the sum of the energy terms:

$$E_T = V_e + T + V_m + V_p$$

$$E_T = -\frac{e^2}{8\pi\epsilon_0\sqrt{a^2-b^2}} \left[\left(2 - \frac{a_0}{a} \right) \ln \frac{a+\sqrt{a^2-b^2}}{a-\sqrt{a^2-b^2}} - 1 \right] = -\frac{e^2}{8\pi\epsilon_0 c'} \left[\left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right].$$

54. The system of claim 53 wherein during bond formation, the electrons undergo a reentrant
20 oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvb} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

The total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}$$

$$E_T = - \left\{ \frac{e^2}{8\pi\epsilon_0 c'} \left[\left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\}$$

55. The system of claim 54 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

The total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}$$

$$E_T = - \left\{ \frac{e^2}{8\pi\epsilon_0 c'} \left[\left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\}$$

10

56. The system of claim 55 wherein the energy of the MO is matched to that of the outermost atomic orbital of a bonding heteroatom in the case where a minimum energy is achieved with a direct bond to the AO.
57. The system of claim 56 wherein the MO is continuous with the AO containing paired electrons that do not participate in the bond, and said paired electrons provide a means for the energy matched MO to form a continuous equipotential energy surface.
58. The system of claim 57 wherein an independent MO is formed such that the AO force balance causes the remaining electrons to be at lower energy and a smaller radius.
59. The system of claim 58 wherein at least two atomic orbital hybridize as a linear combination of electrons at the same energy in order to achieve a bond at an energy minimum.

60. The system of claim 59 wherein at least two atomic orbital hybridize as a linear combination of electrons at the same energy in order to achieve a bond at an energy minimum.

5

61. The system of claim 60 wherein at least two atomic orbitals hybridize as a linear combination of electrons at the same energy in order to achieve a bond at an energy minimum, and the sharing of electrons between two or more such orbitals to form a MO permits the participating hybridized orbitals to decrease in energy through a decrease in the
10 radius of one or more of the participating orbitals.

62. The system of claim 61 where the force constant k' of a H_2 -type ellipsoidal MO due to the equivalent of two point charges of at the foci is given by:

$$k' = \frac{c_1 c_2 2e^2}{4\pi\epsilon_0}$$

15 where c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie and c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of the chemical bond.

63. The system of claim 62 where the distance from the origin of the MO to each focus c' is
20 given by:

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2k' a}} = \sqrt{\frac{a a_0}{2k'}};$$

the internuclear distance is

$$2c' = 2\sqrt{\frac{a a_0}{2k'}};$$

the length of the semiminor axis of the prolate spheroidal MO $b = c$ is given by

25 $b = \sqrt{a^2 - c'^2},$ and

and, the eccentricity, e , is

$$e = \frac{c'}{a}.$$

852

64. The system of claim 63 wherein the potential energy of the two electrons in the central field of the nuclei at the foci is

$$V_e = n_1 c_1 c_2 \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}};$$

The potential energy of the two nuclei is

$$5 \quad V_p = n_1 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}};$$

The kinetic energy of the electrons is

$$T = n_1 c_1 c_2 \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The energy, V_m , of the magnetic force between the electrons is

$$V_m = n_1 c_1 c_2 \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

10 The total energy, E_T , is given by the sum of the energy terms:

$$\begin{aligned} E_T &= V_e + T + V_m + V_p \\ E_T &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] \end{aligned}$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and c_2 is the factor that

15 results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond.

65. The system of claim 64 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the

20 difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

The total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}$$

$$E_T = - \left\{ \frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{c_1 c_2 e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] - n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} \quad 853$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond.

66. The system of claim 65 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$10 \quad \bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

The total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}$$

$$E_T = - \left\{ \frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{c_1 c_2 e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} \right] - n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\}$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond.

67. The system of claim 66 wherein a hybridized shell comprises a linear combination of the electrons of at least two atomic-orbital shells.

68. The system of claim 67 wherein the radius of the hybridized shell is calculated from the Coulombic energy equation by considering that the central field decreases by an integer for

each successive electron of the shell and the total energy of the shell is equal to the total Coulombic energy of the initial AO electrons.

69. The system of claim 68 wherein the total energy $E_T(atom, msp^3)$ (m is the integer of the valence shell) of the AO electrons and the hybridized shell is given by the sum of energies of successive ions of the atom over the n electrons comprising total electrons of the at least two AO shells

70. The system of claim 69 wherein the radius r_{msp^3} of the hybridized shell is given by:

$$r_{msp^3} = \sum_{q=Z-n}^{Z-1} \frac{(Z-q)e^2}{8\pi\epsilon_0 E_T(atom, msp^3)}$$

71. The system of claim 70 wherein the Coulombic energy $E_{Coulomb}(atom, msp^3)$ of the outer electron of the $atom msp^3$ shell is given by

$$E_{Coulomb}(atom, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}}$$

72. The system of claim 71 wherein the during hybridization at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO) and the energy change the promotion to the unpaired state is the magnetic energy $E(magnetic)$ at the initial radius r_n of the AO electron:

$$E(magnetic) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_n)^3} = \frac{8\pi\mu_0 \mu_B^2}{(r_n)^3}$$

73. The system of claim 72 wherein the energy $E(atom, msp^3)$ of the outer electron of the $atom msp^3$ shell is given by the sum of $E_{Coulomb}(atom, msp^3)$ and $E(magnetic)$:

$$E(atom, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_n)^3}$$

74. The system of claim 73 wherein at least two atomic orbitals hybridize as a linear combination of electrons at the same energy in order to achieve a bond at an energy minimum, and the sharing of electrons between two or more such hybridized orbitals to form a MO permits the participating hybridized orbitals to decrease in energy through a decrease in the radius of one or more of the participating orbitals; the total energy of the hybridized orbitals is given by the sum of $E(\text{atom}, msp^3)$ and the next energies of successive ions of the atom over the n electrons comprising total electrons of the at least two initial AO shells; is the sum of the first energy of the atom and the hybridization energy.
75. The system of claim 74 wherein the sharing of electrons between two $\text{atom } msp^3$ HOs to form an atom-atom-bond MO permits each participating hybridized orbital to decrease in radius and energy.
76. The system of claim 75 wherein in order to further satisfy the potential, kinetic, and orbital energy relationships, each $\text{atom } msp^3$ HO donates an excess of 25% per bond of its electron density to the atom-atom-bond MO to form an energy minimum wherein the atom-atom bond comprises one of a single, double, or triple bond.
77. The system of claim 76 wherein the radius of the hybridized shell is calculated from the Coulombic energy equation by considering that the central field decreases by an integer for each successive electron of the shell and the total energy of the shell is equal to the total Coulombic energy of the initial AO electrons and the hybridization energy.
78. The system of claim 77 wherein the total energy $E_T(\text{mol. atom}, msp^3)$ (m is the integer of the valence shell) of the HO electrons is given by the sum of energies of successive ions of the atom over the n electrons comprising total electrons of the at least two initial AO shells and the hybridization energy.
79. The system of claim 78 wherein the radius r_{msp^3} of the hybridized shell is given by:

$$r_{msp^3} = \left(\sum_{q=Z-n}^{Z-1} (Z-q) - s(0.25) \right) \frac{e^2}{8\pi\epsilon_0 E_T(\text{mol. atom}, msp^3)}$$

where $s = 1, 2, 3$ for a single, double, and triple bond, respectively.

80. The system of claim 79 wherein the Coulombic energy $E_{Coulomb}(mol.atom, msp^3)$ of the outer electron of the $atom\ msp^3$ shell is given by

$$5 \quad E_{Coulomb}(mol.atom, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}}$$

81. The system of claim 80 wherein the during hybridization at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO) and the energy change the promotion to the unpaired state is the magnetic energy $E(magnetic)$ at the initial radius r_n of the AO
10 electron:

$$E(magnetic) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_n)^3} = \frac{8\pi\mu_0 \mu_B^2}{(r_n)^3}$$

82. The system of claim 81 wherein the energy $E(mol.atom, msp^3)$ of the outer electron of the $atom\ msp^3$ shell is given by the sum of $E_{Coulomb}(mol.atom, msp^3)$ and $E(magnetic)$:

$$15 \quad E(mol.atom, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_n)^3}$$

83. The system of claim 82 wherein $E_T(atom - atom, msp^3)$, the energy change of each $atom\ msp^3$ shell with the formation of the atom-atom-bond MO is given by the difference between $E(mol.atom, msp^3)$ and $E(atom, msp^3)$.

$$20 \quad E_T(atom - atom, msp^3) = E(mol.atom, msp^3) - E(atom, msp^3)$$

84. The system of claim 83 wherein to meet an energy matching condition for all MOs at all HOs, the energy $E(mol.atom, msp^3)$ of the outer electron of the $atom\ msp^3$ shell of each bonding atom must be the average of $E(mol.atom, msp^3)$ for two different values of s :

$$25 \quad E(mol., msp^3) = \frac{E(mol(s_1), msp^3) + E(mol(s_2), msp^3)}{2} \quad (14.512)$$

85. The system of claim 84 wherein $E_T(atom - atom, msp^3)$, the energy change of each $atom\ msp^3$ shell with the formation of each atom-atom-bond MO, is average of $E_T(atom - atom, msp^3)$ for two different values of s :

$$5 \quad E_T(atom - atom, msp^3) = \frac{E_T(atom - atom(s_1), msp^3) + E_T(atom - atom(s_2), msp^3)}{2}$$

86. The system of claim 85 wherein the radius r_{msp^3} of the $atom\ msp^3$ shell of each bonding atom is given by the Coulombic energy using the initial energy $E_{Coulomb}(atom, msp^3)$ and $E_T(atom - atom, msp^3)$, the energy change of each $atom\ msp^3$ shell with the formation of

10 each atom-atom-bond MO:

$$r_{msp^3} = \frac{-e^2}{8\pi\epsilon_0 a_0 (E_{Coulomb}(atom, msp^3) + E_T(atom - atom, msp^3))} \quad (14.514)$$

87. The system of claim 86 wherein the Coulombic energy $E_{Coulomb}(mol.atom, msp^3)$ of the outer electron of the $atom\ msp^3$ shell is given by

$$15 \quad E_{Coulomb}(mol.atom, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}}$$

88. The system of claim 87 wherein the during hybridization at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO) and the energy change the promotion

20 to the unpaired state is the magnetic energy $E(magnetic)$ at the initial radius r_n of the AO electron:

$$E(magnetic) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_n)^3} = \frac{8\pi\mu_0 \mu_B^2}{(r_n)^3}$$

25 89. The system of claim 88 wherein the energy $E(mol.atom, msp^3)$ of the outer electron of

the *atom msp³* shell is given by the sum of $E_{Coulomb}(mol.atom,msp^3)$ and $E(magnetic)$:

$$E(mol.atom,msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_n)^3}$$

90. The system of claim 89 wherein $E_T(atom - atom,msp^3)$, the energy change of each
5 *atom msp³* shell with the formation of the atom-atom-bond MO is given by the difference between $E(mol.atom,msp^3)$ and $E(atom,msp^3)$.

$$E_T(atom - atom,msp^3) = E(mol.atom,msp^3) - E(atom,msp^3)$$

91. The system of claim 90 wherein $E_{Coulomb}(mol.atom,msp^3)$ is one of is one of
10 $E_{Coulomb}(C_{ethylene},2sp^3)$, $E_{Coulomb}(C_{ethane},2sp^3)$, $E_{Coulomb}(C_{acetylene},2sp^3)$, and
 $E_{Coulomb}(C_{alkane},2sp^3)$;

$$E_{Coulomb}(atom,msp^3) \text{ is one of } E_{Coulomb}(C,2sp^3) \text{ and } E_{Coulomb}(Cl,3sp^3);$$

$$E(mol.atom,msp^3) \text{ is one of } E(C_{ethylene},2sp^3), E(C_{ethane},2sp^3),$$

$$E(C_{acetylene},2sp^3) \text{ and } E(C_{alkane},2sp^3);$$

15 $E(atom,msp^3)$ is one of and $E(C,2sp^3)$ and $E(Cl,3sp^3)$;

$$E_T(atom - atom,msp^3) \text{ is one of } E(C - C,2sp^3), E(C = C,2sp^3), \text{ and}$$

$$E(C \equiv C,2sp^3);$$

$$atom\ msp^3 \text{ is one of } C2sp^3, Cl3sp^3$$

$$E_T(atom - atom(s_1),msp^3) \text{ is } E_T(C - C,2sp^3) \text{ and } E_T(atom - atom(s_2),msp^3) \text{ is}$$

20 $E_T(C = C,2sp^3)$, and

$$r_{msp^3} \text{ is one of } r_{C2sp^3}, r_{ethane2sp^3}, r_{ethylene2sp^3}, r_{acetylene2sp^3}, r_{alkane2sp^3}, \text{ and } r_{Cl3sp^3},$$

92. The system of claim 91 wherein the energy of the MO is matched to that of the outermost
atomic orbital or hybridized orbital of a bonding atom in the case where a minimum energy is
25 achieved with a direct bond to the AO or HO.

93. The system of claim 92 wherein the potential energy of the two electrons in the central field of the nuclei at the foci is

$$V_e = n_1 c_1 c_2 \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}};$$

5 The potential energy of the two nuclei is

$$V_p = n_1 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}};$$

The kinetic energy of the electrons is

$$T = n_1 c_1 c_2 \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The energy, V_m , of the magnetic force between the electrons is

$$10 \quad V_m = n_1 c_1 c_2 \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The total energy, E_T , is given by the sum of the energy terms plus $E(AO)$:

$$\begin{aligned} E_T &= V_e + T + V_m + V_p \\ E_T &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] + E(AO) \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + E(AO) \end{aligned}$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type

15 ellipsoidal MO basis function of a chemical bond of the specie, c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond, and $E(AO)$ is the energy of the at least one atomic orbital to which the MO is energy matched.

20 94. The system of claim 93 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib}, \text{ and}$$

the total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc} + E(AO).$$

95. The system of claim 94 wherein during bond formation, the electrons undergo a reentrant
5 oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

the total energy is

$$10 \quad E_T = V_e + T + V_m + V_p + \bar{E}_{osc} \\ E_T = - \left\{ \frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{c_1 c_2 e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] - n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} + E(AO)$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type
ellipsoidal MO basis function of a chemical bond of the specie, c_2 is the factor that results in
an equipotential energy match of the participating at least two molecular or atomic orbitals of
each chemical bond, and $E(AO)$ is the energy of the at least one atomic orbital to which the
15 MO is energy matched.

96. The system of claim 95 wherein during bond formation, the electrons undergo a reentrant
oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the
difference between the Doppler and average vibrational kinetic energies:

$$20 \quad \bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

the total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}$$

861

$$E_T = - \left\{ \frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{c_1 c_2 e^2}{4\pi\epsilon_0 b^3}}}{m_e c^2}} \right] - n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\} + E(AO)$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond, and $E(AO)$ is the energy of the at least one atomic orbital or hybridized atomic orbital or said orbital or hybridized orbital and the energy change with the formation of the bond by one or more of the orbital electrons to match the energies of the said orbital and the MO.

- 10 97. The system of claim 96 wherein $E(AO)$, the energy of the at least one atomic orbital or hybridized atomic orbital or said orbital or hybridized orbital and the energy change with the formation of the bond by one or more of the orbital electrons to match the energies of the said orbital and the MO, is at least one from the group of

$$E(AO) = E(O2p \text{ shell}) = -E(\text{ionization}; O) = -13.6181 \text{ eV};$$

$$15 \quad E(AO) = E(N2p \text{ shell}) = -E(\text{ionization}; N) = -14.53414 \text{ eV};$$

$$E(AO) = E(C, 2sp^3) = -14.63489 \text{ eV};$$

$$E_T(AO) = E_{\text{Coulomb}}(Cl, 3sp^3) = -14.60295 \text{ eV};$$

$$E_T(AO) = E(\text{ionization}; C) + E(\text{ionization}; C^+);$$

$$E_T(AO) = E(C_{\text{ethane}}, 2sp^3) = -15.35946 \text{ eV};$$

$$20 \quad E_T(AO) = +E(C_{\text{ethylene}}, 2sp^3) - E(C_{\text{ethylene}}, 2sp^3);$$

$$E_T(AO) = E(C, 2sp^3) - 2E_T(C=C, 2sp^3) = -14.63489 \text{ eV} - (-2.26758 \text{ eV});$$

$$E_T(AO) = E(C_{\text{acetylene}}, 2sp^3) - E(C_{\text{acetylene}}, 2sp^3) - E(C_{\text{acetylene}}, 2sp^3) = 16.20002 \text{ eV};$$

$$E_T(AO) = E(C, 2sp^3) - 2E_T(C \equiv C, 2sp^3) = -14.63489 \text{ eV} - (-3.13026 \text{ eV});$$

$$E_T(AO) = E(C_{\text{benzene}}, 2sp^3) - E(C_{\text{benzene}}, 2sp^3);$$

862

$E_T(AO) = E(C, 2sp^3) - E_T(C = C, 2sp^3) = -14.63489 \text{ eV} - (-1.13379 \text{ eV})$, and

$E_T(AO) = E(C_{alkane}, 2sp^3) = -15.56407 \text{ eV}$, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

5

98. The system of claim 97 wherein $c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2k' a}} = \sqrt{\frac{aa_0}{2k'}}$ is substituted into E_T to give

$$\begin{aligned} E_T &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] + E(AO) \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + E(AO) \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2k'}}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2k'}}}{a - \sqrt{\frac{aa_0}{2k'}}} - 1 \right] + E(AO) \end{aligned}$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond, and $E(AO)$ is the energy of the at least one atomic orbital or hybridized atomic orbital to which the MO is energy matched.

99. The system of claim 98 wherein $E(\text{basis energies})$ is given by the sum of a first integer q_1 times the total energy of H_2 and a second integer q_2 times the total energy of H , minus a third integer q_3 times the total energy of $E(AO)$ where the first integer can be 1, 2, 3..., and each of the second and third integers can be 0, 1, 2, 3....

100. The system of claim 99 wherein E_T is set equal to $E(\text{basis energies})$, and the semimajor axis a is solved.

101. The system of claim 100 wherein the semimajor axis a is solved from the equation of the form:

$$\begin{aligned}
 E_T &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] + E(AO) \\
 &= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + E(AO) \\
 &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2k'}}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2k'}}}{a - \sqrt{\frac{aa_0}{2k'}}} - 1 \right] + E(AO) = E(\text{basis energies})
 \end{aligned}$$

102. The system of claims 101 and 63 wherein the distance from the origin of the H_2 -type-
5 ellipsoidal-MO to each focus c' , the internuclear distance $2c'$, and the length of the
semiminor axis of the prolate spheroidal H_2 -type MO $b = c$ are solved from the semimajor
axis a .

103. The system of claims 102 where the number of equivalent bonds of the MO n_1 each
10 comprising an H_2 -type ellipsoidal MO is an integer greater than one.

104. The system of claims 103 where the fraction c_1 of a H_2 -type ellipsoidal MO is 1.
0.75, 0.5, and 0.75/2.

15 105. The system of claims 104 where the factor c_2 of a H_2 -type ellipsoidal MO is given
by one of the list of 1 and the ratio of the ionization energy of at least one atom of the bond
and 13.605804 eV, the Coulombic energy between the electron and proton of H to meet the
equipotential condition of the union of the H_2 -type-ellipsoidal-MO and the AO of the atom,
wherein the calculated and measured values and constants recited in the equations herein can
20 be adjusted, for example, up to $\pm 10\%$, if desired.

106. The system of claim 105 where the factor c_2 of a H_2 -type ellipsoidal MO is
0.936127, the ratio of the ionization energy of N 14.53414 eV and 13.605804 eV,
the Coulombic energy between the electron and proton of H ;

864

0.91771, the ratio of 14.82575 eV , $E_{\text{Coulomb}}(C, 2sp^3)$ and 13.605804 eV , the Coulombic energy between the electron and proton of H ;

0.93172, the ratio of 14.60295 eV , $E_{\text{Coulomb}}(Cl, 3sp^3)$ given by Eq. (13.759), and 13.605804 eV , the Coulombic energy between the electron and proton of H ;

5 0.87495, the ratio of 15.55033 eV , $E_{\text{Coulomb}}(C_{\text{ethane}}, 2sp^3)$ and 13.605804 eV , the Coulombic energy between the electron and proton of H ;

0.85252, the ratio of 15.95955 eV , $E_{\text{Coulomb}}(C_{\text{ethylene}}, 2sp^3)$ and 13.605804 eV , the Coulombic energy between the electron and proton of H ;

0.85252, the ratio of 15.95955 eV , $E_{\text{Coulomb}}(C_{\text{benzene}}, 2sp^3)$ and 13.605804 eV , the
10 Coulombic energy between the electron and proton of H , or

0.86359, the ratio of 15.55033 eV , $E_{\text{Coulomb}}(C_{\text{alkane}}, 2sp^3)$ and 13.605804 eV , the Coulombic energy between the electron and proton of H , wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

15

107. The system of claim 106 where the fraction c_1 of a H_2 -type ellipsoidal MO is such that the orbital energy E for each elliptical cross section of the prolate spheroidal MO is given by the sum of the kinetic T and potential V energies; $E = T + V$ is constant; the orbit is closed such that $T < |V|$; the time average of the kinetic energy, $\langle T \rangle$, for elliptic motion
20 in an inverse-squared field is $1/2$ that of the time average of the magnitude of the potential energy, $|\langle V \rangle|$ ($\langle T \rangle = 1/2 |\langle V \rangle|$), and in the case that the energy of the MO is matched to at least one atomic orbital (AO), $E = T + V$, and for all points on the AO, $|E| = T = 1/2 |V|$.

108. The system of claim 107 where the energy of the MO is matched to at least one
25 atomic orbital (AO) such that $E = T + V$, and for all points on the AO, $|E| = T = 1/2 |V|$.

109. A system of claim 108 of computing and rendering the nature of bound atomic and atomic ionic electrons from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations
30 using a constraint that the bound electron(s) does not radiate under acceleration, comprising:

865

processing means for processing and solving the equations for charge, mass, and current density functions of electron(s) in a selected atom or ion, wherein the equations are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration; and

- 5 a display in communication with the processing means for displaying the current and charge density representation of the electron(s) of the selected atom or ion.

110. The system of claim 109 wherein the physical, Maxwellian solutions of the charge,
10 mass, and current density functions of atoms and atomic ions comprises a solution of the classical wave equation $\left[\nabla^2 - \frac{1}{v^2} \frac{\delta^2}{\delta t^2} \right] \rho(r, \theta, \phi, t) = 0$ which is the equation of motion of the charge.

111. The system of claim 110, wherein the time, radial, and angular solutions of the wave
15 equation are separable.

112. The system of claim 111, wherein the boundary constraint of the wave equation solution is nonradiation according to Maxwell's equations.

20 113. The system of claim 112, wherein a radial function that satisfies the boundary condition is a radial delta function

$$f(r) = \frac{1}{r^2} \delta(r - r_n).$$

114. The system of claim 113, wherein the boundary condition is met for a time harmonic
25 function when the relationship between an allowed radius and the electron wavelength is given by

$$2\pi r_n = \lambda_n,$$

$$\omega = \frac{\hbar}{m_e r^2}, \text{ and}$$

$$v = \frac{\hbar}{m_e r}$$

866

where ω is the angular velocity of each point on the electron surface, v is the velocity of each point on the electron surface, and r is the radius of the electron.

115. The system of claim 114, wherein the spin function is given by the uniform function

5 $Y_0^0(\phi, \theta)$ comprising angular momentum components of $L_{xy} = \frac{\hbar}{4}$ and $L_z = \frac{\hbar}{2}$.

116. The system of claim 115, wherein the atomic and atomic ionic charge and current density functions of bound electrons are described by a charge-density (mass-density) function which is the product of a radial delta function, two angular functions (spherical
10 harmonic functions), and a time harmonic function:

$$\rho(r, \theta, \phi, t) = f(r)A(\theta, \phi, t) = \frac{1}{r^2} \delta(r - r_n)A(\theta, \phi, t); \quad A(\theta, \phi, t) = Y(\theta, \phi)k(t)$$

wherein the spherical harmonic functions correspond to a traveling charge density wave confined to the spherical shell which gives rise to the phenomenon of orbital angular momentum.

15

117. The system of claim 116, wherein based on the radial solution, the angular charge and current-density functions of the electron, $A(\theta, \phi, t)$, must be a solution of the wave equation in two dimensions (plus time),

$$\left[\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] A(\theta, \phi, t) = 0$$

20 where $\rho(r, \theta, \phi, t) = f(r)A(\theta, \phi, t) = \frac{1}{r^2} \delta(r - r_n)A(\theta, \phi, t)$ and $A(\theta, \phi, t) = Y(\theta, \phi)k(t)$

$$\left[\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right)_{r, \phi} + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\partial^2}{\partial \phi^2} \right)_{r, \theta} - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] A(\theta, \phi, t) = 0$$

where v is the linear velocity of the electron.

118. The system of claim 117, wherein the charge-density functions including the time-
25 function factor are

$$l = 0$$

867

$$\rho(r, \theta, \phi, t) = \frac{e}{8\pi r^2} [\delta(r - r_n)] [Y_0^0(\theta, \phi) + Y_l^m(\theta, \phi)]$$

 $l \neq 0$

$$5 \quad \rho(r, \theta, \phi, t) = \frac{e}{4\pi r^2} [\delta(r - r_n)] [Y_0^0(\theta, \phi) + \text{Re}\{Y_l^m(\theta, \phi)e^{i\omega_n t}\}]$$

where $Y_l^m(\theta, \phi)$ are the spherical harmonic functions that spin about the z-axis with angular frequency ω_n with $Y_0^0(\theta, \phi)$ the constant function

$$\text{Re}\{Y_l^m(\theta, \phi)e^{i\omega_n t}\} = P_l^m(\cos\theta)\cos(m\phi + \omega_n t) \text{ where to keep the form of the spherical}$$

10 harmonic as a traveling wave about the z-axis, $\omega_n = m\omega_n$.

119. The system of claim 118, wherein the spin and angular moment of inertia, I, angular momentum, L, and energy, E, for quantum number l are given by

$$l = 0$$

15

$$I_z = I_{spin} = \frac{m_e r_n^2}{2}$$

$$L_z = I\omega_z = \pm \frac{\hbar}{2}$$

$$E_{rotational} = E_{rotational, spin} = \frac{1}{2} \left[I_{spin} \left(\frac{\hbar}{m_e r_n^2} \right)^2 \right] = \frac{1}{2} \left[\frac{m_e r_n^2}{2} \left(\frac{\hbar}{m_e r_n^2} \right)^2 \right] = \frac{1}{4} \left[\frac{\hbar^2}{2I_{spin}} \right]$$

20

$$l \neq 0$$

$$I_{orbital} = m_e r_n^2 \left[\frac{l(l+1)}{l^2 + l + 1} \right]^{\frac{1}{2}}$$

$$L_z = m\hbar$$

$$L_{z \text{ total}} = L_{z \text{ spin}} + L_{z \text{ orbital}}$$

$$E_{\text{rotational, orbital}} = \frac{\hbar^2}{2I} \left[\frac{\ell(\ell+1)}{\ell^2 + 2\ell + 1} \right]$$

$$T = \frac{\hbar^2}{2m_e r_n^2}$$

$$\langle E_{\text{rotational, orbital}} \rangle = 0.$$

- 5 120. The system of claim 119, wherein the force balance equation for one-electron atoms and ions is

$$\frac{m_e v_1^2}{4\pi r_1^2} = \frac{e Ze}{4\pi r_1^2 4\pi \epsilon_0 r_1^2} - \frac{1}{4\pi r_1^2} \frac{\hbar^2}{m_p r_n^3}$$

$$r_1 = \frac{a_H}{Z}$$

where a_H is the radius of the hydrogen atom.

10

121. The system of claim 120, wherein from Maxwell's equations, the potential energy V , kinetic energy T , electric energy or binding energy E_{ele} are

$$V = \frac{-Ze^2}{4\pi \epsilon_0 r_1} = \frac{-Z^2 e^2}{4\pi \epsilon_0 a_H} = -Z^2 \times 4.3675 \times 10^{-18} \text{ J} = -Z^2 \times 27.2 \text{ eV}$$

$$T = \frac{Z^2 e^2}{8\pi \epsilon_0 a_H} = Z^2 \times 13.59 \text{ eV}$$

15 $T = E_{ele} = -\frac{1}{2} \epsilon_0 \int_0^{\eta} E^2 dv \text{ where } E = -\frac{Ze}{4\pi \epsilon_0 r^2}$

$$E_{ele} = -\frac{Z^2 e^2}{8\pi \epsilon_0 a_H} = -Z^2 \times 2.1786 \times 10^{-18} \text{ J} = -Z^2 \times 13.598 \text{ eV},$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

20

122. The system of claim 121, wherein the force balance equation solution of two electron atoms is a central force balance equation with the nonradiation condition is given by

$$\frac{m_e v_2^2}{4\pi r_2^2} = \frac{e (Z-1)e}{4\pi r_2^2 4\pi \epsilon_0 r_2^2} + \frac{1}{4\pi r_2^2} \frac{\hbar^2}{Z m_e r_2^3} \sqrt{s(s+1)}$$

which gives the radius of both electrons as

$$r_2 = r_1 = a_0 \left(\frac{1}{Z-1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)} \right); s = \frac{1}{2}.$$

123. The system of claim 122, wherein the ionization energy for helium, which has no
5 electric field beyond r_1 is given by

$$\text{Ionization Energy}(\text{He}) = -E(\text{electric}) + E(\text{magnetic})$$

where,

$$E(\text{electric}) = -\frac{(Z-1)e^2}{8\pi\epsilon_0 r_1}$$

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_1^3}$$

10 For $3 \leq Z$

$$\text{Ionization Energy} = -\text{Electric Energy} - \frac{1}{Z} \text{Magnetic Energy}.$$

124. The system of claim 123, wherein the electrons of multielectron atoms all exist as
orbitspheres of discrete radii which are given by r_n of the radial Dirac delta function,
15 $\delta(r - r_n)$.

125. The system of claim 124, wherein electron orbitspheres may be spin paired or
unpaired depending on the force balance which applies to each electron wherein the electron
configuration is a minimum of energy.

20

126. The system of claim 125, wherein the minimum energy configurations are given by
solutions to Laplace's equation.

127. The system of claim 126, wherein the electrons of an atom with the same principal
25 and l quantum numbers align parallel until each of the m_l levels are occupied, and then
pairing occurs until each of the m_l levels contain paired electrons.

870

128. The system of claim 127, wherein the electron configuration for one through twenty-electron atoms that achieves an energy minimum is: $1s < 2s < 2p < 3s < 3p < 4s$.
129. The system of claim 128, wherein the corresponding force balance of the central
5 Coulombic, paramagnetic, and diamagnetic forces was derived for each n-electron atom that was solved for the radius of each electron.
130. The system of claim 129, wherein the central Coulombic force is that of a point charge at the origin since the electron charge-density functions are spherically symmetrical
10 with a time dependence that is nonradiative.
131. The system of claim 130, wherein the ionization energies are obtained using the calculated radii in the determination of the Coulombic and any magnetic energies.
- 15 132. The system of claim 131, wherein the general equation for the radii of s electrons is given by

$$r_n = \frac{a_0 \left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \pm a_0 \sqrt{\frac{\left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)^2}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] E r_m \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)}}^2$$

r_m in units of a_0

where positive root must be taken in order that $r_n > 0$;

Z is the nuclear charge, n is the number of electrons,

20 r_m is the radius of the proceeding filled shell(s) given by

$$r_n = \frac{a_0 \left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \pm a_0 \sqrt{\frac{871}{2} \left(\frac{\left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)^2}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] E r_m \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \right)}$$

r_m in units of a_0

for the preceding s shell(s);

$$r_n = \frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \pm a_0 \sqrt{\frac{1}{2} \left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)}$$

r_3 in units of a_0

for the 2p shell, and

$$r_n = \frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} \pm a_0 \sqrt{\frac{1}{2} \left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} \right) r_{12} \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} \right)}$$

r_{12} in units of a_0

for the 3p shell;

the parameter A corresponds to the diamagnetic force, $F_{\text{diamagnetic}}$:

$$\mathbf{F}_{\text{diamagnetic}} = -\frac{\hbar^2}{4m_e r_3^2 r_1} \sqrt{s(s+1)} \mathbf{i}_r;$$

the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{\text{mag } 2}$:

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_1 r_4^2} \sqrt{s(s+1)} \mathbf{i}_r;$$

the parameter C corresponds to the diamagnetic force, $\mathbf{F}_{\text{diamagnetic } 3}$:

$$5 \quad \mathbf{F}_{\text{diamagnetic } 3} = -\frac{1}{Z} \frac{8\hbar^2}{m_e r_{11}^3} \sqrt{s(s+1)} \mathbf{i}_r;$$

the parameter D corresponds to the paramagnetic force, \mathbf{F}_{mag} :

$$\mathbf{F}_{\text{mag}} = \frac{1}{4\pi r_2^2} \frac{1}{Z} \frac{\hbar^2}{m_e r^3} \sqrt{s(s+1)}, \text{ and}$$

the parameter E corresponds to the diamagnetic force, $\mathbf{F}_{\text{diamagnetic } 2}$, due to a relativistic effect

with an electric field for $r > r_n$:

$$10 \quad \mathbf{F}_{\text{diamagnetic } 2} = -\left[\frac{Z-3}{Z-2} \right] \frac{r_1 \hbar^2}{m_e r_3^4} 10\sqrt{3/4} \mathbf{i}_r,$$

$$\mathbf{F}_{\text{diamagnetic } 2} = -\left[\frac{Z-11}{Z-10} \right] \left(1 + \frac{\sqrt{2}}{2} \right) \frac{r_{10} \hbar^2}{m_e r_{11}^4} 10\sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{\text{diamagnetic } 2} = -\left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{2}}{2} + \frac{1}{2} \right) \frac{r_{18} \hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)} \mathbf{i}_r.$$

873

wherein the parameters of atoms filling the 1s, 2s, 3s, and 4s orbitals are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of s Electrons (s state)	Diamag. Force Factor <i>A</i>	Paramag. Force Factor <i>B</i>	Diamag. Force Factor <i>C</i>	Param. ag. Force Factor <i>D</i>	Diamag. Force Factor <i>E</i>
Neutral Atom <i>H</i>	$1s^1$ 1 e	$^2S_{1/2}$	\uparrow 1s	0	0	0	0	0
Neutral Atom <i>He</i>	$1s^2$ 2 e	1S_0	$\uparrow \downarrow$ 1s	0	0	0	1	0
Neutral Atom <i>Li</i>	$2s^1$ 3 e	$^2S_{1/2}$	\uparrow 2s	1	0	0	0	0
Neutral Atom <i>Be</i>	$2s^2$ 4 e	1S_0	$\uparrow \downarrow$ 2s	1	0	0	1	0
Neutral Atom <i>Na</i>	$1s^2 2s^2 2p^6 3s^1$ 11 e	$^2S_{1/2}$	\uparrow 3s	1	0	8	0	0
Neutral Atom <i>Mg</i>	$1s^2 2s^2 2p^6 3s^2$ 12 e	1S_0	$\uparrow \downarrow$ 3s	1	3	12	1	0
Neutral Atom	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$ 19 e	$^2S_{1/2}$	\uparrow 4s	2	0	12	0	0

874

K

Neutral	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	1S_0	$\uparrow \downarrow$						
20 e			4s	1	3	24	1	0	
Atom									
<i>Ca</i>									
1 e Ion	$1s^1$	$^2S_{1/2}$	\uparrow						
			1s	0	0	0	0	0	
2 e Ion	$1s^2$	1S_0	$\uparrow \downarrow$						
			1s	0	0	0	1	0	
3 e Ion	$2s^1$	$^2S_{1/2}$	\uparrow						
			2s	1	0	0	0	1	
4 e Ion	$2s^2$	1S_0	$\uparrow \downarrow$						
			2s	1	0	0	1	1	
11 e Ion	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	\uparrow						
			3s	1	4	8	0	$1 + \frac{\sqrt{2}}{2}$	
12 e Ion	$1s^2 2s^2 2p^6 3s^2$	1S_0	$\uparrow \downarrow$						
			3s	1	6	0	0	$1 + \frac{\sqrt{2}}{2}$	
19 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	\uparrow						
			4s	3	0	24	0	$2 - \sqrt{2}$	
20 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	1S_0	$\uparrow \downarrow$						
			4s	2	0	24	0	$2 - \sqrt{2}$	

133. The system of claim 132, with the radii, r_n , wherein the ionization energy for atoms having an outer s-shell are given by the negative of the electric energy, $E(\text{electric})$, given by:

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{875 (Z - (n-1))e^2}{8\pi\epsilon_0 r_n}$$

except that minor corrections due to the magnetic energy must be included in cases wherein the s electron does not couple to p electrons as given by

$$\text{Ionization Energy}(\text{He}) = -E(\text{electric}) + E(\text{magnetic}) \left(1 - \frac{1}{2} \left(\left(\frac{2}{3} \cos \frac{\pi}{3} \right)^2 + \alpha \right) \right)$$

$$5 \quad \text{Ionization Energy} = -\text{Electric Energy} - \frac{1}{Z} \text{Magnetic Energy}$$

$$\begin{aligned} E(\text{ionization}; \text{Li}) &= \frac{(Z-2)e^2}{8\pi\epsilon_0 r_3} + \Delta E_{\text{mag}} \\ &= 5.3178 \text{ eV} + 0.0860 \text{ eV} = 5.4038 \text{ eV} \end{aligned}$$

$$E(\text{Ionization}) = E(\text{Electric}) + E_T$$

$$\begin{aligned} E(\text{ionization}; \text{Be}) &= \frac{(Z-3)e^2}{8\pi\epsilon_0 r_4} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_4^3} + \Delta E_{\text{mag}}, \text{ and} \\ &= 8.9216 \text{ eV} + 0.03226 \text{ eV} + 0.33040 \text{ eV} = 9.28430 \text{ eV} \end{aligned}$$

$$E(\text{Ionization}) = -\text{Electric Energy} - \frac{1}{Z} \text{Magnetic Energy} - E_T, \text{ wherein the calculated and}$$

10 measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

134. The system of claim 133, wherein the radii and energies of the 2p electrons are solved using the forces given by

$$15 \quad \mathbf{F}_{\text{ele}} = \frac{(Z-n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic}} = -\sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

876

$$\mathbf{F}_{diamagnetic\ 2} = - \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{m_e r_n^4} 10 \sqrt{s(s+1)} \mathbf{i}_r,$$

and the radii r_3 are given by

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right) \right) \left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)}{\pm a_0 \left(\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2 + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2 + 4 \left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)} \right)} \cdot 2$$

r_1 in units of a_0

5 135. The system of claim 134, wherein the electric energy given by

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_n}$$

gives the corresponding ionization energies.

136. The system of claim 135, wherein for each n-electron atom having a central charge of
10 Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right];$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{3}}{4Z} \right) \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right)} \pm a_0 \frac{\left(1 - \frac{\sqrt{3}}{4Z} \right)^2 + 4 \left[\frac{Z-3}{Z-2} \right] r_1 10 \frac{\sqrt{3}}{4}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right)^2 + \left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right)}$$

r_1 in units of a_0

and $n-4$ electrons in an orbitsphere with radius r_n given by

$$r_n = \frac{a_0 \left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right) \pm a_0 \left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2 + 20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{2} ;$$

r_3 in units of a_0

the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $F_{\text{diamagnetic}}$:

878

$$\mathbf{F}_{\text{diamagnetic}} = - \sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r;$$

and the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{\text{mag } 2}$:

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r,$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$5 \quad \mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

879

wherein the parameters of five through ten-electron atoms are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of 2p Electrons (2p state)	Diamagnetic Force Factor A		Paramagnetic Force Factor B	
Neutral 5 e Atom <i>B</i>	$1s^2 2s^2 2p^1$	$^2P_{1/2}^0$	$\begin{array}{ccc} \uparrow & _ & _ \\ 1 & 0 & -1 \end{array}$	2		0	
Neutral 6 e Atom <i>C</i>	$1s^2 2s^2 2p^2$	3P_0	$\begin{array}{ccc} \uparrow & \uparrow & _ \\ 1 & 0 & -1 \end{array}$	$\frac{2}{3}$		0	
Neutral 7 e Atom <i>N</i>	$1s^2 2s^2 2p^3$	$^4S_{3/2}^0$	$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{1}{3}$		1	
Neutral 8 e Atom <i>O</i>	$1s^2 2s^2 2p^4$	3P_2	$\begin{array}{ccc} \uparrow \downarrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	1		2	
Neutral 9 e Atom <i>F</i>	$1s^2 2s^2 2p^5$	$^2P_{3/2}^0$	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{2}{3}$		3	
Neutral 10 e Atom <i>Ne</i>	$1s^2 2s^2 2p^6$	1S_0	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow \\ 1 & 0 & -1 \end{array}$	0		3	
5 e Ion	$1s^2 2s^2 2p^1$	$^2P_{1/2}^0$	$\begin{array}{ccc} \uparrow & _ & _ \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$		1	
6 e Ion	$1s^2 2s^2 2p^2$	3P_0	$\begin{array}{ccc} \uparrow & \uparrow & _ \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$		4	
7 e Ion	$1s^2 2s^2 2p^3$	$^4S_{3/2}^0$	$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$		6	

880

8 e Ion	$1s^2 2s^2 2p^4$	3P_2	$\frac{\uparrow \downarrow}{1}$	$\frac{\uparrow}{0}$	$\frac{\uparrow}{-1}$	$\frac{5}{3}$	6
9 e Ion	$1s^2 2s^2 2p^5$	$^2P_{3/2}^0$	$\frac{\uparrow \downarrow}{1}$	$\frac{\uparrow \downarrow}{0}$	$\frac{\uparrow}{-1}$	$\frac{5}{3}$	9
10 e Ion	$1s^2 2s^2 2p^6$	1S_0	$\frac{\uparrow \downarrow}{1}$	$\frac{\uparrow \downarrow}{0}$	$\frac{\uparrow \downarrow}{-1}$	$\frac{5}{3}$	12

137. The system of claim 136, wherein the ionization energy for the boron atom is given by

$$E(\text{ionization}; B) = \frac{(Z-4)e^2}{8\pi\epsilon_0 r_5} + \Delta E_{\text{mag}}$$

$$= 8.147170901 \text{ eV} + 0.15548501 \text{ eV} = 8.30265592 \text{ eV}$$

5 wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

138. The system of claim 137, wherein the ionization energies for the n-electron atoms
10 having the radii, r_n , are given by the negative of the electric energy, $E(\text{electric})$, given by

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_n}$$

139. The system of claim 138, wherein the radii of the 3p electrons are given using the forces given by

$$15 \quad F_{\text{ele}} = \frac{(Z-n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$F_{\text{diamagnetic}} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$F_{\text{diamagnetic}} = -\left(\frac{2}{3} + \frac{2}{3} + \frac{1}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = -\left(\frac{5}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = (4+4+4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$5 \quad \mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

and the radii r_{12} are given by

$$r_{12} = \frac{a_0 \left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right) \pm a_0 \left(\frac{1}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)^2} + \frac{20\sqrt{3} \left(\left[\frac{Z-12}{Z-11} \right] \left(1 + \frac{\sqrt{2}}{2} \right) r_{10} \right)}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} \right)}{2}$$

r_{10} in units of a_0

140. The system of claim 139, wherein the ionization energies are given by electric energy
10 given by:

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z - (n-1))e^2}{8\pi\epsilon_0 r_n}.$$

141. The system of claim 140, wherein for each n-electron atom having a central charge of
Z times that of the proton and an electron configuration $1s^2 2s^2 2p^6 3s^2 3p^{n-12}$, there are two
15 indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right]$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right) \right)^2 + 4 \left(\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2 + \left(\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2 + 4 \left(\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}} \right)^2 \right)}{2}$$

r_1 in units of a_0

5 three sets of paired indistinguishable electrons in an orbitsphere with radius r_{10} given by:

$$r_{10} = \frac{\left(\frac{a_0}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2 \pm a_0 \left(\frac{1}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2 + 20\sqrt{3} \left(\left[\frac{Z-10}{Z-9} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)^2 + \left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)^2}{2}$$

r_3 in units of a_0

two indistinguishable spin-paired electrons in an orbitsphere with radius r_{12} given by:

$$r_{12} = \frac{\frac{a_0}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z}\right) \frac{\sqrt{3}}{r_{10}}\right)} \pm a_0}{2} \sqrt{\frac{1}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z}\right) \frac{\sqrt{3}}{r_{10}}\right)} + \frac{20\sqrt{3} \left[\frac{Z-12}{Z-11}\right] \left(1 + \frac{\sqrt{2}}{2}\right) r_{10}}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z}\right) \frac{\sqrt{3}}{r_{10}}\right)}}$$

r_{10} in units of a_0

and $n-12$ electrons in a 3p orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z}\right) \frac{\sqrt{3}}{r_{12}}\right)} \pm a_0}{2} \sqrt{\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z}\right) \frac{\sqrt{3}}{r_{12}}\right)} + \frac{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)}\right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2}\right) r_{12}}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z}\right) \frac{\sqrt{3}}{r_{12}}\right)}}$$

r_{12} in units of a_0

5 where the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $F_{\text{diamagnetic}}$:

$$F_{\text{diamagnetic}} = -\sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and the parameter } B \text{ corresponds to the}$$

paramagnetic force, $F_{\text{mag } 2}$:

$$F_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$10 \quad F_{\text{mag } 2} = (4+4+4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

885

wherein the parameters of thirteen through eighteen-electron atoms are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of 3p Electrons (3p state)	Diamagnetic Force Factor A	Paramagnetic Force Factor B
Neutral 13 e Atom <i>Al</i>	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P_{1/2}^0$	$\begin{array}{ccc} \uparrow & \underline{\quad} & \underline{\quad} \\ 1 & 0 & -1 \end{array}$	$\frac{11}{3}$	0
Neutral 14 e Atom <i>Si</i>	$1s^2 2s^2 2p^6 3s^2 3p^2$	3P_0	$\begin{array}{ccc} \uparrow & \uparrow & \underline{\quad} \\ 1 & 0 & -1 \end{array}$	$\frac{7}{3}$	0
Neutral 15 e Atom <i>P</i>	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S_{3/2}^0$	$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	2
Neutral 16 e Atom <i>S</i>	$1s^2 2s^2 2p^6 3s^2 3p^4$	3P_2	$\begin{array}{ccc} \uparrow \downarrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{4}{3}$	1
Neutral 17 e Atom <i>Cl</i>	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P_{3/2}^0$	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{2}{3}$	2
Neutral 18 e Atom <i>Ar</i>	$1s^2 2s^2 2p^6 3s^2 3p^6$	1S_0	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow \\ 1 & 0 & -1 \end{array}$	$\frac{1}{3}$	4
13 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P_{1/2}^0$	$\begin{array}{ccc} \uparrow & \underline{\quad} & \underline{\quad} \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	12

886

14 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^2$	3P_0	$\frac{\uparrow}{1}$	$\frac{\uparrow}{0}$	$\frac{\quad}{-1}$	$\frac{1}{3}$	16
15 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S_{3/2}^0$	$\frac{\uparrow}{1}$	$\frac{\uparrow}{0}$	$\frac{\uparrow}{-1}$	0	24
16 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^4$	3P_2	$\frac{\uparrow \downarrow}{1}$	$\frac{\uparrow}{0}$	$\frac{\uparrow}{-1}$	$\frac{1}{3}$	24
17 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P_{3/2}^0$	$\frac{\uparrow \downarrow}{1}$	$\frac{\uparrow \downarrow}{0}$	$\frac{\uparrow}{-1}$	$\frac{2}{3}$	32
18 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6$	1S_0	$\frac{\uparrow \downarrow}{1}$	$\frac{\uparrow \downarrow}{0}$	$\frac{\uparrow \downarrow}{-1}$	0	40

142. The system of claim 141, wherein the ionization energies for the n-electron 3p atoms are given by electric energy given by:

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z - (n - 1))e^2}{8\pi\epsilon_0 r_n}.$$

5

143. The system of claim 142, wherein the ionization energy for the aluminum atom is given by

$$\begin{aligned} E(\text{ionization}; Al) &= \frac{(Z - 12)e^2}{8\pi\epsilon_0 r_{13}} + \Delta E_{\text{mag}} \\ &= 5.95270 \text{ eV} + 0.031315 \text{ eV} = 5.98402 \text{ eV} \end{aligned}$$

10 wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

144. A system of claim 1 of computing the nature of bound atomic and atomic ionic electrons from physical solutions of the charge, mass, and current density functions of atoms

887

and atomic ions, which solutions are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration, comprising:

processing means for processing and solving the equations for charge, mass, and current density functions of electron(s) in selected atoms or ions, wherein the equations are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration; and

output means for outputting the solutions of the charge, mass, and current density functions of the atoms and atomic ions.

10 145. A system of claim 1 comprising the steps of;

a.) inputting electron functions that are derived from Maxwell's equations using a constraint that the bound electron(s) does not radiate under acceleration;

b.) inputting a trial electron configuration;

c.) inputting the corresponding centrifugal, Coulombic, diamagnetic and paramagnetic forces,

d.) forming the force balance equation comprising the centrifugal force equal to the sum of the Coulombic, diamagnetic and paramagnetic forces;

e.) solving the force balance equation for the electron radii;

f.) calculating the energy of the electrons using the radii and the corresponding electric and magnetic energies;

g.) repeating Steps a-f for all possible electron configurations, and

h.) outputting the lowest energy configuration and the corresponding electron radii for that configuration.

25 146. The system of claim 145, wherein the output is rendered using the electron functions.

147. The system of claim 146, wherein the electron functions are given by at least one of the group comprising:

30 $l = 0$

$$\rho(r, \theta, \phi, t) = \frac{e}{8\pi r^2} [\delta(r - r_n)] [Y_0^0(\theta, \phi) + Y_l^m(\theta, \phi)]$$

$$l \neq 0$$

$$\rho(r, \theta, \phi, t) = \frac{e}{4\pi r^2} [\delta(r - r_n)] \left[Y_0^0(\theta, \phi) + \text{Re} \{ Y_l^m(\theta, \phi) e^{i\omega_n t} \} \right]$$

5

where $Y_l^m(\theta, \phi)$ are the spherical harmonic functions that spin about the z-axis with angular frequency ω_n with $Y_0^0(\theta, \phi)$ the constant function. $\text{Re} \{ Y_l^m(\theta, \phi) e^{i\omega_n t} \} = P_l^m(\cos \theta) \cos(m\phi + \omega_n t)$ where to keep the form of the spherical harmonic as a traveling wave about the z-axis, $\omega_n = m\omega_n$.

10 148. The system of claim 147, wherein the forces are given by at least one of the group comprising:

$$\mathbf{F}_{ele} = \frac{(Z - n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$\mathbf{F}_{ele} = \frac{(Z - (n - 1))e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^2} \frac{1}{Z} \frac{\hbar^2}{m_e r^3} \sqrt{s(s+1)}$$

$$15 \quad \mathbf{F}_{diamagnetic} = -\frac{\hbar^2}{4m_e r_3^2 r_1} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic} = -\left(\frac{2}{3} + \frac{2}{3} + \frac{1}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = -\left(\frac{5}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-3}{Z-2}\right] \frac{r_1 \hbar^2}{m_e r_3^4} 10\sqrt{3/4} \mathbf{i}_r$$

$$20 \quad \mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-n}{Z-(n-1)}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)} \mathbf{i}_r$$

889

$$\mathbf{F}_{\text{diamagnetic } 2} = -\left[\frac{Z-11}{Z-10}\right]\left(1+\frac{\sqrt{2}}{2}\right)\frac{r_{10}\hbar^2}{m_e r_{11}^4}10\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic } 2} = -\left[\frac{Z-n}{Z-(n-1)}\right]\left(1-\frac{\sqrt{2}}{2}+\frac{1}{2}-\frac{\sqrt{2}}{2}+\frac{1}{2}\right)\frac{r_{18}\hbar^2}{m_e r_n^4}10\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic } 3} = -\frac{1}{Z}\frac{8\hbar^2}{m_e r_{11}^3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$5 \quad \mathbf{F}_{\text{mag } 2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_3}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_1 r_4^2}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{\text{mag } 2} = (4+4+4)\frac{1}{Z}\frac{\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r = \frac{1}{Z}\frac{12\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z}\frac{4\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r, \text{ and}$$

$$10 \quad \mathbf{F}_{\text{mag } 2} = \frac{1}{Z}\frac{8\hbar^2}{m_e r_n^2 r_{12}}\sqrt{s(s+1)}\mathbf{i}_r$$

149. The system of claim 148, wherein the radii are given by at least one of the group comprising:

$$r_1 = r_2 = a_o \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right]$$

890

$$r_4 = r_3 = \frac{a_0 \left(1 - \frac{\sqrt{3}}{4} \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right) \pm a_0} \pm a_0 \frac{\left(1 - \frac{\sqrt{3}}{4} \right)^2}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right)^2 + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \frac{\sqrt{3}}{4}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right)}}$$

2

 r_1 in units of a_0

$$r_n = \frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right) \pm a_0} \pm a_0 \frac{\left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2}{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 + \left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)}$$

2

 r_3 in units of a_0

$$r_{10} = \frac{a_0}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right) \pm a_0} \pm a_0 \frac{\left(\frac{1}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2}{20\sqrt{3} \left[\frac{Z-10}{Z-9} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 + \left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)}$$

2

 r_3 in units of a_0

$$r_{11} = \frac{a_0 \left(1 + \frac{8}{Z} \sqrt{\frac{3}{4}} \right)}{(Z-10) - \frac{\sqrt{3}}{4r_{10}}}, r_{10} \text{ in units of } a_0$$

$$r_{12} = \frac{\frac{a_0}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} \pm a_0}{2} \left[\frac{\left(\frac{1}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} \right)^2}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} + \frac{20\sqrt{3} \left[\frac{Z-12}{Z-11} \right] \left(1 + \frac{\sqrt{2}}{2} \right) r_{10}}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} \right] r_{10} \text{ in units of } a_0$$

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} \pm a_0}{2} \left[\frac{\left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} \right)^2}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} + \frac{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} \right) r_{12}}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} \right] r_{12} \text{ in units of } a_0$$

$$r_n = \frac{\frac{a_0 \left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \pm a_0}{2} \left[\frac{\left(\frac{1 + (C-D) \frac{\sqrt{3}}{2Z}}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \right)^2}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} + \frac{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] E r_m}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \right] r_m \text{ in units of } a_0$$

150. The system of claim 149, wherein the electric energy of each electron of radius r_n is given by at least one of the group comprising:

$$E(\text{electric}) = -\frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_n}$$

$$5 \quad \text{Ionization Energy}(\text{He}) = -E(\text{electric}) + E(\text{magnetic}) \left(1 - \frac{1}{2} \left(\left(\frac{2}{3} \cos \frac{\pi}{3} \right)^2 + \alpha \right) \right)$$

$$\text{Ionization Energy} = -\text{Electric Energy} - \frac{1}{Z} \text{Magnetic Energy}$$

$$E(\text{Ionization}) = -\text{Electric Energy} - \frac{1}{Z} \text{Magnetic Energy} - E_T$$

$$E(\text{ionization; Li}) = \frac{(Z-2)e^2}{8\pi\epsilon_0 r_3} + \Delta E_{\text{mag}}$$

$$= 5.3178 \text{ eV} + 0.0860 \text{ eV} = 5.4038 \text{ eV}$$

$$E(\text{ionization; B}) = \frac{(Z-4)e^2}{8\pi\epsilon_0 r_5} + \Delta E_{\text{mag}}$$

$$= 8.147170901 \text{ eV} + 0.15548501 \text{ eV} = 8.30265592 \text{ eV}$$

$$10 \quad E(\text{ionization; Be}) = \frac{(Z-3)e^2}{8\pi\epsilon_0 r_4} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_4^3} + \Delta E_{\text{mag}}$$

$$= 8.9216 \text{ eV} + 0.03226 \text{ eV} + 0.33040 \text{ eV} = 9.28430 \text{ eV}$$

$$E(\text{ionization; Na}) = -\text{Electric Energy} = \frac{(Z-10)e^2}{8\pi\epsilon_0 r_{11}} = 5.12592 \text{ eV}$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

15 151. The system of claim 150, wherein the radii of s electrons are given by

893

$$r_n = \frac{a_0 \left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \pm a_0 \sqrt{\frac{\left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)^2}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} + \frac{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] E r_m}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)}} \quad (5)$$

r_m in units of a_0

where positive root must be taken in order that $r_n > 0$;

Z is the nuclear charge, n is the number of electrons,

r_m is the radius of the proceeding filled shell(s) given by

$$r_m = \frac{a_0 \left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} \pm a_0 \sqrt{\frac{\left(1 + (C-D) \frac{\sqrt{3}}{2Z} \right)^2}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)} + \frac{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] E r_m}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_m} \right)}} \quad (5)$$

r_m in units of a_0

for the preceding s shell(s);

$$r_n = \frac{a_0 \left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right) \pm a_0}{2} \left[\frac{1}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} + \frac{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right]^2$$

r_3 in units of a_0

for the 2p shell, and

$$r_n = \frac{a_0 \left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right) \pm a_0}{2} \left[\frac{1}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} + \frac{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} \right) r_{12}}{\left((Z - (n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_{12}} \right)} \right]^2$$

r_{12} in units of a_0

for the 3p shell;

- 5 the parameter A corresponds to the diamagnetic force, $F_{\text{diamagnetic}}$:

$$F_{\text{diamagnetic}} = -\frac{\hbar^2}{4m_e r_3^2 r_1} \sqrt{s(s+1)} \mathbf{i}_r;$$

the parameter B corresponds to the paramagnetic force, $F_{\text{mag } 2}$:

$$F_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_1 r_4^2} \sqrt{s(s+1)} \mathbf{i}_r;$$

the parameter C corresponds to the diamagnetic force, $F_{\text{diamagnetic } 3}$:

$$10 \quad F_{\text{diamagnetic } 3} = -\frac{1}{Z} \frac{8\hbar^2}{m_e r_{11}^3} \sqrt{s(s+1)} \mathbf{i}_r;$$

the parameter D corresponds to the paramagnetic force, F_{mag} :

$$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^2} \frac{1}{Z} \frac{\hbar^2}{m_e r^3} \sqrt{s(s+1)}, \text{ and}$$

the parameter E corresponds to the diamagnetic force, $\mathbf{F}_{diamagnetic\ 2}$, due to a relativistic effect with an electric field for $r > r_n$:

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-3}{Z-2}\right] \frac{r_1 \hbar^2}{m_e r_3^4} 10\sqrt{3/4} \mathbf{i}_r$$

$$5 \quad \mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-11}{Z-10}\right] \left(1 + \frac{\sqrt{2}}{2}\right) \frac{r_{10} \hbar^2}{m_e r_{11}^4} 10\sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{diamagnetic\ 2} = -\left[\frac{Z-n}{Z-(n-1)}\right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2} - \frac{\sqrt{2}}{2} + \frac{1}{2}\right) \frac{r_{18} \hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)} \mathbf{i}_r.$$

wherein the parameters of atoms filling the 1s, 2s, 3s, and 4s orbitals are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of Electrons (s state)	Dia mag. Forc e Fact or A	Para mag. Forc e Fact or B	Dia mag. Forc e Fact or C	Para mag. Forc e Fact or D	Diamag Force Factor E
Neutral Atom <i>H</i>	1s ¹ 1 e	² S _{1/2}	\uparrow 1s	0	0	0	0	0
Neutral Atom <i>He</i>	1s ² 2 e	¹ S ₀	$\uparrow \downarrow$ 1s	0	0	0	1	0
Neutral Atom <i>Li</i>	2s ¹ 3 e	² S _{1/2}	\uparrow 2s	1	0	0	0	0
Neutral Atom <i>Be</i>	2s ² 4 e	¹ S ₀	$\uparrow \downarrow$ 2s	1	0	0	1	0
Neutral Atom <i>Na</i>	1s ² 2s ² 2p ⁶ 3s ¹ 11 e	² S _{1/2}	\uparrow 3s	1	0	8	0	0
Neutral Atom <i>Mg</i>	1s ² 2s ² 2p ⁶ 3s ² 12 e	¹ S ₀	$\uparrow \downarrow$ 3s	1	3	12	1	0

					897				
Neutral	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	\uparrow						
19 e			4s	2	0	12	0	0	
Atom									
K									
Neutral	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	1S_0	$\uparrow \downarrow$						
20 e			4s	1	3	24	1	0	
Atom									
Ca									
1 e Ion	$1s^1$	$^2S_{1/2}$	\uparrow						
			1s	0	0	0	0	0	
2 e Ion	$1s^2$	1S_0	$\uparrow \downarrow$						
			1s	0	0	0	1	0	
3 e Ion	$2s^1$	$^2S_{1/2}$	\uparrow						
			2s	1	0	0	0	1	
4 e Ion	$2s^2$	1S_0	$\uparrow \downarrow$						
			2s	1	0	0	1	1	
11 e Ion	$1s^2 2s^2 2p^6 3s^1$	$^2S_{1/2}$	\uparrow						
			3s	1	4	8	0	$1 + \frac{\sqrt{2}}{2}$	
12 e Ion	$1s^2 2s^2 2p^6 3s^2$	1S_0	$\uparrow \downarrow$						
			3s	1	6	0	0	$1 + \frac{\sqrt{2}}{2}$	
19 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$	$^2S_{1/2}$	\uparrow						
			4s	3	0	24	0	$2 - \sqrt{2}$	
20 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$	1S_0	$\uparrow \downarrow$						
			4s	2	0	24	0	$2 - \sqrt{2}$	

152. The system of claim 151, with the radii, r_n , wherein the ionization energy for atoms having an outer s-shell are given by the negative of the electric energy, $E(electric)$, given by:

$$E(ionization) = -Electric\ Energy = \frac{(Z - (n-1))e^2}{8\pi\epsilon_0 r_n}$$

except that minor corrections due to the magnetic energy must be included in cases wherein
5 the s electron does not couple to p electrons as given by

$$Ionization\ Energy(He) = -E(electric) + E(magnetic) \left(1 - \frac{1}{2} \left(\left(\frac{2}{3} \cos \frac{\pi}{3} \right)^2 + \alpha \right) \right)$$

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy$$

$$\begin{aligned} E(ionization; Li) &= \frac{(Z-2)e^2}{8\pi\epsilon_0 r_3} + \Delta E_{mag} \\ &= 5.3178\ eV + 0.0860\ eV = 5.4038\ eV \end{aligned}$$

$$E(ionization) = E(Electric) + E_T$$

$$\begin{aligned} 10 \quad E(ionization; Be) &= \frac{(Z-3)e^2}{8\pi\epsilon_0 r_4} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_4^3} + \Delta E_{mag}, \text{ and} \\ &= 8.9216\ eV + 0.03226\ eV + 0.33040\ eV = 9.28430\ eV \end{aligned}$$

$$E(ionization) = -Electric\ Energy - \frac{1}{Z} Magnetic\ Energy - E_T$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

15 153. The system of claim 152, wherein the radii and energies of the 2p electrons are solved using the forces given by

$$F_{ele} = \frac{(Z-n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$F_{diamagnetic} = - \sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$F_{mag\ 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$20 \quad F_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{diamagnetic\ 2} = - \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{m_e r_n^4} 10 \sqrt{s(s+1)} \mathbf{i}_r,$$

and the radii r_3 are given by

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right) \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)} \pm a_0 \frac{\left(\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2 + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2} \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)}$$

r_1 in units of a_0

5

154. The system of claim 153, wherein the electric energy given by

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_n}$$

gives the corresponding ionization energies.

- 10 155. The system of claim 154, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right];$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right) \right)^2}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2} \pm a_0 \frac{\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right)^2}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2} + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2}$$

r_1 in units of a_0

5 and $n-4$ electrons in an orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \pm a_0}{2} \frac{\left(\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2}{20\sqrt{3} \left[\frac{Z-n}{Z-(n-1)} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3} + \frac{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z} \right) \frac{\sqrt{3}}{r_3} \right)}{2}$$

r_3 in units of a_0

901

the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $\mathbf{F}_{\text{diamagnetic}}$:

$$\mathbf{F}_{\text{diamagnetic}} = - \sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r;$$

and the parameter B corresponds to the paramagnetic force, $\mathbf{F}_{\text{mag } 2}$:

$$5 \quad \mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r,$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r.$$

902

wherein the parameters of five through ten-electron atoms are

Atom Type	Electron Configuration	Ground State Term	Orbital Arrangement of 2p Electrons (2p state)	Diamagnetic Force Factor A	Paramagnetic Force Factor B
Neutral 5 e Atom <i>B</i>	$1s^2 2s^2 2p^1$	$^2P_{1/2}^0$	$\begin{array}{ccc} \uparrow & _ & _ \\ 1 & 0 & -1 \end{array}$	2	0
Neutral 6 e Atom <i>C</i>	$1s^2 2s^2 2p^2$	3P_0	$\begin{array}{ccc} \uparrow & \uparrow & _ \\ 1 & 0 & -1 \end{array}$	$\frac{2}{3}$	0
Neutral 7 e Atom <i>N</i>	$1s^2 2s^2 2p^3$	$^4S_{3/2}^0$	$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{1}{3}$	1
Neutral 8 e Atom <i>O</i>	$1s^2 2s^2 2p^4$	3P_2	$\begin{array}{ccc} \uparrow \downarrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	1	2
Neutral 9 e Atom <i>F</i>	$1s^2 2s^2 2p^5$	$^2P_{3/2}^0$	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{2}{3}$	3
Neutral 10 e Atom <i>Ne</i>	$1s^2 2s^2 2p^6$	1S_0	$\begin{array}{ccc} \uparrow \downarrow & \uparrow \downarrow & \uparrow \downarrow \\ 1 & 0 & -1 \end{array}$	0	3
5 e Ion	$1s^2 2s^2 2p^1$	$^2P_{1/2}^0$	$\begin{array}{ccc} \uparrow & _ & _ \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	1
6 e Ion	$1s^2 2s^2 2p^2$	3P_0	$\begin{array}{ccc} \uparrow & \uparrow & _ \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	4
7 e Ion	$1s^2 2s^2 2p^3$	$^4S_{3/2}^0$	$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ 1 & 0 & -1 \end{array}$	$\frac{5}{3}$	6

903

8 e Ion	$1s^2 2s^2 2p^4$	3P_2	$\begin{array}{c} \uparrow \downarrow \\ 1 \end{array}$	$\begin{array}{c} \uparrow \\ 0 \end{array}$	$\begin{array}{c} \uparrow \\ -1 \end{array}$	$\frac{5}{3}$	6
9 e Ion	$1s^2 2s^2 2p^5$	$^2P_{3/2}^0$	$\begin{array}{c} \uparrow \downarrow \\ 1 \end{array}$	$\begin{array}{c} \uparrow \downarrow \\ 0 \end{array}$	$\begin{array}{c} \uparrow \\ -1 \end{array}$	$\frac{5}{3}$	9
10 e Ion	$1s^2 2s^2 2p^6$	1S_0	$\begin{array}{c} \uparrow \downarrow \\ 1 \end{array}$	$\begin{array}{c} \uparrow \downarrow \\ 0 \end{array}$	$\begin{array}{c} \uparrow \downarrow \\ -1 \end{array}$	$\frac{5}{3}$	12

156. The system of claim 155, wherein the ionization energy for the boron atom is given by

$$E(\text{ionization}; B) = \frac{(Z-4)e^2}{8\pi\epsilon_0 r_5} + \Delta E_{\text{mag}}$$

$$= 8.147170901 \text{ eV} + 0.15548501 \text{ eV} = 8.30265592 \text{ eV}$$

5 wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

157. The system of claim 156, wherein the ionization energies for the n-electron atoms having the radii, r_n , are given by the negative of the electric energy, $E(\text{electric})$, given by

$$10 \quad E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z-(n-1))e^2}{8\pi\epsilon_0 r_n}.$$

158. The system of claim 157, wherein the radii of the 3p electrons are given using the forces given by

$$\mathbf{F}_{\text{ele}} = \frac{(Z-n)e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

$$15 \quad \mathbf{F}_{\text{diamagnetic}} = -\sum_m \frac{(\ell+|m|)!}{(2\ell+1)(\ell-|m|)!} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{\text{diamagnetic}} = -\left(\frac{2}{3} + \frac{2}{3} + \frac{1}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = -\left(\frac{5}{3}\right) \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$F_{mag\ 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$F_{mag\ 2} = (4+4+4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$F_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$F_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$5 \quad F_{mag\ 2} = \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

and the radii r_{12} are given by

$$r_{12} = \frac{a_0 \left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right) \pm a_0 \left(\frac{1}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)} \right)^2 + \frac{20\sqrt{3} \left(\left[\frac{Z-12}{Z-11} \right] \left(1 + \frac{\sqrt{2}}{2} \right) r_{10} \right)}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z} \right) \frac{\sqrt{3}}{r_{10}} \right)}}{2}$$

r_{10} in units of a_0

159. The system of claim 158, wherein the ionization energies are given by electric energy
10 given by:

$$E(\text{Ionization}) = -\text{Electric Energy} = \frac{(Z - (n-1))e^2}{8\pi\epsilon_0 r_n}$$

160. The system of claim 159, wherein for each n-electron atom having a central charge of
Z times that of the proton and an electron configuration $1s^2 2s^2 2p^6 3s^2 3p^{n-12}$, there are two
15 indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right]$$

two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right) \right)^2 + \left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2 + 4 \left(\frac{[Z-3]}{[Z-2]} r_1 10 \sqrt{\frac{3}{4}} \right)^2}{2 \left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)}$$

r_1 in units of a_0

5 three sets of paired indistinguishable electrons in an orbitsphere with radius r_{10} given by:

$$r_{10} = \frac{a_0 \left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right) \pm \sqrt{\left(\frac{1}{\left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)} \right)^2 + 20\sqrt{3} \left(\frac{[Z-10]}{[Z-9]} \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)^2}}{2 \left((Z-9) - \left(\frac{5}{24} - \frac{6}{Z} \right) \frac{\sqrt{3}}{r_3} \right)}$$

r_3 in units of a_0

906

two indistinguishable spin-paired electrons in an orbitsphere with radius r_{12} given by:

$$r_{12} = \frac{\frac{a_0}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z}\right) \frac{\sqrt{3}}{r_{10}}\right)} \pm a_0}{2} \sqrt{\frac{1}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z}\right) \frac{\sqrt{3}}{r_{10}}\right)} + \frac{20\sqrt{3} \left(\left[\frac{Z-12}{Z-11}\right] \left(1 + \frac{\sqrt{2}}{2}\right) r_{10}\right)}{\left((Z-11) - \left(\frac{1}{8} - \frac{3}{Z}\right) \frac{\sqrt{3}}{r_{10}}\right)}}^2$$

r_{10} in units of a_0

and $n-12$ electrons in a 3p orbitsphere with radius r_n given by

$$r_n = \frac{\frac{a_0}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z}\right) \frac{\sqrt{3}}{r_{12}}\right)} \pm a_0}{2} \sqrt{\frac{1}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z}\right) \frac{\sqrt{3}}{r_{12}}\right)} + \frac{20\sqrt{3} \left(\left[\frac{Z-n}{Z-(n-1)}\right] \left(1 - \frac{\sqrt{2}}{2} + \frac{1}{2}\right) r_{12}\right)}{\left((Z-(n-1)) - \left(\frac{A}{8} - \frac{B}{2Z}\right) \frac{\sqrt{3}}{r_{12}}\right)}}^2$$

r_{12} in units of a_0

5 where the positive root must be taken in order that $r_n > 0$;

the parameter A corresponds to the diamagnetic force, $F_{\text{diamagnetic}}$:

$$F_{\text{diamagnetic}} = -\sum_m \frac{(\ell + |m|)!}{(2\ell + 1)(\ell - |m|)!} \frac{\hbar^2}{4m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and the parameter } B \text{ corresponds to}$$

the paramagnetic force, $F_{\text{mag } 2}$:

$$F_{\text{mag } 2} = \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$10 \quad F_{\text{mag } 2} = (4 + 4 + 4) \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r = \frac{1}{Z} \frac{12\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{4\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r, \text{ and}$$

$$\mathbf{F}_{mag\ 2} = \frac{1}{Z} \frac{8\hbar^2}{m_e r_n^2 r_{12}} \sqrt{s(s+1)} \mathbf{i}_r,$$

5

wherein the parameters of thirteen to eighteen-electron atoms are

Atom Type	Electron Configuration	Ground State Term	908 Orbital Arrangement of 3p Electrons (3p state)			Diamagnetic Force Factor A	Paramagnetic Force Factor B
			\uparrow 1	$\underline{\hspace{0.5cm}}$ 0	$\underline{\hspace{0.5cm}}$ -1		
Neutral Atom <i>Al</i>	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P_{1/2}^0$	\uparrow 1	$\underline{\hspace{0.5cm}}$ 0	$\underline{\hspace{0.5cm}}$ -1	$\frac{11}{3}$	0
Neutral Atom <i>Si</i>	$1s^2 2s^2 2p^6 3s^2 3p^2$	3P_0	\uparrow 1	\uparrow 0	$\underline{\hspace{0.5cm}}$ -1	$\frac{7}{3}$	0
Neutral Atom <i>P</i>	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S_{3/2}^0$	\uparrow 1	\uparrow 0	\uparrow -1	$\frac{5}{3}$	2
Neutral Atom <i>S</i>	$1s^2 2s^2 2p^6 3s^2 3p^4$	3P_2	$\uparrow \downarrow$ 1	\uparrow 0	\uparrow -1	$\frac{4}{3}$	1
Neutral Atom <i>Cl</i>	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P_{3/2}^0$	$\uparrow \downarrow$ 1	$\uparrow \downarrow$ 0	\uparrow -1	$\frac{2}{3}$	2
Neutral Atom <i>Ar</i>	$1s^2 2s^2 2p^6 3s^2 3p^6$	1S_0	$\uparrow \downarrow$ 1	$\uparrow \downarrow$ 0	$\uparrow \downarrow$ -1	$\frac{1}{3}$	4
13 e Ion <i>Ar</i>	$1s^2 2s^2 2p^6 3s^2 3p^1$	$^2P_{1/2}^0$	\uparrow 1	$\underline{\hspace{0.5cm}}$ 0	$\underline{\hspace{0.5cm}}$ -1	$\frac{5}{3}$	12

14 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^2$	3P_0	$\overset{909}{\uparrow}$	\uparrow	$\underline{\quad}$	$\frac{1}{3}$	16
			1	0	-1		
15 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^3$	$^4S_{3/2}^0$	\uparrow	\uparrow	\uparrow	0	24
			1	0	-1		
16 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^4$	3P_2	$\uparrow \downarrow$	\uparrow	\uparrow	$\frac{1}{3}$	24
			1	0	-1		
17 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^5$	$^2P_{3/2}^0$	$\uparrow \downarrow$	$\uparrow \downarrow$	\uparrow	$\frac{2}{3}$	32
			1	0	-1		
18 e Ion	$1s^2 2s^2 2p^6 3s^2 3p^6$	1S_0	$\uparrow \downarrow$	$\uparrow \downarrow$	$\uparrow \downarrow$	0	40
			1	0	-1		

161. The system of claim 160 wherein the bond comprises a H_2 -type MO with two paired electron wherein the formation of the H_2 -type MO by the contribution of an electron from each participating atom results in a diamagnetic force between the remaining atomic electrons and the H_2 -type MO which causes the H_2 -type MO to move to greater principal axes than would result with the Coulombic force alone.

162. The system of claim 161 wherein the integer increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining atomic electrons of each atom decrease the radius of the corresponding shell such that the energy minimum is achieved that is lower than that of the reactant atoms.

163. The system of claim 162 wherein the general equation for the central Coulomb force on the outer-most shell (nth where $n = Z - 1$) electron due to the nucleus and the inner electrons is given by:

$$F_{ele} = \frac{2e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

for $r > r_{n-1}$.

164. The system of claim 163 wherein the general equation for $F_{\text{diamagnetic}}$ due to the p -orbital contribution is given by

$$5 \quad F_{\text{diamagnetic}} = -A \frac{\hbar^2}{12m_e r_n^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r$$

165. The system of claim 164 wherein the general equation for $F_{\text{mag } 2}$ is given by

$$F_{\text{mag } 2} = B \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

10 166. The system of claim 165 wherein the general equation for $F_{\text{diamagnetic } 2}$ due to the binding of the p -orbital electron having an electric field of +1 outside of its radius is given by

$$F_{\text{diamagnetic } 2} = -\left[\frac{1}{2}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{m_e r_n^4} 10 \sqrt{s(s+1)} \mathbf{i}_r$$

167. The system of claim 166 wherein the general equation for $F_{\text{mag } 3}$, due to the
15 contribution of a $2p$ electron from each binding atom in the formation of the σ MO that gives rise to a paramagnetic force on the remaining two $2p$ electrons that pair, is given by

$$F_{\text{mag } 3} = C \frac{\hbar^2}{4m_e r_n^3} \sqrt{s(s+1)} \mathbf{i}_r$$

168. The system of claim 167 wherein the parameters A , B , and C are zero or a positive
20 integer such that the resulting energy of the molecule is minimized and the electron angular momentum is conserved in the formation of the specie.

169. The system of claim 168 wherein the radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric and diamagnetic and paramagnetic
25 forces:

911

$$\frac{m_e v_n^2}{r_n} = \left(\frac{2e^2}{4\pi\epsilon_0 r_n^2} - A \frac{\hbar^2}{12m_e r_n^2 r_3} \sqrt{s(s+1)} + B \frac{\hbar^2}{Zm_e r_n^2 r_3} \sqrt{s(s+1)} - \left[\frac{1}{2} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_n^4 m_e} 10 \sqrt{s(s+1)} + C \frac{\hbar^2}{4m_e r_n^3} \sqrt{s(s+1)} \right)$$

170. The system of claim 169 wherein the radius of the 2p shell is calculated by equating the outward centrifugal force to the sum of the electric and diamagnetic and paramagnetic

5 forces where the velocity given by $v_n = \frac{\hbar}{m_e r_n}$ and $s = \frac{1}{2}$ gives:

$$\frac{\hbar^2}{m_e r_n^3} - C \frac{\hbar^2}{4m_e r_n^3} \sqrt{\frac{3}{4}} = \frac{2e^2}{4\pi\epsilon_0 r_n^2} - A \frac{\hbar^2}{12m_e r_n^2 r_3} \sqrt{\frac{3}{4}} + B \frac{\hbar^2}{Zm_e r_n^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{1}{2} \right] \left(1 - \frac{\sqrt{2}}{2} \right) \frac{r_3 \hbar^2}{r_n^4 m_e} 10 \sqrt{\frac{3}{4}}$$

171. The system of claim 170 wherein the general equation for the radius of the shell of the remaining electrons is given by

$$r_6 = \frac{a_0 \left(1 - C \frac{\sqrt{3}}{8} \right)}{\left(2 - \left(\frac{A}{12} - \frac{B}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \pm a_0 \sqrt{\frac{\left(\frac{1 - C \frac{\sqrt{3}}{8}}{\left(2 - \left(\frac{A}{12} - \frac{B}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)} \right)^2}{20\sqrt{3} \left(\left[\frac{1}{2} \right] \left(1 - \frac{\sqrt{2}}{2} \right) r_3 \right)} + \frac{\left(2 - \left(\frac{A}{12} - \frac{B}{Z} \right) \frac{\sqrt{3}}{2r_3} \right)}{2}} \quad r_3 \text{ in units of } a_0$$

172. The system of claim 171 wherein the radii r_3 are given by

$$r_4 = r_3 = \frac{\pm a_0 \left(\frac{1 - \frac{\sqrt{3}}{4}}{Z} \right) \left[(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right] \pm \frac{\left(\frac{1 - \frac{\sqrt{3}}{4}}{Z} \right)^2 + 4 \left[\frac{Z-3}{Z-2} \right] r_1 \frac{\sqrt{3}}{4}}{\left[(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right] \left[(Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right]}}{2}$$

r_1 in units of a_0

173. The system of claim 172, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two
- 5 indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{3}}{Z(Z-1)} \right];$$

and two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\pm a_0 \left(\frac{a_0 \left(1 - \frac{\sqrt{3}}{4} \right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right)} \right)^2 + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1 10 \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{3}}{r_1} \right)}}{2}$$

r_1 in units of a_0

174. The system of claim 173 wherein the sum $E_T(\text{molecule}, AOs)$ of the Coulombic energy change of the AO electrons of both atoms of a bond is given by using the initial radius r_n of each atom and the final radius r_{n-1} of the binding shell of each atom and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell:

$$E_T(\text{molecule}, AOs) = \sum_{m=n_1-3}^{n_1-2} \frac{(Z-m)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_{n_1-1}} - \frac{1}{r_{n_1}} \right) + \sum_{m=n_2-3}^{n_2-2} \frac{(Z-m)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_{n_2-1}} - \frac{1}{r_{n_2}} \right)$$

where the subscript designates atom 1 and atom 2 of the bond.

10

175. The system of claim 174 wherein the forces on the electrons of the MO are the Coulombic force:

$$F_{Coulomb} = \frac{e^2}{8\pi\epsilon_0 ab^2} D i_z;$$

the spin pairing force:

$$\mathbf{F}_{spin-pairing} = \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi$$

the diamagnetic force:

$$\mathbf{F}_{diamagneticMO1} = -\frac{n_e \hbar^2}{4m_e a^2 b^2} D \mathbf{i}_\xi$$

where n_e is the total number of electrons that interact with the binding σ -MO electron, the
5 force on the pairing electron of the σ MO:

$$\mathbf{F}_{diamagneticMO2} = -\frac{1}{Z} \frac{|L| \hbar}{2m_e a^2 b^2} D \mathbf{i}_\xi$$

where $|L|$ is the magnitude of the angular momentum of each atom at a focus that is the
source of the diamagnetism at the σ -MO, and the centrifugal force:

$$\mathbf{F}_{centrifugalMO} = -\frac{\hbar^2}{m_e a^2 b^2} D \mathbf{i}_\xi, \text{ wherein the force balance of the centrifugal force equated}$$

10 to the Coulombic and magnetic forces is solved for the length of the semimajor axis.

176. The system of claim 175 wherein the force balance equation for the σ -MO with
 $n_e = 2$ and $|L| = \hbar$ is

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z}\right) \frac{\hbar^2}{2m_e a^2 b^2} D$$

15 $a = \left(2 + \frac{1}{Z}\right) a_0$

177. The system of claim 176 wherein the force balance equation for the σ -MO with
 $n_e = 2$ and $|L| = \sqrt{\frac{3}{4}} \hbar$ is

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(\frac{3}{2} + \frac{\sqrt{\frac{3}{4}}}{Z}\right) \frac{\hbar^2}{2m_e a^2 b^2} D$$

20 $a = \left(\frac{5}{2} + \frac{\sqrt{\frac{3}{4}}}{Z}\right) a_0$

178. The system of claim 177 wherein the force balance equation for the σ -MO $n_e = 2$, and $|L| = \hbar$:

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(\frac{5}{2} + \frac{1}{2Z} \right) \frac{\hbar^2}{2m_e a^2 b^2} D$$

5
$$a = \left(\frac{7}{2} + \frac{1}{2Z} \right) a_0$$

179. The system of claim 178 wherein the Coulombic force is

$$\mathbf{F}_{\text{Coulomb}} = \frac{e^2}{8\pi\epsilon_0 a b^2} D \mathbf{i}_\xi;$$

the spin pairing force is

10
$$\mathbf{F}_{\text{spin-pairing}} = \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi$$

the force on the electrons of the MO due to two paired electrons in at least one shell with $n_e = 2$ is

$$\mathbf{F}_{\text{diamagneticMO1}} = -\frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi;$$

the force $\mathbf{F}_{\text{diamagneticMO2}}$ is given by the sum of the contributions over the components of

15 angular momentum $|L|$ of the atoms at the foci acting on the electrons of the σ -MO:

$$\mathbf{F}_{\text{diamagneticMO2}} = -\sum_{i,j} \frac{|L_i| \hbar}{Z_j 2m_e a^2 b^2} D \mathbf{i}_\xi, \text{ and}$$

the centrifugal force is

$$\mathbf{F}_{\text{centrifugalMO}} = -\frac{\hbar^2}{m_e a^2 b^2} D \mathbf{i}_\xi.$$

20 180. The system of claim 179 wherein the force balance equation for the σ -MO of the

carbon nitride radical comprising carbon with charge $Z_1 = 6$ and $|L_1| = \hbar$ and $|L_2| = \sqrt{\frac{3}{4}}\hbar$ and

nitrogen with $Z_2 = 7$ and $|L_3| = \hbar$ is

916

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D$$

$$a = \left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) a_0$$

181. The system of claim 180 wherein the Coulombic force is

$$5 \quad \mathbf{F}_{Coulomb} = \frac{e^2}{8\pi\epsilon_0 a b^2} D \mathbf{i}_\xi;$$

the spin pairing force is

$$\mathbf{F}_{spin-pairing} = \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi$$

the force on the electrons of the MO due to two paired electrons in at least one shell with $n_{e_1} = 2$ is

$$10 \quad \mathbf{F}_{diamagneticMO1} = -\frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi;$$

the force $\mathbf{F}_{diamagneticMO2}$ is given by the sum of the contributions over the components of angular momentum $|L|$ of the atoms at the foci acting on the electrons of the σ -MO:

$$\mathbf{F}_{diamagneticMO2} = -\sum_{i,j} \frac{|L_i| \hbar}{Z_j 2m_e a^2 b^2} D \mathbf{i}_\xi, \text{ and}$$

the centrifugal force is

$$15 \quad \mathbf{F}_{centrifugalMO} = -\frac{\hbar^2}{m_e a^2 b^2} D \mathbf{i}_\xi.$$

182. The system of claim 181 wherein the force balance equation for the σ -MO of the

carbon nitride radical comprising carbon with charge $Z_1 = 6$ and $|L_1| = \hbar$ and $|L_2| = \sqrt{\frac{3}{4}}\hbar$ and

nitrogen with $Z_2 = 7$ and $|L_3| = \hbar$ is

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D$$

$$a = \left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) a_0$$

183. The system of claim 182 wherein the Coulombic force is

$$5 \quad \mathbf{F}_{Coulomb} = \frac{e^2}{8\pi\epsilon_0 a b^2} D \mathbf{i}_\xi;$$

the spin pairing force is

$$\mathbf{F}_{spin-pairing} = \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi$$

the force on the electrons of the MO due to two paired electrons in at least one shell with $n_e = 2$ is

$$10 \quad \mathbf{F}_{diamagneticMO1} = -\frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi;$$

the force $\mathbf{F}_{diamagneticMO2}$ is given by the sum of the contributions over the components of angular momentum $|L|$ of the atoms at the foci acting on the electrons of the σ -MO, and the sum of the contributions from atom 1 with $Z = Z_1$ and atom 2 with $Z = Z_2$ with $|L_i| = \hbar$ is

$$\mathbf{F}_{diamagneticMO2} = -\left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi, \text{ and}$$

15 the centrifugal force is

$$\mathbf{F}_{centrifugalMO} = -\frac{\hbar^2}{m_e a^2 b^2} D \mathbf{i}_\xi.$$

184. The system of claim 183 wherein force balance equation for the σ -MO of the carbon monoxide molecule is

$$20 \quad \frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \left(\frac{1}{Z_1} + \frac{1}{Z_2} \right) \right) \frac{\hbar^2}{2m_e a^2 b^2} D$$

$$a = \left(2 + \frac{1}{Z_1} + \frac{1}{Z_2} \right) a_0$$

185. The system of claim 184 wherein the Coulombic force is

$$\mathbf{F}_{\text{Coulomb}} = \frac{e^2}{8\pi\epsilon_0 ab^2} D \mathbf{i}_\xi;$$

5 the spin pairing force is

$$\mathbf{F}_{\text{spin-pairing}} = \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi$$

the force on the electrons of the MO due to two paired electrons in at least one shell with $n_e = 2$ is

$$\mathbf{F}_{\text{diamagneticMO1}} = -\frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi;$$

10 the force $\mathbf{F}_{\text{diamagneticMO2}}$ of the nitric oxide radical comprising nitrogen with charge $Z_1 = 7$ and

$|L_1| = \hbar$ and $|L_2| = \sqrt{\frac{3}{4}}\hbar$ and oxygen with $Z_2 = 8$ and $|L_3| = \hbar$ given by the corresponding sum of the contributions is

$$\mathbf{F}_{\text{diamagneticMO2}} = -\left(\frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_\xi, \text{ and}$$

the centrifugal force is

15
$$\mathbf{F}_{\text{centrifugalMO}} = -\frac{\hbar^2}{m_e a^2 b^2} D \mathbf{i}_\xi.$$

186. The system of claim 185 wherein the general force balance equation for the σ -MO of the nitric oxide radical is the same as that of CN:

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 ab^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D$$

$$a = \left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{1}{Z_2} \right) a_0$$

where $Z_1 = 7$ and $Z_2 = 8$ for NO and $Z_1 = 6$ and $Z_2 = 7$ for CN .

187. The system of claims 186 and 63 wherein the distance from the origin of the H_2 -type-
5 ellipsoidal-MO to each focus c' , the internuclear distance $2c'$, and the length of the
semiminor axis of the prolate spheroidal H_2 -type MO $b = c$ are solved from the semimajor
axis a .

188. The system of claim 187 wherein the potential energy of the two electrons in the
10 central field of the nuclei at the foci is

$$V_e = n_1 c_1 c_2 \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}};$$

The potential energy of the two nuclei is

$$V_p = n_1 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}};$$

The kinetic energy of the electrons is

$$15 \quad T = n_1 c_1 c_2 \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The energy, V_m , of the magnetic force between the electrons is

$$V_m = n_1 c_1 c_2 \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

the total energy, E_T , is given by the sum of the energy terms plus $E(AO)$:

$$E_T = V_e + T + V_m + V_p$$

$$20 \quad E_T = -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right]$$

$$= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right]$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond, and $E(AO)$ is the energy of the at least one atomic orbital to which the
5 MO is energy matched.

189. The system of claim 188 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$10 \quad \bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib}, \text{ and}$$

the total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}.$$

190. The system of claim 189 wherein during bond formation, the electrons undergo a
15 reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

the total energy is

$$20 \quad E_T = V_e + T + V_m + V_p + \bar{E}_{osc}$$

$$E_T = - \left\{ \frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{c_1 c_2 e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] - n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\}$$

where R is b or a , n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond, and $E(AO)$ is the energy of the at least one atomic orbital to
25 which the MO is energy matched.

191. The system of claim 190 wherein the bond comprises a H_2 -type MO with four paired electron wherein the formation of the H_2 -type MO by the contribution of two electrons from each participating atom results in a diamagnetic force between the remaining atomic electrons and the H_2 -type MO which causes the H_2 -type MO to move to greater principal axes than
5 would result with the Coulombic force alone.

192. The system of claim 191 wherein the integer increase of the central field and the resulting increased Coulombic as well as magnetic central forces on the remaining atomic electrons of each atom decrease the radius of the corresponding shell such that the energy
10 minimum is achieved that is lower than that of the reactant atoms.

193. The system of claim 192 wherein the general equation for the central Coulomb force on the outer-most shell (n th where $n = Z - 2$) electron due to the nucleus and the inner electrons is given by:

$$15 \quad F_{ele} = \frac{3e^2}{4\pi\epsilon_0 r_n^2} \mathbf{i}_r$$

for $r > r_{n-1}$.

194. The system of claim 193 wherein the general equation for $F_{diamagnetic}$ due to the p -orbital contribution is given by

$$20 \quad F_{diamagnetic} = -A \frac{\hbar^2}{12m_e r_n^2 r_3} \sqrt{\frac{3}{4}} \mathbf{i}_r$$

195. The system of claim 194 wherein the general equation for $F_{mag\ 2}$ is given by

$$F_{mag\ 2} = B \frac{1}{Z} \frac{\hbar^2}{m_e r_n^2 r_3} \sqrt{s(s+1)} \mathbf{i}_r$$

25

196. The system of claim 195 wherein the general equation for $F_{diamagnetic\ 2}$ due to the binding of the p -orbital electron having an electric field of +2 outside of its radius is given by

922

$$\mathbf{F}_{\text{diamagnetic } 2} = -\left[\frac{2}{3}\right]\left(1 - \frac{\sqrt{2}}{2}\right)\frac{r_3\hbar^2}{m_e r_n^4} 10\sqrt{s(s+1)}\mathbf{i}_r$$

197. The system of claim 196 wherein the general equation for $\mathbf{F}_{\text{mag } 3}$, due to the contribution of a $2p$ electron from each binding atom in the formation of the σ MO that gives rise to a paramagnetic force on the remaining two $2p$ electrons that pair, is given by

$$\mathbf{F}_{\text{mag } 3} = C \frac{\hbar^2}{4m_e r_n^3} \sqrt{s(s+1)}\mathbf{i}_r$$

198. The system of claim 197 wherein the parameters A , B , and C are zero or a positive integer such that the resulting energy of the molecule is minimized and the electron angular momentum is conserved in the formation of the specie.

199. The system of claim 198 wherein the radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric and diamagnetic and paramagnetic forces:

$$\frac{m_e v_n^2}{r_n} = \left(\frac{3e^2}{4\pi\epsilon_0 r_n^2} - A \frac{\hbar^2}{12m_e r_n^2 r_3} \sqrt{s(s+1)} + B \frac{\hbar^2}{Zm_e r_n^2 r_3} \sqrt{s(s+1)} - \left[\frac{2}{3}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{r_n^4 m_e} 10\sqrt{s(s+1)} + C \frac{\hbar^2}{4m_e r_n^3} \sqrt{s(s+1)} \right) \quad (13.626)$$

200. The system of claim 199 wherein the radius of the $2p$ shell is calculated by equating the outward centrifugal force to the sum of the electric and diamagnetic and paramagnetic

forces where the velocity given by $v_n = \frac{\hbar}{m_e r_n}$ and $s = \frac{1}{2}$ gives:

$$\frac{\hbar^2}{m_e r_n^3} - C \frac{\hbar^2}{4m_e r_n^3} \sqrt{\frac{3}{4}} = \frac{3e^2}{4\pi\epsilon_0 r_n^2} - A \frac{\hbar^2}{12m_e r_n^2 r_3} \sqrt{\frac{3}{4}} + B \frac{\hbar^2}{Zm_e r_n^2 r_3} \sqrt{\frac{3}{4}} - \left[\frac{2}{3}\right] \left(1 - \frac{\sqrt{2}}{2}\right) \frac{r_3 \hbar^2}{r_n^4 m_e} 10\sqrt{\frac{3}{4}}$$

201. The system of claim 200 wherein the general equation for the radius of the shell of the remaining electrons is given by

$$r_6 = \frac{a_0 \left(1 - C \frac{\sqrt{3}}{8}\right)}{\left(3 - \left(\frac{A}{12} - \frac{B}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)} \pm a_0 \sqrt{\frac{\left(\frac{1 - C \frac{\sqrt{3}}{8}}{\left(3 - \left(\frac{A}{12} - \frac{B}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)}\right)^2 + \frac{20\sqrt{3} \left[\frac{2}{3}\right] \left(1 - \frac{\sqrt{2}}{2}\right) r_3}{\left(3 - \left(\frac{A}{12} - \frac{B}{Z}\right) \frac{\sqrt{3}}{2r_3}\right)}}}{2}$$

r_3 in units of a_0

202. The system of claim 201 wherein the radii r_3 are given by

$$r_4 = r_3 = \frac{\left(\frac{a_0 \left(1 - \frac{\sqrt{3}}{4Z}\right)}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z}\right) \frac{\sqrt{3}}{r_1} \right)} \pm a_0 \sqrt{\frac{\left(\frac{1 - \frac{\sqrt{3}}{4Z}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z}\right) \frac{\sqrt{3}}{r_1} \right)} \right)^2 + 4 \frac{\left[\frac{Z-3}{Z-2} \right] r_1^{10} \sqrt{\frac{3}{4}}}{\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z}\right) \frac{\sqrt{3}}{r_1} \right)}}}{2} \right)}{2}$$

r_1 in units of a_0

5

203. The system of claim 202, wherein for each n-electron atom having a central charge of Z times that of the proton and an electron configuration $1s^2 2s^2 2p^{n-4}$, there are two indistinguishable spin-paired electrons in an orbitsphere with radii r_1 and r_2 both given by:

924

$$r_1 = r_2 = a_0 \left[\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)} \right];$$

and two indistinguishable spin-paired electrons in an orbitsphere with radii r_3 and r_4 both given by:

$$r_4 = r_3 = \frac{\left(a_0 \left(1 - \frac{\sqrt{\frac{3}{4}}}{Z} \right) \right)^2 + 4 \left(\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2 + \left(\left((Z-3) - \left(\frac{1}{4} - \frac{1}{Z} \right) \frac{\sqrt{\frac{3}{4}}}{r_1} \right)^2 + 4 \left(\left[\frac{Z-3}{Z-2} \right] r_1^{10} \sqrt{\frac{3}{4}} \right) \right) \right)}{2}$$

r_1 in units of a_0

5

204. The system of claim 203 wherein the force balance of a double-bond MO corresponds to that of a second pair of two electrons binding to a molecular ion having $+2e$ at each focus and a first bound pair such that the forces are the same as those of a molecule ion having $+e$ at each focus.

10

205. The system of claim 204 wherein the forces on the electrons of the MO are the Coulombic force:

$$\mathbf{F}_{Coulomb} = \frac{e^2}{8\pi\epsilon_0 ab^2} D\mathbf{i}_\xi;$$

the spin pairing force:

$$F_{spin-pairing} = \frac{\hbar^2}{2m_e a^2 b^2} D\mathbf{i}_\xi;$$

the diamagnetic force:

$$F_{diamagneticMO1} = -\frac{n_e \hbar^2}{4m_e a^2 b^2} D\mathbf{i}_\xi$$

where n_e is the total number of electrons that interact with the binding σ -MO electron;

5 the force on the pairing electron of the σ MO:

$$F_{diamagneticMO2} = -\frac{1}{Z} \frac{|L| \hbar}{2m_e a^2 b^2} D\mathbf{i}_\xi$$

where $|L|$ is the magnitude of the angular momentum of each atom at a focus that is the source of the diamagnetism at the σ -MO, and

the centrifugal force:

$$10 \quad F_{centrifugalMO} = -\frac{\hbar^2}{m_e a^2 b^2} D\mathbf{i}_\xi, \text{ and}$$

the force balance of the centrifugal force equated to the Coulombic and magnetic forces is solved for the length of the semimajor axis.

206. The system of claim 205 wherein the forces on the electrons of the NO_2 MO are the
15 Coulombic force:

$$F_{Coulomb} = \frac{e^2}{8\pi\epsilon_0 ab^2} D\mathbf{i}_\xi;$$

the spin pairing force:

$$F_{spin-pairing} = \frac{\hbar^2}{2m_e a^2 b^2} D\mathbf{i}_\xi;$$

the diamagnetic force $F_{diamagneticMO1}$ for each σ -MO of the NO_2 molecule due to the
20 two paired electrons in the $O2p$ shell with $n_e = 2$:

$$F_{diamagneticMO1} = \frac{\hbar^2}{2m_e a^2 b^2} D\mathbf{i}_\xi$$

which is also the corresponding force of NO ;

$F_{diamagneticMO2}$ of the nitrogen dioxide molecule comprising nitrogen with charge $Z_1 = 7$

and $|L_1| = \hbar$ and $|L_2| = \sqrt{\frac{3}{4}}\hbar$ and the two oxygen atoms, each with $Z_2 = 8$ and $|L_3| = \hbar$ given

by the corresponding sum of the contributions:

$$F_{\text{diamagneticMO}_2} = \left(\frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{2}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D \mathbf{i}_z$$

which is also the corresponding force of *NO* and *CN* except the term due to oxygen is twice
5 that of *NO* due to the two oxygen atoms of *NO*₂, and

the centrifugal force

$$F_{\text{centrifugalMO}} = -\frac{\hbar^2}{m_e a^2 b^2} D \mathbf{i}_z, \text{ and}$$

the force balance of the centrifugal force equated to the Coulombic and magnetic
forces is solved for the length of the semimajor axis.

10

207. The system of claim 206 wherein the force balance equation for the σ -MOs of *NO*₂
with $Z_1 = 7$ and $Z_2 = 8$ is

$$\frac{\hbar^2}{m_e a^2 b^2} D = \frac{e^2}{8\pi\epsilon_0 a b^2} D + \frac{\hbar^2}{2m_e a^2 b^2} D - \left(1 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{2}{Z_2} \right) \frac{\hbar^2}{2m_e a^2 b^2} D$$

15

$$a = \left(2 + \frac{1}{Z_1} + \frac{\sqrt{\frac{3}{4}}}{Z_1} + \frac{2}{Z_2} \right) a_0$$

208. The system of claims 207 and 63 wherein the distance from the origin of the *H*₂-type-
ellipsoidal-MO to each focus c' , the internuclear distance $2c'$, and the length of the
semiminor axis of the prolate spheroidal *H*₂-type MO $b = c$ are solved from the semimajor

20 axis a .

209. The system of claim 208 wherein the sum $E_T(\text{molecule}, AOs)$ of the Coulombic energy change of the AO electrons of the participating atoms or ions of the bonds of the specie is the sum of the contributions over all such atoms or ions given by using the initial radius r_n of the atom or ion and the final radius r_{n-2} of the binding shell of the atom or ion
 5 and by considering that the central Coulombic field decreases by an integer for each successive electron of the shell:

$$E_T(\text{molecule}, AOs) = \sum_{m=n_1-4}^{n_1-3} \frac{(Z-m)e^2}{8\pi\epsilon_0} \left(\frac{1}{r_{n_1-2}} - \frac{1}{r_{n_1}} \right)$$

where the subscript designates the exemplary atom 1 of the bond.

- 10 210. The system of claim 209 wherein the sum $E_T(\text{molecule}, AOs)$ of the Coulombic energy change of the AO electron of a participating atom of a bond wherein all of the electrons of the shell of the AO are contributed to the bond is given by the sum of the corresponding ionization energies of the AO electrons.

- 15 211. The system of claim 210 wherein energy of the double bond MO is match to the participating AO s and the potential energy of the four electrons in the central field of the nuclei at the foci is

$$V_e = 2^2 n_1 c_1 c_2 \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}};$$

The potential energy of the two nuclei is

20
$$V_p = 2^2 n_1 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}};$$

The kinetic energy of the electrons is

$$T = 2n_1 c_1 c_2 \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The energy, V_m , of the magnetic force between the electrons is

$$V_m = 2^2 n_1 c_1 c_2 \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

- 25 The total energy, E_T , is given by the sum of the energy terms plus $E(AO)$:

$$E_T = V_e + T + V_m + V_p$$

$$E_T = -\frac{2^2 n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 (2) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] + n_1 E(AO)$$

$$= -\frac{2^2 n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 (2) \ln \frac{a + c'}{a - c'} - 1 \right] + n_1 E(AO)$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond, and $E(AO)$ is the energy of the at least one atomic orbital to which the MO is energy matched.

212. The system of claim 211 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib}, \text{ and}$$

the total energy is

$$E_T = V_e + T + V_m + V_p + E(AO) + \bar{E}_{osc}.$$

15

213. The system of claim 212 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$E_T = -2^2 \frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 (2) \ln \frac{a + c'}{a - c'} - 1 \right] + n_1 E(AO) + \bar{E}_{osc}$$

20 where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond, and $E(AO)$ is the energy of the at least one atomic orbital to which the MO is energy matched.

25

214. The system of claim 213 wherein the energy components of are V_e , V_p , T , V_m , and E_T , except that the terms based on charge are multiplied by four and the kinetic energy term is multiplied by two due to each σ -MO double bond.

5 215. The system of claim 214 wherein the potential energy of the four electrons of double bond in the central field of the nuclei at the foci is

$$V_e = 2^2 n_1 c_1 c_2 \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}};$$

The potential energy of the two nuclei is

$$V_p = 2^2 n_1 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}};$$

10 The kinetic energy of the electrons is

$$T = 2n_1 c_1 c_2 \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The energy, V_m , of the magnetic force between the electrons is

$$V_m = 2^2 n_1 c_1 c_2 \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The total energy, E_T , is given by the sum of the energy terms:

$$15 \quad E_T = V_e + T + V_m + V_p$$

$$\begin{aligned} E_T &= -\frac{2^2 n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 (2) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] \\ &= -\frac{2^2 n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 (2) \ln \frac{a + c'}{a - c'} - 1 \right] \end{aligned}$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic
20 orbitals of each chemical bond.

216. The system of claim 215 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib}, \text{ and}$$

5 the total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}.$$

217. The system of claim 216 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is
10 the difference between the Doppler and average vibrational kinetic energies:

$$E_T = -\frac{2^2 n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 (2) \ln \frac{a+c'}{a-c'} - 1 \right] + \bar{E}_{osc}$$

n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of
15 each chemical bond.

218. The system of claim 217 wherein the total energy of the specie is the sum of the total energy of the components comprising the energy contribution of the MO formed between the participating atoms, the change in the energy of the AOs or HOs upon forming the bond, the
20 change in magnetic energy with bond formation, and the energy of oscillation in the transition state.

219. The system of claim 218 wherein the total energy of the specie is the sum over all of the component groups wherein the total energy of each said group is the sum of the total
25 energy of the components comprising the energy contribution of the MO formed between the participating atoms, the change in the energy of the AOs or HOs upon forming the bond, the change in magnetic energy with bond formation, and the energy of oscillation in the transition state.

30 220. The system of claim 219 wherein the change in magnetic energy is given by

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_n^3} = \frac{8\pi\mu_0 \mu_B^2}{r_n^3}$$

where r_n^3 is the radius of the atom that reacts to form the bond.

221. The system of claim 220 wherein the bond energy of the molecular or molecular ion
5 is difference in the energy of the total energy of the starting species and the total energy of the specie.

222. The system of claim 221 wherein the bond energy of a specific bond of the molecular
or molecular ion is difference in the energy of the sum of the energies of the atoms and any
10 change in energy of any groups formed with the starting atoms and the total energy of the bond of the specie.

223. The system of claim 222 wherein the total energy of a molecule, $E_T(\text{molecule})$, is
given by the sum of:
15 the sum of the energies of the electrons donated to each bond,
the sum of the energies of electrons of at least one other atom donated to the bonds,
the sum over the participating atoms of each AO contribution due to the decrease in
radius with the formation of each bond, and
the σ MO energy contribution per bond.

20

224. The system of claim 223 wherein the total energy of a molecule, $E_T(\text{molecule})$, is
given by the sum of:
the sum of the energies of the electrons donated to each bond such that all of the
electrons of the shell are donated,
25 the sum of the energies of electrons of at least one other atom donated to the bonds,
the sum over the participating atoms of each AO contribution due to the decrease in
radius with the formation of each bond, and
the σ MO energy contribution per bond.

225. The system of claim 223 wherein the total energy of CO_2 , $E_T(CO_2)$, is given by the sum of $E(\text{ionization}; C)$ and $E(\text{ionization}; C^+)$, the sum of the energies of the first and second electrons of carbon donated to each double bond, the sum of $E(\text{ionization}; O)$ and two times $E(\text{ionization}; O^+)$, the energies of the first and second electrons of oxygen
 5 donated to the double bonds, two times $E_T(O, 2p)$, the $O2p$ AO contribution due to the decrease in radius with the formation of each bond and two times $E_T(C=O, \sigma)$, the σ MO contribution:

$$\begin{aligned}
 E_T(CO_2) &= \left(E(\text{ionization}; C) + E(\text{ionization}; C^+) + E(\text{ionization}; O) \right. \\
 &\quad \left. + 2E(\text{ionization}; O^+) + 2E_T(O, 2p) + 2E_T(C=O, \sigma) \right) \\
 &= \left(11.26030 \text{ eV} + 24.38332 \text{ eV} + 13.61806 \text{ eV} \right. \\
 &\quad \left. + 2(35.11730 \text{ eV}) + 2(-32.12759 \text{ eV}) \right. \\
 &\quad \left. - 2 \left(\frac{4e^2}{8\pi\epsilon_0 \sqrt{\frac{2aa_0}{3}}} \left(\left(\frac{3}{2} \right) \ln \frac{a + \sqrt{\frac{2aa_0}{3}}}{a - \sqrt{\frac{2aa_0}{3}}} - 1 \right) \right) \right) \\
 &= \left(11.26030 \text{ eV} + 24.38332 \text{ eV} + 13.61806 \text{ eV} \right. \\
 &\quad \left. + 2(35.11730 \text{ eV}) + 2(-32.12759 \text{ eV}) - 2(55.25423 \text{ eV}) \right) \\
 &= -55.26841 \text{ eV}
 \end{aligned}$$

10

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

15 226. The system of claim 225 wherein the total energy of a molecule, $E_{T+osc}(\text{molecule})$ is given by the sum of $E_T(\text{molecule})$ and \bar{E}_{osc} .

227. The system of claim 226 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is
 20 the difference between the Doppler and average vibrational kinetic energies:

933

$$\bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_{hv} \sqrt{\frac{2\bar{E}_K}{m_e c^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right), \text{ and}$$

where the angular frequency of the reentrant oscillation in the transition state is

$$\omega = \sqrt{\frac{c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3}}{m_e}};$$

the kinetic energy, E_K , is given by Planck's:

$$5 \quad \bar{E}_K = \hbar\omega = \hbar \sqrt{\frac{c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3}}{m_e}};$$

the Doppler energy of the electrons of the reentrant orbit is

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{m_e c^2}} = E_{hv} \sqrt{\frac{2\hbar \sqrt{\frac{c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3}}{m_e}}}{m_e c^2}}$$

\bar{E}_{osc} is given by the sum of and \bar{E}_{Kvib} , the vibrational energy:

$$\bar{E}_{osc}(\text{molecule}) = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_{hv} \sqrt{\frac{2\hbar \sqrt{\frac{c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3}}{m_e}}}{m_e c^2}} + E_{vib} \right)$$

- 10 where R is b or a , n_1 is the number of equivalent bonds of the MO, c_{BO} is the bond-order factor which is 1 for a single bond, 4 for a double bond, and 9 for a triplet bond, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond.

15

228. The system of claim 227 wherein E_{hv} is given by $E_T(\text{molecule})/2$ in the case of a double bond such that

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = E_T(molecule)/2 \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}.$$

229. The system of claim 228 wherein E_{hv} of a molecule having n_1 bonds is given by $E_T(molecule)/n_1$ such that

$$5 \quad \bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_T(molecule)/n_1 \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right).$$

230. The system of claim 229 wherein E_{hv} of a molecule having n_1 bonds is given by $E_T(H_2)$ such that

$$10 \quad \bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_T(H_2) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right).$$

where $E_T(H_2) = -31.63537 \text{ eV}$ is the total energy of the hydrogen molecule, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

15 231. The system of claim 230 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_{hv} \sqrt{\frac{2\bar{E}_K}{m_e c^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right), \text{ and}$$

where the angular frequency of the reentrant oscillation in the transition state is determined
20 by the force between the central field and the electrons in the transition state; said force and its derivative are given by

$$f(R) = -c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3}$$

and

$$f'(a) = 2c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3}$$

935

such that the angular frequency of the oscillation is given by

$$\omega = \sqrt{\frac{\left[\frac{-3}{a} f(a) - f'(a) \right]}{m_e}} = \sqrt{\frac{k}{m_e}} = \sqrt{\frac{c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3}}{m_e}}$$

where R is b or a , c_{BO} is the bond-order factor which is 1 for a single bond, 4 for a double bond, and 9 for a triplet bond, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and c_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of each chemical bond.

232. The system of claim 231 wherein the nucleus of the B atom and the nucleus of the A atom comprise the foci of each H_2 -type ellipsoidal MO of the $A-B$ bond and the parameters of the point of intersection of each H_2 -type ellipsoidal MO and the A -atom AO are determined from the polar equation of the ellipse:

$$r = r_0 \frac{1+e}{1+e \cos \theta'}$$

233. The system of claim 232 wherein the radius of the A shell is r_A , and the polar radial coordinate of the ellipse and the radius of the A shell are equal at the point of intersection such that

$$r_A = (a - c') \frac{1 + \frac{c'}{a}}{1 + \frac{c'}{a} \cos \theta'}, \text{ and}$$

the polar angle θ' at the intersection point is given by

$$\theta' = \cos^{-1} \left(\frac{a}{c'} \left((a - c') \frac{1 + \frac{c'}{a}}{a_0} - 1 \right) \right)$$

20

234. The system of claim 233 wherein the angle θ_{AAO} the radial vector of the A AO makes with the internuclear axis is

$$\theta_{AAO} = 180^\circ - \theta'$$

235. The system of claim 234 wherein the distance from the point of intersection of the orbitals to the internuclear axis must be the same for both component orbitals such that the angle $\omega t = \theta_{H_2MO}$ between the internuclear axis and the point of intersection of each H_2 -type ellipsoidal MO with the A radial vector obeys the following relationship:

$$5 \quad r_A \sin \theta_{AAO} = b \sin \theta_{H_2MO}$$

such that

$$\theta_{H_2MO} = \sin^{-1} \frac{r_a \sin \theta_{AAO}}{b};$$

the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$10 \quad d_{H_2MO} = a \cos \theta_{H_2MO}, \text{ and}$$

the distance d_{AAO} along the internuclear axis from the origin of the A atom to the point of intersection of the orbitals is given by

$$d_{AAO} = c' - d_{H_2MO}.$$

15 236. The system of claim 235 where the bond angle is determined from the zero energy condition of the total energy of the potential bond between any pair of terminal atoms.

237. The system of claim 236 where the force constant k' of a H_2 -type ellipsoidal MO due to the equivalent of two point charges of at the foci is given by:

$$20 \quad k' = \frac{c_1 c_2 2e^2}{4\pi\epsilon_0}$$

where c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie and c_2 is the factor that results in an equipotential energy match of the participating at least two atomic orbitals of the chemical bond.

25 238. The system of claim 237 where the distance from the origin of the MO to each focus c' is given by:

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2k' a}} = \sqrt{\frac{aa_0}{2k'}};$$

the internuclear distance is

$$2c' = 2\sqrt{\frac{aa_0}{2k'}};$$

the length of the semiminor axis of the prolate spheroidal MO $b = c$ is given by

$$b = \sqrt{a^2 - c'^2}, \text{ and}$$

and, the eccentricity, e , is

$$5 \quad e = \frac{c'}{a}.$$

239. The system of claim 238 wherein the potential energy of the two electrons in the central field of the nuclei at the foci is

$$V_e = c_1 c_2 \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}};$$

10 The potential energy of the two nuclei is

$$V_p = \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}};$$

The kinetic energy of the electrons is

$$T = c_1 c_2 \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The energy, V_m , of the magnetic force between the electrons is

$$15 \quad V_m = c_1 c_2 \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}}, \text{ and}$$

The total energy, E_T , is given by the sum of the energy terms:

$$\begin{aligned} E_T &= V_e + T + V_m + V_p \\ E_T &= -\frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] \\ &= -\frac{e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] \end{aligned}$$

where c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of
 20 the specie, and c_2 is the factor that results in an equipotential energy match of the
 participating at least two molecular or atomic orbitals of each chemical bond.

240. The system of claim 239 wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the difference between the Doppler and average vibrational kinetic energies:

$$\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib} = (V_e + T + V_m + V_p) \sqrt{\frac{2\bar{E}_K}{Mc^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}, \text{ and}$$

5 The total energy is

$$E_T = V_e + T + V_m + V_p + \bar{E}_{osc}$$

$$E_T = - \left\{ \frac{e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{c_1 c_2 e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] - \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right\}$$

where c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and c_2 is the factor that results in an equipotential energy match of the
 10 participating at least two molecular or atomic orbitals of each chemical bond.

241. The system of claim 240 wherein the vibrational energy $\frac{1}{2} \hbar \sqrt{\frac{k}{\mu}}$ is given by

$$+ \frac{1}{2} \hbar \sqrt{\frac{\frac{c_1 e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 (a+c')^3}}{\mu}}$$

where μ is the reduced mass of the nuclei.

15

242. The system of claim 241 wherein the energy components of are V_e , V_p , T , V_m , and E_T , except that the terms based on charge are multiplied by four and the kinetic energy term is multiplied by two due to a σ -MO double bond.

20 243. The system of claim 242 wherein $c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2k' a}} = \sqrt{\frac{aa_0}{2k'}}$ is substituted into the

equation for E_T which is set equal to zero, and the semimajor axis is solved.

244. The system of claims 243 and 63 wherein the distance from the origin of the H_2 -type-ellipsoidal-MO to each focus c' , the internuclear distance $2c'$, and the length of the semiminor axis of the prolate spheroidal H_2 -type MO $b = c$ are solved from the semimajor axis a .

5

245. The system of claim 244 wherein with $2c'_{A-B}$ defined as the internuclear distance of each $A-B$ bond and $C = 2c'_{B-B}$ defined as the internuclear distance of the two terminal B atoms, the bond angle between the $A-B$ bonds is given by the law of cosines is

$$\theta = \cos^{-1} \left(\frac{2(2c'_{A-B})^2 - (2c'_{B-B})^2}{2(2c'_{A-B})^2} \right)$$

10

246. The system of claim 245 wherein the specie comprises more than two bonds with one $A-B$ bond along an axis defined as the vertical or z-axis and any two $A-B$ bonds form an isosceles triangle; the angle of the bonds from the defined axis is determined from the geometrical relationships:

15 the distance $d_{origin-B}$ from the origin to the nucleus of a terminal B atom is given by

$$d_{origin-B} = \frac{2c'_{B-B}}{2 \sin 60^\circ};$$

the height along the z-axis from the origin to A nucleus d_{height} is given by

$$d_{height} = \sqrt{(2c'_{A-B})^2 - (d_{origin-B})^2}, \text{ and}$$

the angle θ_v of each $A-B$ bond from the z-axis is given by

20
$$\theta_v = \tan^{-1} \left(\frac{d_{origin-B}}{d_{height}} \right).$$

247. The system of claim 246 wherein the vibrational energies are determined by the forces between the central field and the electrons and those between the nuclei; said electron-central-field force and its derivative are given by

25
$$f(R) = -c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3}$$

and

$$f'(a) = 2c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3};$$

said nuclear repulsion force and its derivative are given by

$$f(2c') = \frac{e^2}{8\pi\epsilon_0 (2c')^2}$$

and

$$5 \quad f'(2c') = -\frac{e^2}{4\pi\epsilon_0 (2c')^3}$$

such that the angular frequency of the oscillation is given by

$$\omega = \sqrt{\frac{\left[\frac{-3}{a} f(a) - f'(a) \right]}{\mu}} = \sqrt{\frac{k}{m_e}} = \sqrt{\frac{c_{BO} \frac{c_1 c_2 e^2}{4\pi\epsilon_0 R^3} - \frac{e^2}{8\pi\epsilon_0 (2c')^2}}{\mu}}$$

where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the nuclei, R is b or a , c_{BO} is the bond-order

factor which is 1 for a single bond, 4 for a double bond, and 9 for a triplet bond, c_1 is the
 10 fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie, and
 c_2 is the factor that results in an equipotential energy match of the participating at least two
 molecular or atomic orbitals of each chemical bond.

248. The system of claim 247 wherein the transition-state vibrational energy, $E_{vib}(0)$,
 15 given by Planck's equation is:

$$E_{vib}(0) = \hbar\omega$$

249. The system of claim 248 wherein the energy $\tilde{\nu}_\nu$ of state ν is given by

$$\tilde{\nu}_\nu = \nu\omega_0 - \nu(\nu-1)\omega_0 x_0, \quad \nu = 0, 1, 2, 3 \dots$$

20 where

$$\omega_0 x_0 = \frac{\hbar c \omega_0^2}{4D_0}$$

ω_0 is the frequency of the $\nu = 1 \rightarrow \nu = 0$ transition, and D_0 is the bond dissociation energy.

250. The system of claim 249 wherein ω_0 is given by

$$\omega_0 = E_{vib}(0) - 2\omega_0 x_0, \text{ and}$$

$$\omega_0 = E_{vib}(0) - 2 \frac{hc\omega_0^2}{4D_0}$$

such that

$$5 \quad \omega_0^2 + \frac{2D_0}{hc} \omega_0 - \frac{2D_0}{hc} E_{vib}(0) = 0, \text{ and}$$

$$\omega_0 = \frac{-\frac{2D_0}{hc} \pm \sqrt{\left(\frac{2D_0}{hc}\right)^2 + 4 \frac{2D_0}{hc} E_{vib}(0)}}{2}.$$

251. The system of claim 250 wherein B_e , the rotational parameter, for $A-B$ is given by:

$$B_e = \frac{\hbar^2}{2I_e hc}$$

10 where $I = \mu r^2$, $r = 2c'$, and μ is the reduced mass.

252. The system of claim 251 wherein the vibrational energy levels of the $A-A$ and $A-B$ bonds of the specie are solved as sets of coupled atomic harmonic oscillators wherein each atom of a chain of bonds is further coupled to at least one additional harmonic
15 oscillators by using the Lagrangian, the differential equation of motion, and the eigenvalue solutions wherein the spring constants are derived from the central forces.

253. The system of claim 252 wherein the vibrational energy levels of the $C-C$ bonds of $C_n H_{2n+2}$ are solved as $n-1$ sets of coupled carbon harmonic oscillators wherein each carbon
20 is further coupled to two or three equivalent H harmonic oscillators by using the Lagrangian, the differential equation of motion, and the eigenvalue solutions wherein the spring constants are derived from the central forces.

254. The system of claim 253 wherein $E_D(C_n H_{2n+2})$, the total bond dissociation energy of
25 $C_n H_{2n+2}$, is given as the sum of the energy components due to the two methyl groups, $n-2$ methylene groups, and $n-1$ $C-C$ bonds.

255. The system of claim 254 wherein the total bond dissociation energy of C_nH_{2n+2} is given by

$$E_D(C_nH_{2n+2}) = E_D(C-C)_{n-1} + 2E_{D_{alkane}}(^{12}CH_3) + (n-2)E_{D_{alkane}}(^{12}CH_2) \\ = (n-1)(4.32754 \text{ eV}) + 2(12.49186 \text{ eV}) + (n-2)(7.83016 \text{ eV})$$

wherein the calculated and measured values and constants recited in the equations herein can
5 be adjusted, for example, up to $\pm 10\%$, if desired.

256. The system of claim 255 wherein the exact solution for the dimensional parameters, charge density functions, and energies of molecules are determined using the results for the determination of C_nH_{2n+2} and the functional groups as basis sets that are linearly combined.
10

257. The system of claim 256 wherein one or more of the hydrogen atoms of the solution for C_nH_{2n+2} are substituted with one or more of the previously solved functional groups or derivative functional groups to give the parameters of a desired molecule.

15 258. The system of claim 257 wherein the parameters of a given molecule are given by energy matching each group to C_nH_{2n+2} .

259. The system of claim 258 wherein substitution of one or more H 's of C_nH_{2n+2} with functional groups from the list of CH_3 , other C_nH_{2n+2} groups, $H_2C=CH_2$, $HC\equiv CH$, F ,
20 Cl , O , OH , NH , NH_2 , CN , NO , NO_2 , CO , CO_2 , and C_6H_6 give the solutions of branched alkanes, alkenes, and alkynes, alkyl halides, ethers, alcohols, amides, amines, nitriles, alkyl nitrosos, alkyl nitrates, aldehydes, ketones, carboxylic acids, esters, and substituted aromatics.

25 260. A composition of matter comprising a plurality of atoms, the improvement comprising a novel property or use discovered by calculation of at least one of a bond distance between two of the atoms,
a bond angle between three of the atoms, and
a bond energy between two of the atoms,
30 orbital intercept distances and angles,

charge-density functions of atomic, hybridized, and molecular orbitals,
the bond distance, bond angle, and bond energy being calculated from physical solutions of
the charge, mass, and current density functions of atoms and atomic ions, which solutions are
derived from Maxwell's equations using a constraint that a bound electron(s) does not radiate
5 under acceleration.

261. A composition of matter according to claim 260, wherein the novel property is a new
pharmaceutical use.

10 262. A composition of matter according to claim 261 that is novel, wherein the novel
property is stability at room temperature of a new arrangement of atoms or ions.

263. A novel composition of matter discovered by calculating at least one of
a bond distance between two of the atoms,
15 a bond angle between three of the atoms, and
a bond energy between two of the atoms,
orbital intercept distances and angles,
charge-density functions of atomic, hybridized, and molecular orbitals,
the bond distance, bond angle, and bond energy being calculated from physical solutions of
20 the charge, mass, and current density functions of atoms and atomic ions, which solutions are
derived from Maxwell's equations using a constraint that a bound electron(s) does not radiate
under acceleration.

264. A system of determining at least one of
25 a bond distance between two of the atoms,
a bond angle between three of the atoms, and
a bond energy between two of the atoms,
orbital intercept distances and angles,
charge-density functions of atomic, hybridized, and molecular orbitals,
30 wherein the bond distance, bond angle, and bond energy being calculated from physical
solutions of the charge, mass, and current density functions of atoms and atomic ions, which
solutions are derived from Maxwell's equations using a constraint that the bound electron(s)
does not radiate under acceleration.

265. The system of claim 264, further comprising discovering a new composition of matter.
- 5 266. The system of claim 1, wherein the nature of said specie includes the nature of a chemical bond thereof.
267. The system of claim 1, wherein at least one bond angle between three atoms is computed and displayed.
- 10 268. The system of claim 1, wherein at least one bond distance between two atoms is computed and displayed.
269. The system of claim 1, wherein at least one geometric component is computed and
15 displayed.
270. The system of claim 1, wherein at least one potential energy between an electron and a nucleus is computed.
- 20 271. The system of claim 1, wherein at least one potential energy between two nuclei is computed.
272. The system of claim 1, wherein at least one kinetic energy of an electron is computed.
- 25 273. The system of claim 1, wherein at least one magnetic energy between electrons is computed.
274. The system of claim 1, wherein at least one total energy of a bond is computed.
- 30 275. The system of claim 1, wherein at least one change in atomic energy between atoms due to bonding is computed.

276. The system of claim 1, further comprising at least one database of functional groups and at least one database of molecules, molecular ions, and/or molecular radicals, characterized in that the functional groups, molecules, molecular ions, and molecular radicals having the nature of their chemical bonds computed by the Maxwellian solutions of charge, mass, and current density, or both databases being combined into one database or split into multiple databases, and the processing means being in communication with the databases or combined database.

277. The system of claim 1, further comprising at least one database of functional groups having the nature of their chemical bonds computed by the Maxwellian solutions of charge, mass, and current density.

278. The system of claim 276, further comprising means for selecting a desired first functional group, characterized in that the selected functional group is displayed with open bonds being displayed, means for selecting an open bond, means for selecting a second functional group to be bound to the first functional group, characterized in that the second functional group is displayed with open bonds being displayed, means for selecting an open bond on the second functional group, and means for combining the selected bonds, whereby at least a portion of a desired molecule is displayed.

20

279. The system of claim 276, further comprising selecting and combining functional groups until a desired molecule is displayed.

280. The system of claim 277, further comprising means for selecting a desired first functional group, characterized in that the selected functional group is displayed with open bonds being displayed, means for selecting an open bond, means for selecting a second functional group to be bound to the first functional group, characterized in that the second functional group is displayed with open bonds being displayed, means for selecting an open bond on the second functional group, and means for combining the selected bonds, whereby at least a portion of a desired molecule is displayed.

30

281. The system of claim 277, further comprising selecting and combining functional groups until a desired molecule is displayed.

282. The system of claim 1, further comprising a means for inputting the chemical structure of a desired molecule and parsing the inputted molecule into functional groups.
- 5 283. The system of claim 282, further comprising a database of functional groups having the nature of their chemical bonds computed by the Maxwellian solutions of charge, mass, and current density, characterized in that the processing means combines the functional groups and the desired molecule is displayed.
- 10 284. The system of claim 282, wherein the chemical structure is imputed using a Simplified Molecular Input Line Entry System.
285. The system of claim 283, wherein the chemical structure is imputed using a Simplified Molecular Input Line Entry System.
- 15 286. The system of claim 282, wherein the Simplified Molecular Input Line Entry System is SMILES.
287. The system of claim 283, wherein the Simplified Molecular Input Line Entry System
20 is SMILES.
288. The system according to claim 1, wherein the force generalized constant k' of a H_2 -type ellipsoidal MO due to the equivalent of two point charges at the foci is given by:
- $$k' = \frac{C_1 C_2 2e^2}{4\pi\epsilon_0} \quad (15.1)$$
- 25 where C_1 is the fraction of the H_2 -type ellipsoidal molecular orbital basis function of a chemical bond of the specie and C_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of the chemical bond.

289. The system according to claim 1, wherein the distance from the origin of the MO to each focus c' is given by:

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2C_1 C_2 a}} = \sqrt{\frac{aa_0}{2C_1 C_2}} \quad (15.2)$$

5

the internuclear distance is

$$2c' = 2 \sqrt{\frac{aa_0}{2C_1 C_2}} \quad (15.3)$$

the length of the semiminor axis of the prolate spheroidal MO $b = c$ is given by

$$b = \sqrt{a^2 - c'^2} \quad (15.4)$$

and, the eccentricity, e , is

$$e = \frac{c'}{a} \quad (15.5).$$

15

290. The system according to claim 289, wherein a potential energy of the electrons in the central field of the nuclei at the foci is

$$V_e = n_1 c_1 c_2 \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (15.6)$$

where n_1 is the number of equivalent bonds of the MO for functional groups and in the case of independent MOs not in contact with the bonding atoms, the terms based on charge are multiplied by c_{BO} , the bond-order factor, which is 1 for a single bond, 4 for an independent double bond and 9 for an independent triplet bond.

291. The system according to claim 1, wherein the potential energy of the two nuclei is

$$V_p = n_1 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \quad (15.7).$$

292. The system according to claim 1, wherein the kinetic energy of the electrons is

$$T = n_1 c_1 c_2 \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (15.8)$$

where n_1 is the number of equivalent bonds of the MO for functional groups and in the case of independent MOs not in contact with the bonding atoms, the terms based on charge are multiplied by c_{BO} , the bond-order factor, which is 1 for a single bond, 4 for an independent double bond and 9 for an independent triplet bond.

293. The system according to claim 1, wherein the energy, V_m , of the magnetic force between the electrons is

$$V_m = n_1 c_1 c_2 \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (15.9)$$

10 where n_1 is the number of equivalent bonds of the MO for functional groups and in the case of independent MOs not in contact with the bonding atoms, the terms based on charge are multiplied by c_{BO} , the bond-order factor, which is 1 for a single bond, 4 for an independent double bond and 9 for an independent triplet bond.

15 294. The system according to claim 1, wherein total energy of the H_2 -type prolate spheroidal MO, $E_T(H_2MO)$, is given by the sum of the energy terms:

$$E_T(H_2MO) = V_e + T + V_m + V_p \quad (15.10)$$

$$E_T(H_2MO) = -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] \\ = -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] \quad (15.11)$$

20 where n_1 is the number of equivalent bonds of the MO for functional groups and in the case of independent MOs not in contact with the bonding atoms, the terms based on charge are multiplied by c_{BO} , the bond-order factor, which is 1 for a single bond, 4 for an independent double bond and 9 for an independent triplet bond.

295. The system according to claim 1, wherein the total energy $E_T(atom, msp^3)$ (m is the integer of the valence shell) of the AO electrons and the hybridized shell is given by the sum of energies of successive ions of the atom over the n electrons comprising total electrons of the at least one AO shell according to the formula:

$$5 \quad E_T(atom, msp^3) = -\sum_{m=1}^n IP_m \quad (15.12)$$

where IP_m is the m th ionization energy (positive) of the atom.

296. The system according to claim 295, wherein the radius r_{msp^3} of the hybridized shell is given by:

$$10 \quad r_{msp^3} = \sum_{q=Z-n}^{Z-1} \frac{-(Z-q)e^2}{8\pi\epsilon_0 E_T(atom, msp^3)} \quad (15.13).$$

297. The system according to claim 295, wherein the Coulombic energy

$E_{Coulomb}(atom, msp^3)$ of the outer electron of the $atom msp^3$ shell is given by

$$E_{Coulomb}(atom, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} \quad (15.14).$$

15

298. The system of claim 296, wherein in the case that during hybridization at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ at the initial radius r of the AO electron:

$$20 \quad E(magnetic) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r^3} = \frac{8\pi\mu_0 \mu_B^2}{r^3} \quad (15.15)$$

then, the energy $E(atom, msp^3)$ of the outer electron of the $atom msp^3$ shell is given by the sum of $E_{Coulomb}(atom, msp^3)$ and $E(magnetic)$:

$$E(atom, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r^3} \quad (15.16).$$

25

299. The system according to claim 295, wherein the total energy $E_T(mol.atom,msp^3)$ (m is the integer of the valence shell) of the HO electrons is given by the sum of energies of successive ions of the atom over the n electrons comprising total electrons of the at least one initial AO shell and the hybridization energy:

$$5 \quad E_T(mol.atom,msp^3) = E(atom,msp^3) - \sum_{m=2}^n IP_m \quad (15.17)$$

where IP_m is the m th ionization energy (positive) of the atom and the sum of $-IP_1$ plus the hybridization energy is $E(atom,msp^3)$.

10 300. The system of claim 299, wherein the radius r_{msp^3} of the hybridized shell is given by:

$$r_{msp^3} = \left(\sum_{q=Z-n}^{Z-1} (Z-q) - s(0.25) \right) \frac{-e^2}{8\pi\epsilon_0 E_T(mol.atom,msp^3)} \quad (15.18)$$

where $s = 1, 2, 3$ for a single, double, and triple bond, respectively.

301. The system of claim 299, wherein the Coulombic energy $E_{Coulomb}(mol.atom,msp^3)$ of
15 the outer electron of the $atom\ msp^3$ shell is given by

$$E_{Coulomb}(mol.atom,msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} \quad (15.19).$$

302. The system of claim 298, wherein in the case that during hybridization at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change
20 for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ at the initial radius r of the AO electron given by Eq. (15.15). Then, the energy $E(mol.atom,msp^3)$ of the outer electron of the $atom\ msp^3$ shell is given by the sum of $E_{Coulomb}(mol.atom,msp^3)$ and $E(magnetic)$:

$$E(mol.atom,msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r^3} \quad (15.20)$$

$E_T(\text{atom} - \text{atom}, msp^3)$, the energy change of each $\text{atom } msp^3$ shell with the formation of the atom-atom-bond MO is given by the difference between $E(\text{mol. atom}, msp^3)$ and $E(\text{atom}, msp^3)$:

$$E_T(\text{atom} - \text{atom}, msp^3) = E(\text{mol. atom}, msp^3) - E(\text{atom}, msp^3) \quad (15.21).$$

5

303. The system of claim 295, wherein $E_{\text{Coulomb}}(\text{mol. atom}, msp^3)$ is one of:

$E_{\text{Coulomb}}(C_{\text{ethylene}}, 2sp^3)$, $E_{\text{Coulomb}}(C_{\text{ethane}}, 2sp^3)$, $E_{\text{Coulomb}}(C_{\text{acetylene}}, 2sp^3)$, and

$E_{\text{Coulomb}}(C_{\text{alkane}}, 2sp^3)$;

$E_{\text{Coulomb}}(\text{atom}, msp^3)$ is one of $E_{\text{Coulomb}}(C, 2sp^3)$ and $E_{\text{Coulomb}}(Cl, 3sp^3)$;

10 $E(\text{mol. atom}, msp^3)$ is one of $E(C_{\text{ethylene}}, 2sp^3)$, $E(C_{\text{ethane}}, 2sp^3)$,

$E(C_{\text{acetylene}}, 2sp^3)$ $E(C_{\text{alkane}}, 2sp^3)$;

$E(\text{atom}, msp^3)$ is one of $E(C, 2sp^3)$ and $E(Cl, 3sp^3)$;

$E_T(\text{atom} - \text{atom}, msp^3)$ is one of $E(C - C, 2sp^3)$, $E(C = C, 2sp^3)$, and

$E(C \equiv C, 2sp^3)$;

15

$\text{atom } msp^3$ is one of $C2sp^3$, $Cl3sp^3$

$E_T(\text{atom} - \text{atom}(s_1), msp^3)$ is $E_T(C - C, 2sp^3)$ and $E_T(\text{atom} - \text{atom}(s_2), msp^3)$ is

$E_T(C = C, 2sp^3)$, and

r_{msp^3} is one of r_{C2sp^3} , $r_{\text{ethane } 2sp^3}$, $r_{\text{ethylene } 2sp^3}$, $r_{\text{acetylene } 2sp^3}$, $r_{\text{alkane } 2sp^3}$, and r_{Cl3sp^3} .

In the case of the $C2sp^3$ HO, the initial parameters (Eqs. (14.142-14.146)) are

$$20 \quad r_{2sp^3} = \sum_{n=2}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0 (e148.25751 \text{ eV})} = \frac{10e^2}{8\pi\epsilon_0 (e148.25751 \text{ eV})} = 0.91771a_0 \quad (15.22)$$

$$E_{\text{Coulomb}}(C, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.91771a_0} = -14.82575 \text{ eV} \quad (15.23)$$

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} = \frac{8\pi\mu_0 \mu_B^2}{(0.84317a_0)^3} = 0.19086 \text{ eV} \quad (15.24)$$

$$\begin{aligned} E(C, 2sp^3) &= \frac{-e^2}{8\pi\epsilon_0 r_{2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} \\ &= -14.82575 \text{ eV} + 0.19086 \text{ eV} \\ &= -14.63489 \text{ eV} \end{aligned} \tag{15.25}$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

5 304. The system of claim 300, wherein

$$\sum_{q=Z-n}^{Z-1} (Z-q) = 10 \tag{15.26}$$

Equations (14.147) and (15.17) give

$$E_T(mol.atom,msp^3) = E_T(C_{ethane},2sp^3) = -151.61569 \text{ eV} \tag{15.27}$$

and using Eqs. (15.18-15.28), the final values of r_{C2sp^3} , $E_{Coulomb}(C2sp^3)$, and $E(C2sp^3)$, and

10 the resulting $E_T\left(C^{BO}-C,C2sp^3\right)$ of the MO due to charge donation from the HO to the MO

where $C^{BO}-C$ refers to the bond order of the carbon-carbon bond for different values of the parameter s are given in Table 15.1.

Table 15.1:

15

MO Bond Order (BO)	s 1	s 2	$r_{C2sp^3}(a_0)$ Final	$E_{Coulomb}(C2sp^3)(\text{eV})$ Final	$E(C2sp^3)$ (eV) Final	$E_T\left(C^{BO}-C,C2sp^3\right)$ (eV)
I	1	0	0.87495	-15.55033	-15.35946	-0.72457
II	2	0	0.85252	-15.95955	-15.76868	-1.13379
III	3	0	0.83008	-16.39089	-16.20002	-1.56513
IV	4	0	0.80765	-16.84619	-16.65532	-2.02043

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

305. The system of claim 295, wherein a minimum-energy bond with the constraint that it must meet the energy matching condition for all MOs at all HOs or AOs, the energy $E(\text{mol.atom}, msp^3)$ of the outer electron of the *atom msp³* shell of each bonding atom is the average of $E(\text{mol.atom}, msp^3)$ for two different values of s :

$$5 \quad E(\text{mol.atom}, msp^3) = \frac{E(\text{mol.atom}(s_1), msp^3) + E(\text{mol.atom}(s_2), msp^3)}{2} \quad (15.28).$$

306. The system of claim 305, wherein in the case, $E_T(\text{atom} - \text{atom}, msp^3)$, the energy change of each *atom msp³* shell with the formation of each atom-atom-bond MO, is average for two different values of s :

$$10 \quad E_T(\text{atom} - \text{atom}, msp^3) = \frac{E_T(\text{atom} - \text{atom}(s_1), msp^3) + E_T(\text{atom} - \text{atom}(s_2), msp^3)}{2} \quad (15.29).$$

307. The system of claim 305, wherein a first MO and its HOs comprising a linear combination of bond orders and a second MO that shares a HO with the first. In addition to the mutual HO, the second MO comprises another AO or HO having a single bond order or a mixed bond order, in order for the two MOs to be energy matched, the bond order of the second MO and its HOs or its HO and AO is a linear combination of the terms corresponding to the bond order of the mutual HO and the bond order of the independent HO or AO, and in general, $E_T(\text{atom} - \text{atom}, msp^3)$, the energy change of each *atom msp³* shell with the formation of each atom-atom-bond MO, is a weighted linear sum for different values of s that matches the energy of the bonded MOs, HOs, and AOs:

$$E_T(\text{atom} - \text{atom}, msp^3) = \sum_{n=1}^N c_{s_n} E_T(\text{atom} - \text{atom}(s_n), msp^3) \quad (15.30)$$

where c_{s_n} is the multiple of the BO of s_n . The radius r_{msp^3} of the *atom msp³* shell of each bonding atom is given by the Coulombic energy using the initial energy $E_{\text{Coulomb}}(\text{atom}, msp^3)$ and $E_T(\text{atom} - \text{atom}, msp^3)$, the energy change of each *atom msp³* shell with the formation of each atom-atom-bond MO:

$$r_{msp^3} = \frac{954}{8\pi\epsilon_0 a_0 \left(E_{Coulomb}(atom, msp^3) + E_T(atom - atom, msp^3) \right)} \quad (15.31)$$

where $E_{Coulomb}(C2sp^3) = -14.825751 \text{ eV}$.

5 308. The system of claim 307, wherein the Coulombic energy $E_{Coulomb}(mol.atom, msp^3)$ of the outer electron of the $atom msp^3$ shell is given by Eq. (15.19).

309. The system of claim 308, wherein in the case that during hybridization, at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change
10 for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ (Eq. (15.15)) at the initial radius r of the AO electron. Then, the energy $E(mol.atom, msp^3)$ of the outer electron of the $atom msp^3$ shell is given by the sum of $E_{Coulomb}(mol.atom, msp^3)$ and $E(magnetic)$ (Eq. (15.20)).

15 310. The system of claim 308, wherein, $E_T(atom - atom, msp^3)$, the energy change of each $atom msp^3$ shell with the formation of the atom-atom-bond MO is given by the difference between $E(mol.atom, msp^3)$ and $E(atom, msp^3)$ given by Eq. (15.21).

20 311. The system of claim 310, wherein using the equation (15.23) for $E_{Coulomb}(C, 2sp^3)$ in equation (15.31), the single bond order energies given by Eqs. (15.18-15.27) and shown in Table 15.1, and the linear combination energies (Eqs. (15.28-15.30)), the parameters of linear combinations of bond orders and linear combinations of mixed bond orders are given in Table 15.2, wherein the calculated and measured values and constants recited in
25 the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

312. The system of claim 295, wherein the radius r_{mol2sp^3} of the $C2sp^3$ HO of a carbon atom of a given specie is calculated using Eq. (14.514) by considering

$\sum E_{T_{mol}}(MO, 2sp^3)$, the total energy donation to each bond with which it participates in bonding.

5 313. The system of claim 1, wherein equation for the radius is given by

$$\begin{aligned} r_{mol2sp^3} &= \frac{-e^2}{8\pi\epsilon_0 \left(E_{Coulomb}(C, 2sp^3) + \sum E_{T_{mol}}(MO, 2sp^3) \right)} \\ &= \frac{e^2}{8\pi\epsilon_0 \left(e14.825751 \text{ eV} + \sum |E_{T_{mol}}(MO, 2sp^3)| \right)} \end{aligned} \quad (15.32).$$

314. The system of 295, wherein the Coulombic energy $E_{Coulomb}(mol.atom, msp^3)$ of the outer electron of the *atom msp³* shell is given by Eq. (15.19).

10

315. The system of claim 295, in the case that during hybridization, at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ (Eq. (15.15)) at the initial radius r of the AO electron. Then, the energy $E(mol.atom, msp^3)$ of the outer
15 electron of the *atom msp³* shell is given by the sum of $E_{Coulomb}(mol.atom, msp^3)$ and $E(magnetic)$ (Eq. (15.20)).

20

316. The system of claim 315, wherein for the $C2sp^3$ HO of each methyl group of an alkane contributes -0.92918 eV (Eq. (14.513)) to the corresponding single $C-C$ bond; the corresponding $C2sp^3$ HO radius is given by Eq. (14.514). The $C2sp^3$ HO of each methylene group of C_nH_{2n+2} contributes -0.92918 eV to each of the two corresponding $C-C$ bond MOs, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

25 317. The system of claim 316, wherein the radius (Eq. (15.32)), the Coulombic energy (Eq. (15.19)), and the energy (Eq. (15.20)) of each alkane methylene group are

956

$$\begin{aligned}
 r_{alkaneC_{methylene}2sp^3} &= \frac{-e^2}{8\pi\epsilon_0 \left(E_{Coulomb}(C, 2sp^3) + \sum E_{T_{alkane}}(methylene\ C-C, 2sp^3) \right)} \\
 &= \frac{e^2}{8\pi\epsilon_0 (e14.825751\ eV + e0.92918\ eV + e0.92918\ eV)} \\
 &= 0.81549a_0
 \end{aligned}$$

(15.33)

$$E_{Coulomb}(C_{methylene}2sp^3) = \frac{-e^2}{8\pi\epsilon_0 (0.81549a_0)} = -16.68412\ eV \quad (15.34)$$

$$E(C_{methylene}2sp^3) = \frac{-e^2}{8\pi\epsilon_0 (0.81549a_0)} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (0.84317a_0)^3} = -16.49325\ eV \quad (15.35)$$

5 wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

318. The system of claim 316, wherein in the determination of the parameters of functional groups, heteroatoms bonding to $C2sp^3$ HOs to form MOs are energy matched to the
 10 $C2sp^3$ HOs, the radius and the energy parameters of a bonding heteroatom are given by the same equations as those for $C2sp^3$ HOs.

319. The system of claim 318, wherein using Eqs. (15.15), (15.19-15.20), (15.24), and (15.32) in a generalized fashion, the final values of the radius of the HO or AO,
 15 $r_{Atom.HO.AO}$, $E_{Coulomb}(mol.atom,msp^3)$, and $E(C_{mol}2sp^3)$ are calculated using $\sum E_{T_{group}}(MO, 2sp^3)$, the total energy donation to each bond with which an atom participates in bonding corresponding to the values of $E_T\left(C^{BO}-C, C2sp^3\right)$ of the MO due to charge donation from the AO or HO to the MO given in Tables 15.1 and 15.2 and the final values of $r_{Atom.HO.AO}$, $E_{Coulomb}(mol.atom,msp^3)$, and $E(C_{mol}2sp^3)$ calculated using
 20 the values of $E_T\left(C^{BO}-C, C2sp^3\right)$ given in Tables 15.1 and 15.2 are shown in Tables 15.3A and 15.3B in the specification, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

320. The system of claim 319, wherein the energy of the MO is matched to each of the participating outermost atomic or hybridized orbitals of the bonding atoms wherein the energy match includes the energy contribution due to the AO or HO's donation of charge
5 to the MO.

321. The system of claim 320, wherein the force constant k' (Eq. (15.1)) is used to determine the ellipsoidal parameter c' (Eq. (15.2)) of the each H_2 -type-ellipsoidal-MO in terms of the central force of the foci, c' is substituted into the energy equation (from Eq. (15.11))) which is set equal to n_1 times the total energy of H_2 where n_1 is the number of
10 equivalent bonds of the MO and the energy of H_2 , -31.63536831 eV , Eq. (11.212) is the minimum energy possible for a prolate spheroidal MO, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

15

322. The system of claim 321, wherein the energy equation and the relationship between the axes, the dimensions of the MO are solved, the energy equation has the semimajor axis a as its only parameter, the solution of the semimajor axis a allows for the solution of the other axes of each prolate spheroid and eccentricity of each MO (Eqs. (15.3-15.5)),
20 and the parameter solutions then allow for the component and total energies of the MO to be determined.

323. The system of claim 1, wherein the total energy, $E_T(H_2MO)$, is given by the sum of the energy terms (Eqs. (15.6-15.11)) plus $E_T(AO/HO)$:

25 $E_T(H_2MO) = V_e + T + V_m + V_p + E_T(AO/HO) \quad (15.36)$

$$E_T(H_2MO) = -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] + E_T(AO/HO)$$

$$= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + E_T(AO/HO)$$

(15.37)

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type

ellipsoidal MO basis function of a chemical bond of the group, c_2 is the factor that results in an equipotential energy match of the participating at least two atomic orbitals of each chemical bond, and $E_T(AO/HO)$ is the total energy comprising the difference of the energy $E(AO/HO)$ of at least one atomic or hybrid orbital to which the MO is energy matched and
 5 any energy component $\Delta E_{H_2MO}(AO/HO)$ due to the AO or HO's charge donation to the MO.

324. The system of claim 323, wherein

$$E_T(AO/HO) = E(AO/HO) - \Delta E_{H_2MO}(AO/HO) \quad (15.38)$$

10 325. The system of claim 324, wherein as specific examples, $E_T(AO/HO)$ is one from the group of

$$E_T(AO/HO) = E(O2p \text{ shell}) = -E(\text{ionization}; O) = -13.6181 \text{ eV};$$

$$E_T(AO/HO) = E(N2p \text{ shell}) = -E(\text{ionization}; N) = -14.53414 \text{ eV};$$

$$E_T(AO/HO) = E(C, 2sp^3) = -14.63489 \text{ eV};$$

15 $E_T(AO/HO) = E_{Coulomb}(Cl, 3sp^3) = -14.60295 \text{ eV};$

$$E_T(AO/HO) = E(\text{ionization}; C) + E(\text{ionization}; C^+);$$

$$E_T(AO/HO) = E(C_{ethane}, 2sp^3) = -15.35946 \text{ eV};$$

$$E_T(AO/HO) = +E(C_{ethylene}, 2sp^3) - E(C_{ethylene}, 2sp^3);$$

$$E_T(AO/HO) = E(C, 2sp^3) - 2E_T(C = C, 2sp^3) = -14.63489 \text{ eV} - (-2.26758 \text{ eV});$$

20 $E_T(AO/HO) = E(C_{acetylene}, 2sp^3) - E(C_{acetylene}, 2sp^3) - E(C_{acetylene}, 2sp^3) = 16.20002 \text{ eV};$

$$E_T(AO/HO) = E(C, 2sp^3) - 2E_T(C \equiv C, 2sp^3) = -14.63489 \text{ eV} - (-3.13026 \text{ eV});$$

$$E_T(AO/HO) = E(C_{benzene}, 2sp^3) - E(C_{benzene}, 2sp^3);$$

$$E_T(AO/HO) = E(C, 2sp^3) - E_T(C = C, 2sp^3) = -14.63489 \text{ eV} - (-1.13379 \text{ eV}), \text{ and}$$

$$E_T(AO/HO) = E(C_{alkane}, 2sp^3) = -15.56407 \text{ eV}, \text{ wherein the calculated and measured}$$

25 values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

326. The system of claim 1, wherein to solve the bond parameters and energies,

$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2C_1 C_2 a}} = \sqrt{\frac{aa_0}{2C_1 C_2}}$ (Eq. (15.2)) is substituted into $E_T(H_2MO)$ to give

$$\begin{aligned} E_T(H_2MO) &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] + E_T(AO/HO) \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + E_T(AO/HO) \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T(AO/HO) \end{aligned}$$

(15.39)

5 wherein the total energy is set equal to $E(\text{basis energies})$ which in the most general case is given by the sum of a first integer n_1 times the total energy of H_2 minus a second integer n_2 times the total energy of H , minus a third integer n_3 times the valence energy of $E(AO)$ (e.g. $E(N) = -14.53414 \text{ eV}$) where the first integer can be 1, 2, 3..., and each of the second and third integers can be 0, 1, 2, 3....

10

327. The system of claim 326, wherein

$$E(\text{basis energies}) = n_1 (-31.63536831 \text{ eV}) - n_2 (-13.605804 \text{ eV}) - n_3 E(AO)$$

(15.40)

15 in the case that the MO bonds two atoms other than hydrogen, $E(\text{basis energies})$ is n_1 times the total energy of H_2 where n_1 is the number of equivalent bonds of the MO and the energy of H_2 , -31.63536831 eV , Eq. (11.212) is the minimum energy possible for a prolate spheroidal MO:

$$E(\text{basis energies}) = n_1 (-31.63536831 \text{ eV}) \quad (15.41)$$

$E_T(H_2MO)$, is set equal to $E(\text{basis energies})$, and the semimajor axis a is solved.

20

328. The system of claims 289 or 326, wherein the semimajor axis a is solved from the equation of the form:

$$-\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T(AO/HO) = E(\text{basis energies}) \quad (15.42)$$

The distance from the origin of the H_2 -type-ellipsoidal-MO to each focus c' , the internuclear distance $2c'$, and the length of the semiminor axis of the prolate spheroidal H_2 -type MO

5 $b = c$ are solved from the semimajor axis a using Eqs. (15.2-15.4).

329. The system of claim 328, wherein the component energies are given by Eqs. (15.6-15.9) and (15.39).

10 330. The system of claim 323, wherein the total energy of the MO of the functional group, $E_T(MO)$, is the sum of the total energy of the components comprising the energy contribution of the MO formed between the participating atoms and $E_T(atom - atom, msp^3.AO)$, the change in the energy of the AOs or HOs upon forming the bond.

15

331. The system of claim 330, wherein from Eqs. (15.39-15.40), $E_T(MO)$ is

$$E_T(MO) = E(\text{basis energies}) + E_T(atom - atom, msp^3.AO) \quad (15.43)$$

332. The system of claim 331, wherein during bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the sum of the Doppler, \bar{E}_D , and average vibrational kinetic energies, \bar{E}_{Kvib} :

20

$$\bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_{tr} \sqrt{\frac{2\bar{E}_K}{m_e c^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (15.44)$$

where n_1 is the number of equivalent bonds of the MO, k is the spring constant of the equivalent harmonic oscillator, and μ is the reduced mass.

25

333. The system of claim 332, wherein the angular frequency of the reentrant oscillation in the transition state corresponding to \bar{E}_D is determined by the force between the central

field and the electrons in the transition state.

334. The system of claim 333, wherein the force and its derivative are given by

$$f(R) = -c_{BO} \frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3} \quad (15.45)$$

5 and

$$f'(a) = 2c_{BO} \frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3} \quad (15.46)$$

such that the angular frequency of the oscillation in the transition state is given by

$$\omega = \sqrt{\frac{\left[\frac{-3}{a} f(a) - f'(a) \right]}{m_e}} = \sqrt{\frac{k}{m_e}} = \sqrt{\frac{c_{BO} \frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3}}{m_e}} \quad (15.47)$$

where R is the semimajor axis a or the semiminor axis b depending on the eccentricity of
 10 the bond that is most representative of the oscillation in the transition state, c_{BO} is the bond-order factor which is 1 for a single bond and when the MO comprises n_1 equivalent single bonds as in the case of functional groups, c_{BO} is 4 for an independent double bond as in the case of the CO_2 and NO_2 molecules and 9 for an independent triplet bond, C_{1o} is the fraction of the H_2 -type ellipsoidal MO basis function of the oscillatory transition state of a
 15 chemical bond of the group, and C_{2o} is the factor that results in an equipotential energy match of the participating at least two atomic orbitals of the transition state of the chemical bond. Typically, $C_{1o} = C_1$ and $C_{2o} = C_2$, the kinetic energy, E_K , corresponding to \bar{E}_D is given by Planck's equation for functional groups:

$$\bar{E}_K = \hbar\omega = \hbar \sqrt{\frac{\frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3}}{m_e}} \quad (15.48)$$

20

335. The system of claim 334, wherein the Doppler energy of the electrons of the reentrant orbit is

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{m_e c^2}} = E_{hv} \sqrt{\frac{2\hbar \sqrt{\frac{\frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3}}{m_e}}}{m_e c^2}} \quad (15.49)$$

\bar{E}_{osc} given by the sum of \bar{E}_D and \bar{E}_{Kvib} is

$$\bar{E}_{osc} (group) = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_{hv} \sqrt{\frac{2\hbar \sqrt{\frac{C_{1o}C_{2o}e^2}{4\pi\epsilon_0 R^3}}}{m_e c^2}} + E_{vib} \right) \quad (15.50)$$

E_{hv} of a group having n_1 bonds is given by $E_T(MO)/n_1$ such that

$$\bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_T(MO)/n_1 \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (15.51)$$

5 $E_{T+osc}(Group)$ is given by the sum of $E_T(MO)$ (Eq. (15.42)) and \bar{E}_{osc} (Eq. (15.51)):

$$E_{T+osc}(Group) = E_T(MO) + \bar{E}_{osc}$$

$$= \left(\begin{aligned} & -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1C_2}}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1C_2}}}{a - \sqrt{\frac{aa_0}{2C_1C_2}}} - 1 \right] \\ & + E_T(AO/HO) + E_T(atom - atom, msp^3.AO) \end{aligned} \right) \left(1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{1o}C_{2o}e^2}{4\pi\epsilon_0 R^3}}}{m_e c^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)$$

$$= \left(E(basis\ energies) + E_T(atom - atom, msp^3.AO) \right) \left(1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{1o}C_{2o}e^2}{4\pi\epsilon_0 R^3}}}{m_e c^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right)$$

(15.52).

336. The system of claim 335, wherein the total energy of the functional group $E_T(group)$ is the sum of the total energy of the components comprising the energy contribution of the MO formed between the participating atoms, $E(basis\ energies)$, the change in the energy of the AOs or HOs upon forming the bond ($E_T(atom - atom, msp^3.AO)$), the energy of
- 10

oscillation in the transition state, and the change in magnetic energy with bond formation,

$$E_{mag}.$$

337. The system of claim 336, wherein from Eq. (15.52), the total energy of the group

5 $E_T(\text{Group})$ is

$$E_T(\text{Group}) = \left(E(\text{basis energies}) + E_T(\text{atom} - \text{atom}, msp^3 AO) \right) \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{1o}C_{2o}e^2}{4\pi\epsilon_o R^3}}}{m_e c^2}} \right] + n_1 \bar{E}_{Kvib} + E_{mag} \quad (15.53).$$

10 338. The system of claim 337, wherein the change in magnetic energy E_{mag} which arises due to the formation of unpaired electrons in the corresponding fragments relative to the bonded group is given by

$$E_{mag} = c_3 \frac{2\pi\mu_o e^2 \hbar^2}{m_e^2 r^3} = c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} \quad (15.54)$$

where r^3 is the radius of the atom that reacts to form the bond and c_3 is the number of

15 electron pairs.

$$E_T(\text{Group}) = \left(E(\text{basis energies}) + E_T(\text{atom} - \text{atom}, msp^3 AO) \right) \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{1o}C_{2o}e^2}{4\pi\epsilon_o R^3}}}{m_e c^2}} \right] + n_1 \bar{E}_{Kvib} + c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} \quad (15.55).$$

339. The system of claim 338, wherein the total bond energy of the group $E_D(\text{Group})$ is the
20 negative difference of the total energy of the group (Eq. (15.55)) and the total energy of the starting species given by the sum of $c_4 E_{initial}(c_4 AO/HO)$ and $c_5 E_{initial}(c_5 AO/HO)$:

$$E_D(\text{Group}) = - \left(E(\text{basis energies}) + E_T(\text{atom} - \text{atom}, msp^3 \text{ AO}) \right) \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 R^3}}}{m_e c^2}} \right] + n_1 \bar{E}_{Kvlb} + c_3 \frac{8\pi\mu_o \mu_B^2}{r_n^3} - (c_4 E_{initial}(\text{AO/HO}) + c_5 E_{initial}(c_5 \text{ AO/HO})) \quad (15.56).$$

340. The system of claim 336, wherein in the case of organic molecules, the atoms of the functional groups are energy matched to the $C2sp^3$ HO such that

$$E(\text{AO/HO}) = -14.63489 \text{ eV} \quad (15.57)$$

For examples of E_{mag} from previous sections:

$$E_{mag}(C2sp^3) = c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} = c_3 \frac{8\pi\mu_o \mu_B^2}{(0.91771a_0)^3} = c_3 0.14803 \text{ eV} \quad (15.58)$$

$$E_{mag}(O2p) = c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} = c_3 \frac{8\pi\mu_o \mu_B^2}{a_0^3} = c_3 0.11441 \text{ eV} \quad (15.59)$$

$$E_{mag}(N2p) = c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} = c_3 \frac{8\pi\mu_o \mu_B^2}{(0.93084a_0)^3} = c_3 0.14185 \text{ eV} \quad (15.60).$$

341. The system of claim 340, wherein in the general case of the solution of an organic functional group, the geometric bond parameters are solved from the semimajor axis and the relationships between the parameters by first using Eq. (15.42) to arrive at a , the remaining parameters are determined using Eqs. (15.1-15.5), the energies are given by Eqs. (15.52-15.59), and to meet the equipotential condition for the union of the H_2 -type-ellipsoidal-MO and the HO or AO of the atom of a functional group, the factor c_2 of a H_2 -type ellipsoidal MO in principal Eqs. (15.42) and (15.52) may given by

(i) one:

$$c_2 = 1 \quad (15.61)$$

(ii) the ratio that is less than one of 13.605804 eV , the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243), and the magnitude of the

Coulombic energy of the participating AO or HO of the atom, $E_{Coulomb}(MO.atom,msp^3)$

given by Eqs. (15.19) and (15.31-15.32). For $|E_{Coulomb}(MO.atom,msp^3)| > 13.605804 \text{ eV}$:

$$c_2 = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{A-B \text{ AorBsp}^3}}} = \frac{13.605804 \text{ eV}}{|E_{Coulomb}(MO.atom,msp^3)|} \quad (15.62)$$

For $|E_{Coulomb}(MO.atom,msp^3)| < 13.605804 \text{ eV}$:

$$c_2 = \frac{\frac{e^2}{8\pi\epsilon_0 r_{A-B \text{ AorBsp}^3}}}{\frac{e^2}{8\pi\epsilon_0 a_0}} = \frac{|E_{Coulomb}(MO.atom,msp^3)|}{13.605804 \text{ eV}} \quad (15.63)$$

(iii) the ratio that is less than one of 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243), and the magnitude of the valence energy, $E(valence)$, of the participating AO or HO of the atom where $E(valence)$ is the ionization energy or $E(MO.atom,msp^3)$ given by Eqs. (15.20) and (15.31-15.32). For $|E(valence)| > 13.605804 \text{ eV}$:

$$c_2 = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{A-B \text{ AorBsp}^3}}} = \frac{13.605804 \text{ eV}}{|E(valence)|} \quad (15.64)$$

For $|E(valence)| < 13.605804 \text{ eV}$:

$$c_2 = \frac{\frac{e^2}{8\pi\epsilon_0 r_{A-B \text{ AorBsp}^3}}}{\frac{e^2}{8\pi\epsilon_0 a_0}} = \frac{|E(valence)|}{13.605804 \text{ eV}} \quad (15.65)$$

15

(iv) the ratio that is less than one of the magnitude of the Coulombic energy of the participating AO or HO of a first atom, $E_{Coulomb}(MO.atom,msp^3)$ given by Eqs. (15.19) and (15.31-15.32), and the magnitude of the valence energy, $E(valence)$, of the participating AO or HO of a second atom to which the first is energy matched where $E(valence)$ is the

ionization energy or $E(MO.atom,msp^3)$ given by Eqs. (15.20) and (15.31-15.32). For

$|E_{Coulomb}(MO.atom,msp^3)| > E(valence)$:

$$c_2 = \frac{|E(valence)|}{|E_{Coulomb}(MO.atom,msp^3)|} \quad (15.66)$$

For $|E_{Coulomb}(MO.atom,msp^3)| < E(valence)$:

$$c_2 = \frac{|E_{Coulomb}(MO.atom,msp^3)|}{|E(valence)|} \quad (15.67)$$

(v) the ratio that is less than one of the magnitude of the valence-level energies, $E_n(valence)$, of the AO or HO of the n th participating atom of two that are energy matched where $E(valence)$ is the ionization energy or $E(MO.atom,msp^3)$ given by Eqs. (15.20) and (15.31-15.32):

$$c_2 = \frac{E_1(valence)}{E_2(valence)} \quad (15.68)$$

(vi) the factor that is the ratio of the hybridization factor $c_2(1)$ of the valence AO or HO of a first atom and the hybridization factor $c_2(2)$ of the valence AO or HO of a second atom to which the first is energy matched where $c_2(n)$ is given by Eqs. (15.62-15.68); alternatively c_2 is the hybridization factor $c_2(1)$ of the valence AOs or HOs a first pair of atoms and the hybridization factor $c_2(2)$ of the valence AO or HO a third atom or second pair to which the first two are energy matched:

$$c_2 = \frac{c_2(1)}{c_2(2)} \quad (15.69)$$

20

(vii) the factor that is the product of the hybridization factor $c_2(1)$ of the valence AO or HO of a first atom and the hybridization factor $c_2(2)$ of the valence AO or HO of a second atom to which the first is energy matched where $c_2(n)$ is given by Eqs. (15.62-15.69); alternatively c_2 is the hybridization factor $c_2(1)$ of the valence AOs or HOs a first pair of

atoms and the hybridization factor $c_2(2)$ of the valence AO or HO a third atom or second pair to which the first two are energy matched:

$$c_2 = c_2(1)c_2(2) \quad (15.70)$$

The hybridization factor c_2 corresponds to the force constant k (Eqs. (11.65) and (13.58)).

- 5 In the case that the valence or Coulombic energy of the AO or HO is less than 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243), then C_2 corresponding to k' (Eq. (15.1)) is given by Eqs. (15.62-15.70).

342. The system of claim 341, wherein specific examples of the factors c_2 and C_2 of a
10 H_2 -type ellipsoidal MO of Eq. (15.51) given in following sections are

$$c_2(C2sp^3HO \text{ to } F) = \frac{E(C, 2sp^3)}{E(F)} c_2(C2sp^3HO) = \frac{-14.63489 \text{ eV}}{-17.42282 \text{ eV}} (0.91771) = 0.77087;$$

$$C_2(C2sp^3HO \text{ to } Cl) = \frac{E(Cl)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-12.96764 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.81317;$$

$$C_2(C2sp^3HO \text{ to } Br) = \frac{E(Br)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-11.81381 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.74081;$$

$$C_2(C2sp^3HO \text{ to } I) = \frac{E(I)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-10.45126 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.65537;$$

15 $c_2(C2sp^3HO \text{ to } O) = \frac{E(O)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-13.61806 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.85395;$

$$c_2(H \text{ to } 1^\circ N) = \frac{E(N)}{E(C, 2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.35946 \text{ eV}} = 0.94627;$$

$$c_2(C2sp^3HO \text{ to } N) = \frac{E(N)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-14.53414 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.91140;$$

$$c_2(H \text{ to } 2^\circ N) = \frac{E(N)}{E(C, 2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.56407 \text{ eV}} = 0.93383;$$

$$C_2(S3p \text{ to } H) = \frac{E(S, 3p)}{E(H)} = \frac{-10.36001 \text{ eV}}{-13.60580 \text{ eV}} = 0.76144;$$

20 $C_2(C2sp^3HO \text{ to } S) = \frac{E(S)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-10.36001 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.64965;$

968

$$\begin{aligned}
 c_2(O \text{ to } S3sp^3 \text{ to } C2sp^3HO) &= \frac{E(O)}{E(S)} c_2(C2sp^3HO) \\
 &= \frac{-13.61806 \text{ eV}}{-10.36001 \text{ eV}} (0.91771); \\
 &= 1.20632
 \end{aligned}$$

$$c_2(S3sp^3) = \frac{E_{Coulomb}(S3sp^3)}{E(H)} = \frac{-11.57099 \text{ eV}}{-13.60580 \text{ eV}} = 0.85045;$$

$$C_2(C2sp^3HO \text{ to } S3sp^3) = \frac{E(S3sp^3)}{E(C, 2sp^3)} c_2(S3sp^3) = \frac{-11.52126 \text{ eV}}{-14.63489 \text{ eV}} (0.85045) = 0.66951;$$

$$\begin{aligned}
 C_2(S3sp^3 \text{ to } O \text{ to } C2sp^3HO) &= \frac{E(S, 3sp^3)}{E(O, 2p)} c_2(C2sp^3HO) \\
 &= \frac{-11.52126 \text{ eV}}{-13.61806 \text{ eV}} (0.91771); \\
 &= 0.77641
 \end{aligned}$$

$$\begin{aligned}
 c_2(O \text{ to } N2p \text{ to } C2sp^3HO) &= \frac{E(O)}{E(N)} c_2(C2sp^3HO) \\
 &= \frac{-13.61806 \text{ eV}}{-14.53414 \text{ eV}} (0.91771); \\
 &= 0.85987
 \end{aligned}$$

5

$$c_2(N2p \text{ to } O2p) = \frac{c_2(C2sp^3HO \text{ to } N)}{c_2(C2sp^3HO \text{ to } O)} = \frac{0.91140}{0.85395} = 1.06727;$$

$$C_2(\text{benzene}C2sp^3HO) = c_2(\text{benzene}C2sp^3HO) = \frac{13.605804 \text{ eV}}{15.95955 \text{ eV}} = 0.85252;$$

$$\begin{aligned}
 c_2(\text{aryl}C2sp^3HO \text{ to } O) &= \frac{E(O)}{E(C, 2sp^3)} c_2(\text{aryl}C2sp^3HO) \\
 &= \frac{-13.61806 \text{ eV}}{-14.63489 \text{ eV}} (0.85252); \\
 &= 0.79329
 \end{aligned}$$

$$c_2(H \text{ to anline } N) = \frac{E(N)}{E(C, 2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.76868 \text{ eV}} = 0.92171,$$

$$\begin{aligned}
 c_2(\text{aryl}C2sp^3HO \text{ to } N) &= \frac{E(N)}{E(C, 2sp^3)} c_2(\text{aryl}C2sp^3HO) \\
 &= \frac{-14.53414 \text{ eV}}{-14.63489 \text{ eV}} (0.85252) \\
 &= 0.84665
 \end{aligned}$$

10

$$\begin{aligned}
 c_2(H \text{ to anline } N) &= \frac{E(N)}{E(C, 2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.76868 \text{ eV}} = 0.92171; \\
 c_2(arylC2sp^3HO \text{ to } N) &= \frac{E(N)}{E(C, 2sp^3)} c_2(arylC2sp^3HO) \\
 &= \frac{-14.53414 \text{ eV}}{-14.63489 \text{ eV}} (0.85252) \quad , \quad \text{and} \\
 &= 0.84665 \\
 C_2(S3p \text{ to aryl-type } C2sp^3HO) &= \frac{E(S, 3p)}{E(C, 2sp^3)} = \frac{-10.36001 \text{ eV}}{-15.76868 \text{ eV}} = 0.65700.
 \end{aligned}$$

5 wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

343. The system of claim 1, wherein the parameters of the point of intersection of each H_2 -type ellipsoidal MO and the A -atom AO are determined from the polar equation of
10 the ellipse:

$$r = r_0 \frac{1+e}{1+e \cos \theta'} \quad (15.71).$$

344. The system of claim 343, wherein the radius of the A shell is r_A , and the polar radial coordinate of the ellipse and the radius of the A shell are equal at the point of intersection
15 such that

$$r_A = (a - c') \frac{1 + \frac{c'}{a}}{1 + \frac{c'}{a} \cos \theta'} \quad (15.72).$$

345. The system of claim 344, wherein the polar angle θ' at the intersection point is given by

$$20 \quad \theta' = \cos^{-1} \left(\frac{a}{c'} \left((a - c') \frac{1 + \frac{c'}{a}}{a_0} - 1 \right) \right) \quad (15.73).$$

346. The system of claim 345, wherein the angle θ_{AAO} the radial vector of the A AO makes with the internuclear axis is

$$\theta_{AAO} = 180^\circ - \theta' \quad (15.74).$$

347. The system of claim 345, wherein the distance from the point of intersection of the orbitals to the internuclear axis is the same for both component orbitals such that the angle $\omega t = \theta_{H_2MO}$ between the internuclear axis and the point of intersection of each H_2 -type ellipsoidal MO with the A radial vector obeys the following relationship:

$$r_A \sin \theta_{AAO} = b \sin \theta_{H_2MO} \quad (15.75)$$

such that

$$\theta_{H_2MO} = \sin^{-1} \frac{r_a \sin \theta_{AAO}}{b} \quad (15.76).$$

10

348. The system of claim 347, wherein the distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{H_2MO} = a \cos \theta_{H_2MO} \quad (15.77).$$

- 15 349. The system of claim 347, wherein the distance d_{AAO} along the internuclear axis from the origin of the A atom to the point of intersection of the orbitals is given by

$$d_{AAO} = c' - d_{H_2MO} \quad (15.78).$$

- 20 350. The system of claim 1, wherein in ACB MO comprising a linear combination of $C-A$ -bond and $C-B$ -bond MOs where C is the general central atom and a bond is possible between the A and B atoms of the $C-A$ and $C-B$ bonds, the $\angle ACB$ bond angle is determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal A and B atoms is zero.

- 25 351. The system of claim 350, wherein the force constant k' of a H_2 -type ellipsoidal MO due to the equivalent of two point charges of at the foci is given by:

$$k' = \frac{C_1 C_2 2e^2}{4\pi\epsilon_0} \quad (15.79)$$

where C_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the specie which is 0.75 (Eq. (13.59)) for a terminal $A-H$ (A is H or other atom) and 1

otherwise and C_2 is the factor that results in an equipotential energy match of the participating at least two atomic orbitals of the chemical bond and is equal to the corresponding factor of Eqs. (15.42) and (15.52).

- 5 352. The system of claim 351, wherein the distance from the origin of the MO to each focus c' of the $A-B$ ellipsoidal MO is given by:

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2C_1 C_2 a}} = \sqrt{\frac{aa_0}{2C_1 C_2}} \quad (15.80).$$

353. The system of claim 351, wherein the internuclear distance is

10 $2c' = 2 \sqrt{\frac{aa_0}{2C_1 C_2}} \quad (15.81).$

354. The system of claim 351, wherein the length of the semiminor axis of the prolate spheroidal $A-B$ MO $b = c$ is given by Eq. (15.4).

- 15 355. The system of claim 351, wherein the component energies and the total energy, $E_T(H_2MO)$, of the $A-B$ bond are given by the energy equations (Eqs. (11.207-11.212), (11.213-11.217), and (11.239)) of H_2 except that the terms based on charge are multiplied by c_{BO} , the bond-order factor which is 1 for a single bond and when the MO comprises n_1 equivalent single bonds as in the case of functional groups. c_{BO} is 4 for an
20 independent double bond as in the case of the CO_2 and NO_2 molecules.

356. The system of claim 355, wherein the kinetic energy term is multiplied by c'_{BO} which is 1 for a single bond, 2 for a double bond, and 9/2 for a triple bond, the electron energy terms are multiplied by c_1 , the fraction of the H_2 -type ellipsoidal MO basis function of a
25 terminal chemical bond which is 0.75 (Eq. (13.233)) for a terminal $A-H$ (A is H or other atom) and 1 otherwise.

357. The system of claim 355, wherein the electron energy terms are further multiplied by c'_2 , the hybridization or energy-matching factor that results in an equipotential energy match of the participating at least two atomic orbitals of each terminal bond.

5 358. The system of claim 350, wherein when $A-B$ comprises atoms other than H , $E_T(\text{atom-atom}, msp^3.AO)$, the energy component due to the AO or HO's charge donation to the terminal MO, is added to the other energy terms to give $E_T(H_2MO)$:

$$E_T(H_2MO) = \frac{-e^2}{8\pi\epsilon_0 c'} \left[c_1 c'_2 \left(2c_{BO} - c'_{BO} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] + E_T(\text{atom-atom}, msp^3.AO)$$

(15.82).

10

359. The system of claim 350, the radiation reaction force in the case of the vibration of $A-B$ in the transition state corresponds to the Doppler energy, E_D , given by Eq. (11.181) that is dependent on the motion of the electrons and the nuclei.

15 360. The system of claim 359, wherein the total energy that includes the radiation reaction of the $A-B$ MO is given by the sum of $E_T(H_2MO)$ (Eq. (15.82)) and \bar{E}_{asc} given Eqs. (11.213-11.220), (11.231-11.236), and (11.239-11.240).

361. The system of claim 360, wherein the total energy $E_T(A-B)$ of the $A-B$ MO
20 including the Doppler term is

$$E_T(A-B) = \left[\left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[c_1 c'_2 \left(2c_{BO} - c'_{BO} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] + E_T(\text{atom-atom}, msp^3.AO) \right) \right] \left[1 + \sqrt{\frac{c_{BO} \frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 a^3}}{m_e c^2}} \right] + \frac{1}{2} \hbar \sqrt{\frac{c_{BO} \frac{c_1 c'_2 e^2}{8\pi\epsilon_0 a^3} - \frac{c_{BO} e^2}{8\pi\epsilon_0 (a+c')^3}}{\mu}}$$

(15.83)

where C_{10} is the fraction of the H_2 -type ellipsoidal MO basis function of the oscillatory transition state of the $A-B$ bond which is 0.75 (Eq. (13.233)) in the case of H bonding to a
25 central atom and 1 otherwise, C_{20} is the factor that results in an equipotential energy match of

the participating at least two atomic orbitals of the transition state of the chemical bond, and

$\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the nuclei given by Eq. (11.154).

362. The system of claim 361, wherein to match the boundary condition that the total
5 energy of the $A-B$ ellipsoidal MO is zero, $E_T(A-B)$ given by Eq. (15.83) is set equal to zero and substitution of Eq. (15.81) into Eq. (15.83) gives

$$0 = \left[\left(\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \left[c_1 c_2' \left(2c_{BO} - c_{BO}' \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T(\text{atom} - \text{atom}, msp^3, AO) \right) \right. \\ \left. + \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 a^3}} \frac{c_{BO}}{2\hbar \sqrt{\frac{m_e}{m_e c^2}}} + \frac{1}{2} \hbar \sqrt{\frac{c_{BO} \frac{c_1 c_2' e^2}{8\pi\epsilon_0 a^3} - \frac{c_{BO} e^2}{8\pi\epsilon_0 \left(a + \sqrt{\frac{aa_0}{2C_1 C_2}} \right)^3}}{\mu}} \right] \quad (15.84).$$

- 10 363. The system of claim 362, wherein the vibrational energy-term of Eq. (15.84) is determined by the forces between the central field and the electrons and those between the nuclei (Eqs. (11.141-11.145)).

364. The system of claim 362, wherein the electron-central-field force and its derivative
15 are given by

$$f(a) = -c_{BO} \frac{c_1 c_2' e^2}{4\pi\epsilon_0 a^3} \quad (15.85)$$

and

$$f'(a) = 2c_{BO} \frac{c_1 c_2' e^2}{4\pi\epsilon_0 a^3} \quad (15.86).$$

- 20 365. The system of claim 364, wherein the nuclear repulsion force and its derivative are given by

$$f(a+c') = \frac{e^2}{8\pi\epsilon_0 (a+c')^2} \quad (15.87)$$

and

$$f'(a+c') = -\frac{e^2}{4\pi\epsilon_0(a+c')^3} \quad (15.88)$$

such that the angular frequency of the oscillation is given by

$$\omega = \sqrt{\frac{\left[\frac{-3}{a}f(a) - f'(a)\right]}{\mu}} = \sqrt{\frac{k}{m_e}} = \sqrt{\frac{c_{BO} \frac{c_1 c_2' e^2}{4\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0(a+c')^2}}{\mu}} \quad (15.89).$$

5

366. The system of claim 365, wherein since both terms of $\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib}$ are small due to the large values of a and c' , an approximation of Eq. (15.84) which is evaluated to determine the bond angles of functional groups is given by

$$0 = \left[\left(\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \left[c_1 c_2' \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T(\text{atom} - \text{atom}, msp^3, AO) \right) \right. \\ \left. \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{c_1 e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} + \frac{1}{2} \hbar \sqrt{\frac{\frac{c_1 e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 \left(a + \sqrt{\frac{aa_0}{2C_1 C_2}} \right)^3}}{\mu}} \right] \right] \quad (15.90).$$

10

367. The system of claim 366, wherein from the energy relationship given by Eq. (15.90) and the relationship between the axes given by Eqs. (15.2-15.5), the dimensions of the $A-B$ MO can be solved.

15

368. The system of claim 367, wherein Eq. (15.90) is solved by the reiterative technique using a computer.

369. The system of claim 366, wherein a factor c_2 of a given atom in the determination of c_2' for calculating the zero of the total $A-B$ bond energy is given by Eqs. (15.62-15.65).

20

370. The system of claim 369, wherein in the case of a $H-H$ terminal bond of an alkyl or alkenyl group, c'_2 is the ratio of c_2 of Eq. (15.62) for the $H-H$ bond which is one and c_2 of the carbon of the corresponding $C-H$ bond:

$$c'_2 = \frac{1}{c_2(C2sp^3)} = \frac{13.605804 \text{ eV}}{E_{Coulomb}(C-H \text{ } C2sp^3)} \quad (15.91)$$

- 5 wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

371. The system of claim 366, wherein in the case of the determination of the bond angle of the ACH MO comprising a linear combination of $C-A$ -bond and $C-H$ -bond MOs
10 where A and C are general, C is the central atom, and c_2 for an atom is given by Eqs. (15.62-15.70), c'_2 of the $A-H$ terminal bond is the ratio of c_2 of the A atom for the $A-H$ terminal bond and c_2 of the C atom of the corresponding $C-H$ bond:

$$c'_2 = \frac{c_2(A(A-H)msp^3)}{c_2(C(C-H)(msp^3))} \quad (15.92).$$

- 15 372. The system of claim 366, wherein in the case of the determination of the bond angle of the COH MO of an alcohol comprising a linear combination of $C-O$ -bond and $O-H$ -bond MOs where C , O , and H are carbon, oxygen, and hydrogen, respectively, c'_2 of the $C-H$ terminal bond is 0.91771 since the oxygen and hydrogen atoms are at the Coulomb potential of a proton and an electron (Eqs. (1.236) and (10.162),
20 respectively) that is energy matched to the $C2sp^3$ HO, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

373. The system of claim 366, wherein in the determination of the hybridization factor c'_2
25 of Eq. (15.90) from Eqs. (15.62-15.70), the Coulombic energy, $E_{Coulomb}(MO.atom, msp^3)$, or the energy, $E(MO.atom, msp^3)$, the radius $r_{A-B \text{ } AorBsp^3}$ of the A or B AO or HO of the heteroatom of the $A-B$ terminal bond MO such as the $C2sp^3$ HO of a terminal $C-C$

bond is calculated using Eq. (15.32) by considering $\sum E_{T_{mol}}(MO, 2sp^3)$, the total energy donation to each bond with which it participates in bonding as it forms the terminal bond.

374. The system of claim 373, wherein the Coulombic energy $E_{Coulomb}(MO.atom, msp^3)$ of
5 the outer electron of the *atom msp³* shell is given by Eq. (15.19).

375. The system of claim 374, wherein in the case that during hybridization, at least one of
the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change
for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ (Eq. (15.15))
10 at the initial radius r of the AO electron, and the energy $E(MO.atom, msp^3)$ of the outer
electron of the *atom msp³* shell is given by the sum of $E_{Coulomb}(MO.atom, msp^3)$ and
 $E(magnetic)$ (Eq. (15.20)).

376. The system of claim 365, wherein in the specific case of the terminal bonding of two
15 carbon atoms, the c_2 factor of each carbon given by Eq. (15.62) is determined using the
Coulombic energy $E_{Coulomb}(C-C C2sp^3)$ of the outer electron of the *C2sp³* shell given
by Eq. (15.19) with the radius $r_{C-C C2sp^3}$ of each *C2sp³* HO of the terminal *C-C* bond
calculated using Eq. (15.32) by considering $\sum E_{T_{mol}}(MO, 2sp^3)$, the total energy donation
to each bond with which it participates in bonding as it forms the terminal bond including
20 the contribution of the methylene energy, 0.92918 eV (Eq. (14.513)), corresponding to
the terminal *C-C* bond. The corresponding $E_T(atom-atom, msp^3.AO)$ in Eq. (15.90) is
 $E_T(C-C C2sp^3) = -1.85836 \text{ eV}$, wherein the calculated and measured values and
constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if
desired.

25

377. The system of claim 366, wherein in the case that the terminal atoms are carbon or
other heteroatoms, the terminal bond comprises a linear combination of the HOs or AOs;
thus, c'_2 is the average of the hybridization factors of the participating atoms
corresponding to the normalized linear sum:

977

$$c'_2 = \frac{1}{2} (c'_2(\text{atom } 1) + c'_2(\text{atom } 2)) \quad (15.93)$$

In the exemplary cases of $C-C$, $O-O$, and $N-N$ where C is carbon:

$$c'_2 = \frac{1}{2} \left(\frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{A-A A_1 AO/HO}}} + \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{A-A A_2 AO/HO}}} \right) \quad (15.94)$$

$$= \frac{1}{2} \left(\frac{13.605804 \text{ eV}}{E_{\text{Coulomb}}(A-A A_1 AO/HO)} + \frac{13.605804 \text{ eV}}{E_{\text{Coulomb}}(A-A A_2 AO/HO)} \right)$$

In the exemplary cases of $C-N$, $C-O$, and $C-S$,

$$5 \quad c'_2 = \frac{1}{2} \left(\frac{13.605804 \text{ eV}}{E_{\text{Coulomb}}(C-B C2sp^3)} + c_2(C \text{ to } B) \right) \quad (15.95)$$

where C is carbon and $c_2(C \text{ to } B)$ is the hybridization factor of Eqs. (15.52) and (15.84) that matches the energy of the atom B to that of the atom C in the group.

378. The system of claim 363, wherein the corresponding $E_T(\text{atom} - \text{atom}, msp^3.AO)$

10 term in Eq. (15.90) depends on the hybridization and bond order of the terminal atoms in the molecule, but typical values matching those used in the determination of the bond energies (Eq. (15.56)) are

$$E_T(C-O C2sp^3.O2p) = -1.44915 \text{ eV}; E_T(C-O C2sp^3.O2p) = -1.65376 \text{ eV};$$

$$E_T(C-N C2sp^3.N2p) = -1.44915 \text{ eV}; E_T(C-S C2sp^3.S2p) = -0.72457 \text{ eV};$$

$$15 \quad E_T(O-O O2p.O2p) = -1.44915 \text{ eV}; E_T(O-O O2p.O2p) = -1.65376 \text{ eV};$$

$$E_T(N-N N2p.N2p) = -1.44915 \text{ eV}; E_T(N-O N2p.O2p) = -1.44915 \text{ eV};$$

$$E_T(F-F F2p.F2p) = -1.44915 \text{ eV}; E_T(Cl-Cl Cl3p.Cl3p) = -0.92918 \text{ eV};$$

$$E_T(Br-Br Br4p.Br4p) = -0.92918 \text{ eV}; E_T(I-I I5p.I5p) = -0.36229 \text{ eV};$$

$$E_T(C-F C2sp^3.F2p) = -1.85836 \text{ eV}; E_T(C-Cl C2sp^3.Cl3p) = -0.92918 \text{ eV};$$

$$20 \quad E_T(C-Br C2sp^3.Br4p) = -0.72457 \text{ eV}; E_T(C-I C2sp^3.I5p) = -0.36228 \text{ eV}, \text{ and}$$

$$E_T(O-Cl O2p.Cl3p) = -0.92918 \text{ eV}, \text{ wherein the calculated and measured values and}$$

constants recited in the equations herein can be adjusted, for example, up to + 10%, if desired.

379. The system of claim 366, wherein in the case that the terminal bond is $X-X$ where X is a halogen atom, c_1 is one, and c'_2 is the average (Eq. (15.93)) of the hybridization factors of the participating halogen atoms given by Eqs. (15.62-15.63) where

5 $E_{Coulomb}(MO.atom,msp^3)$ is determined using Eq. (15.32) and

$$E_{Coulomb}(MO.atom,msp^3) = 13.605804 \text{ eV for } X = I.$$

380. The system of claim 379, wherein the factor C_1 of Eq. (15.90) is one for all halogen atoms.

10

381. The system of claim 379, wherein the factor C_2 of fluorine is one since it is the only halogen wherein the ionization energy is greater than that 13.605804 eV , the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243), wherein the calculated and measured values and constants recited in the equations herein

15 can be adjusted, for example, up to $\pm 10\%$, if desired.

382. The system of claim 379, wherein for each of the halogens, Cl , Br , and I , C_2 is the hybridization factor of Eq. (15.52) given by Eq. (15.70) with $c_2(1)$ being that of the halogen given by Eq. (15.68) that matches the valence energy of X ($E_1(valence)$) to that

20 of the $C2sp^3$ HO ($E_2(valence) = -14.63489 \text{ eV}$, Eq. (15.25)) and to the hybridization of $C2sp^3$ HO ($c_2(2) = 0.91771$, Eq. (13.430)), wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

25 383. The system of claim 382, wherein $E_r(atom - atom,msp^3.AO)$ of Eq. (15.90) is the maximum for the participating atoms which is -1.44915 eV , -0.92918 eV , -0.92918 eV , and -0.33582 eV for F , Cl , Br , and I , respectively, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

30

384. The system of claim 366, wherein in the case that the terminal bond is $C-X$ where C is a carbon atom and X is a halogen atom, the factors c_1 and C_1 of Eq. (15.90) are one for all halogen atoms.
- 5 385. The system of claim 384, wherein for $X = F$, c'_2 is the average (Eq. (15.95)) of the hybridization factors of the participating carbon and F atoms where c_2 for carbon is given by Eq. (15.62) and c_2 for fluorine matched to carbon is given by Eq. (15.70) with $c_2(1)$ for the fluorine atom given by Eq. (15.68) that matches the valence energy of F ($E_1(\text{valence}) = -17.42282 \text{ eV}$) to that of the $C2sp^3$ HO ($E_2(\text{valence}) = -14.63489 \text{ eV}$,
10 Eq. (15.25)) and to the hybridization of $C2sp^3$ HO ($c_2(2) = 0.91771$, Eq. (13.430)), wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.
386. The system of claim 385, wherein the factor C_2 of fluorine is one since it is the only
15 halogen wherein the ionization energy is greater than that 13.605804 eV , the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243), wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.
- 20 387. The system of claim 385, wherein for each of the other halogens, Cl , Br , and I , c'_2 is the hybridization factor of the participating carbon atom since the halogen atom is energy matched to the carbon atom.
388. The system of claim 387, wherein the C_2 of the terminal-atom bond matches that
25 used to determine the energies of the corresponding $C-X$ -bond MO.
389. The system of claim 388, wherein C_2 is the hybridization factor of Eq. (15.52) given by Eq. (15.70) with $c_2(1)$ for the halogen atom given by Eq. (15.68) that matches the valence energy of X ($E_1(\text{valence})$) to that of the $C2sp^3$ HO
30 ($E_2(\text{valence}) = -14.63489 \text{ eV}$, Eq. (15.25)) and to the hybridization of $C2sp^3$ HO

980

($c_2(2) = 0.91771$, Eq. (13.430)), wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

5 390. The system of claim 385, wherein $E_T(\text{atom} - \text{atom}, \text{msp}^3 \text{AO})$ of Eq. (15.90) is the maximum for the participating atoms which is -1.85836 eV , -0.92918 eV , -0.72457 eV , and -0.33582 eV for F , Cl , Br , and I , respectively, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

10

391. The system of claim 385, wherein in the case that the terminal bond is $H - X$ corresponding to the angle of the atoms $H CX$ where C is a carbon atom and X is a halogen atom, the factors c_1 and C_1 of Eq. (15.90) are 0.75 for all halogen atoms.

15 392. The system of claim 385, wherein for $X = F$, c'_2 is given by Eq. (15.69) with c_2 of the participating carbon and F atoms given by Eq. (15.62) and Eq. (15.65), respectively.

393. The system of claim 392, wherein the factor C_2 of fluorine is one since it is the only halogen wherein the ionization energy is greater than that 13.605804 eV , the magnitude
20 of the Coulombic energy between the electron and proton of H given by Eq. (1.243), wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

394. The system of claim 393, wherein for each of the other halogens, Cl , Br , and I , c'_2
25 is also given by Eq. (15.69) with c_2 of the participating carbon given by Eq. (15.62) and c_2 of the participating X atom given by $c_2 = 0.91771$ (Eq. (13.430)) since the X atom is energy matched to the $C2sp^3$ HO.

395. The system of claim 394, wherein C_2 is given by Eq. (15.65) for the corresponding
30 atom X where C_2 matches the energy of the atom X to that of H .

396. The system of claim 366, wherein the distance between the two atoms A and B of the general molecular group ACB when the total energy of the corresponding $A-B$ MO is zero, the corresponding bond angle can be determined from the law of cosines:

$$s_1^2 + s_2^2 - 2s_1s_2\cosine\theta = s_3^2 \quad (15.96).$$

5

397. The system of claim 396, wherein with $s_1 = 2c'_{C-A}$, the internuclear distance of the $C-A$ bond, $s_2 = 2c'_{C-B}$, the internuclear distance of each $C-B$ bond, and $s_3 = 2c'_{A-B}$, the internuclear distance of the two terminal atoms, the bond angle $\theta_{\angle ACB}$ between the $C-A$ and $C-B$ bonds is given by

$$10 \quad (2c'_{C-A})^2 + (2c'_{C-B})^2 - 2(2c'_{C-A})(2c'_{C-B})\cosine\theta = (2c'_{A-B})^2 \quad (15.97)$$

$$\theta_{\angle ACB} = \cos^{-1} \left(\frac{(2c'_{C-A})^2 + (2c'_{C-B})^2 - (2c'_{A-B})^2}{2(2c'_{C-A})(2c'_{C-B})} \right) \quad (15.98).$$

398. The system of claim 397, wherein the structure $C_b C_a (O_a) O_b$ wherein C_a is bound to C_b , O_a , and O_b , the three bonds are coplanar and two of the angles are known, say θ_1 and θ_2 , then the third θ_3 can be determined geometrically:

$$15 \quad \theta_3 = 360 - \theta_1 - \theta_2 \quad (15.99)$$

399. The system of claim 397, wherein in the general case that two of the three coplanar bonds are equivalent and one of the angles is known, say θ_1 , then the second and third can be determined geometrically:

$$20 \quad \theta_2 = \theta_3 = \frac{(360 - \theta_1)}{2} \quad (15.100).$$

400. The system of claim 1, wherein in the general case where the group comprises three $A-B$ bonds having B as the central atom at the apex of a pyramidal structure formed by the three bonds with the A atoms at the base in the xy -plane.

25

401. The system of claim 400, wherein the C_{3v} axis centered on B is defined as the vertical or z -axis, and any two $A-B$ bonds form an isosceles triangle, and the angle of

the bonds and the distances from and along the z-axis are determined from the geometrical relationships given by Eqs. (13.412-13.416):

the distance $d_{\text{origin}-B}$ from the origin to the nucleus of a terminal B atom is given by

$$d_{\text{origin}-B} = \frac{2c'_{B-B}}{2 \sin 60^\circ} \quad (15.101)$$

5 the height along the z-axis from the origin to the A nucleus d_{height} is given by

$$d_{\text{height}} = \sqrt{(2c'_{A-B})^2 - (d_{\text{origin}-B})^2}, \text{ and } (15.102)$$

the angle θ_v of each $A-B$ bond from the z-axis is given by

$$\theta_v = \tan^{-1} \left(\frac{d_{\text{origin}-B}}{d_{\text{height}}} \right) \quad (15.103).$$

10 402. The system of claim 401, wherein in the case where the central atom B is further bound to a fourth atom C and the $B-C$ bond is along the z-axis. Then, the bond $\theta_{\angle ABC}$ given by Eq. (14.206) is

$$\theta_{\angle ABC} = 180 - \theta_v \quad (15.104).$$

15

403. The system of claim 400, wherein in the plane defined by a general ACA MO comprising a linear combination of two $C-A$ -bond MOs where C is the central atom, the dihedral angle $\theta_{\angle BC/ACA}$ between the ACA -plane and a line defined by a third bond with C , specifically that corresponding to a $C-B$ -bond MO, is calculated from the bond
20 angle $\theta_{\angle ACA}$ and the distances between the A , B , and C atoms.

404. The system of claim 403, wherein the distance d_1 along the bisector of $\theta_{\angle ACA}$ from C to the internuclear-distance line between A and A , $2c'_{A-A}$, is given by

$$d_1 = 2c'_{C-A} \cos \frac{\theta_{\angle ACA}}{2} \quad (15.105)$$

25 where $2c'_{C-A}$ is the internuclear distance between A and C .

405. The system of claim 404, wherein the atoms A , A , and B define the base of a pyramid.

406. The system of claim 405, wherein the pyramidal angle $\theta_{\angle ABA}$ can be solved from the internuclear distances between A and A , $2c'_{A-A}$, and between A and B , $2c'_{A-B}$, using the law of cosines (Eq. (15.98)):

$$\theta_{\angle ABA} = \cos^{-1} \left(\frac{(2c'_{A-B})^2 + (2c'_{A-B})^2 - (2c'_{A-A})^2}{2(2c'_{A-B})(2c'_{A-B})} \right) \quad (15.106).$$

407. The system of claim 406, wherein the distance d_2 along the bisector of $\theta_{\angle ABA}$ from B to the internuclear-distance line $2c'_{A-A}$, is given by

$$d_2 = 2c'_{A-B} \cos \frac{\theta_{\angle ABA}}{2} \quad (15.107).$$

408. The system of claim 407, wherein the lengths d_1 , d_2 , and $2c'_{C-B}$ define a triangle wherein the angle between d_1 and the internuclear distance between B and C , $2c'_{C-B}$, is the dihedral angle $\theta_{\angle BCIACA}$ that can be solved using the law of cosines (Eq. (15.98)):

$$\theta_{\angle BCIACA} = \cos^{-1} \left(\frac{d_1^2 + (2c'_{C-B})^2 - d_2^2}{2d_1(2c'_{B-C})} \right) \quad (15.108).$$

409. The system of claim 1, wherein the specie are solved using the solutions of organic chemical functional groups as basis elements wherein the structures and energies where linearly added to achieve the molecular solutions, each functional group can be treated as a building block to form any desired molecular solution from the corresponding linear combination, each functional group element was solved using the atomic orbital and hybrid orbital spherical orbitalsphere solutions bridged by molecular orbitals comprised of the H_2 -type prolate spheroidal solution given in the Nature of the Chemical Bond of Hydrogen-Type Molecules section, the energy of each MO was matched at the HO or AO by matching the hybridization and total energy of the MO to the AOs and HOs, the energy E_{mag} (e.g. given by Eq. (15.58)) for a $C2sp^3$ HO and Eq.(15.59) for an $O2p$ AO) was subtracted for each set of unpaired electrons created by bond breakage.

410. The system of claim 409, wherein the bond energy is not equal to the component energy of each bond as it exists in the specie, although, they are close.
411. The system of claim 409, wherein the total energy of each group is its contribution to the total energy of the specie as a whole.
412. The system of claim 409, wherein the determination of the bond energies for the creation of the separate parts must take into account the energy of the formation of any radicals and any redistribution of charge density within the pieces and the corresponding energy change with bond cleavage.
413. The system of claim 409, wherein the vibrational energy in the transition state is dependent on the other groups that are bound to a given functional group, which will effect the functional-group energy, however because the variations in the energy based on the balance of the molecular composition are typically of the order of a few hundreds of electron volts at most, they are neglected.
414. The system of claim 409, wherein the energy of each functional-group MO bonding to a given carbon HO is independently matched to the HO by subtracting the contribution to the change in the energy of the HO from the total MO energy given by the sum of the MO contributions and $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (13.428)).
415. The system of claim 409, wherein the intercept angles are determined from Eqs. (15.71-15.78) using the final radius of the HO of each atom.
416. The system of claim 409, wherein a final carbon-atom radius is determined using Eqs. (15.32) wherein the sum of the energy contributions of each atom to all the MOs in which it participates in bonding is determined.
417. The system of claim 416, wherein the final radius is used in Eqs. (15.19) and (15.20) to calculate the final valence energy of the HO of each atom at the corresponding final radius.

418. The system of claim 417, wherein the radius of any bonding heteroatom that contributes to a MO is calculated in the same manner, and the energy of its outermost shell is matched to that of the MO by the hybridization factor between the carbon-HO energy and the energy of the heteroatomic shell.
- 5
419. The system of claim 416, wherein the donation of electron density to the AOs and HOs reduces the energy.
420. The system of claim 419, wherein the donation of the electron density to the MO's at
10 each AO or HO is that which causes the resulting energy to be divided equally between the participating AOs or HOs to achieve energy matching.
421. The system of claim 1, wherein the molecular solutions are used to design synthetic pathways and predict product yields based on equilibrium constants calculated from the
15 heats of formation.
422. The system of claim 1, wherein the new stable compositions of matter are predicted as well as the structures of combinatorial chemistry reactions.
- 20 423. The system of claim 1, wherein pharmaceutical applications include the ability to graphically or computationally render the structures of drugs that permit the identification of the biologically active parts of the specie to be identified from the common spatial charge-density functions of a series of active species.
- 25 424. The system of claim 1, wherein novel drugs are designed according to geometrical parameters and bonding interactions with the data of the structure of the active site of the drug.
- 30 425. The system of claim 1, wherein to calculate conformations, folding, and physical properties, the exact solutions of the charge distributions in any given specie are used to calculate the fields, and from the fields, the interactions between groups of the same specie or between groups on different species are calculated wherein the interactions are distance and relative orientation dependent.

426. The system of claim 425, the fields and interactions can be determined using a finite-element-analysis approach of Maxwell's equations.

427. The system of claim 1, wherein in the case where three sets of $C = C$ -bond MOs
 5 form bonds between the two carbon atoms of each molecule to form a six-member ring such that the six resulting bonds comprise eighteen paired electrons, and each bond comprises a linear combination of two MOs wherein each comprises two $C2sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the $C2sp^3$ HOs:

$$\left(\begin{array}{l} 3(2 C2sp^3 + 0.75 H_2 MO) \rightarrow 3 \left(\overset{4e}{C=C} \right) - \text{ethylene-type-bond MO} \\ \rightarrow 6 \left(\overset{3e}{C=C} \right) - \text{bond MO of benzene} \end{array} \right) \quad (15.142).$$

10

428. The system of claim 427, wherein the linear combination of each H_2 -type ellipsoidal MO with each $C2sp^3$ HO further comprises an excess 25% charge-density contribution per bond from each $C2sp^3$ HO to the $C = C$ -bond MO to achieve an energy minimum.

15 429. The system of claim 427, wherein the dimensional parameters of each bond $C = C$ -bond are determined using Eqs. (15.42) and (15.1-15.5) in a form that are the same equations as those used to determine the same parameters of the $C = C$ -bond MO of ethylene (Eqs. (14.242-14.268)) while matching the boundary conditions of the structure of benzene.

20

430. The system of claim 427, wherein hybridization with 25% electron donation to each $C = C$ -bond gives rise to the $C_{benzene}2sp^3$ HO-shell Coulombic energy

$$E_{Coulomb} \left(C_{benzene}, 2sp^3 \right) \text{ given by Eq. (14.245).}$$

25 431. The system of claim 427, wherein to meet the equipotential condition of the union of the six $C2sp^3$ HOs, c_2 and C_2 of Eq. (15.42) for the aromatic $\overset{3e}{C=C}$ -bond MO is given by Eq. (15.62) as the ratio of 15.95955 eV, the magnitude of $E_{Coulomb} \left(C_{benzene}, 2sp^3 \right)$ (Eq.

(14.245)), and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H (Eq. (1.243)):

$$C_2(\text{benzene}C2sp^3HO) = c_2(\text{benzene}C2sp^3HO) = \frac{13.605804 \text{ eV}}{15.95955 \text{ eV}} = 0.85252 \quad (15.143)$$

wherein the calculated and measured values and constants recited in the equations herein can
5 be adjusted, for example, up to $\pm 10\%$, if desired.

432. The system of claim 427, wherein the energies of each $C=C$ ^{3e} bond of benzene are determined using the same equations as those of ethylene (Eqs. (14.251-14.253) and (14.319-14.333) with the parameters of benzene.

10

433. The system of claim 427, wherein ethylene serves as a basis element for the $C=C$ ^{3e} bonding of benzene wherein each of the six $C=C$ ^{3e} bonds of benzene comprises (0.75)(4) = 3 electrons according to Eq. (15.142).

15 434. The system of claim 427, wherein the total energy of the bonds of the eighteen electrons of the $C=C$ ^{3e} bonds of benzene, $E_T(C_6H_6, C=C)$ ^{3e}, is given by (6)(0.75) times $E_{T+osc}(C=C)$ (Eq. (14.492)), the total energy of the $C=C$ ^{3e}-bond MO of benzene including the Doppler term, minus eighteen times $E(C, 2sp^3)$ (Eq. (14.146)), the initial energy of each $C2sp^3$ HO of each C that forms the $C=C$ ^{3e} bonds of bond order two.

20

435. The system of claim 427, wherein the total energy of the six $C=C$ ^{3e} bonds of benzene with three electron per aromatic bond given by Eq. (14.493) is

$$\begin{aligned} E_T(C_6H_6, C=C) &= (6)(0.75)E_{T+osc}(C=C) - (6)(3)E(C, 2sp^3) \\ &= (6)(0.75)(-66.05796 \text{ eV}) - 18(-14.63489 \text{ eV}) \quad (15.144) \\ &= -297.26081 \text{ eV} - (-263.42798 \text{ eV}) \\ &= -33.83284 \text{ eV} \end{aligned}$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

436. The system of claim 427, wherein the results of benzene can be generalized to the class of aromatic and heterocyclic compounds. E_{hv} of an aromatic bond is given by $E_T(H_2)$ (Eqs. (11.212) and (14.486)), the maximum total energy of each H_2 -type MO such that

$$\bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(-31.63536831 \text{ eV} \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (15.145),$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

437. The system of claim 435, wherein the factor of 0.75 corresponding to the three electrons per aromatic bond of bond order two given in the Benzene Molecule (C_6H_6) section modifies Eqs. (15.52-15.56).

438. The system of claim 437, wherein the multiplication of the total energy given by Eq. (15.55) by $f_1 = 0.75$ with the substitution of Eq. (15.145) gives the total energy of the aromatic bond:

$$E_T(\text{Group}) = f_1 \left(\begin{array}{l} E(\text{basis energies}) + E_T(\text{atom-atom,msp}^3 \text{ AO}) \\ -31.63536831 \text{ eV} \sqrt{\frac{2\hbar \sqrt{\frac{C_{10}C_{20}e^2}{4\pi\epsilon_0 R^3}}}{m_e} + n_1 \bar{E}_{Kvib} + c_3 \frac{8\pi\mu_0\mu_B^2}{r^3}} \end{array} \right) \quad (15.146)$$

wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

439. The system of claim 427, wherein the total bond energy of the aromatic group $E_D(\text{Group})$ is the negative difference of the total energy of the group (Eq. (15.146)) and the

total energy of the starting species given by the sum of $c_4 E_{initial}(c_4 AO / HO)$ and $c_5 E_{initial}(c_5 AO / HO)$:

$$E_D(Group) = - \left[f_1 \left(\begin{array}{c} E(basis\ energies) + E_T(atom - atom, msp^3 AO) \\ -31.63536831\ eV \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 R^3}}}{m_e}} + n_1 \bar{E}_{Kvib} + c_3 \frac{8\pi\mu_0 \mu_B^2}{r^3} \end{array} \right) - (c_4 E_{initial}(AO / HO) + c_5 E_{initial}(c_5 AO / HO)) \right]$$

(15.147), wherein the calculated and measured values and constants recited in the 5 equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

440. The system of claim 427, wherein benzene is considered as comprising chemical bonds between six CH radicals wherein each radical comprises a chemical bond between carbon and hydrogen atoms, energy components of V_e , V_p , T , V_m , and E_T are the same as those of the hydrogen carbide radical, except that $E_T(C = C, 2sp^3) = -1.13379\ eV$ (Eq. (14.247)) is subtracted from $E_T(CH)$ of Eq. (13.495) to match the energy of each $C - H$ -bond MO to the decrease in the energy of the corresponding $C2sp^3$ HO, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

15

441. The system of claim 440, wherein in the corresponding generalization of the aromatic CH group, the geometrical parameters are determined using Eq. (15.42) and Eqs. (15.1-15.5) with $E_T(atom - atom, msp^3 AO) = -1.13379\ eV$ wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

20

442. The system of claim 440, wherein the total energy of the benzene $C - H$ -bond MO, $E_{T_{benzene}}(C - H)$, given by Eq. (14.467) is the sum of $0.5 E_T(C = C, 2sp^3)$, the energy change of each $C2sp^3$ shell per single bond due to the decrease in radius with the

formation of the corresponding $C=C$ -bond MO (Eq. (14.247)), and $E_{T_{benzene}}^{3e}(CH)$, the σ MO contribution given by Eq. (14.441). In the corresponding generalization of the aromatic CH group, the energy parameters are determined using Eqs. (15.146-15.147) with $f_1 = 1$ and $E_T(atom - atom, msp^3 AO) = \frac{-1.13379 \text{ eV}}{2}$, wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

443. The system of claim 442, wherein the energy contribution to the single aromatic CH bond is one half that of the $C=C$ double bond contribution, which matches the energies of the CH and $C=C$ aromatic groups, conserves the electron number with the equivalent charge density as that of $s = 1$ in Eqs. (15.18-15.21), and further gives a minimum energy for the molecule.

444. The system of claim 443, wherein the breakage of the aromatic $C=C$ bonds to give CH groups creates unpaired electrons in these fragments that corresponds to $c_3 = 1$ in Eq. (15.56) with E_{mag} given by Eq. (15.58).

445. The system of claim 444, wherein each of the $C-H$ bonds of benzene comprises two electrons according to Eq. (14.439).

20

446. The system of claim 445, wherein from the energy of each $C-H$ bond, $-E_{D_{benzene}}(^{12}CH)$ (Eq. (14.477)), the total energy of the twelve electrons of the six $C-H$ bonds of benzene, $E_T(C_6H_6, C-H)$, given by Eq. (14.494) is

$$E_T(C_6H_6, C-H) = (6)(-E_{D_{benzene}}(^{12}CH)) = 6(-3.90454 \text{ eV}) = -23.42724 \text{ eV}$$

(15.148), wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

25

447. The system of claim 440, wherein the total bond dissociation energy of benzene,

$E_D(C_6H_6)$, given by Eq. (14.495) is the negative sum of $E_T\left(C_6H_6, C \equiv C\right)$ (Eq.

(14.493)) and $E_T(C_6H_6, C-H)$ (Eq. (14.494)):

$$\begin{aligned} E_D(C_6H_6) &= -\left[E_T\left(C_6H_6, C \equiv C\right) + E_T(C_6H_6, C-H)\right] \\ &= -\left((-33.83284 \text{ eV}) + (-23.42724 \text{ eV})\right) \quad (15.149) \\ &= 57.2601 \text{ eV} \end{aligned}$$

5 wherein the calculated and measured values and constants recited in the equations herein can be adjusted, for example, up to $\pm 10\%$, if desired.

448. The system of claim 447, wherein using the parameters given in Tables 15.214 and 15.216 in the general equations (Eqs. (15.42), (15.1-15.5), and (15.146-15.147))

10 reproduces the results for benzene given in the Benzene Molecule (C_6H_6) section as shown in Tables 15.214 and 15.216.

449. A system of computing and rendering the nature of at least one specie selected from a group of diatomic molecules having at least one atom that is other than hydrogen,

15 polyatomic molecules, molecular ions, polyatomic molecular ions, or molecular radicals, or any functional group therein, comprising physical, Maxwellian solutions of charge, mass, and current density functions of said specie, said system comprising:

processing means for processing physical, Maxwellian equations representing charge, mass, and current density functions of said specie to produce at least one Maxwellian

20 solution; and

an output means for outputting the Maxwellian solution.

450. The system of claim 449, further comprising a data transfer system for inputting

numerical data into or out of a computational components and storage components of the

25 main system.

451. The system of claim 449, further comprising a spreadsheet containing solutions of the bond parameters with output in a standard spreadsheet format.

452. The system of claim 451, further comprising a data-handling program to transfer data from the spreadsheets into the main program.

5 453. The system of claim 449, wherein output may be at least one of graphical, simulation, text, and numerical data.

454. The system of claim 453, wherein the output may be the calculation of at least one of:
10 (1) a bond distance between two of the atoms; (2) a bond angle between three of the atoms; (3) a bond energy between two of the atoms; (4) orbital intercept distances and angles; and (5) charge-density functions of atomic, hybridized, and molecular orbitals, wherein the bond distance, bond angle, and bond energy are calculated from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that a bound
15 electron(s) does not radiate under acceleration

455. The system of claim 449, wherein the charge, current, energy, and geometrical parameters are output to be inputs to other programs that can be used in further applications.

20

456. The system of claim 455, wherein the data of heats of formation can be input to another program to be used to predict stability (existence of compounds) equilibrium constants and to predict synthetic pathways.

25 457. The system of claim 456, wherein novel composition of matters can be discovered by calculating at least one of a bond distance between two of the atoms, a bond angle between three of the atoms, and a bond energy between two of the atoms, orbital intercept distances and angles, charge-density functions of atomic, hybridized, and molecular orbitals, the bond distance, bond angle, and bond energy being calculated from physical
30 solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that a bound electron(s) does not radiate under acceleration.

993

458. The system of claim 456, wherein the charge and current density functions can be used to predict the electric and magnetic fields of the species to determine other properties due to the interaction of the fields between species.

5 459. The system of claim 458, wherein finite-element analysis is used to predict or calculate the interaction and resulting properties, such as the freezing point, boiling point, density, viscosity, and refractive index.

10 460. The system of claim 449, wherein the output data can be used to give thermodynamic, spectroscopic, and other properties, aid in drug design and other applications with or without direct visualization.

15 461. The system of claim 449, wherein the data can be input into other programs of the system, which calculate thermodynamic and other properties, or performs a simulation, such as a chemical reaction or molecular dynamics.

462. A method of using any of the systems or compositions of matter of claims 1-461.

463. A use of any system or composition of matter of claims 1-461.

Fig.1

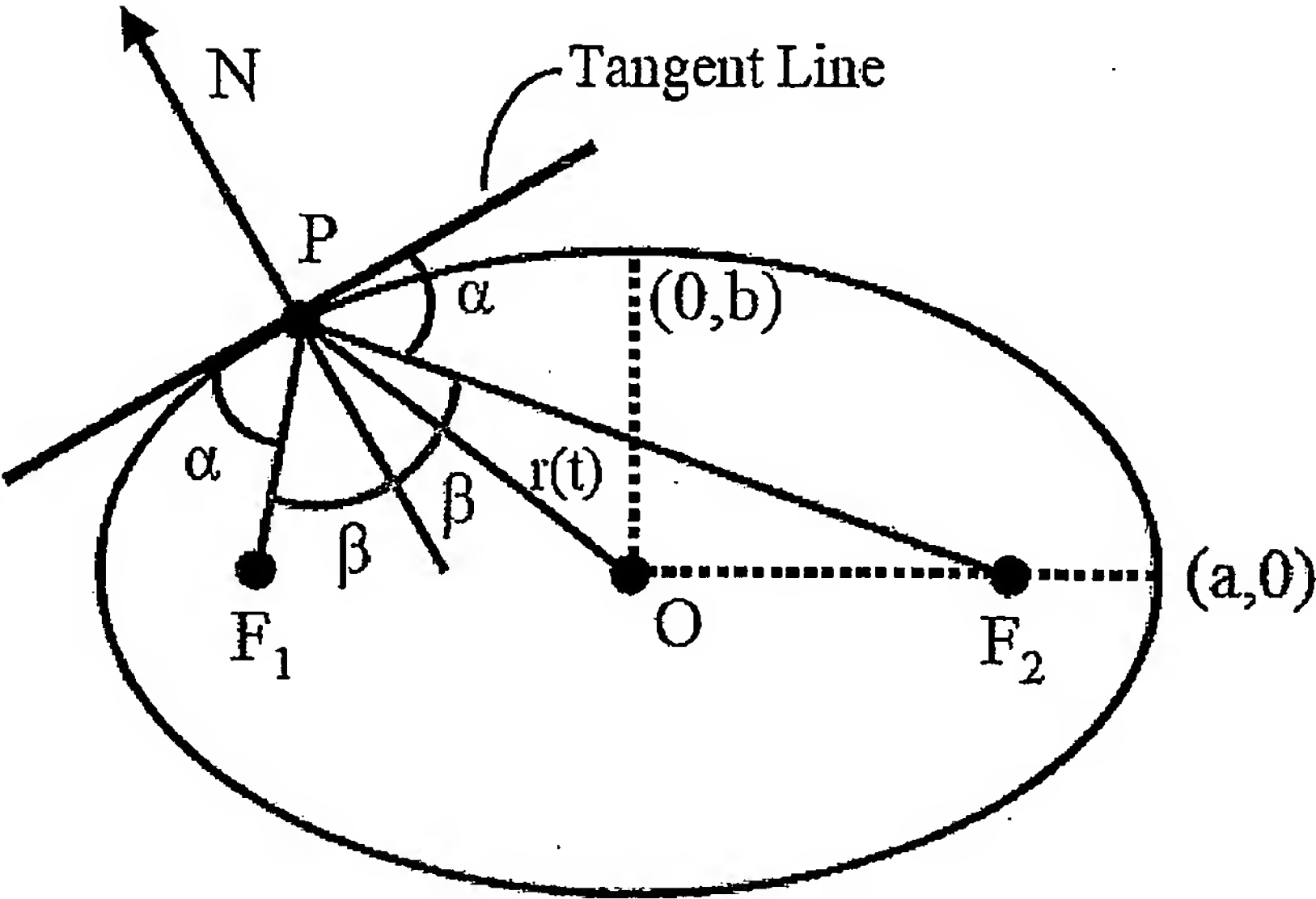
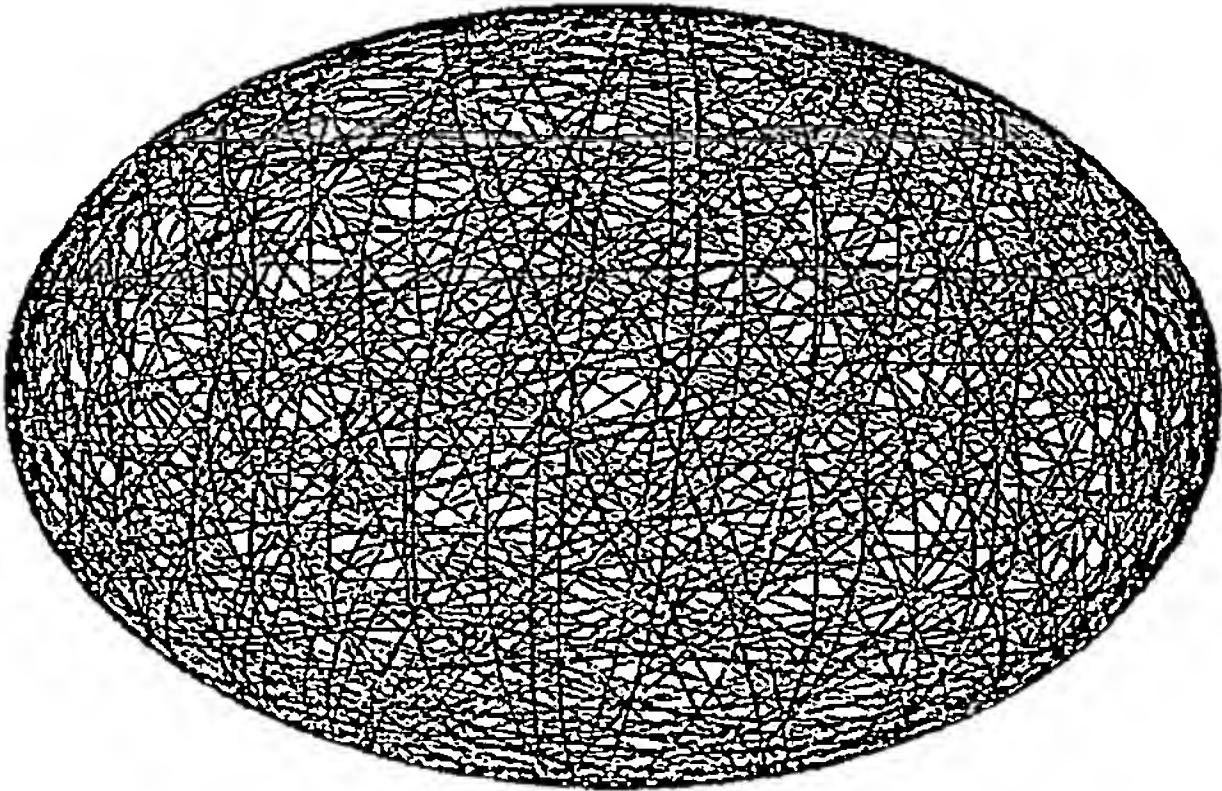


Fig.2



2/51
Fig.3

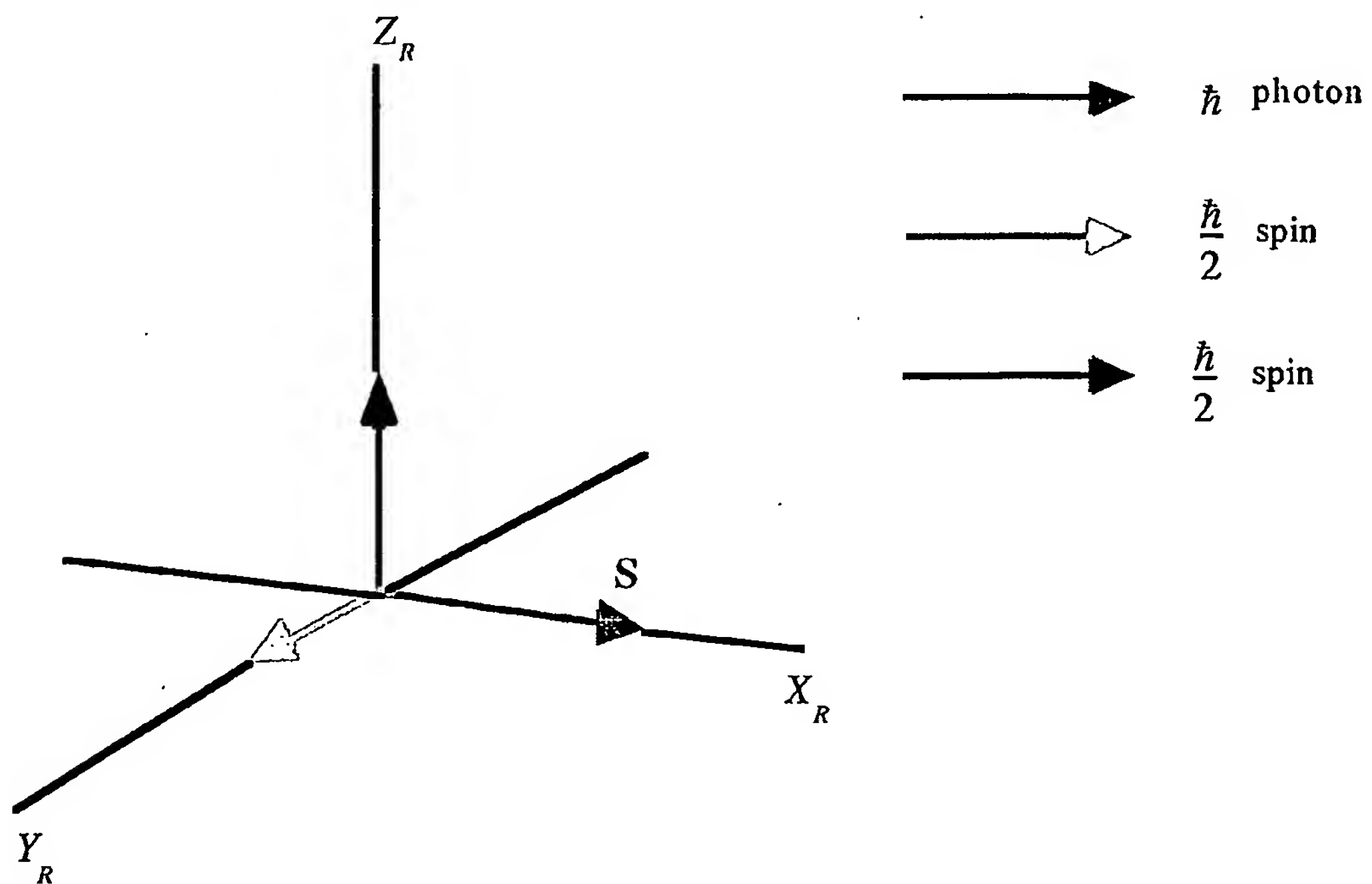


Fig.4

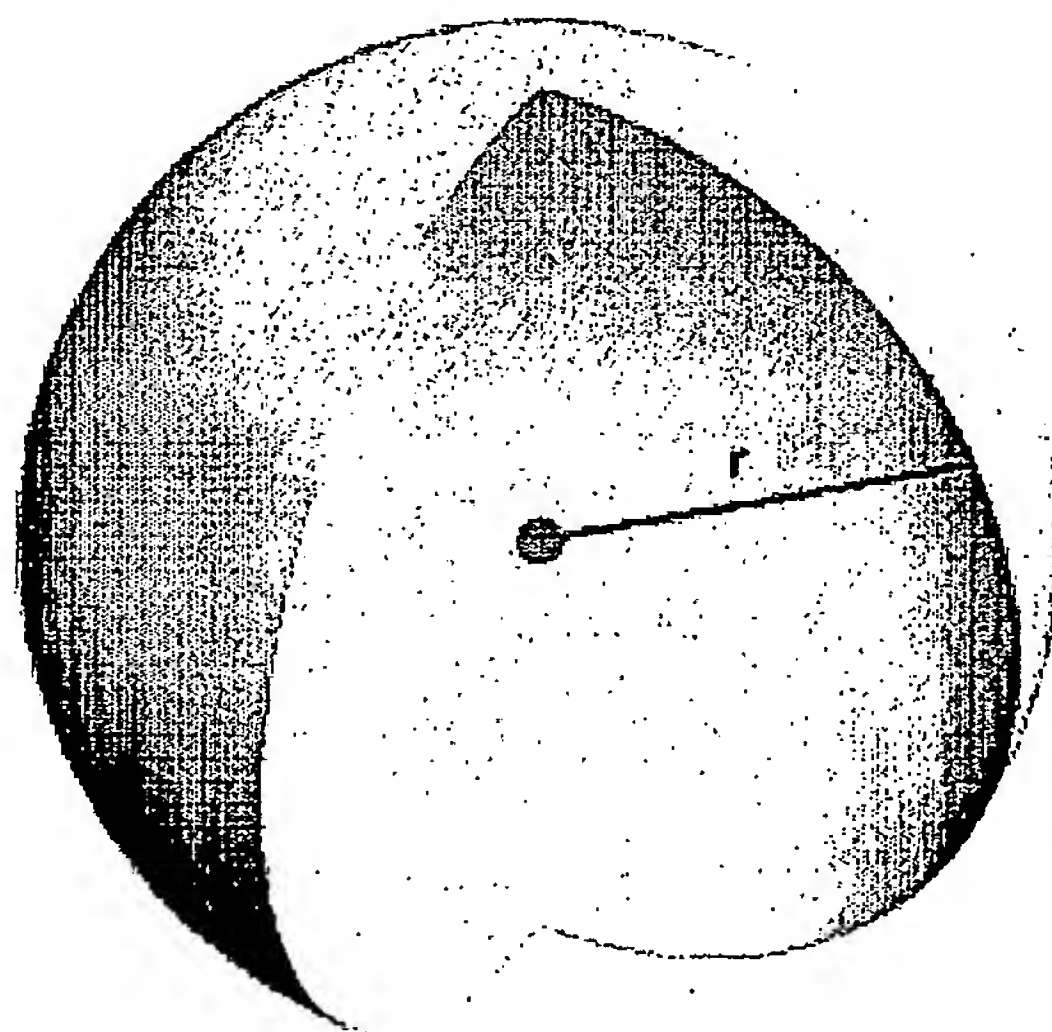
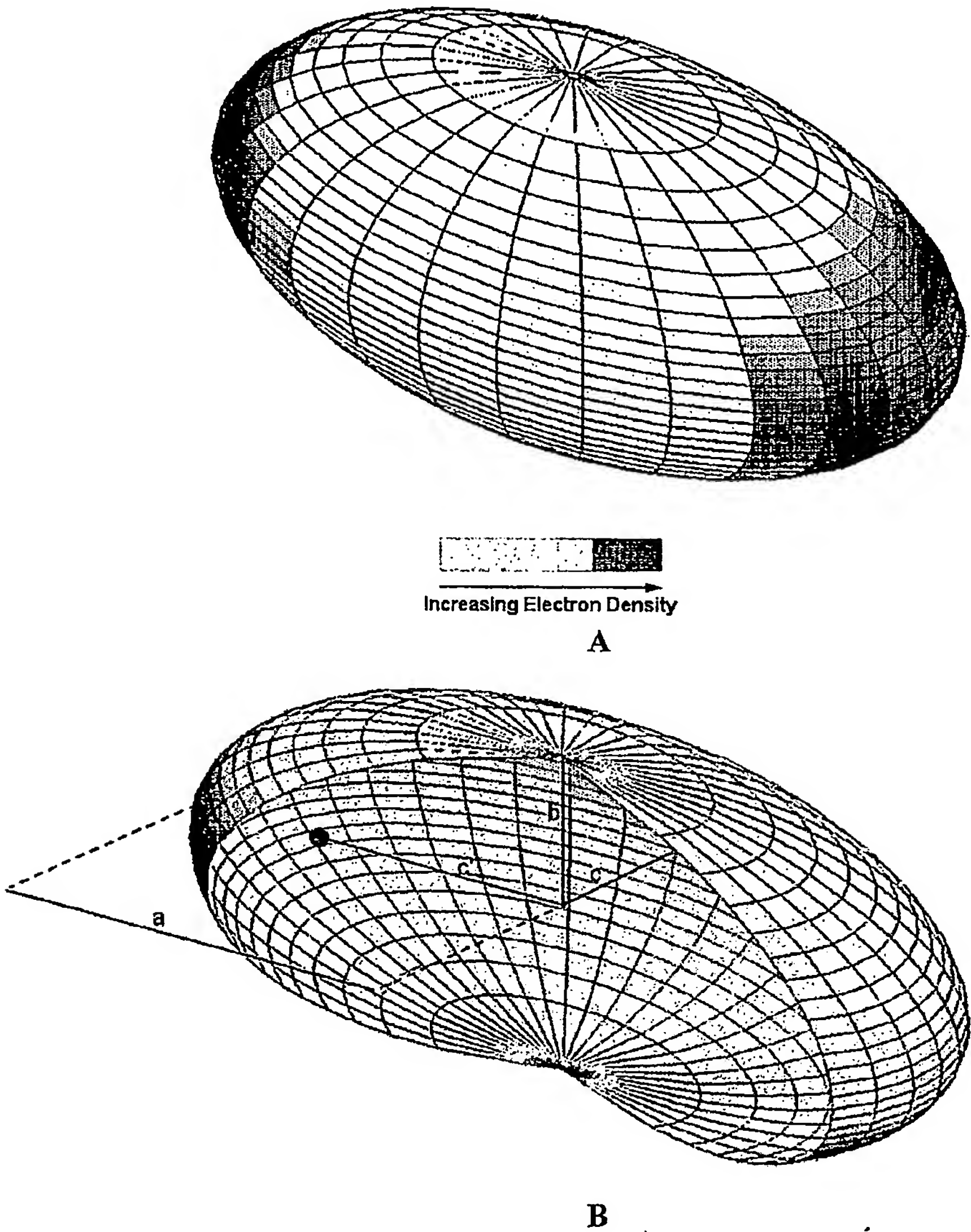
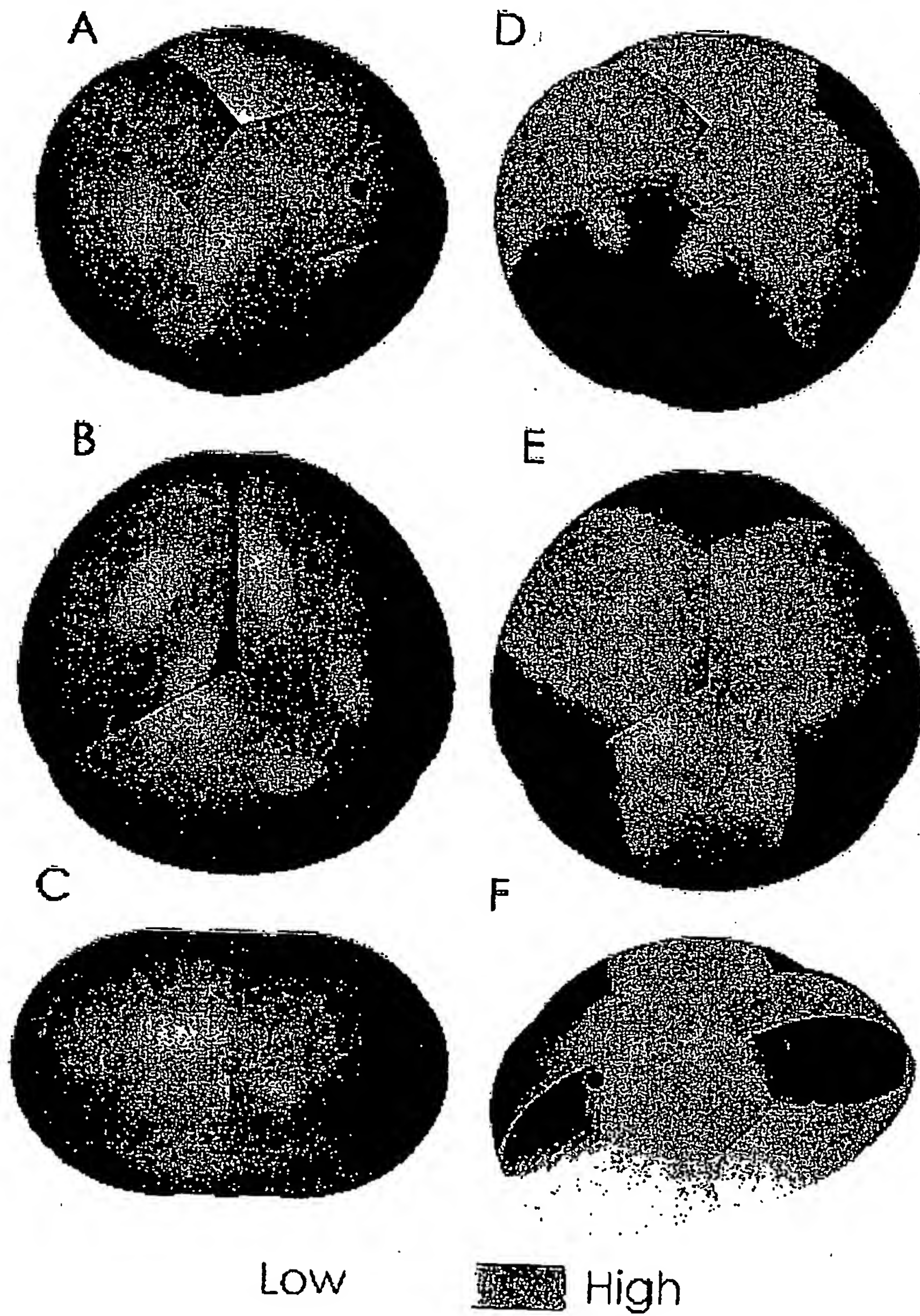


Fig.5



4/51
Fig. 6



5/51

Fig.7

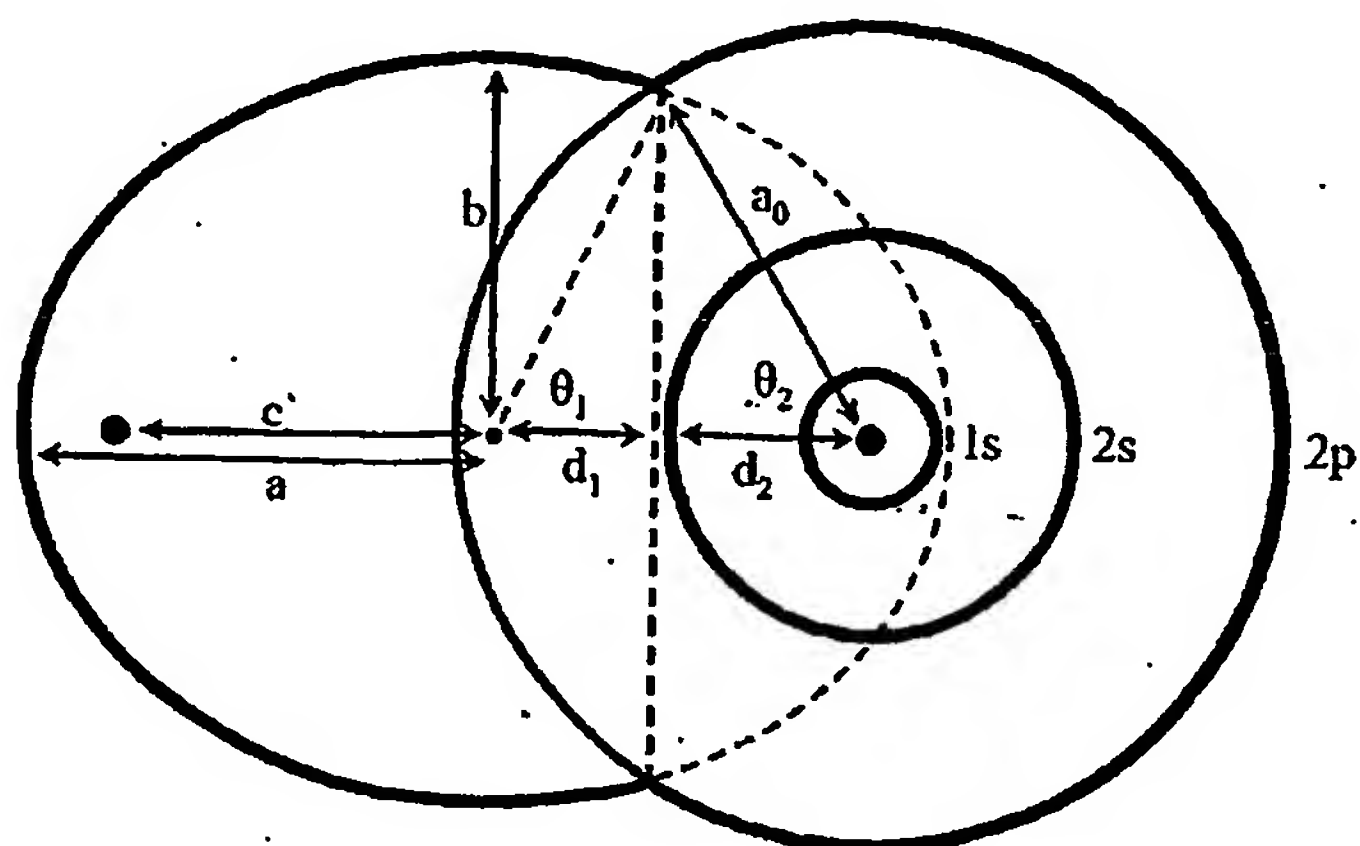
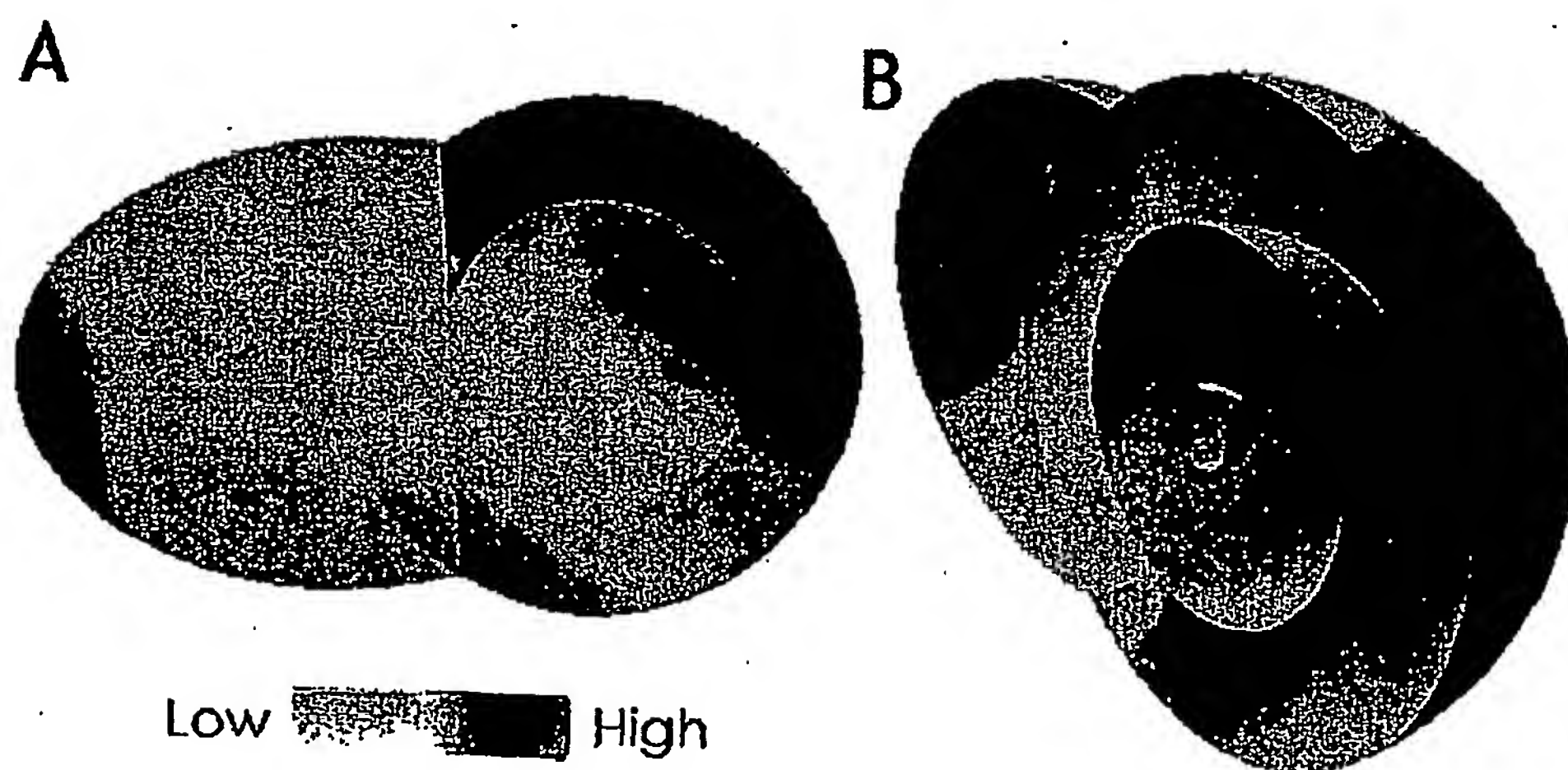


Fig.8



6/51
Fig.9

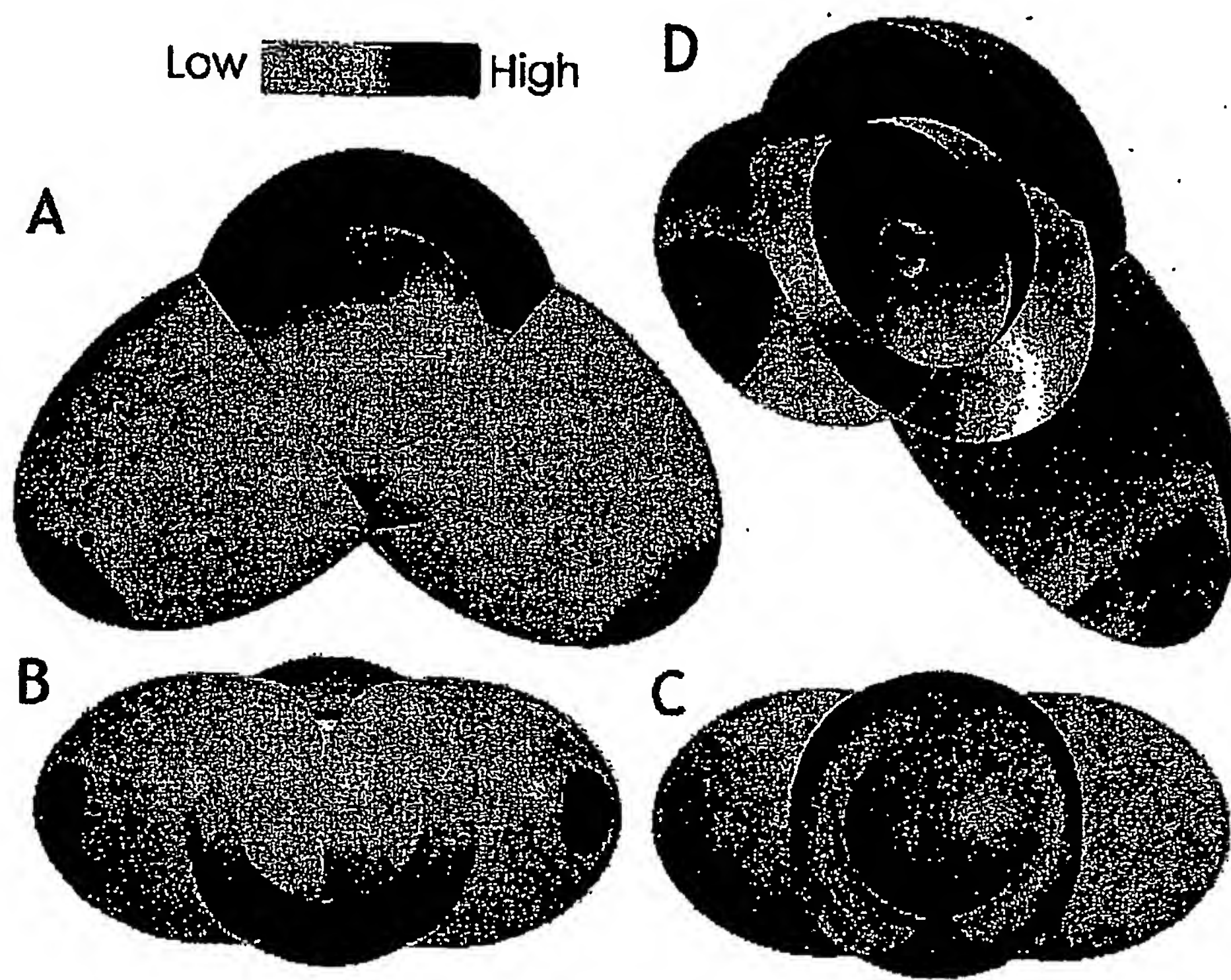
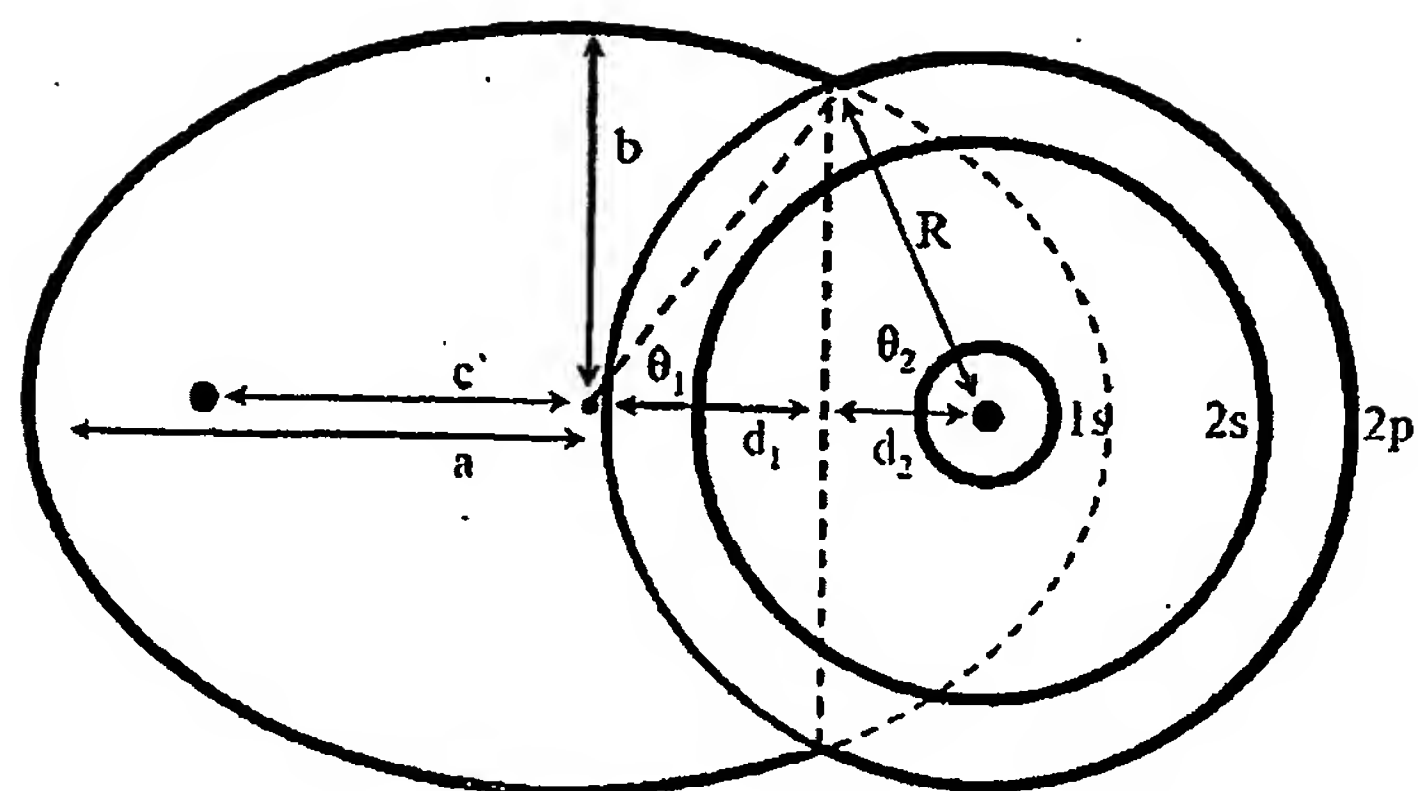


Fig.10



7/51

Fig.11

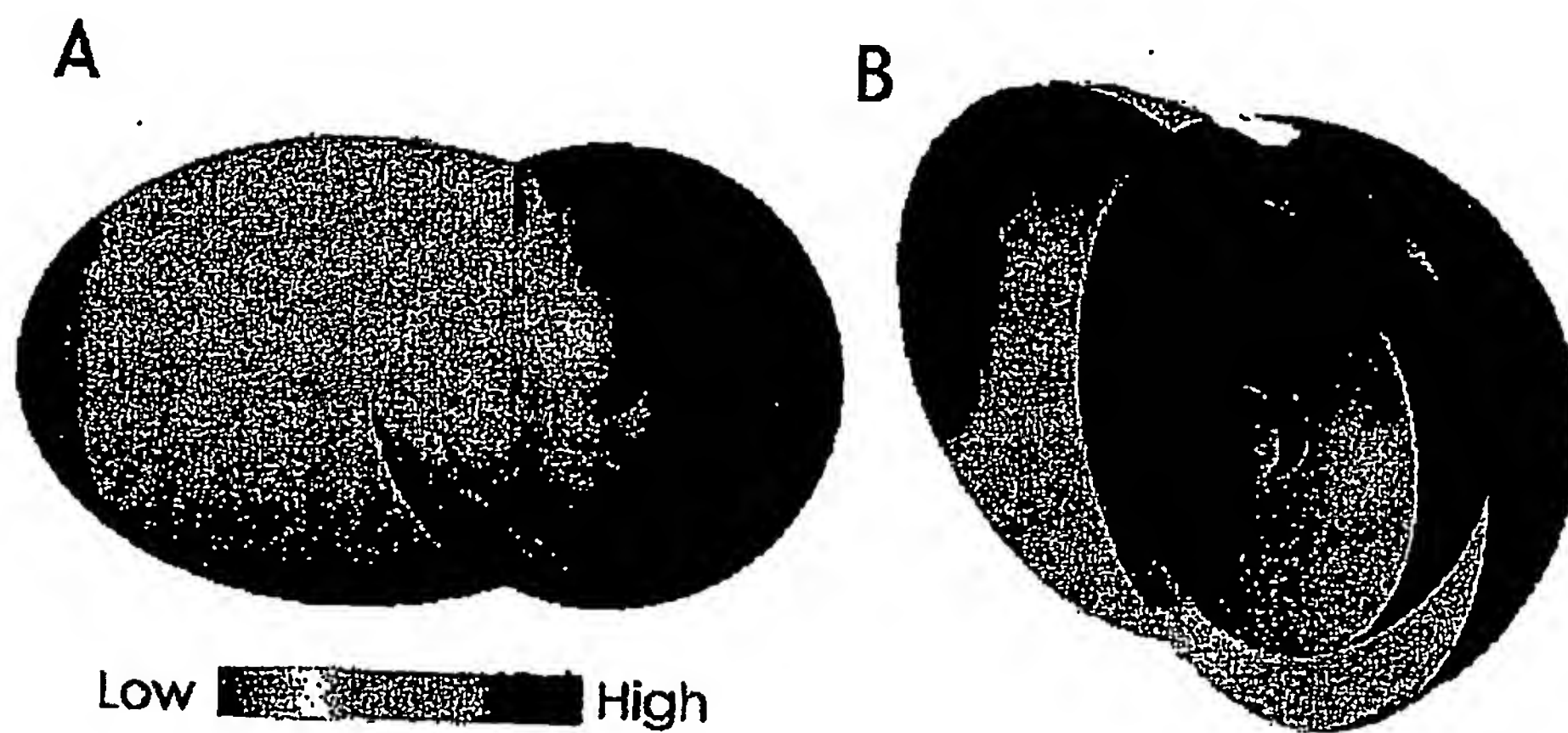
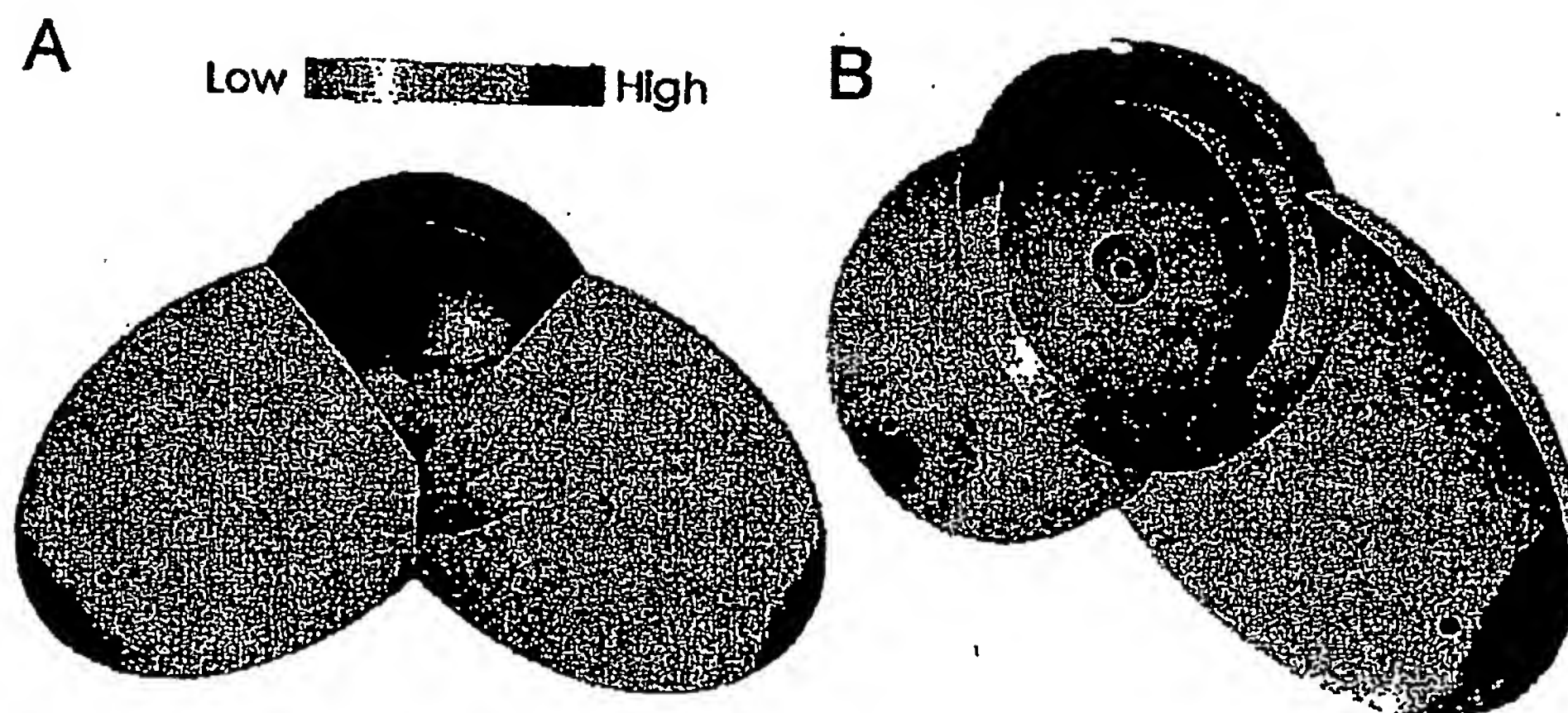
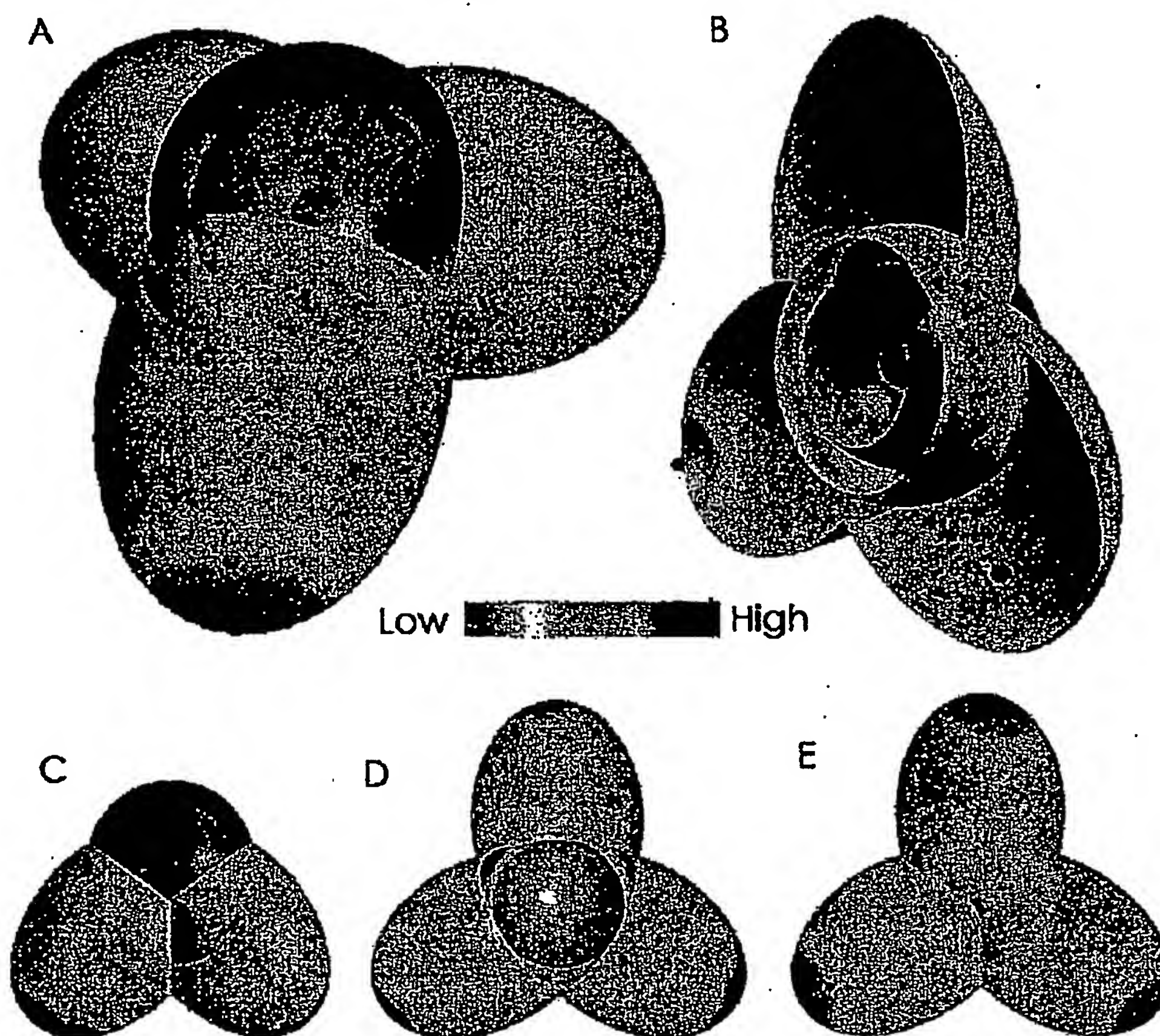


Fig.12



8/51
Fig.13



9/51

Fig.14

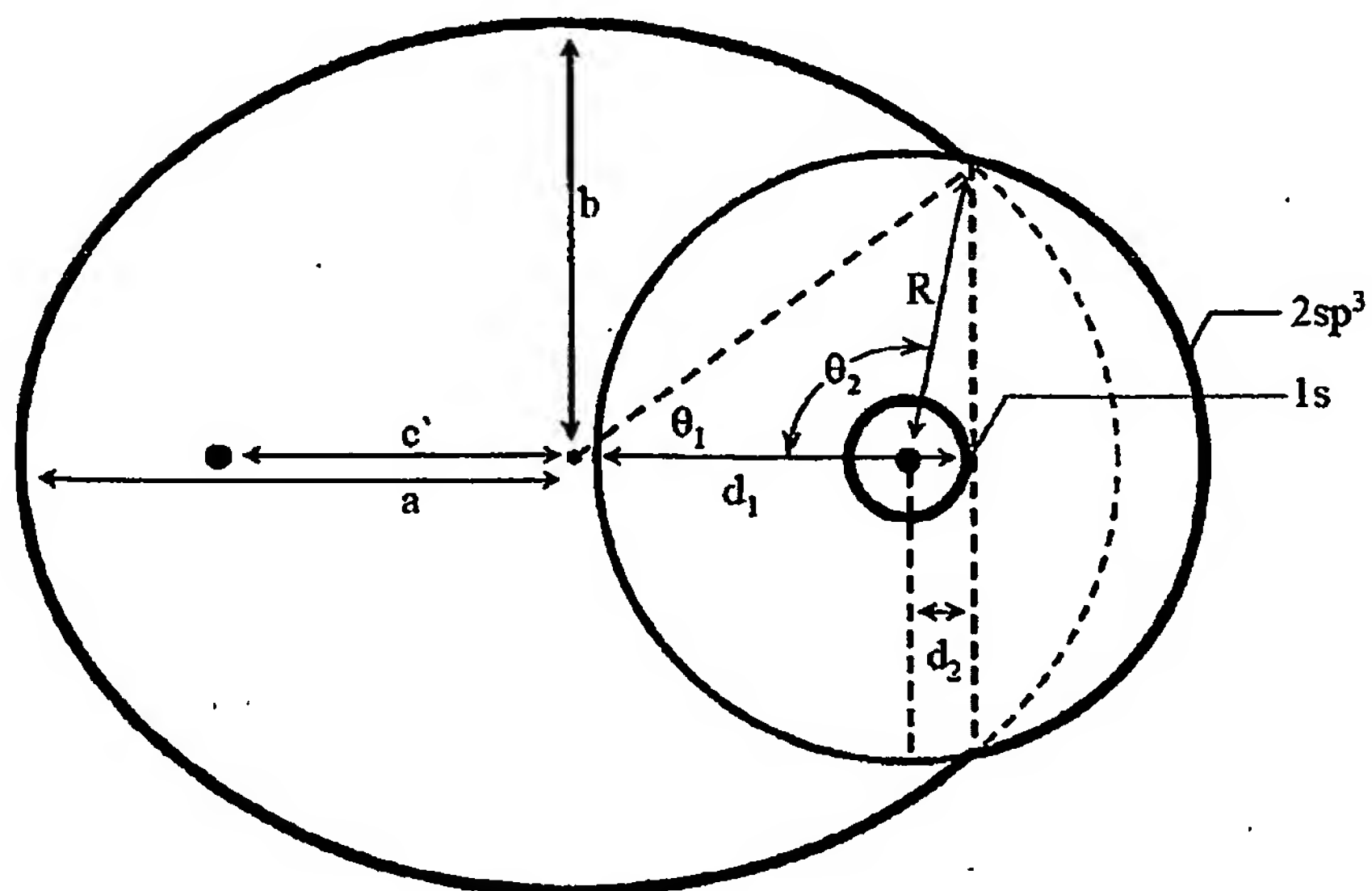
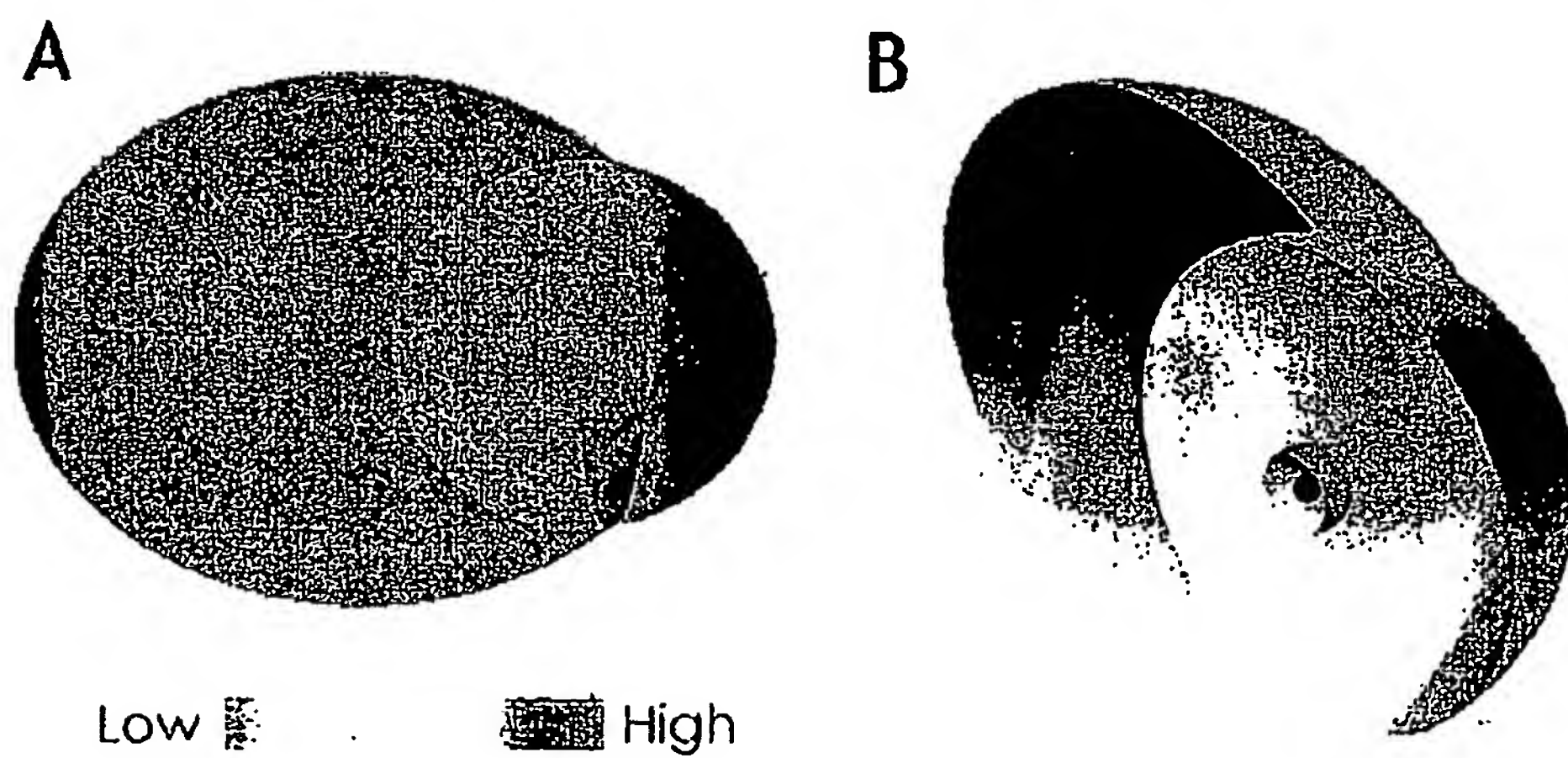


Fig.15



10/51
Fig.16

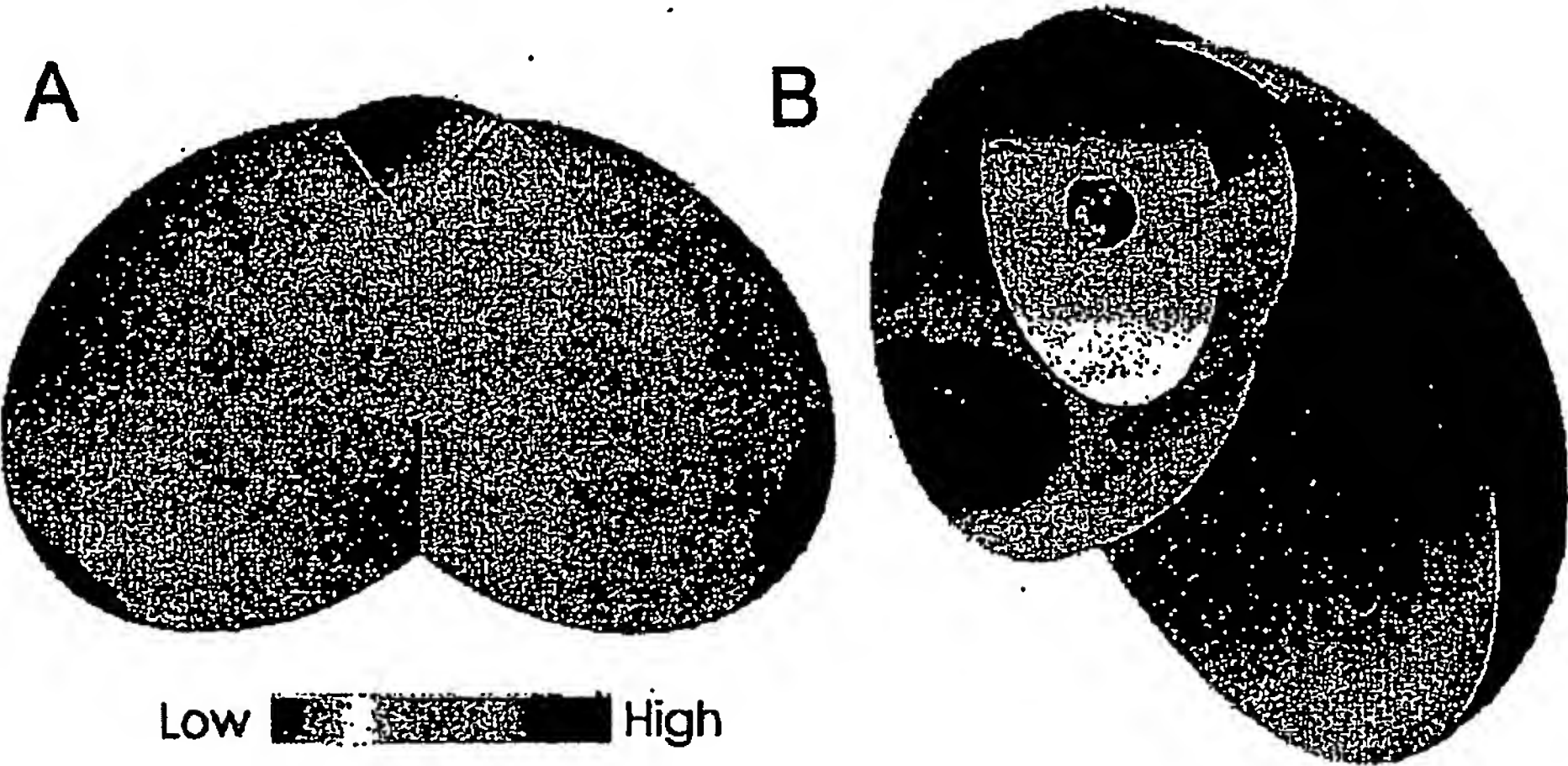


Fig.17

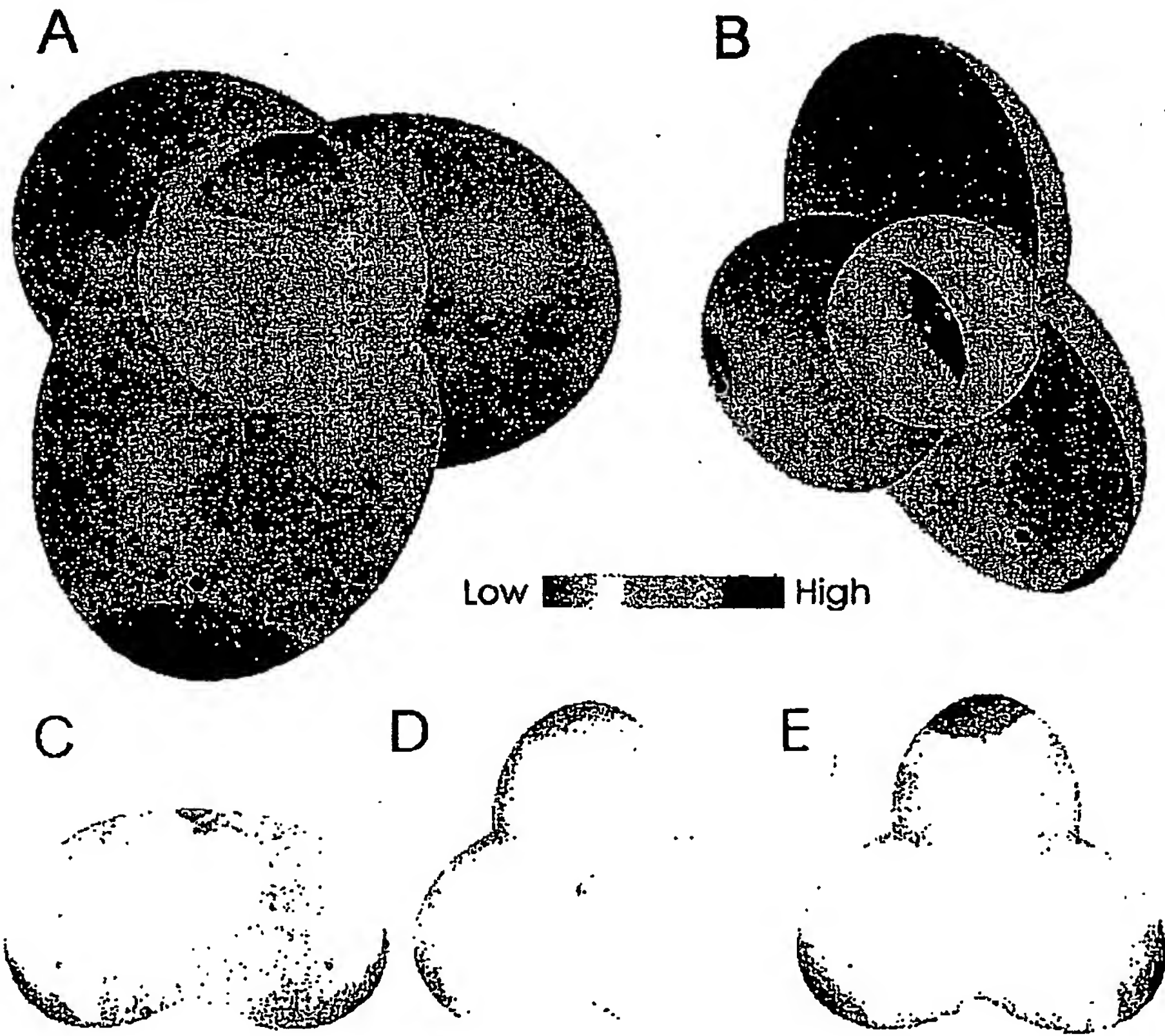
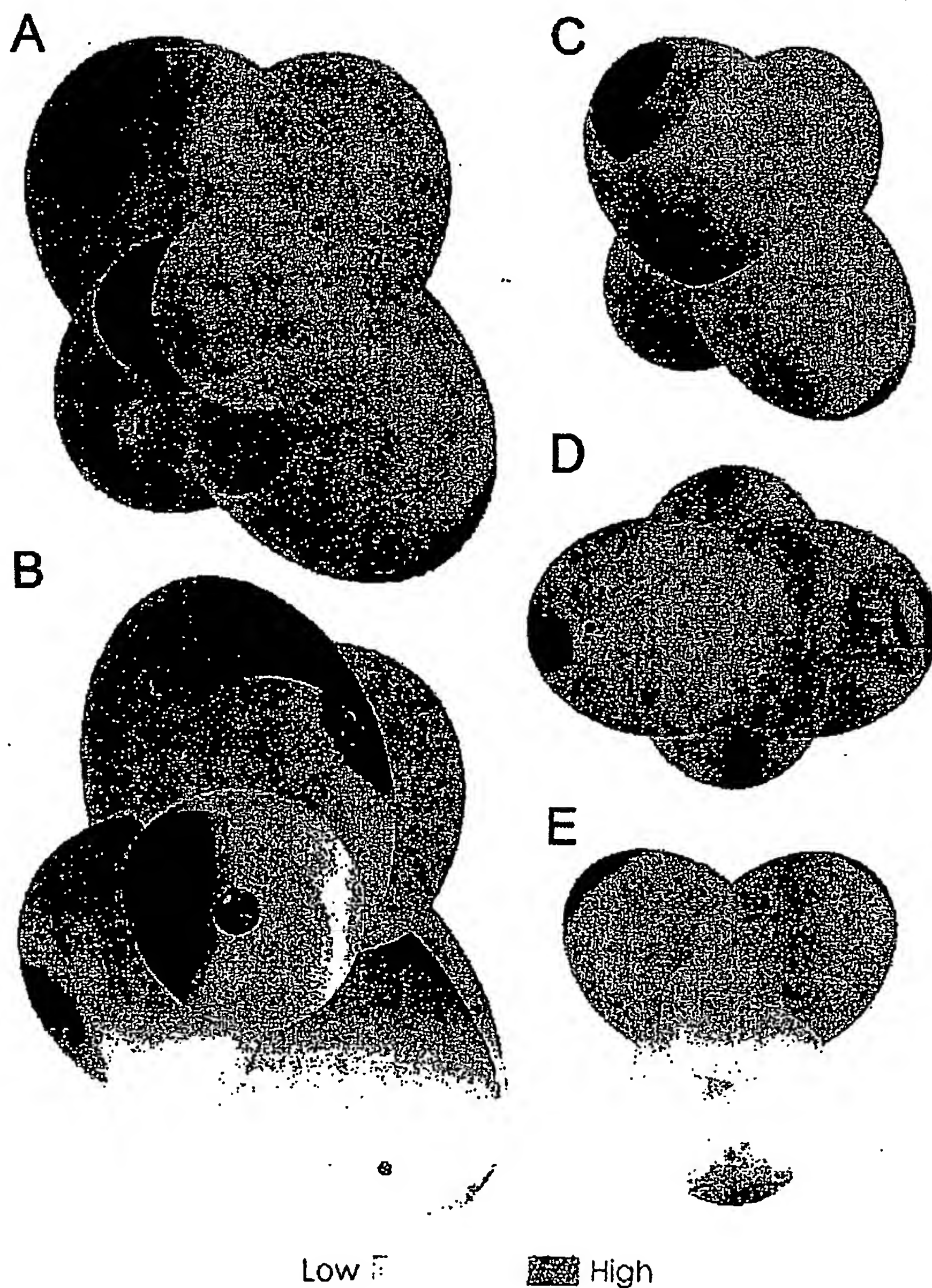


Fig.18



12/51
Fig.19

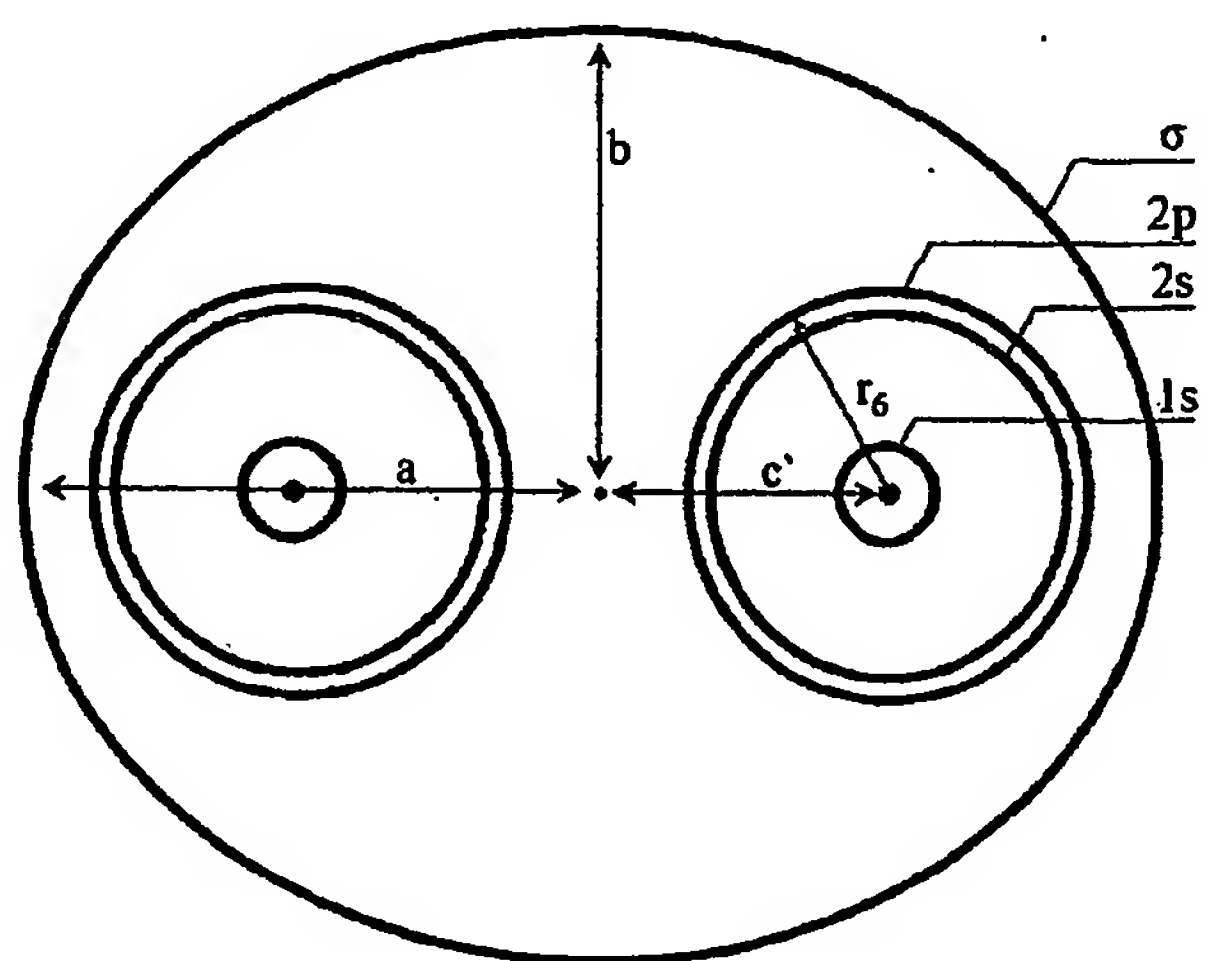
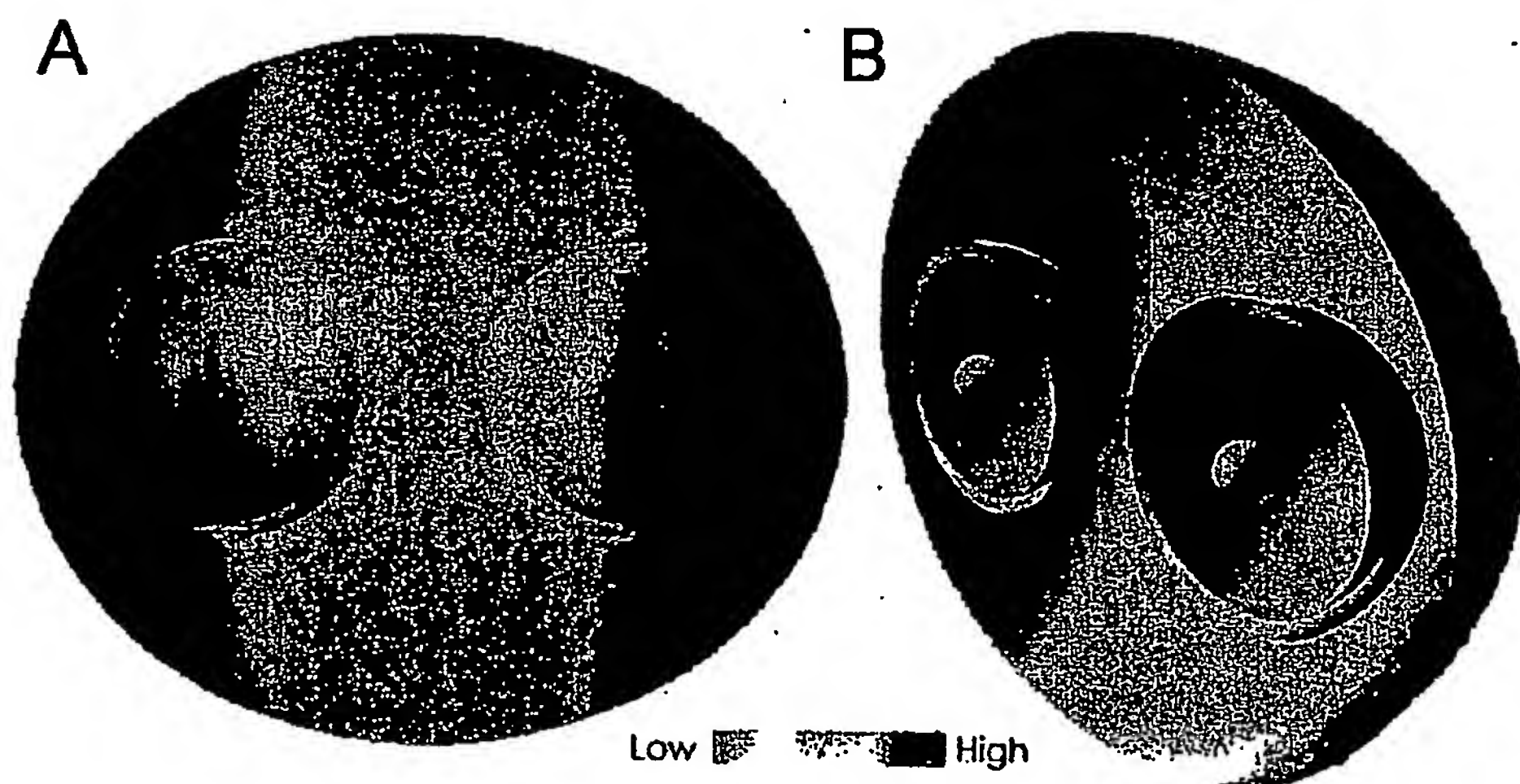


Fig.20



13/51

Fig.21

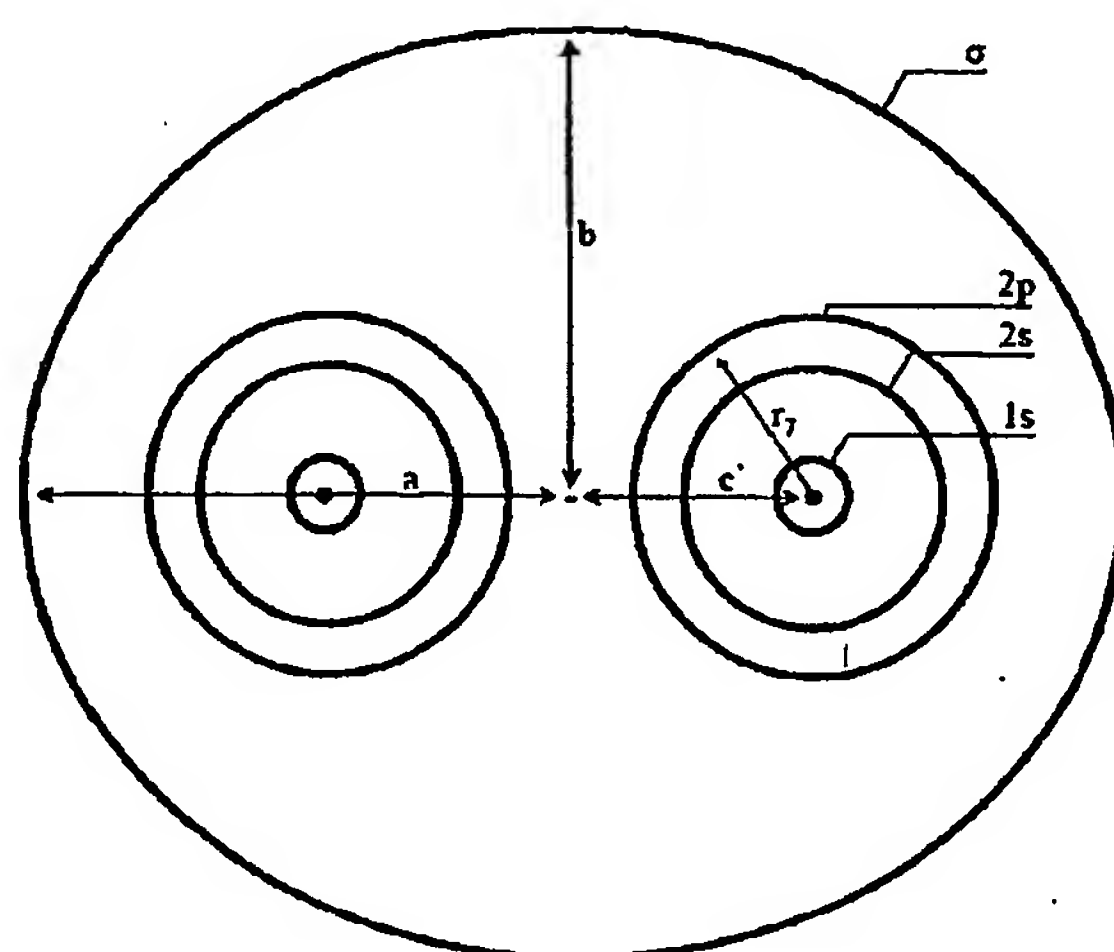
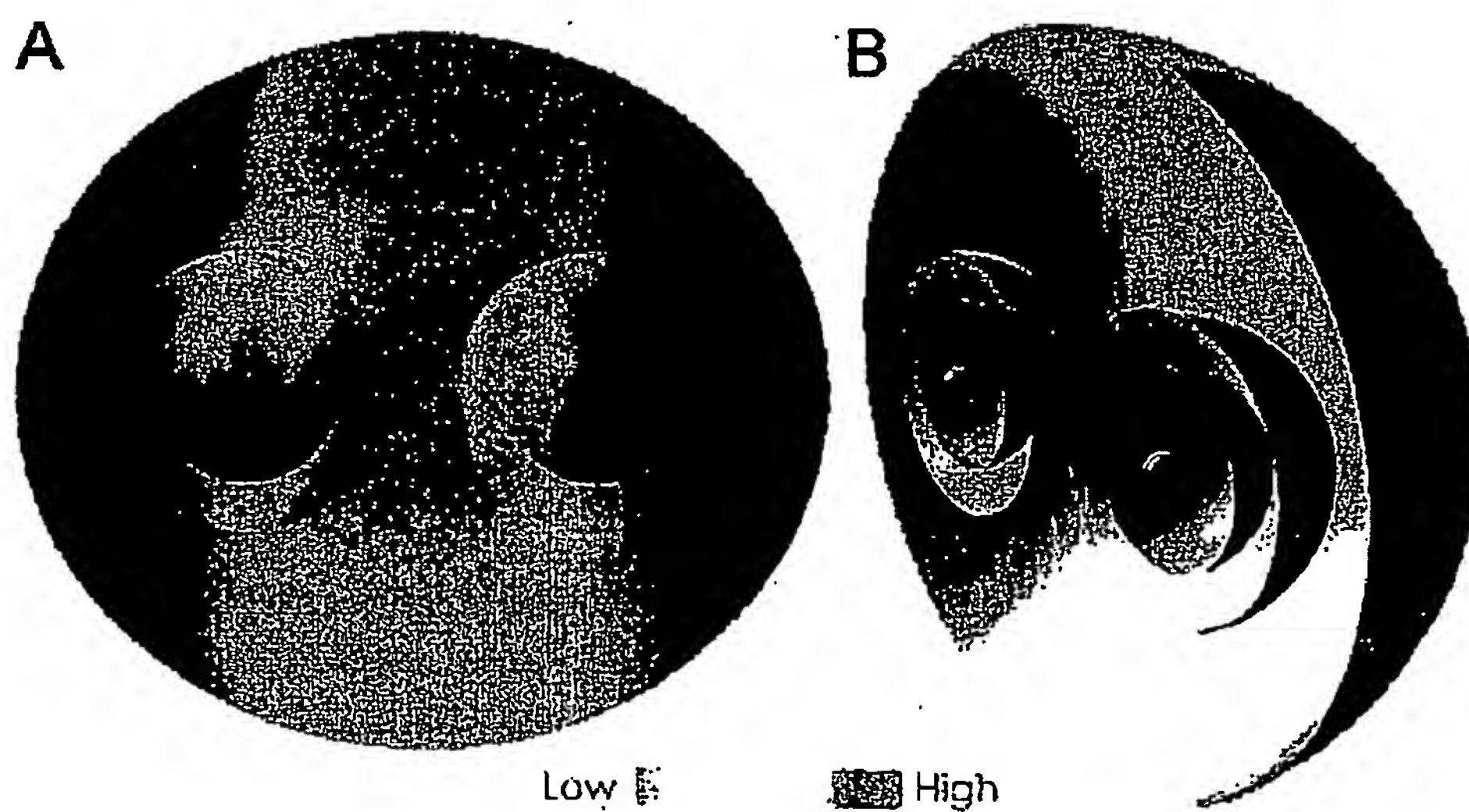


Fig.22



14/51
Fig.23

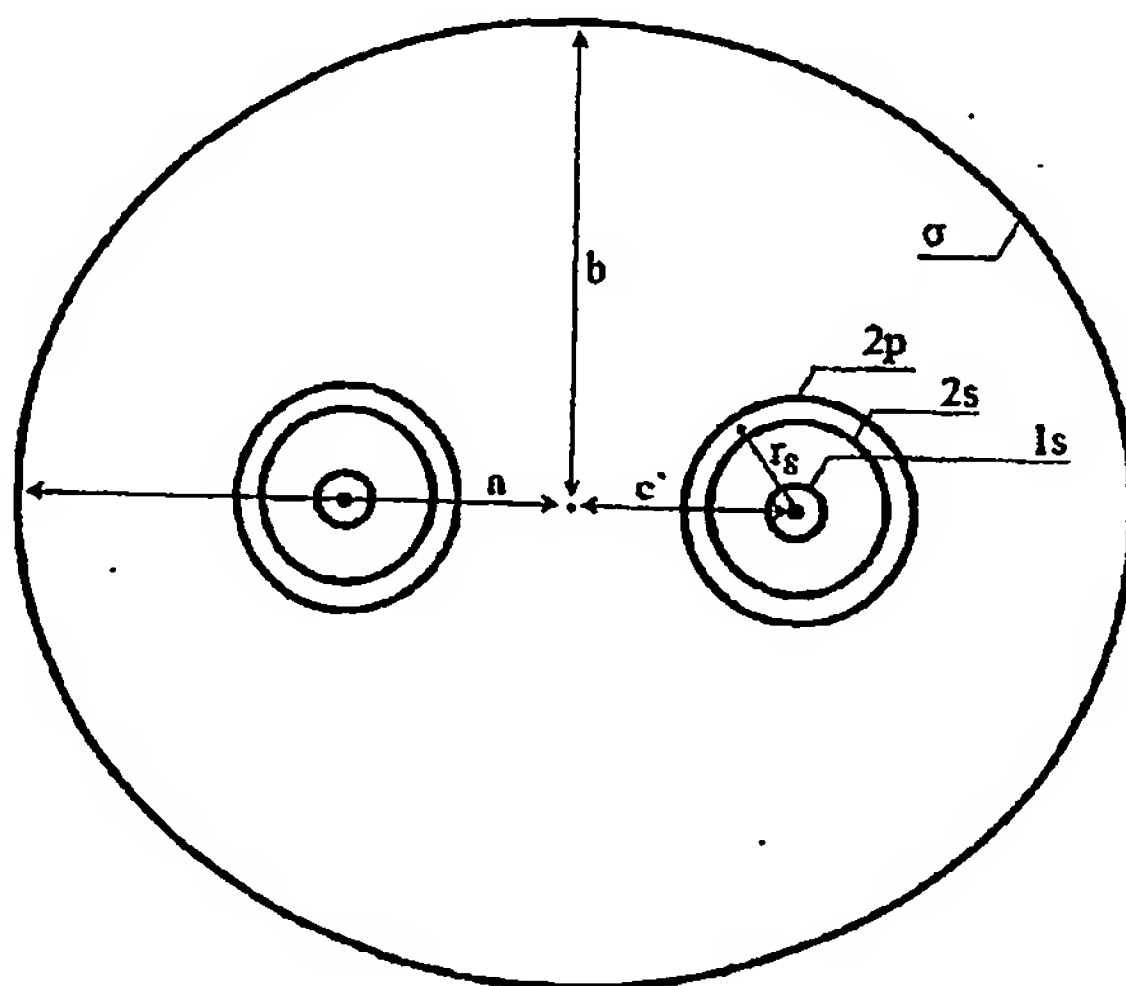
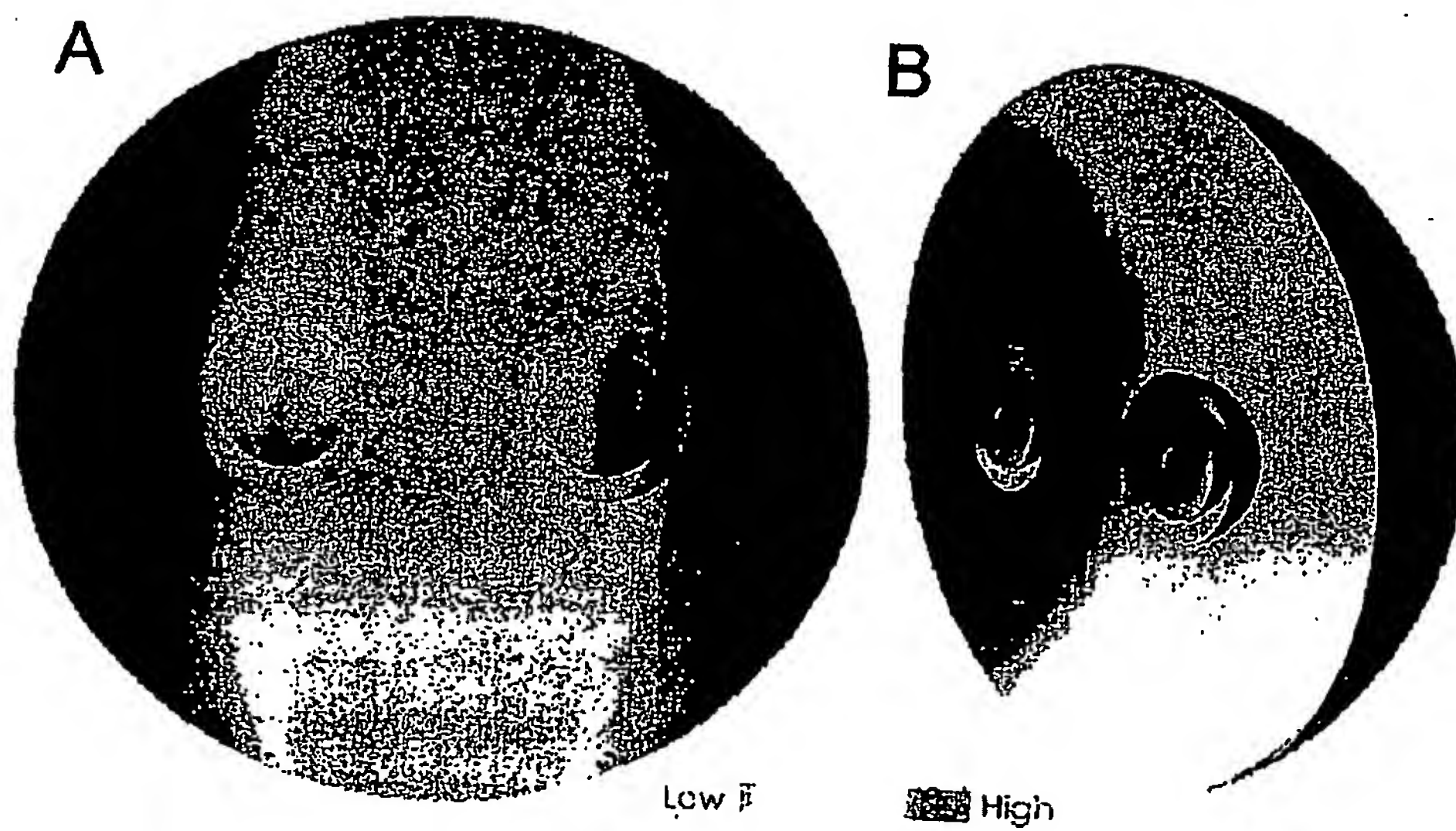


Fig.24



15/51

Fig.25

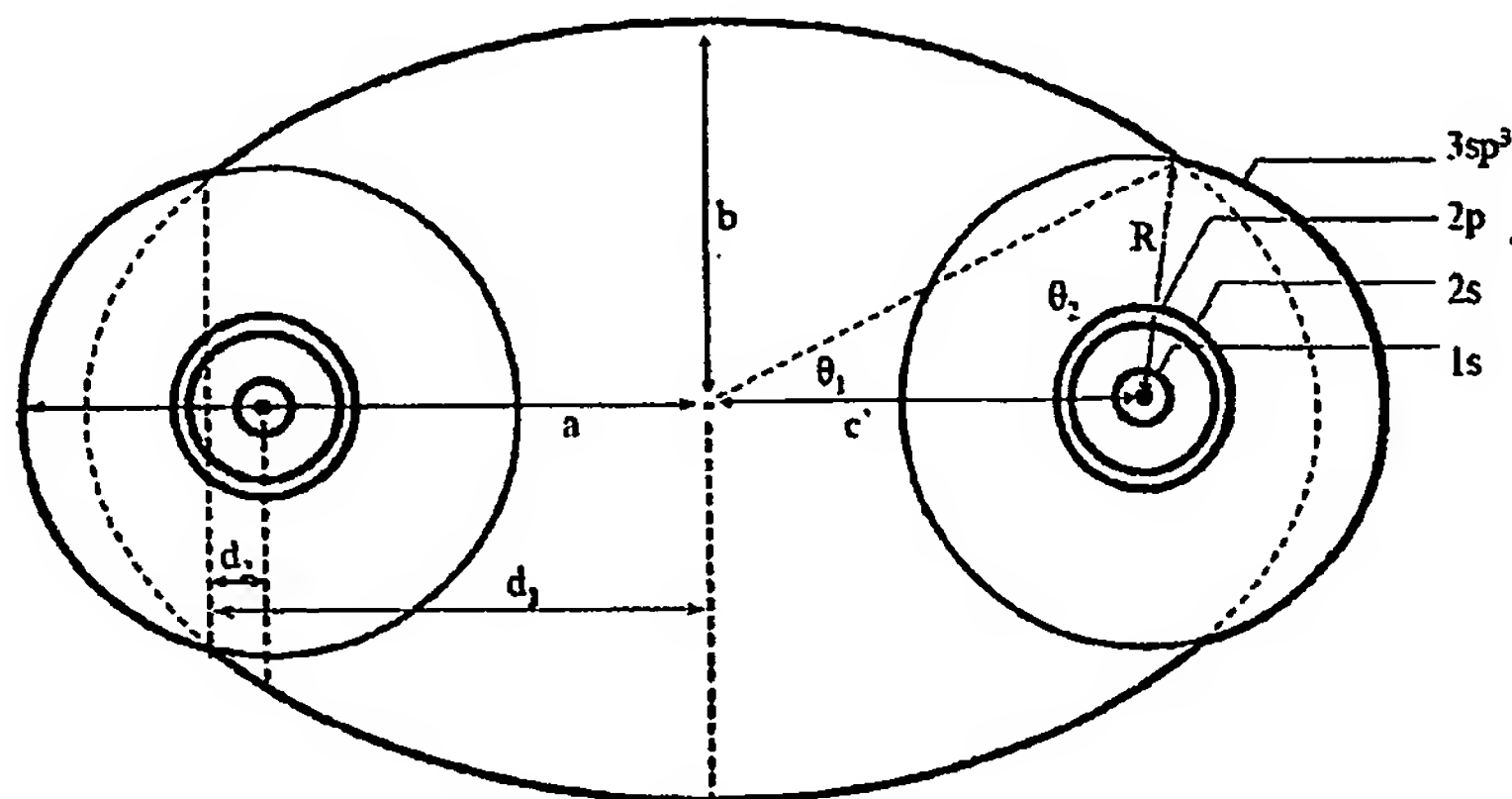
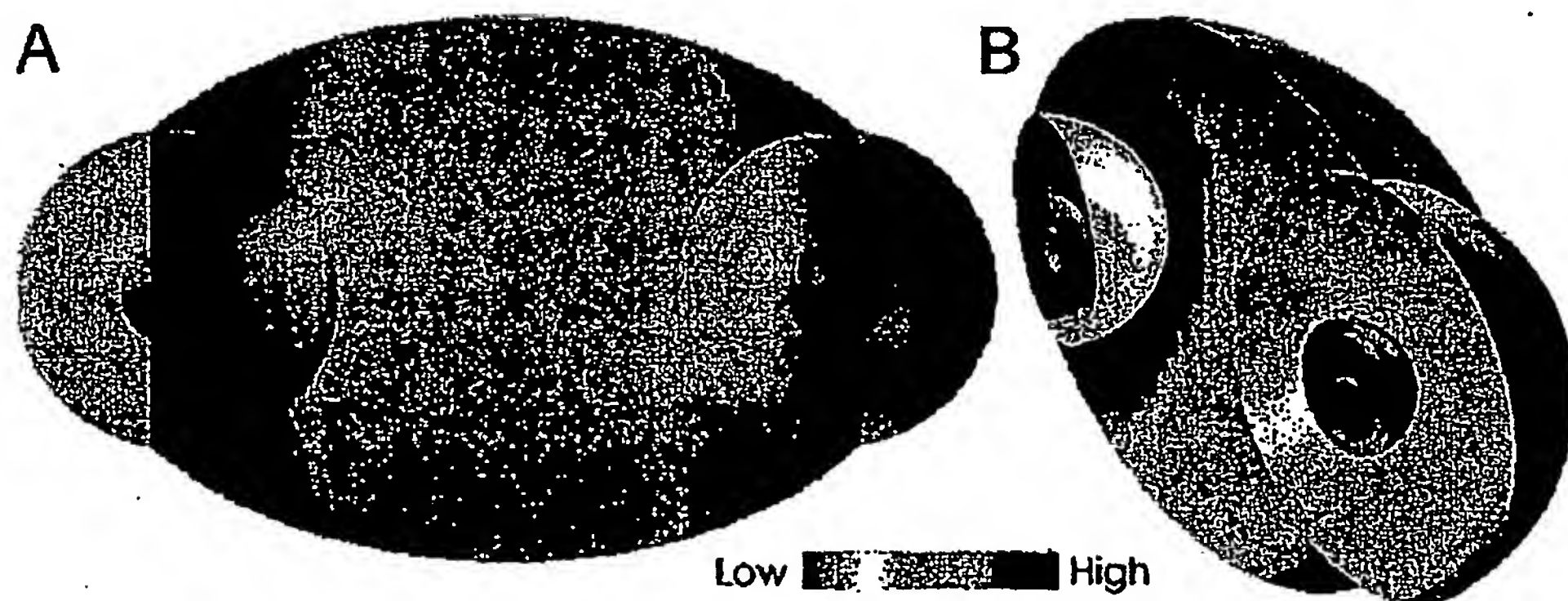


Fig.26



16/51

Fig.27

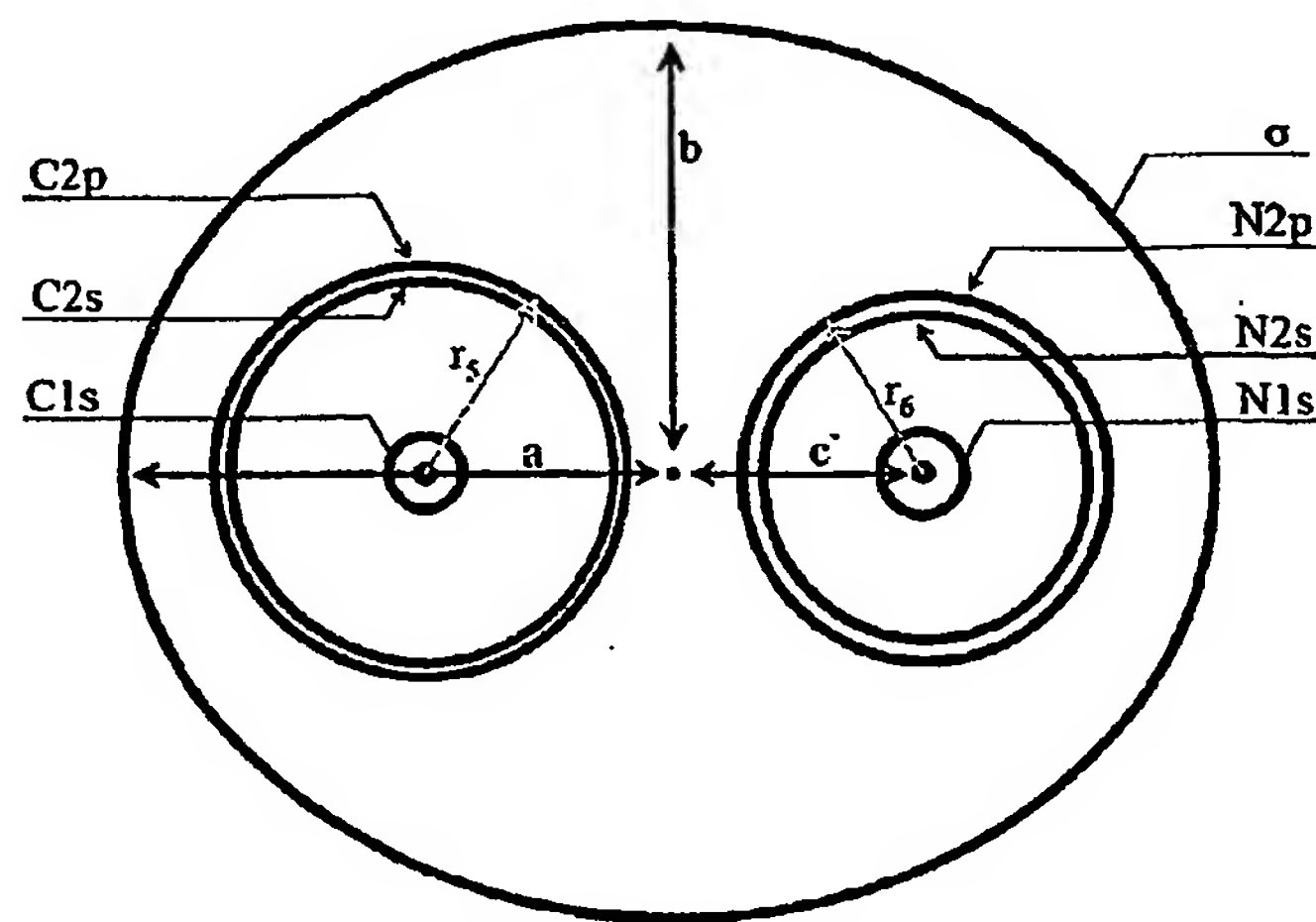
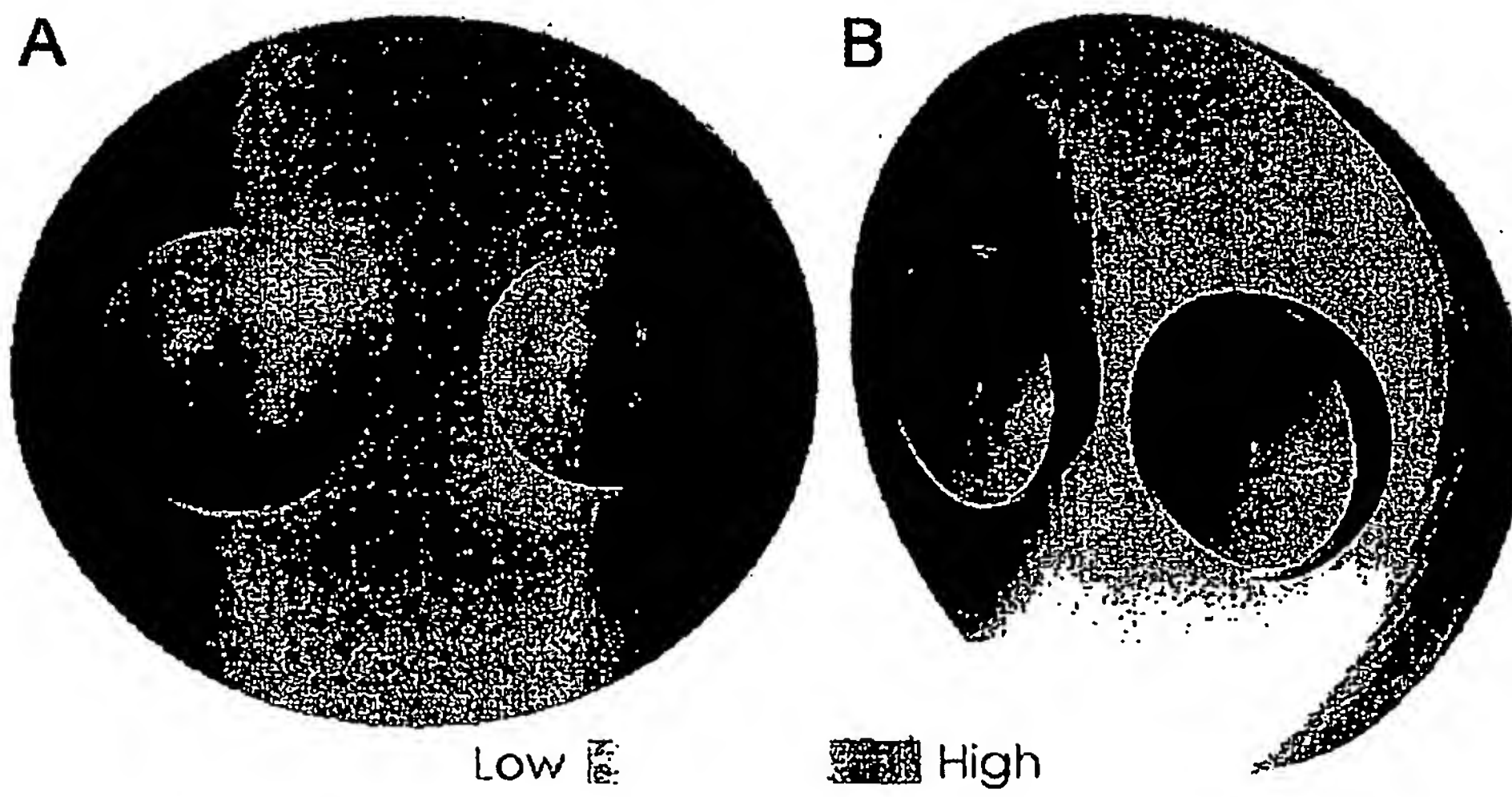


Fig.28



17/51

Fig.29

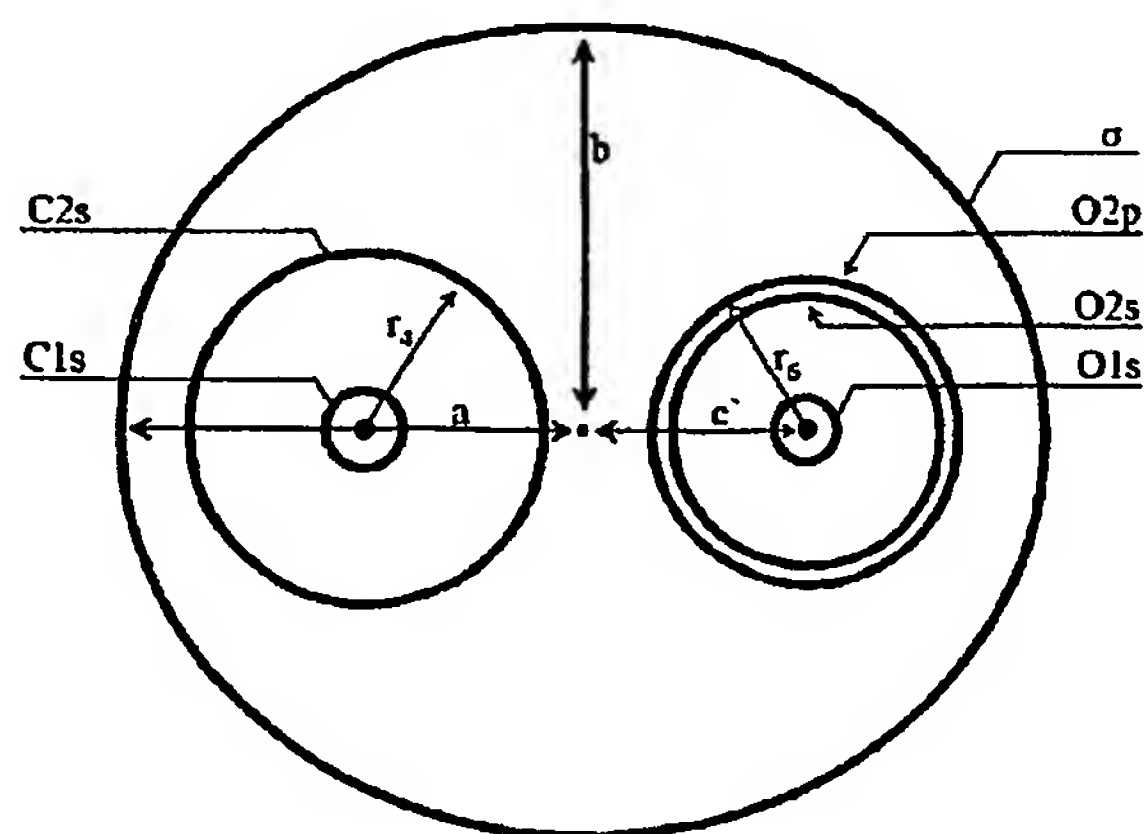
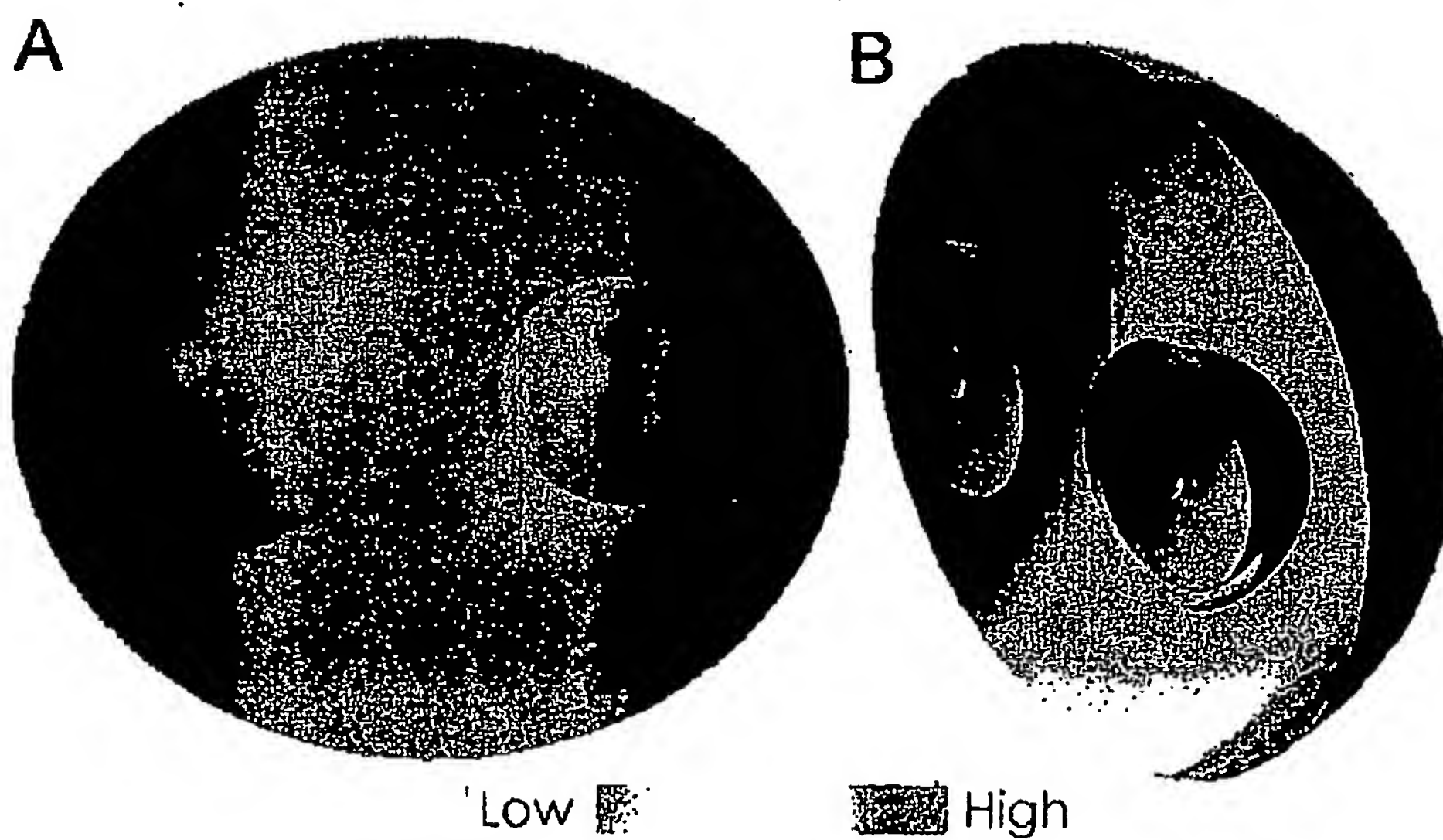


Fig.30



18/51
Fig.31

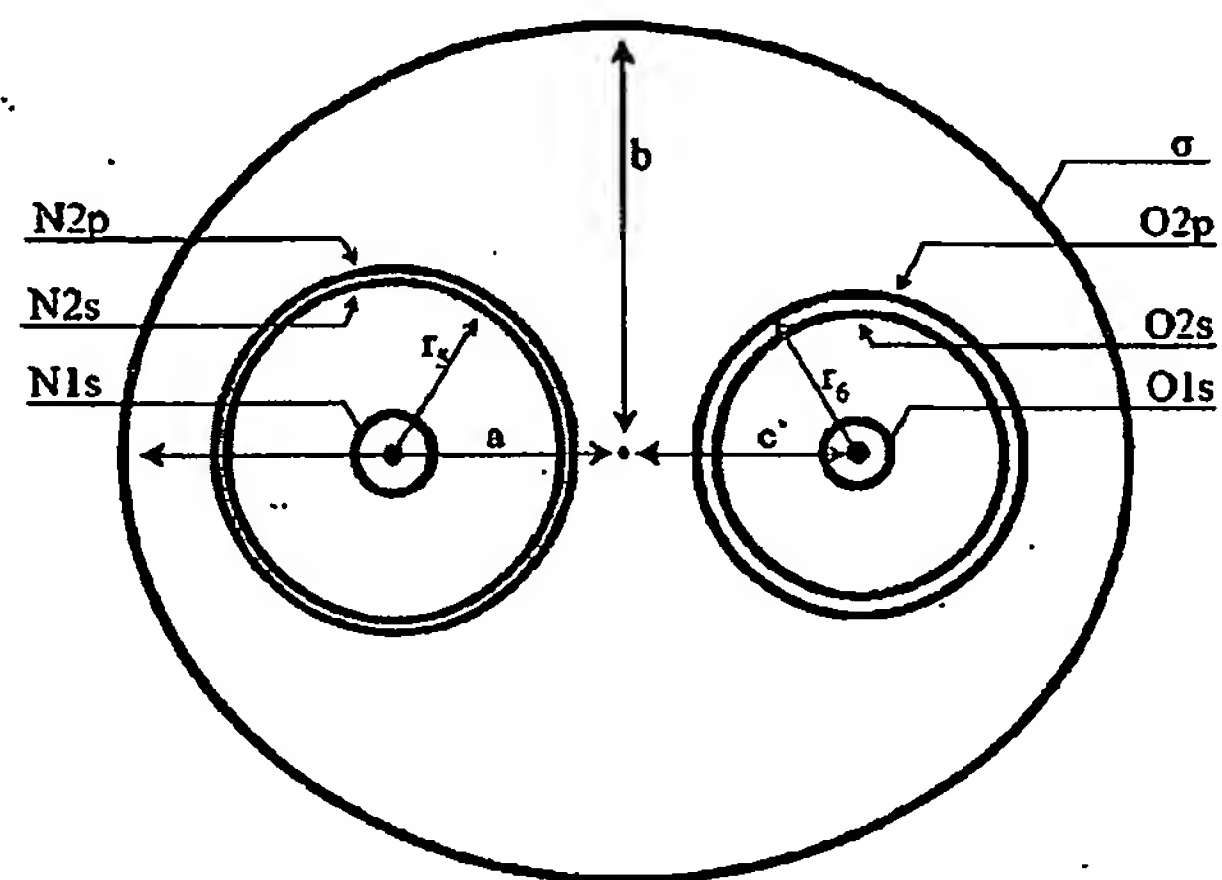
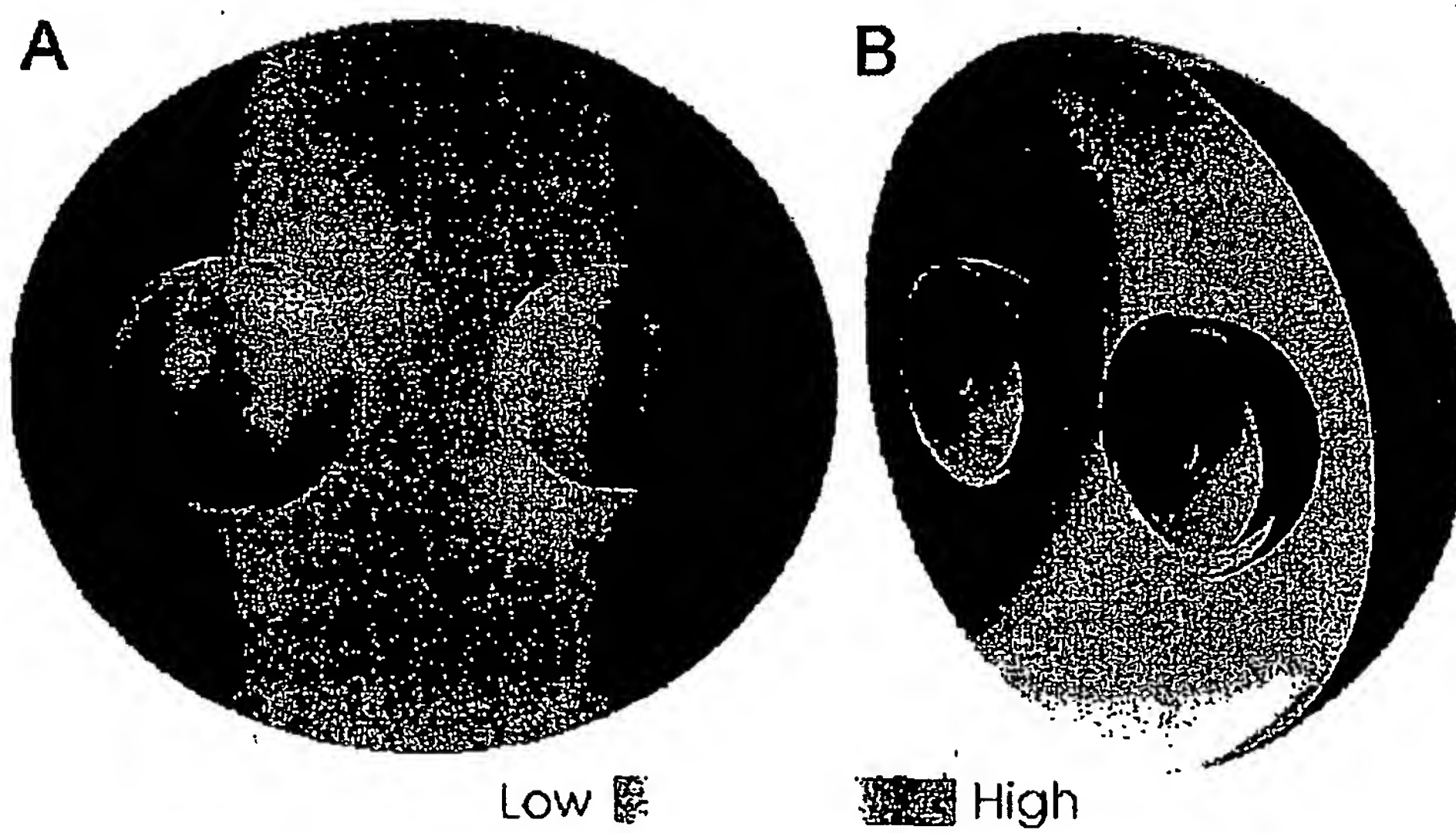


Fig.32



19/51

Fig.33

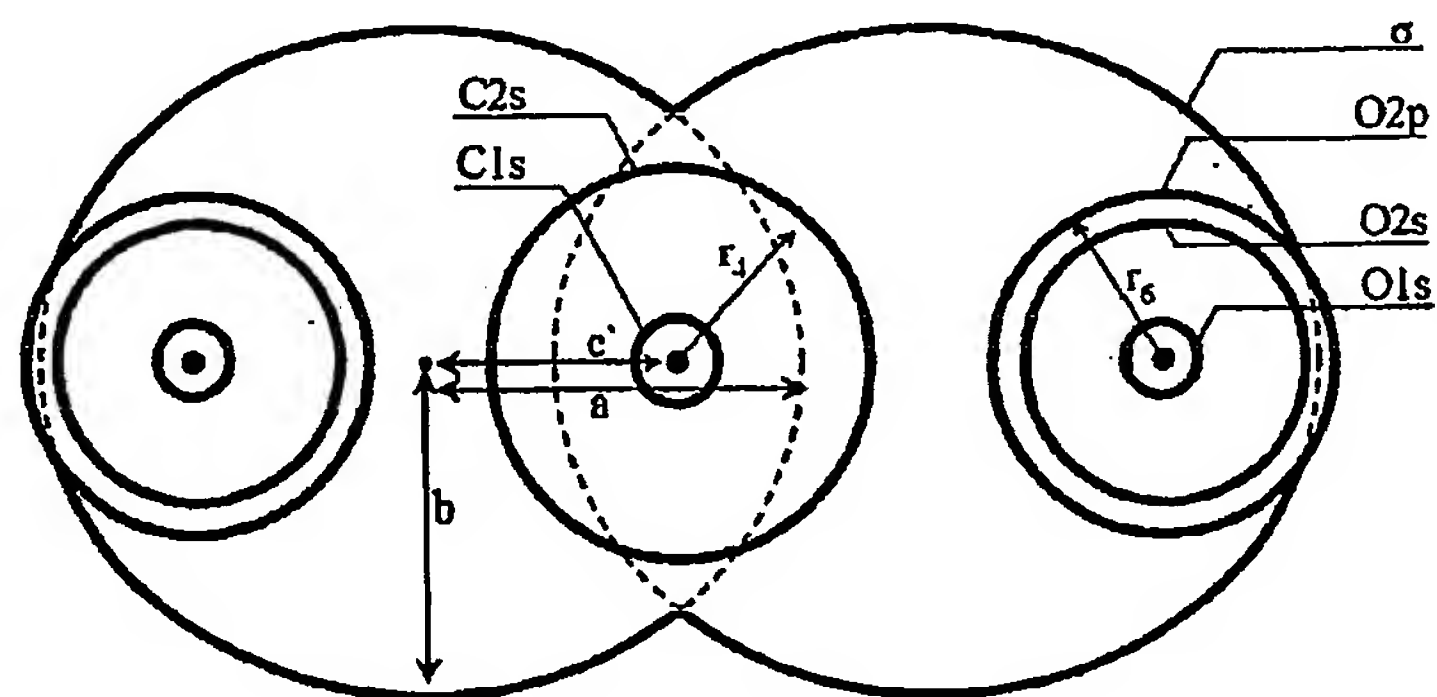
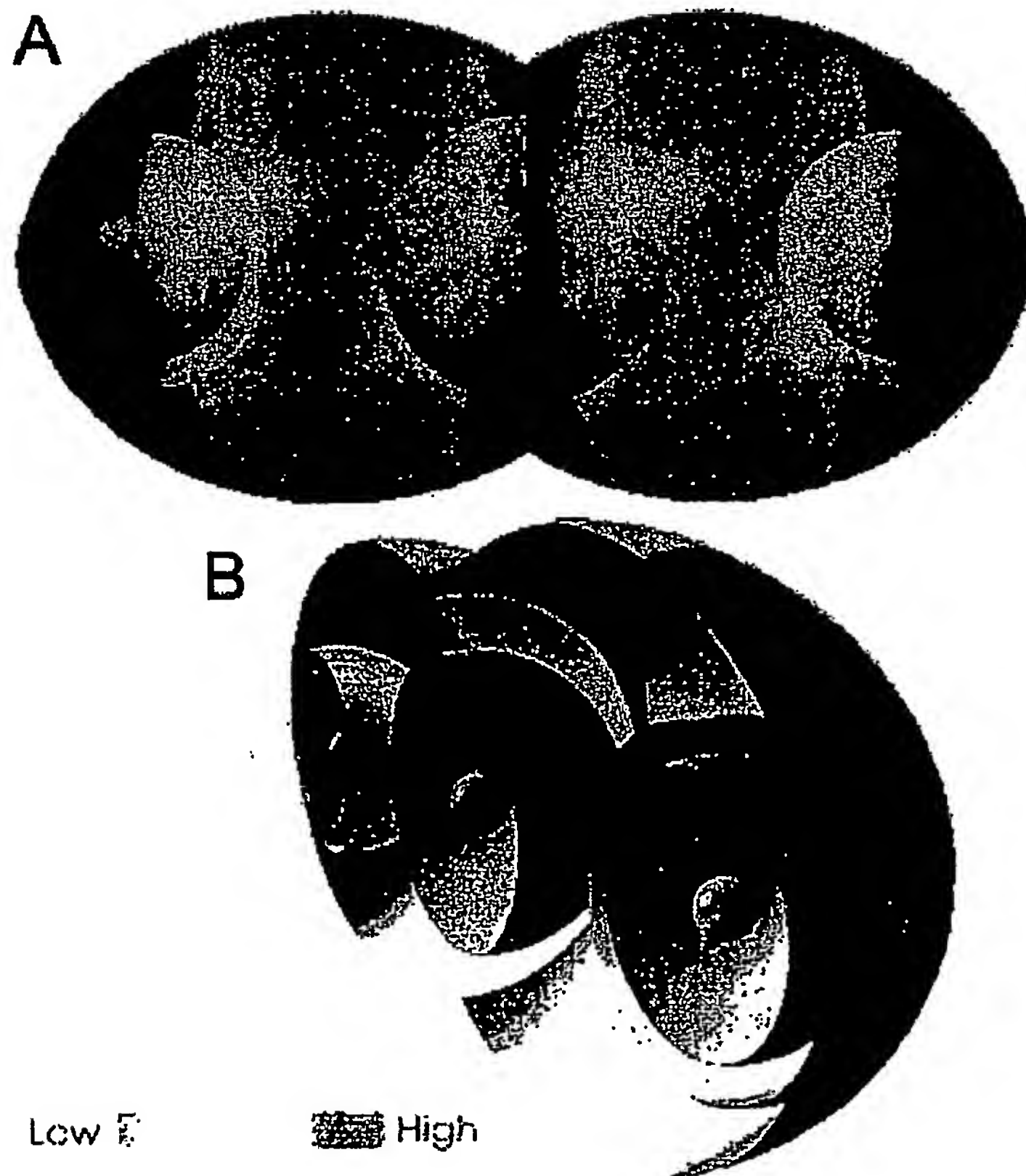


Fig.34



21/51
Fig.37

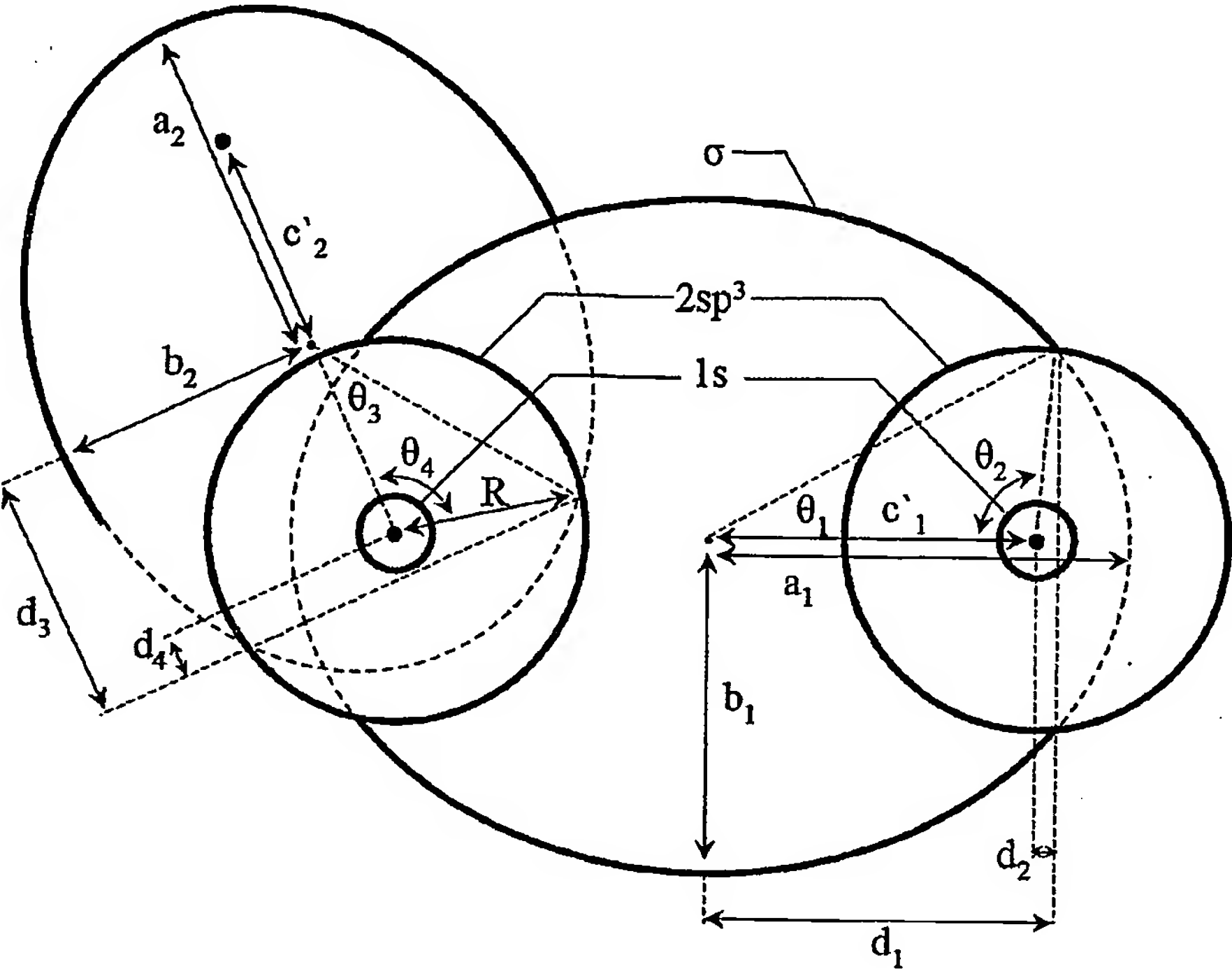
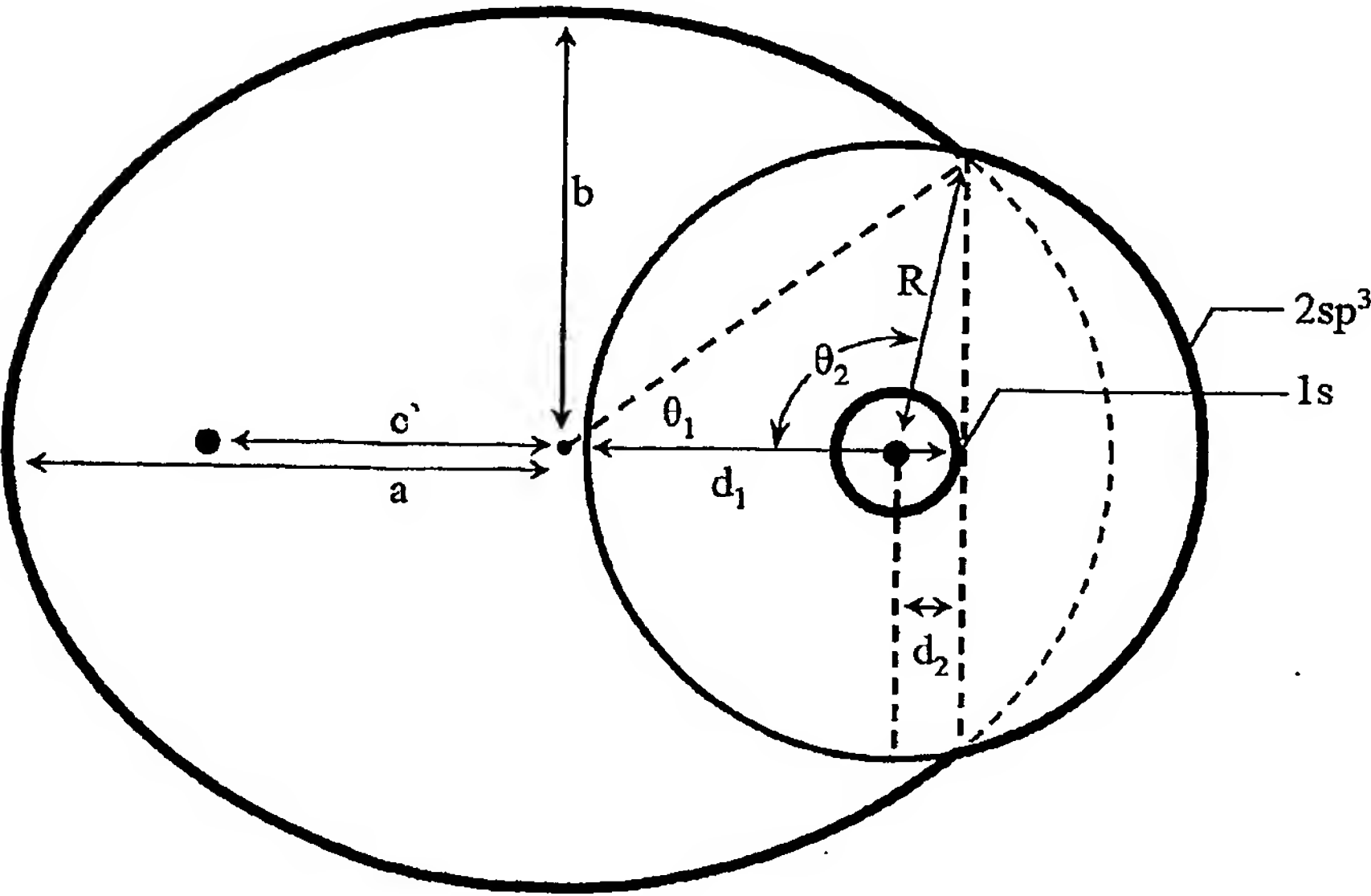
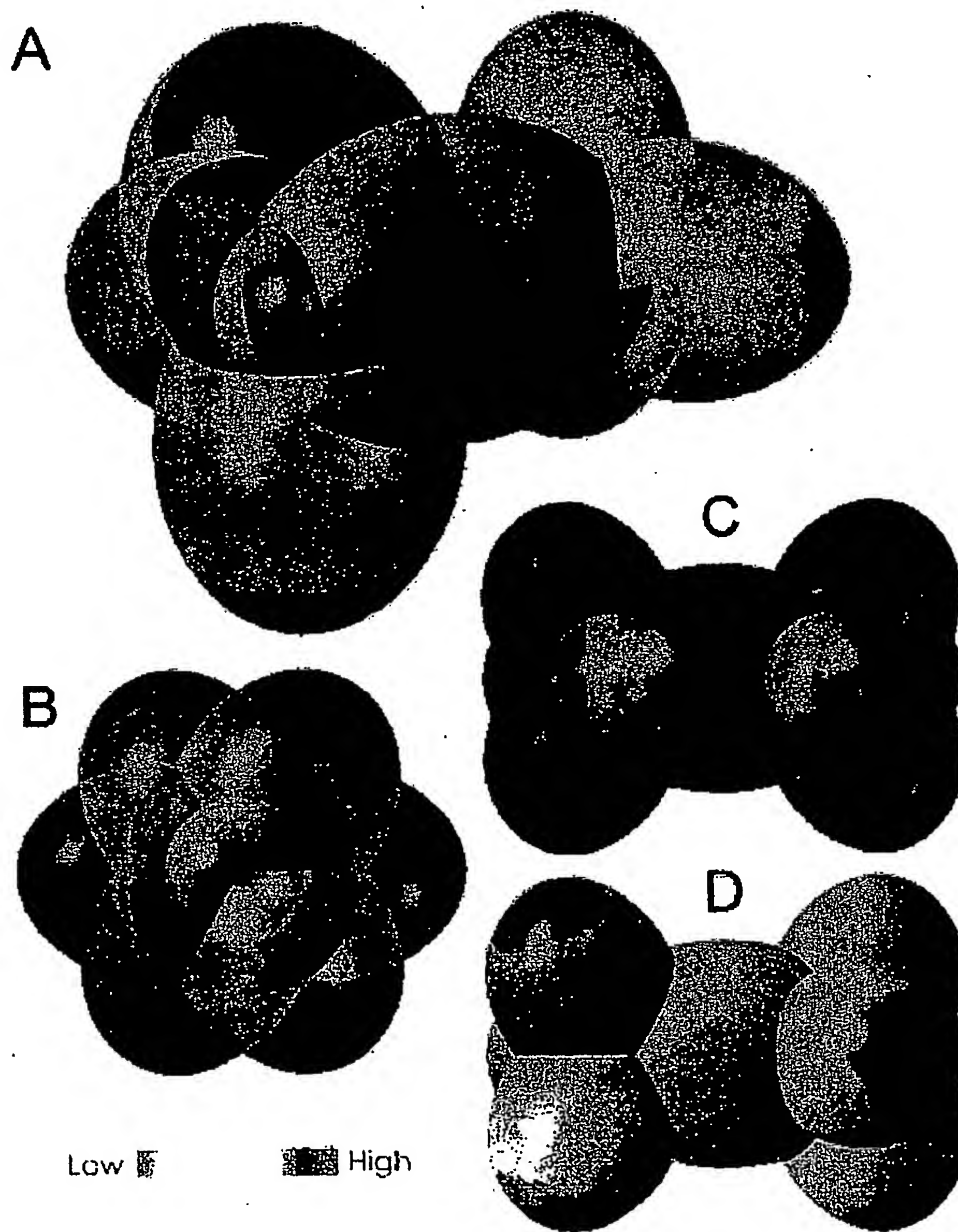


Fig.38



22/51

Fig.39



23/51
Fig.40

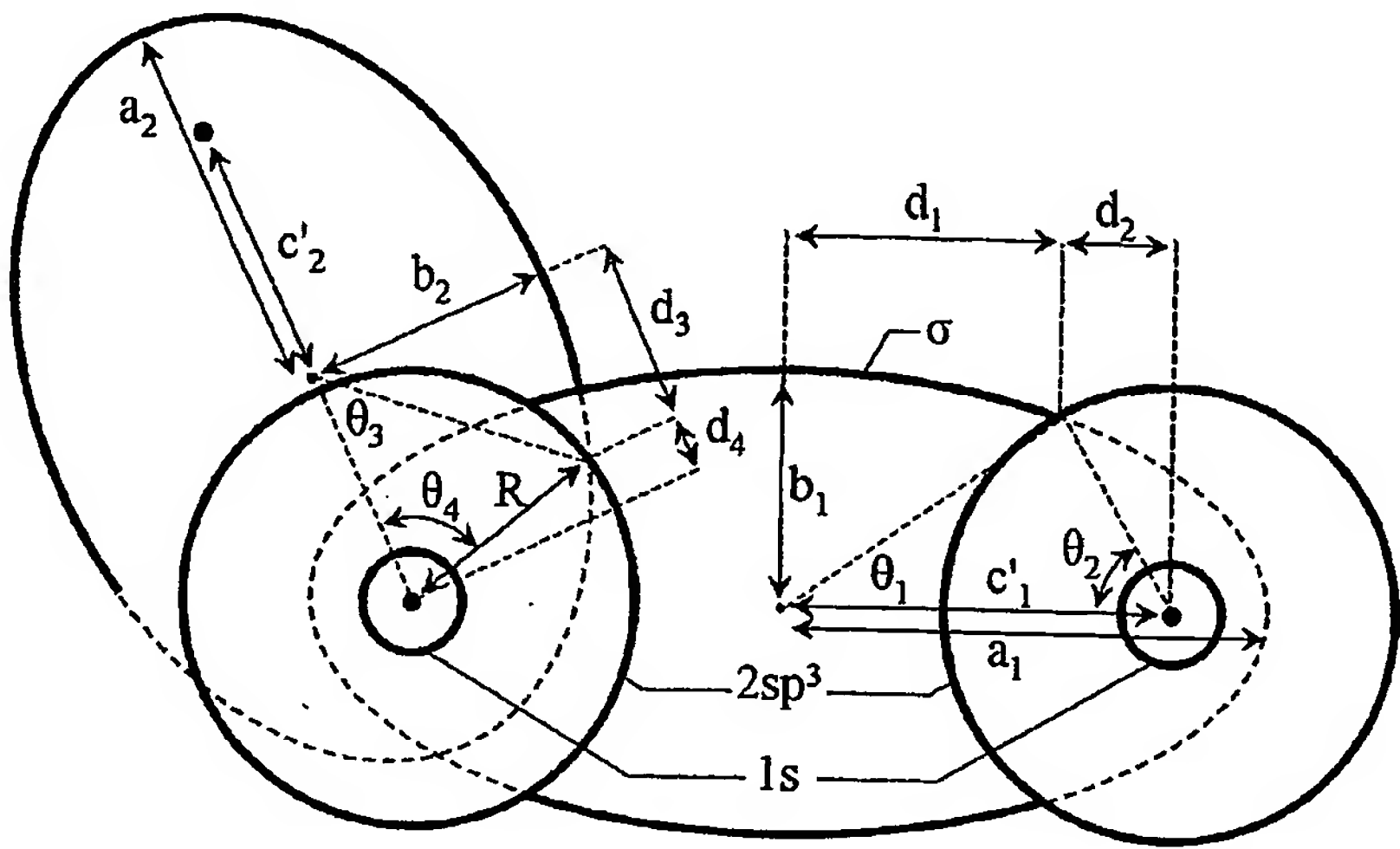
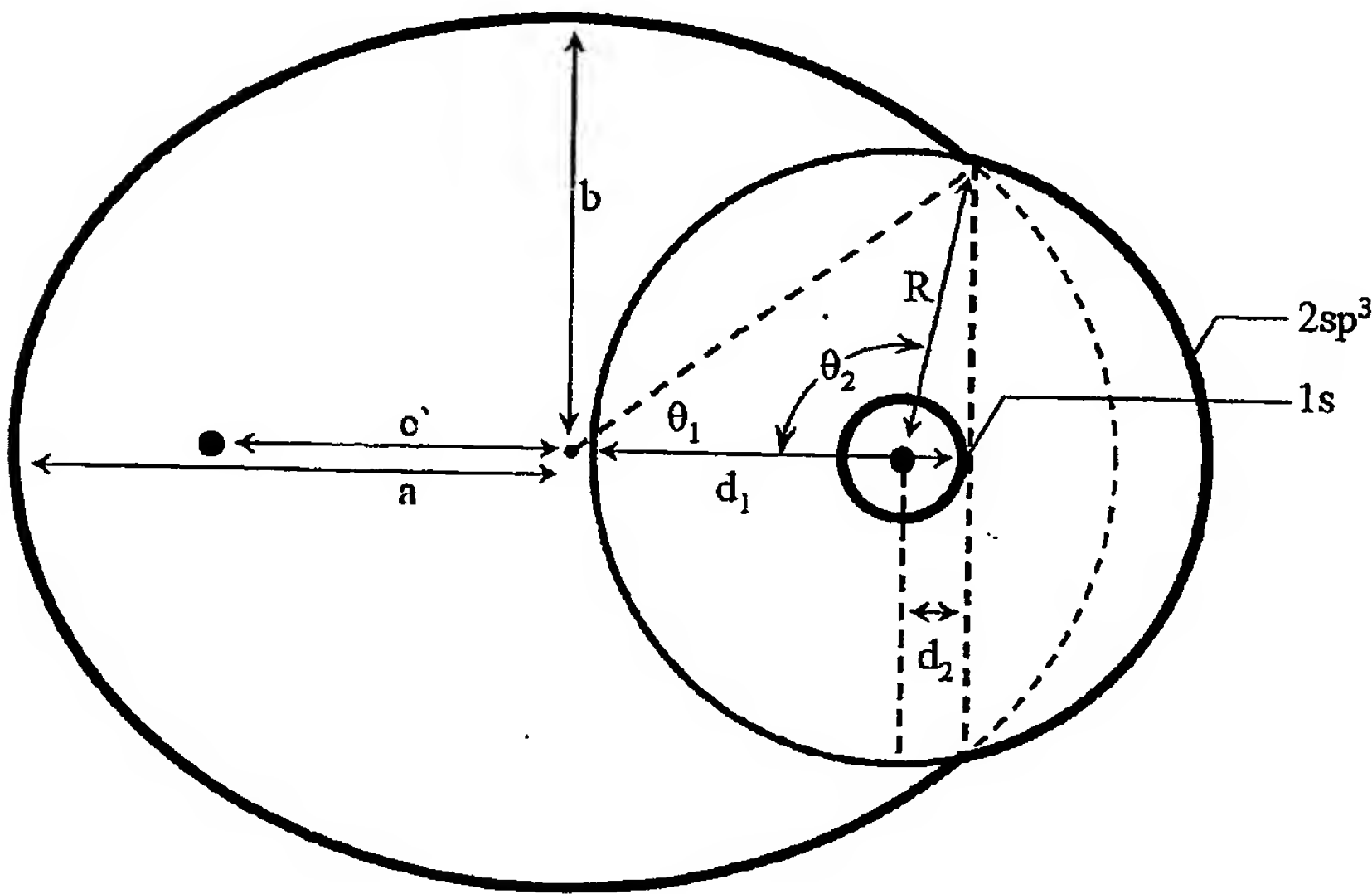
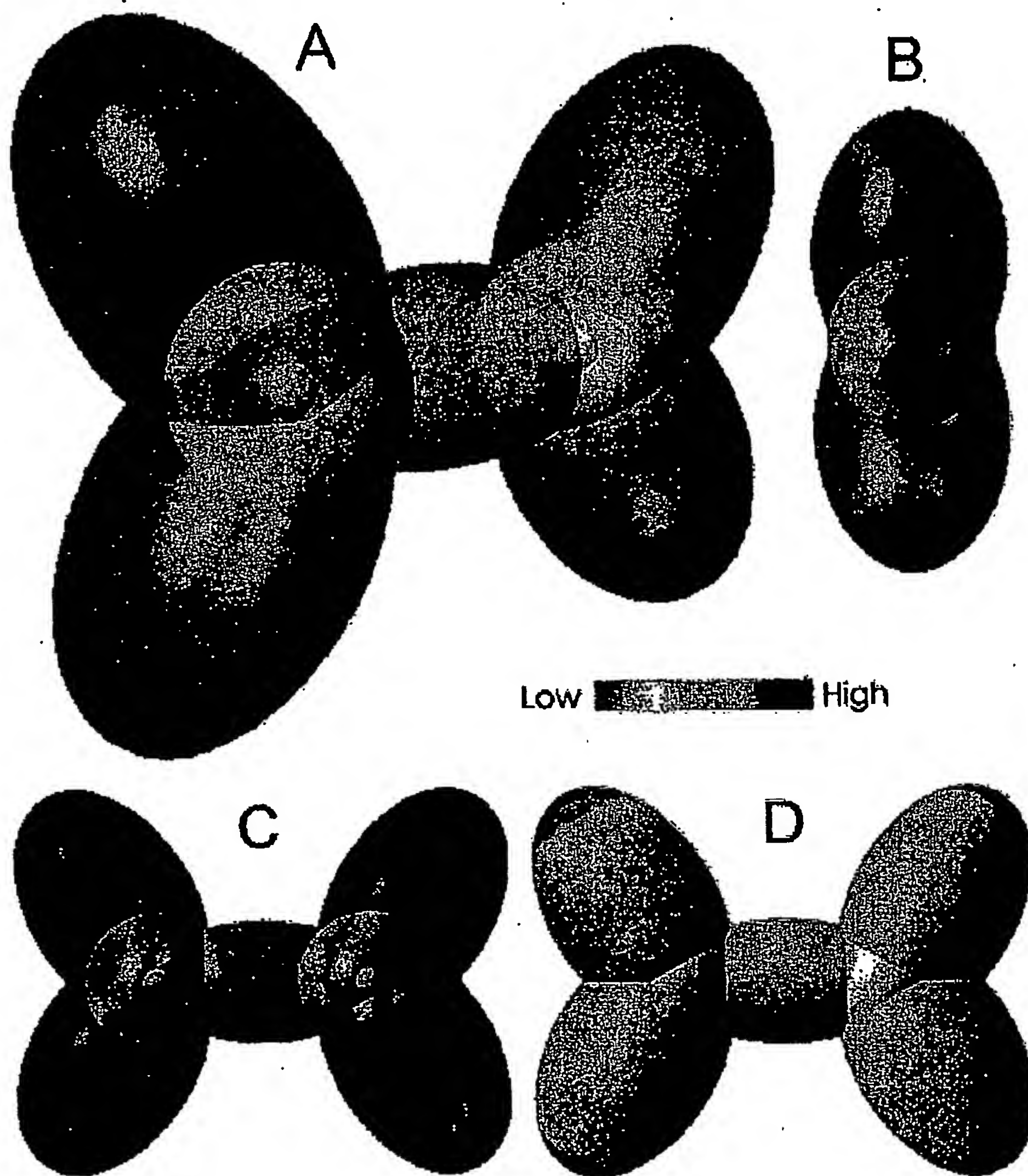


Fig.41



24/51

Fig.42



25/51

Fig.43

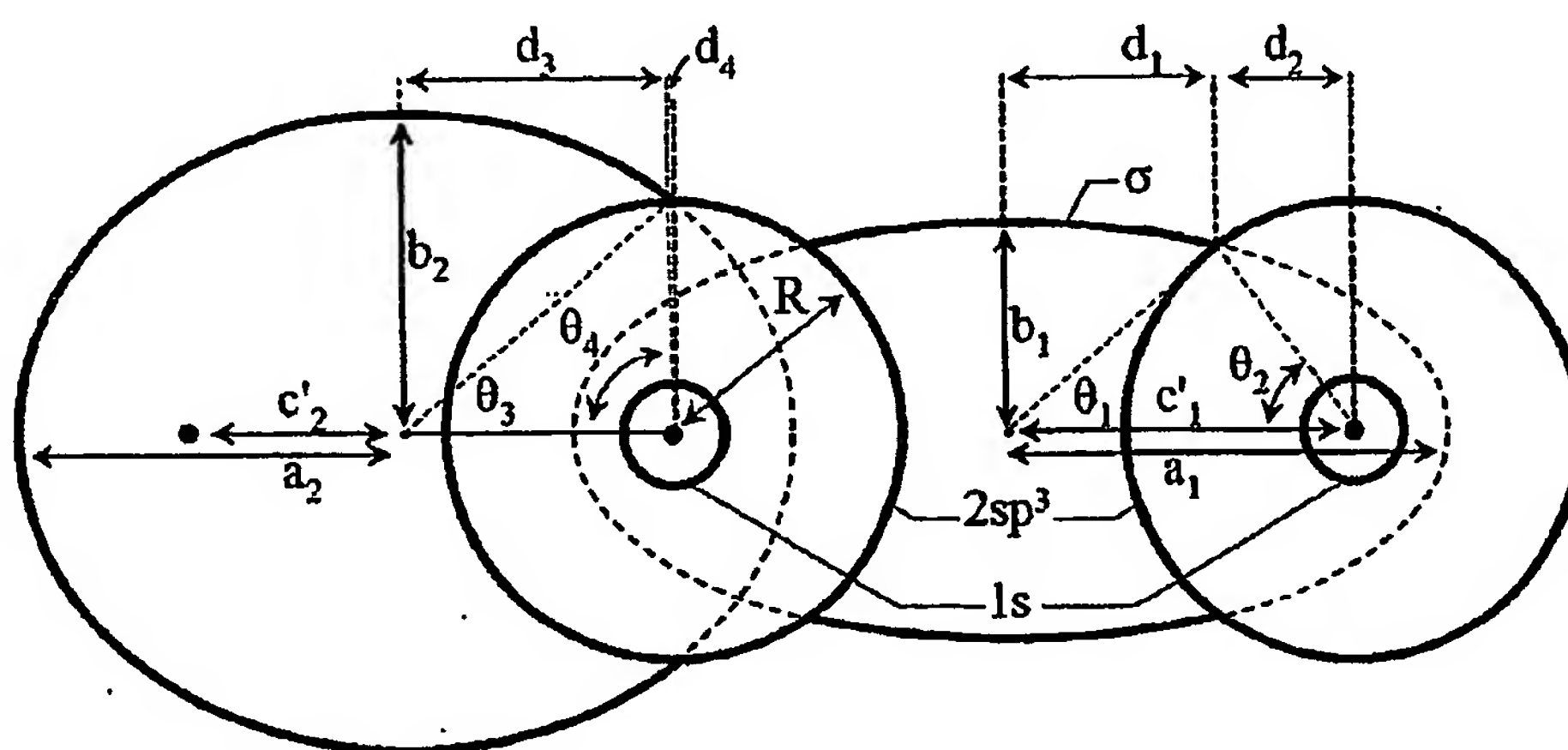
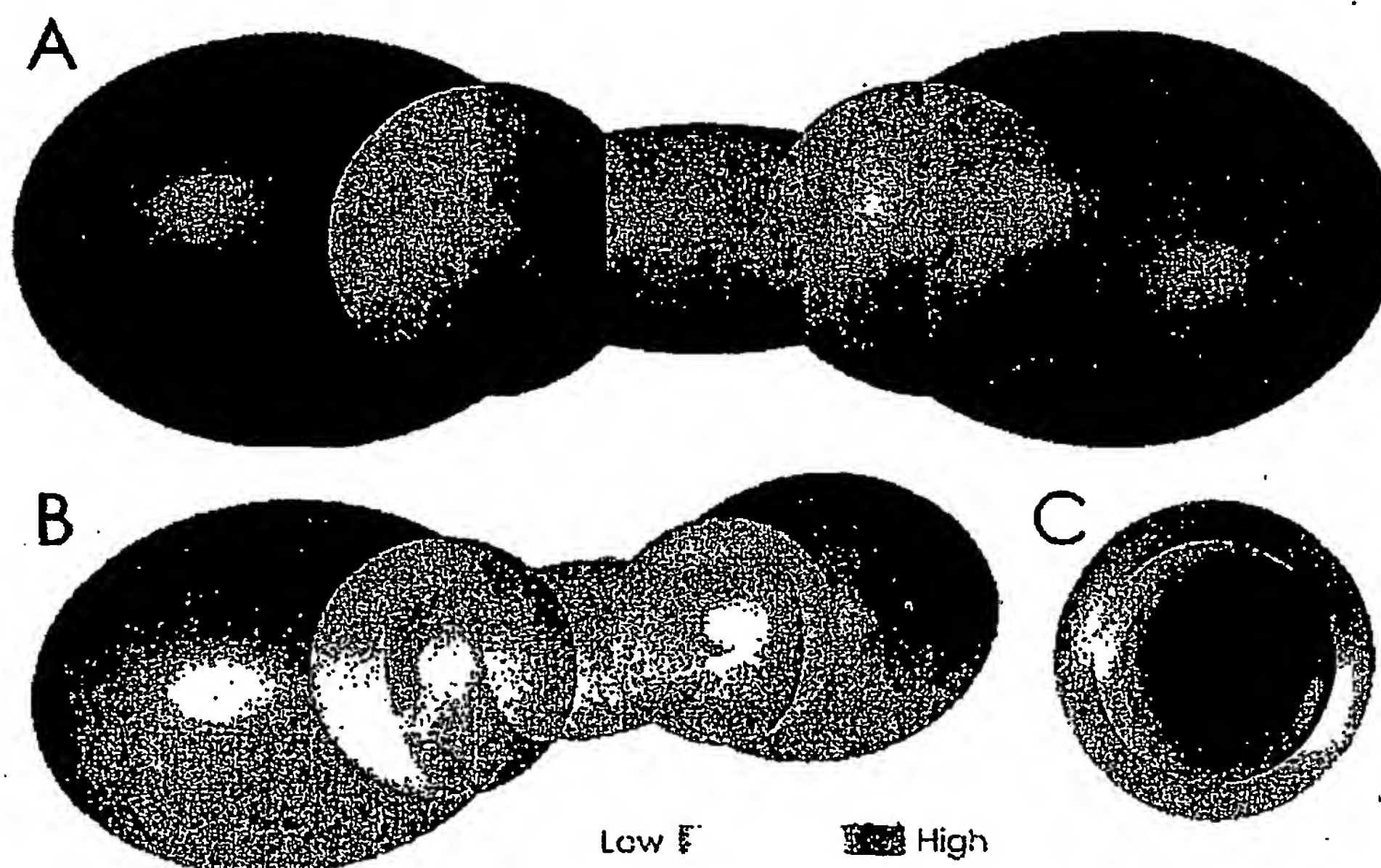


Fig.44



26/51
Fig.45

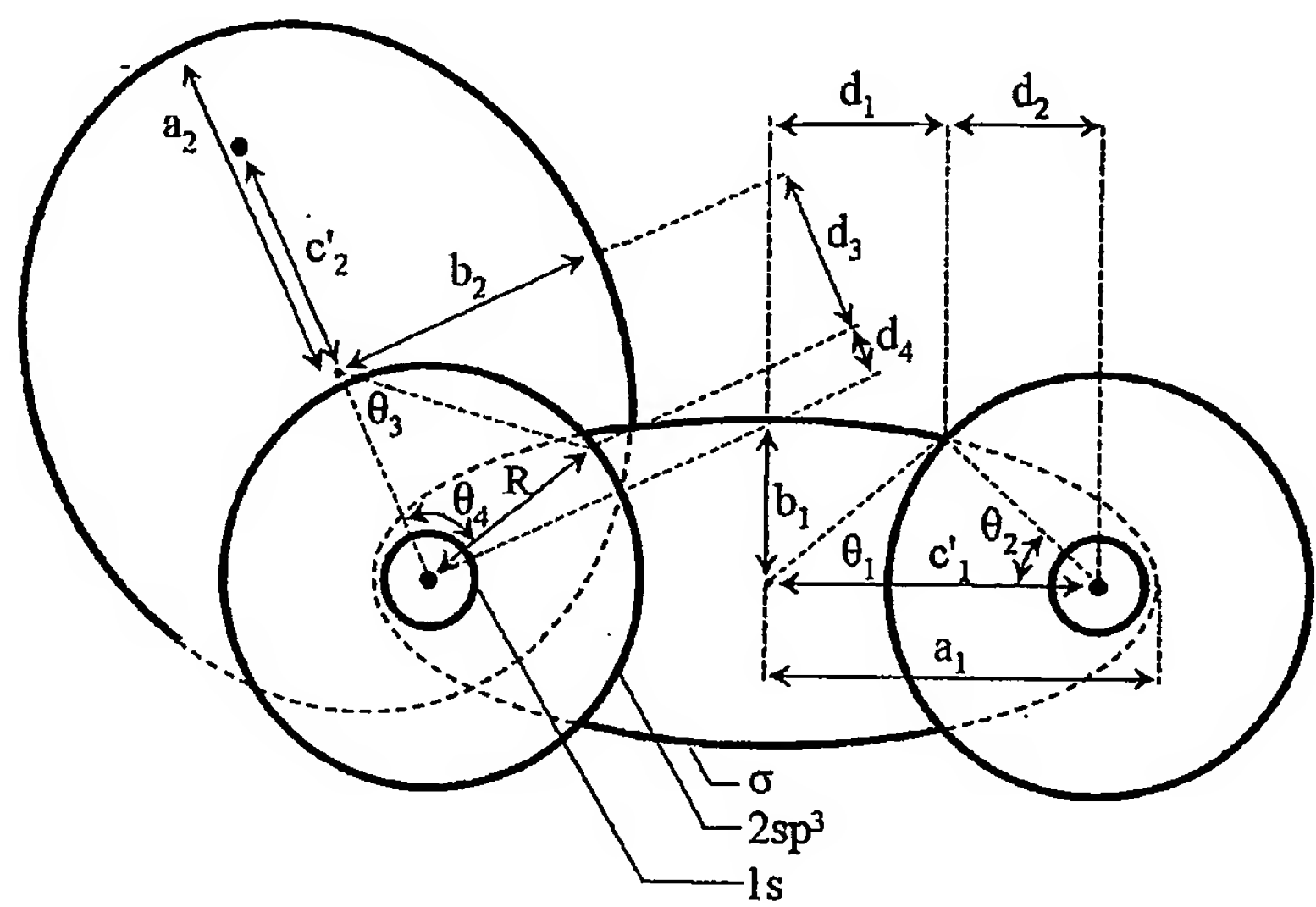
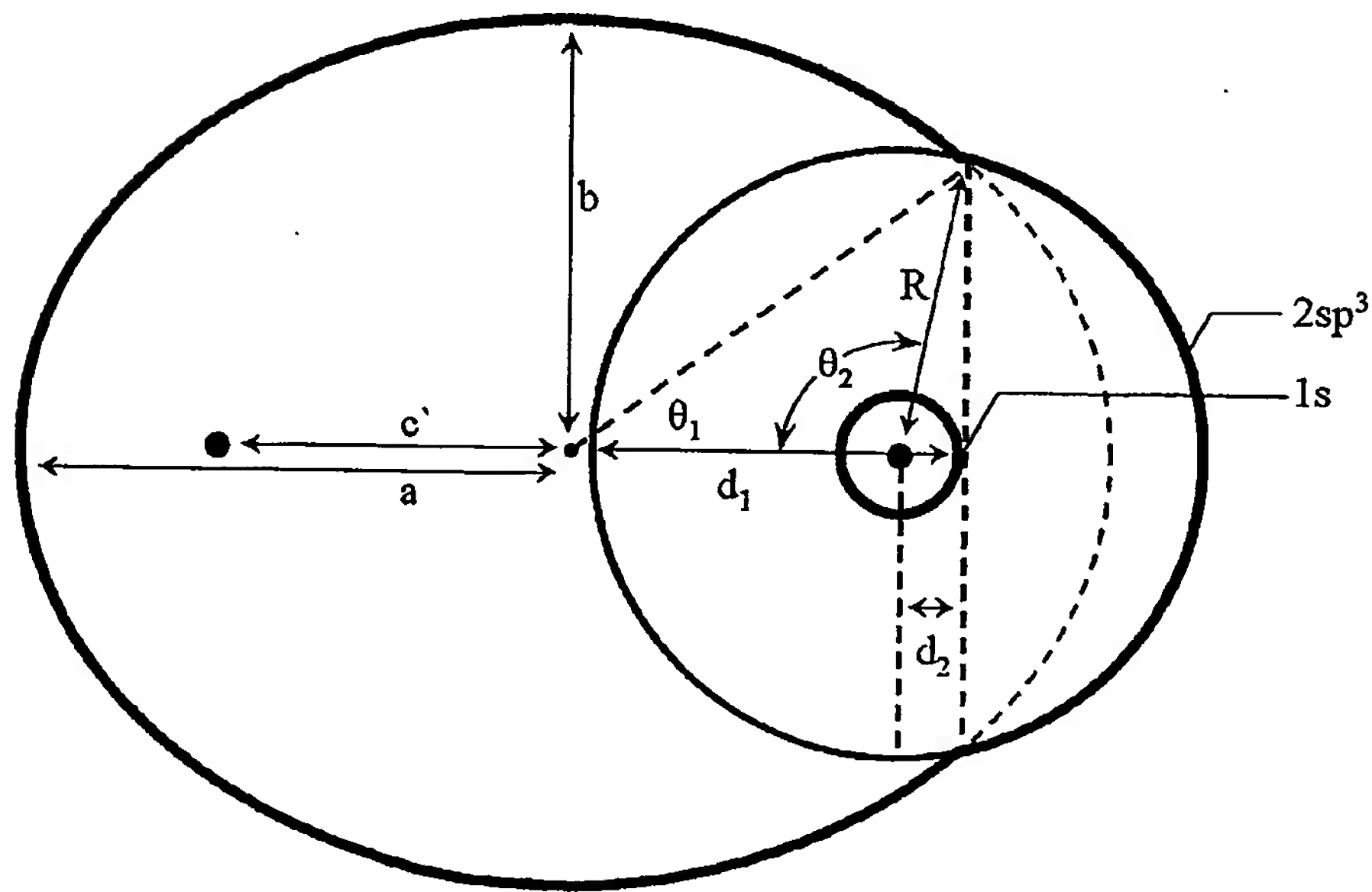
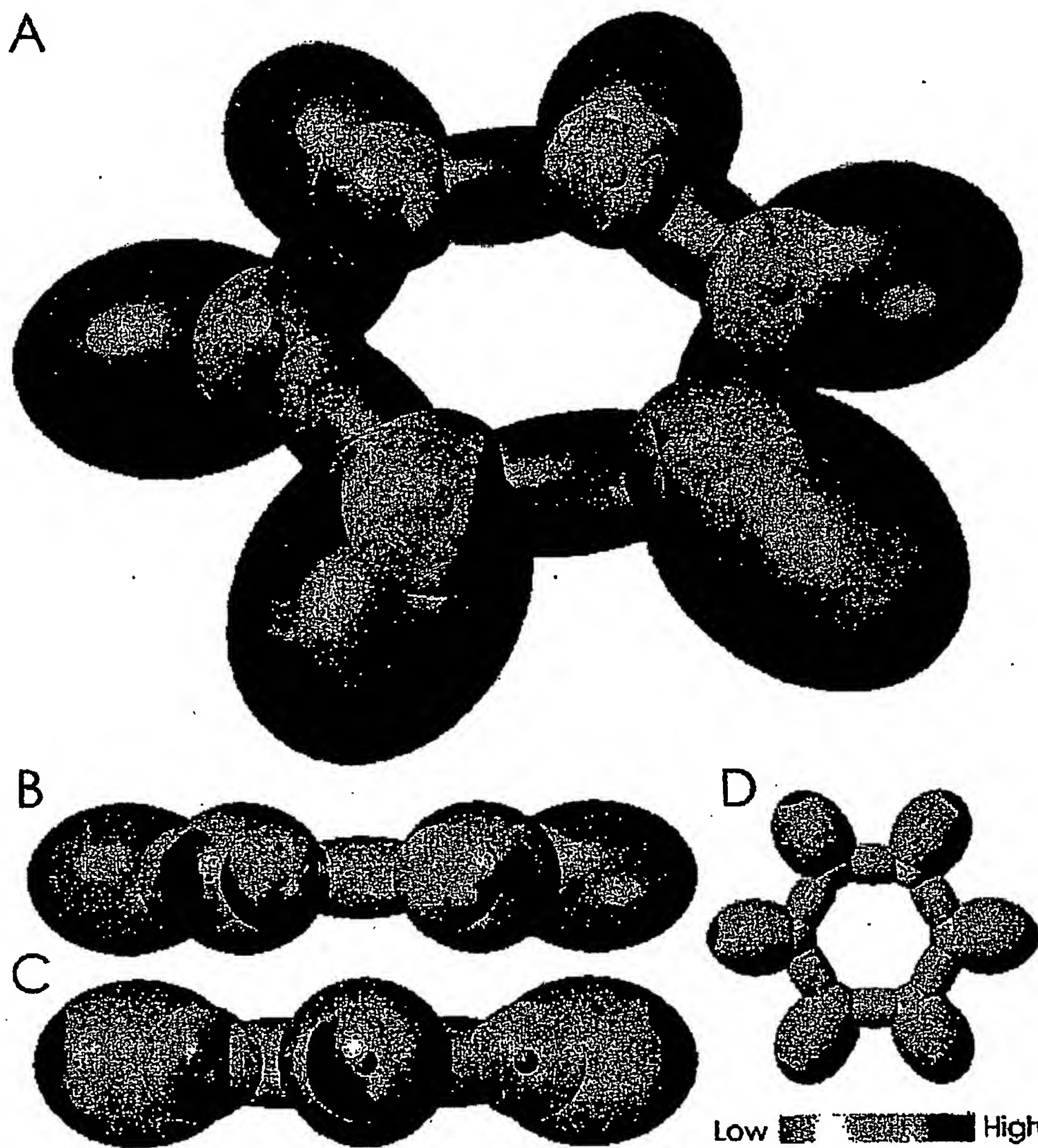


Fig.46

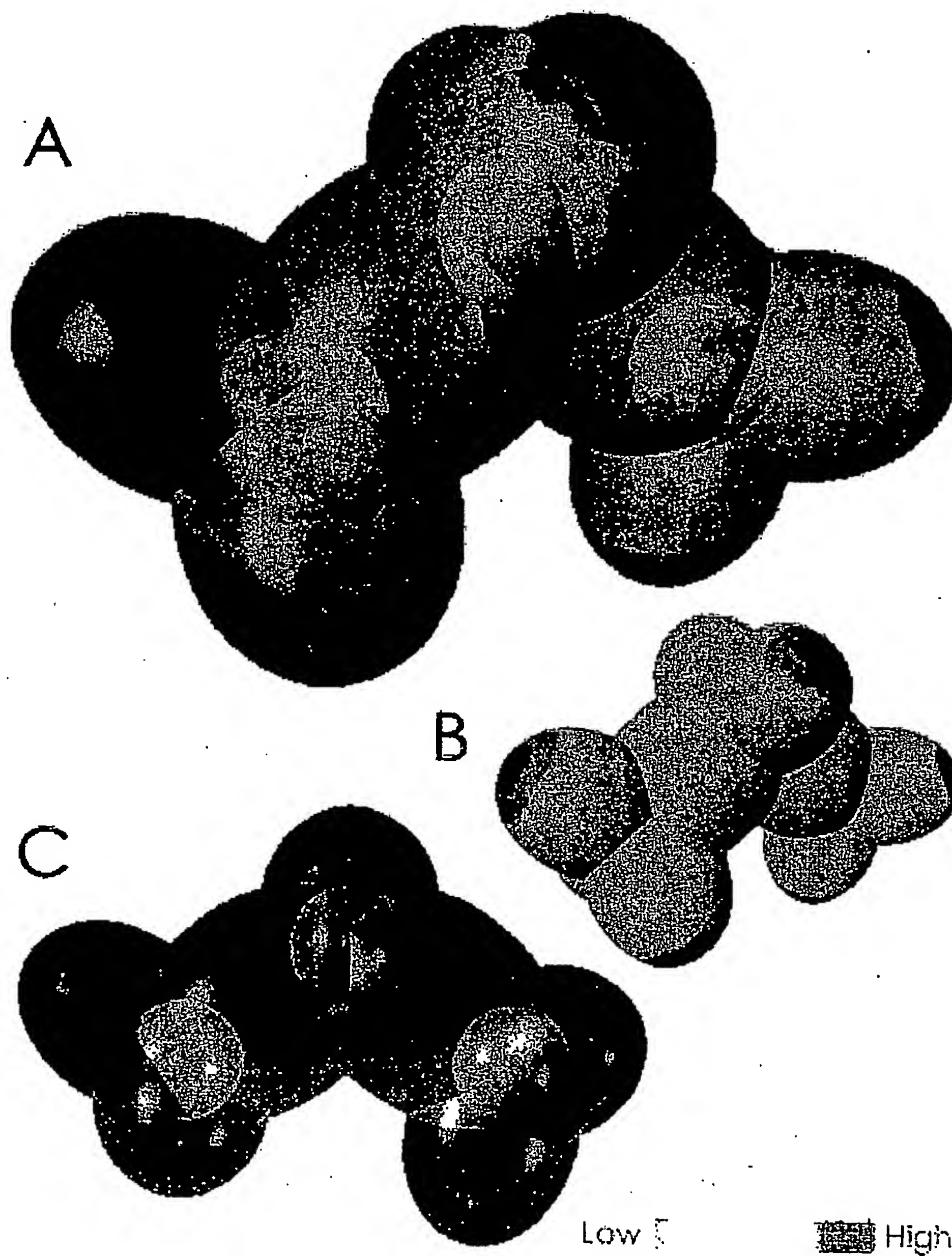


27/51

Fig.47

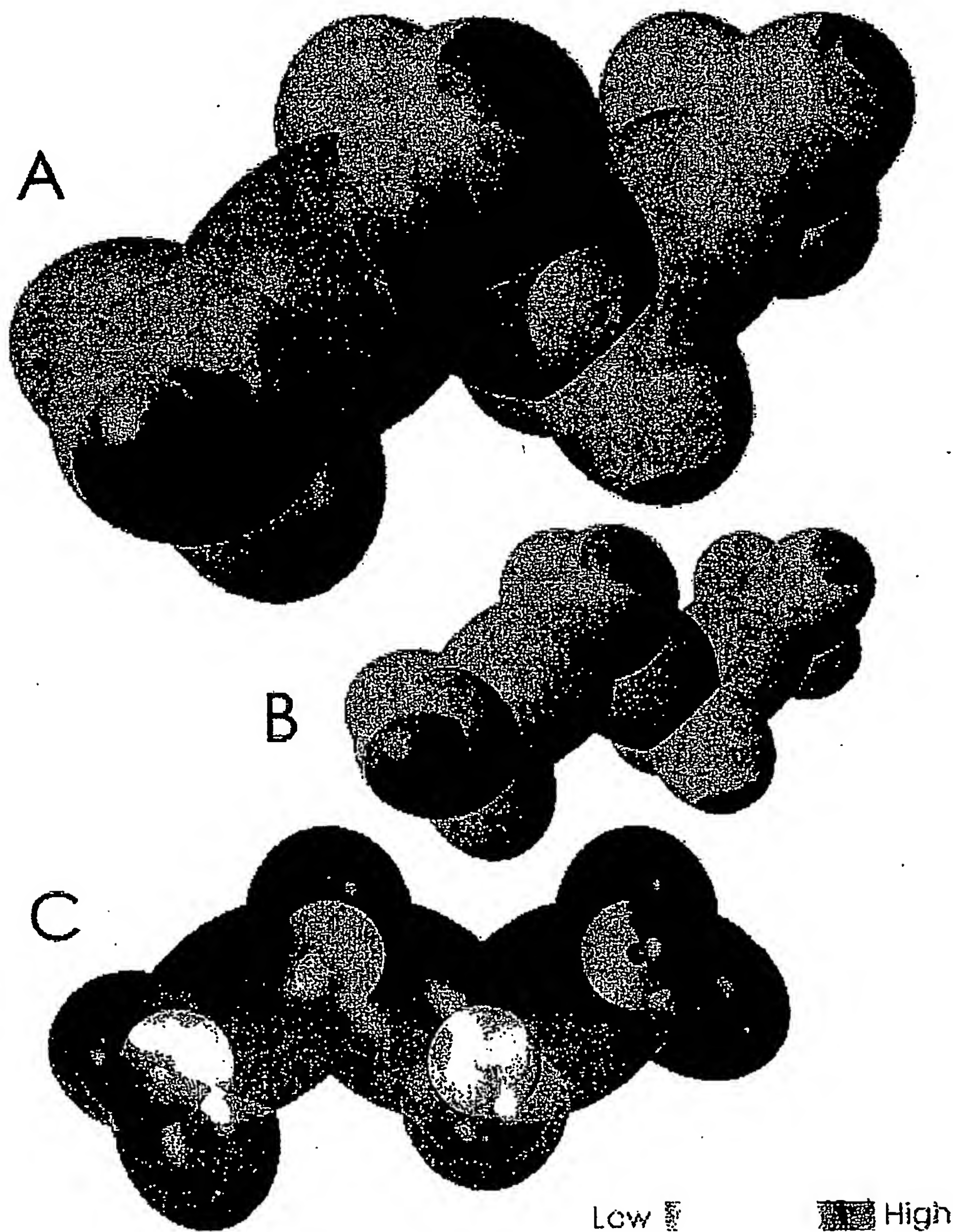


29/51
Fig.50



30/51

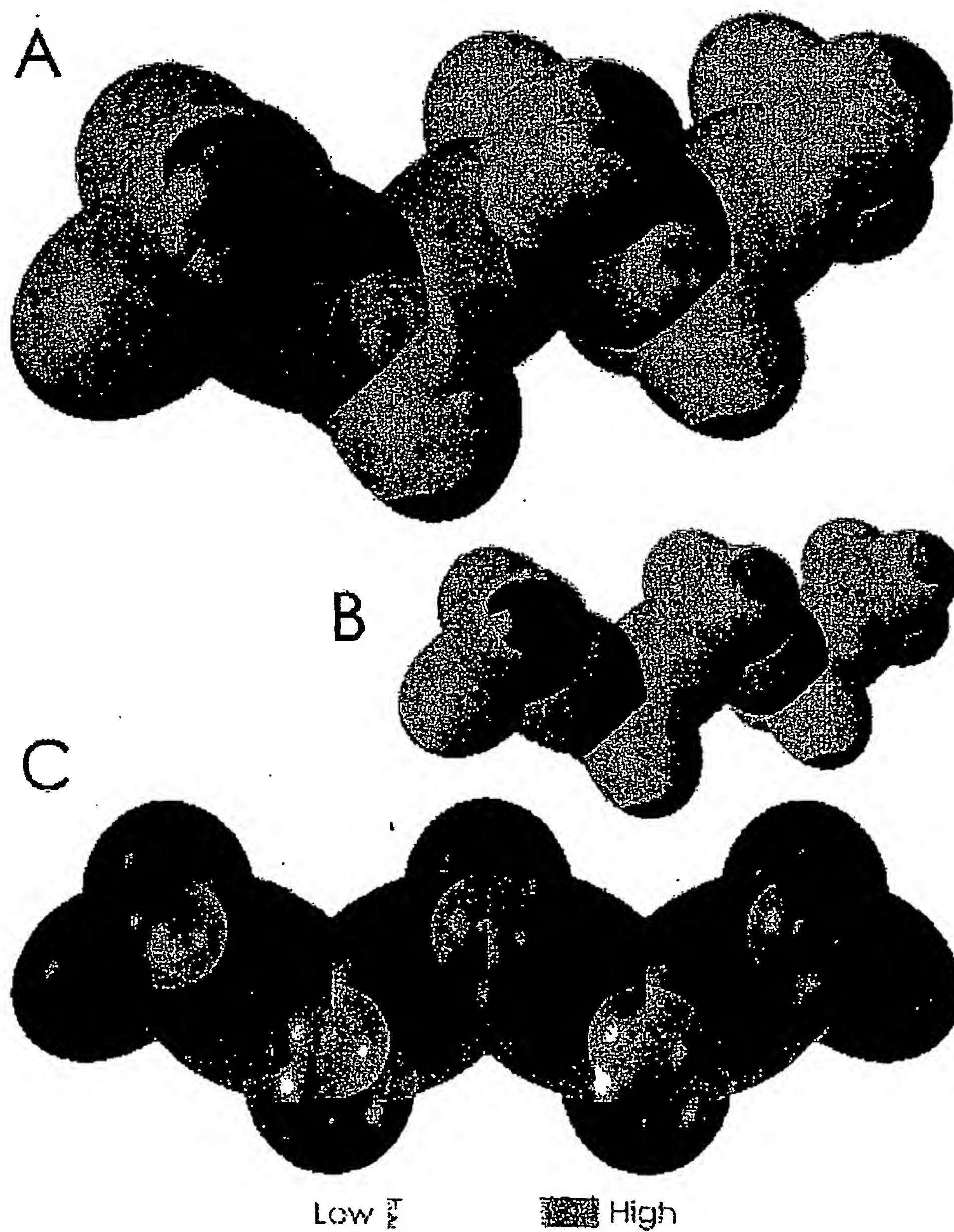
Fig.51



SUBSTITUTE SHEET (RULE 26)

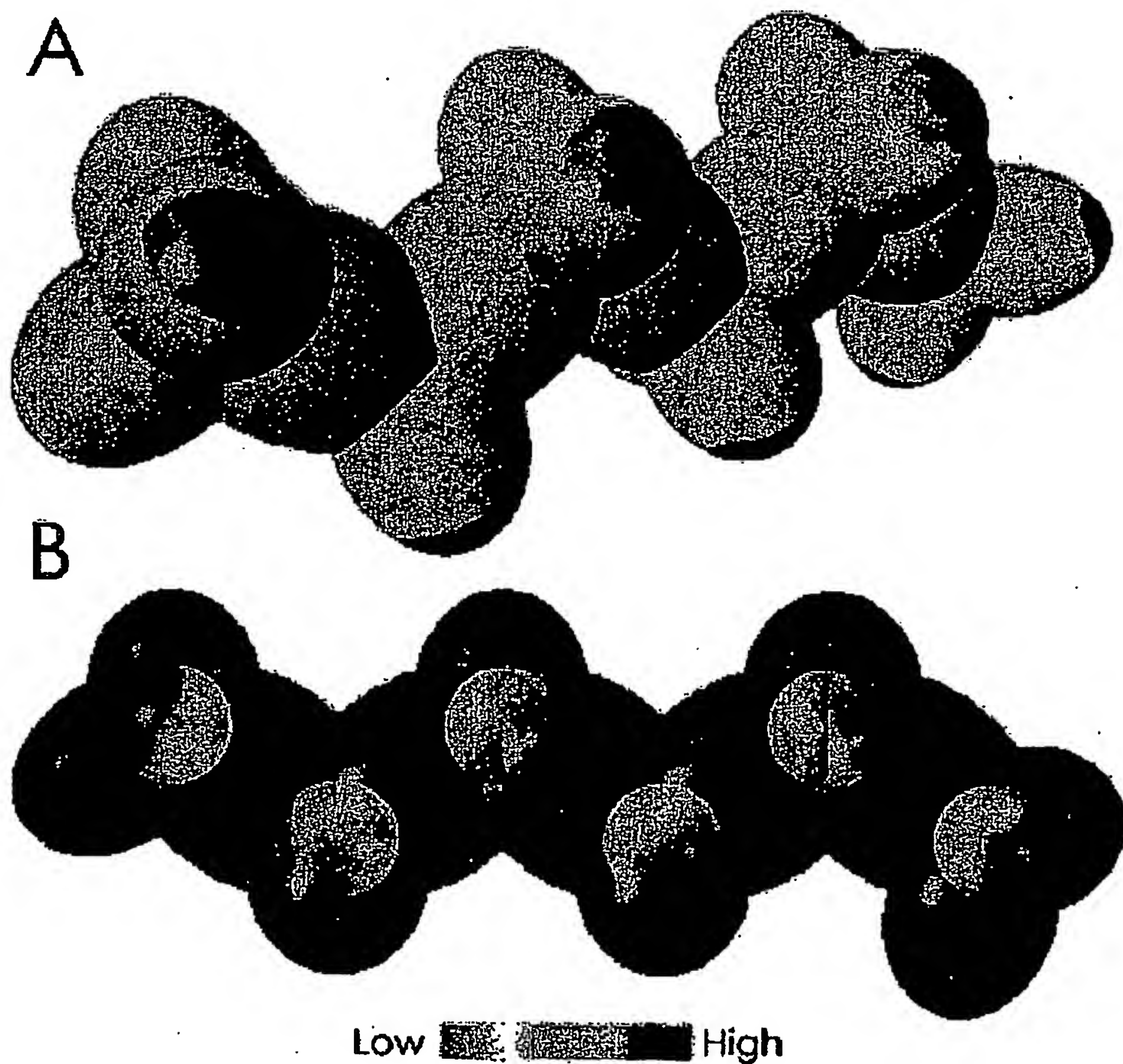
31/51

Fig.52



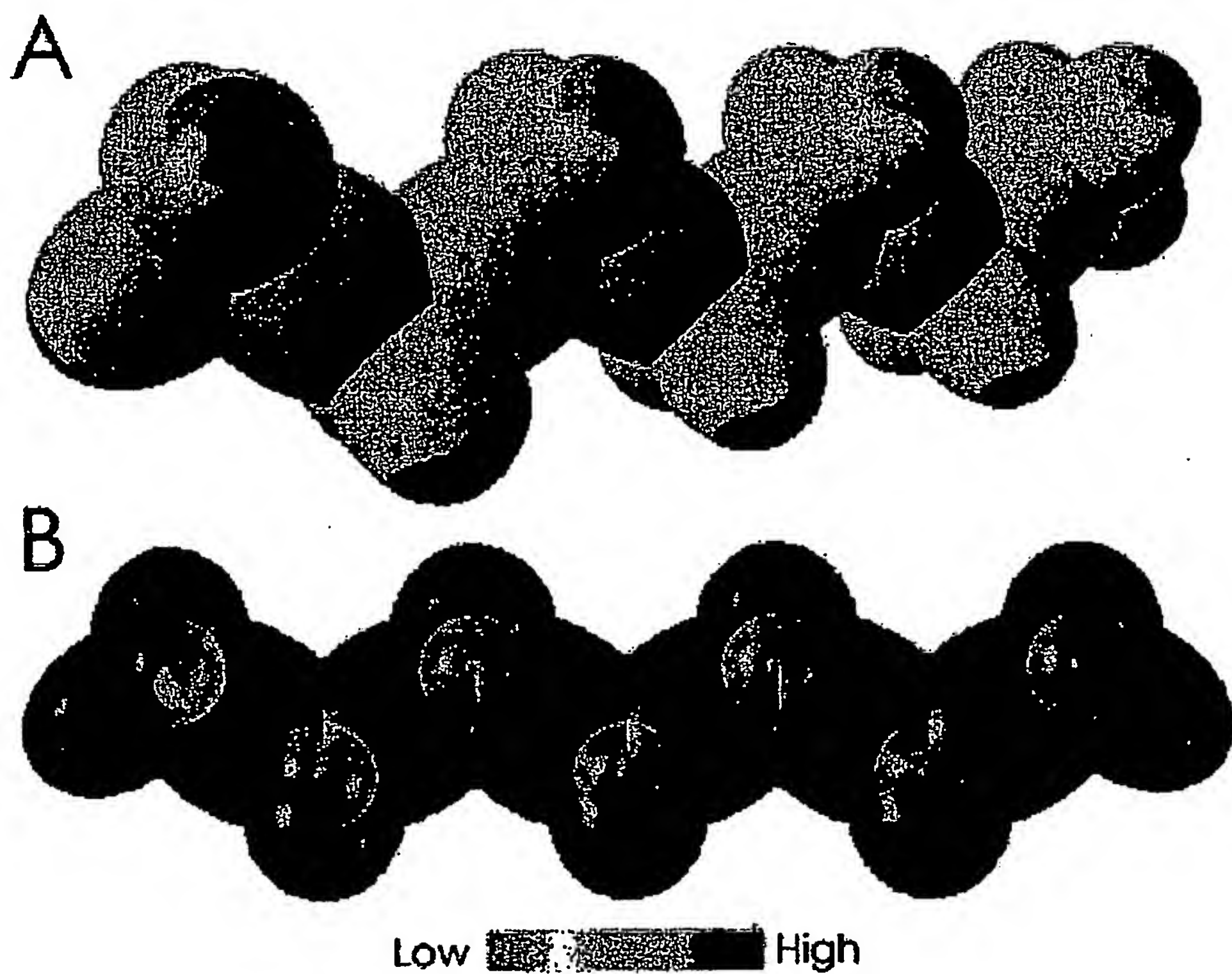
32/51

Fig.53

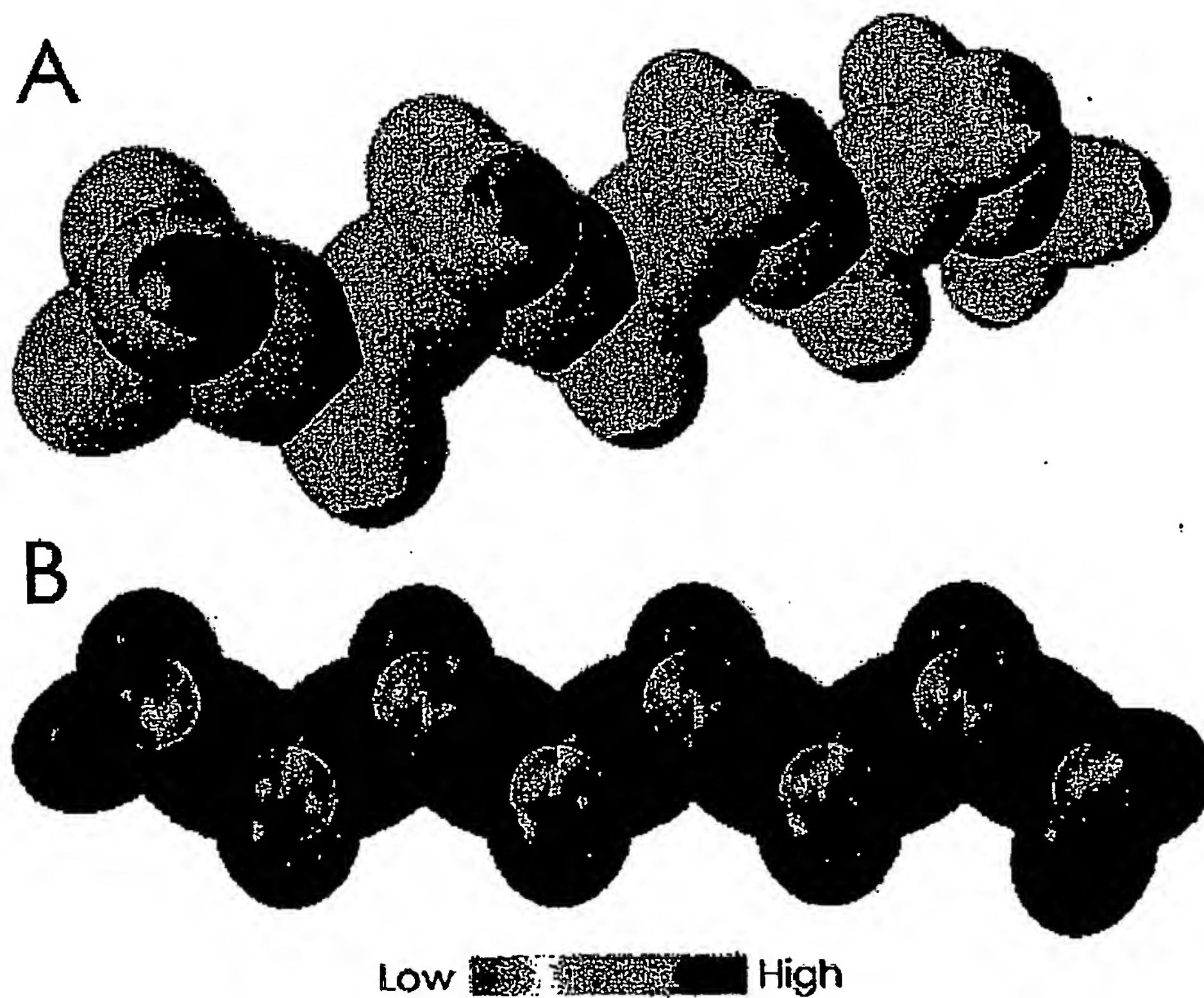


33/51

Fig.54

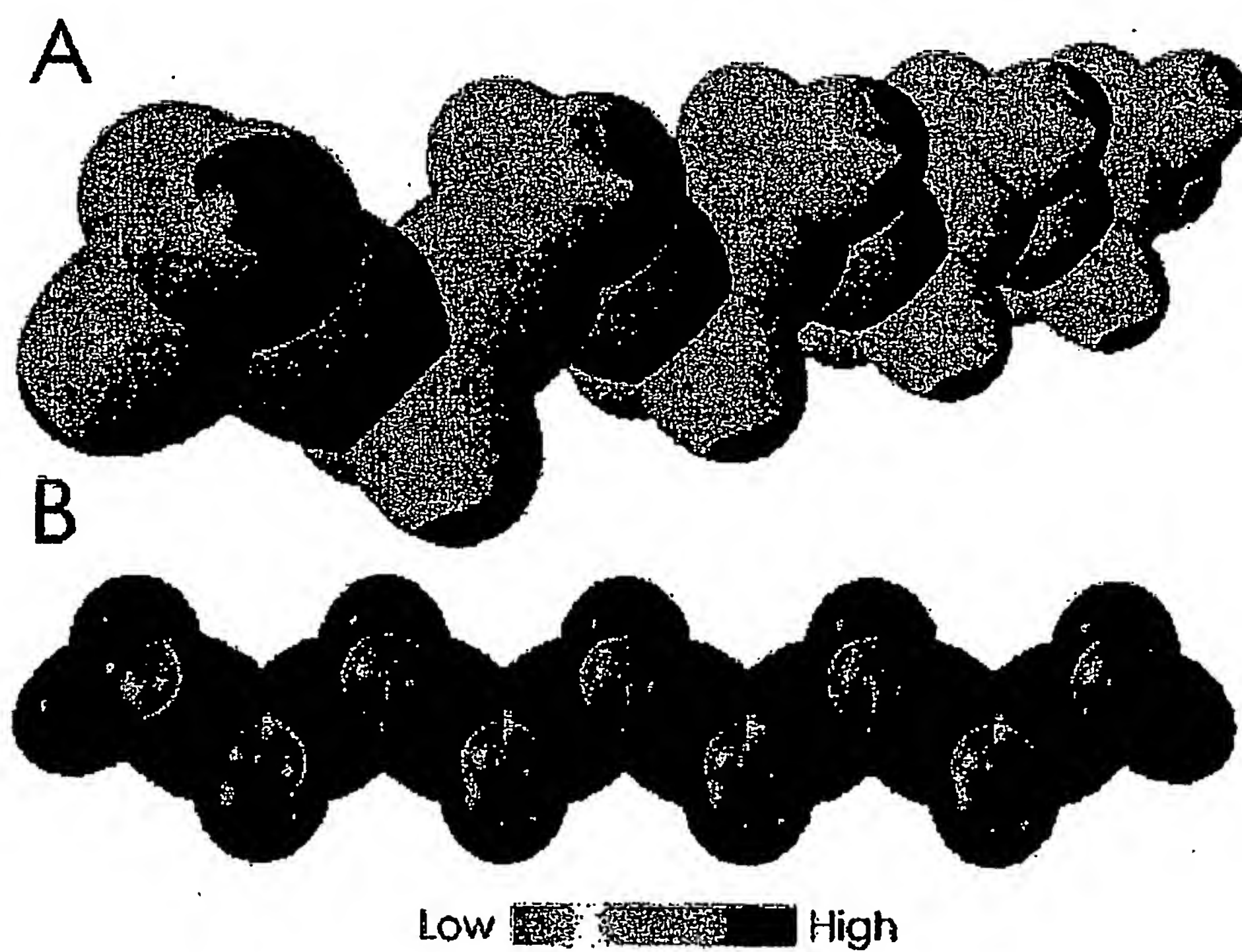


34/51
Fig.55



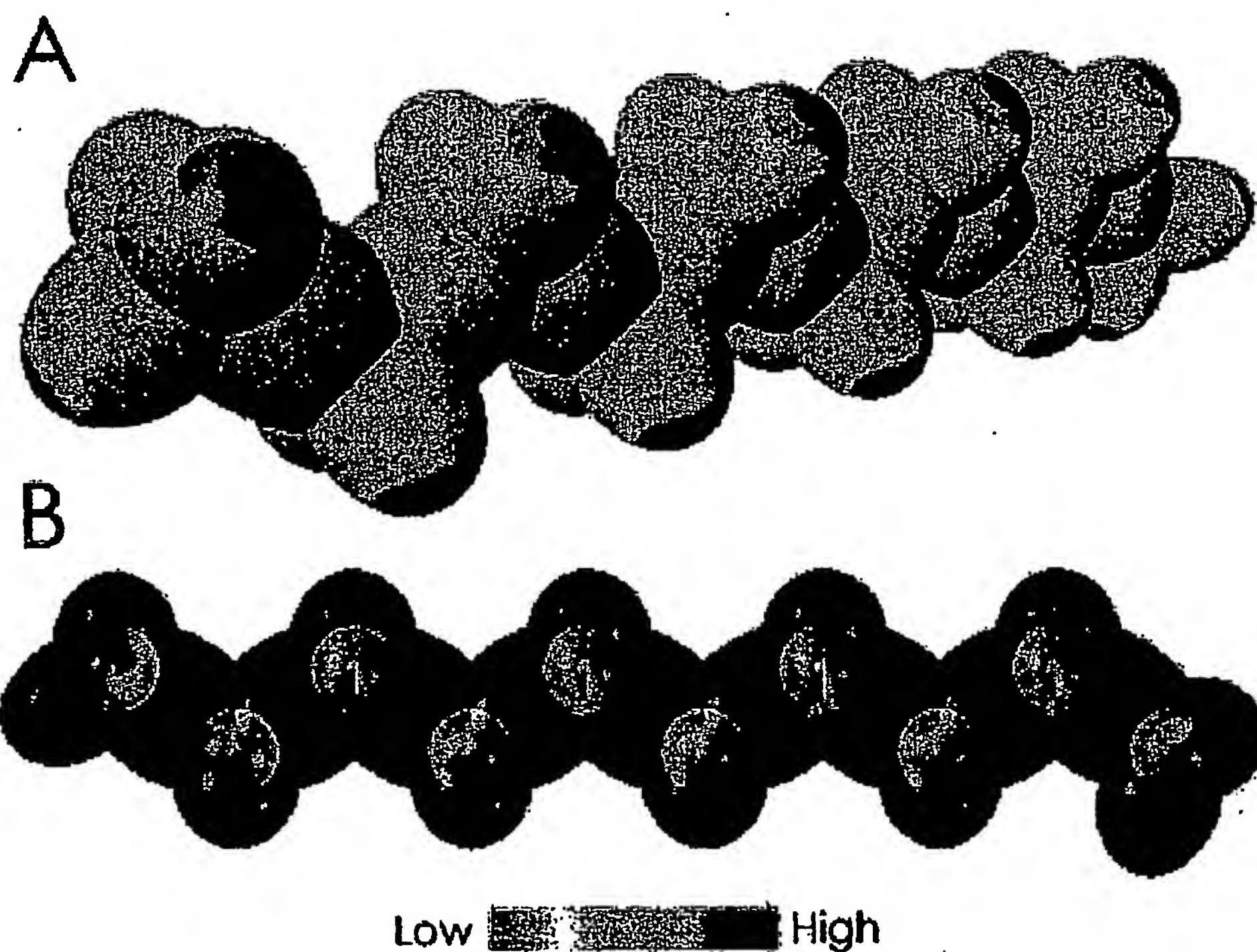
35/51

Fig. 56



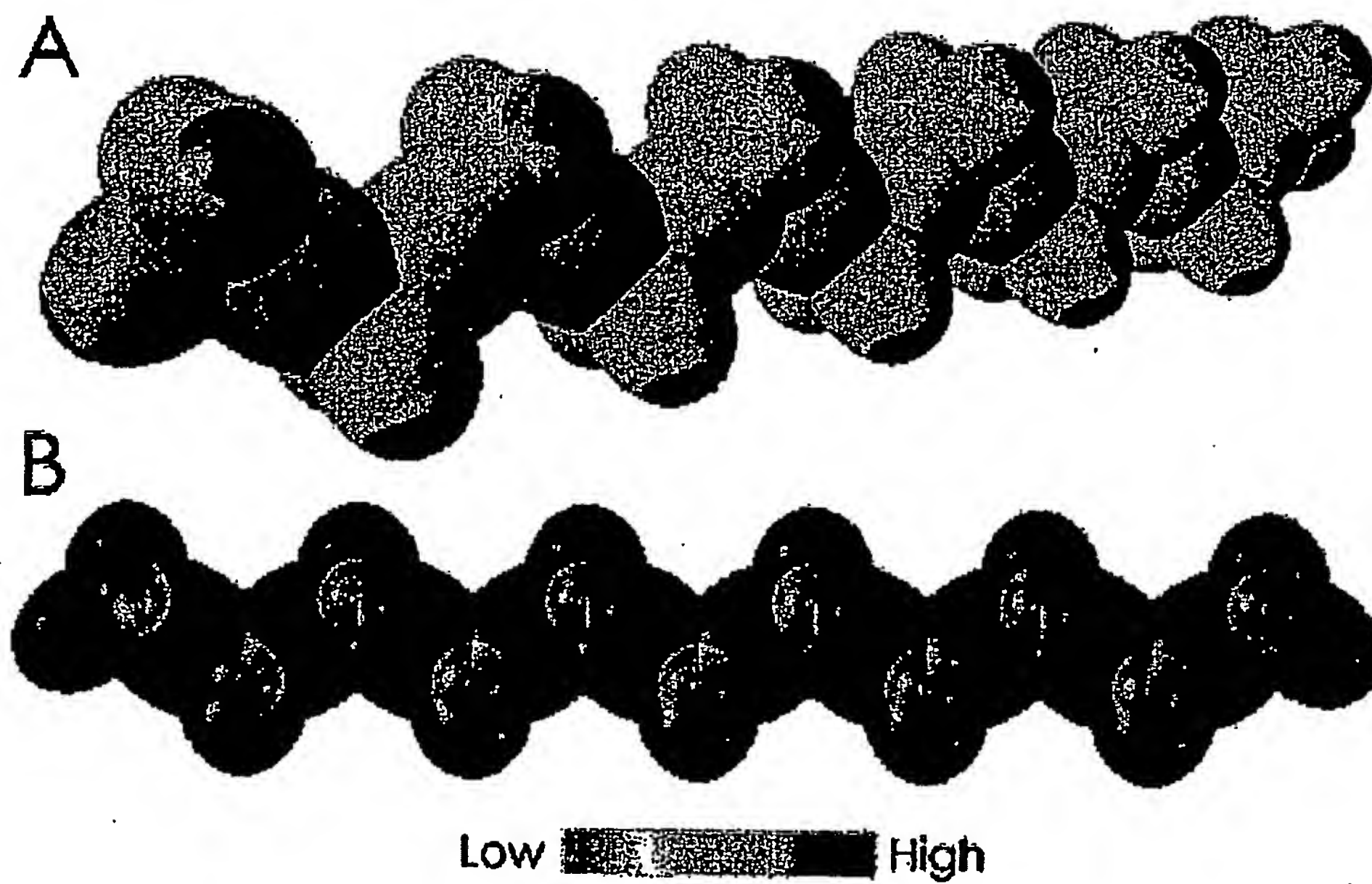
36/51

Fig.57



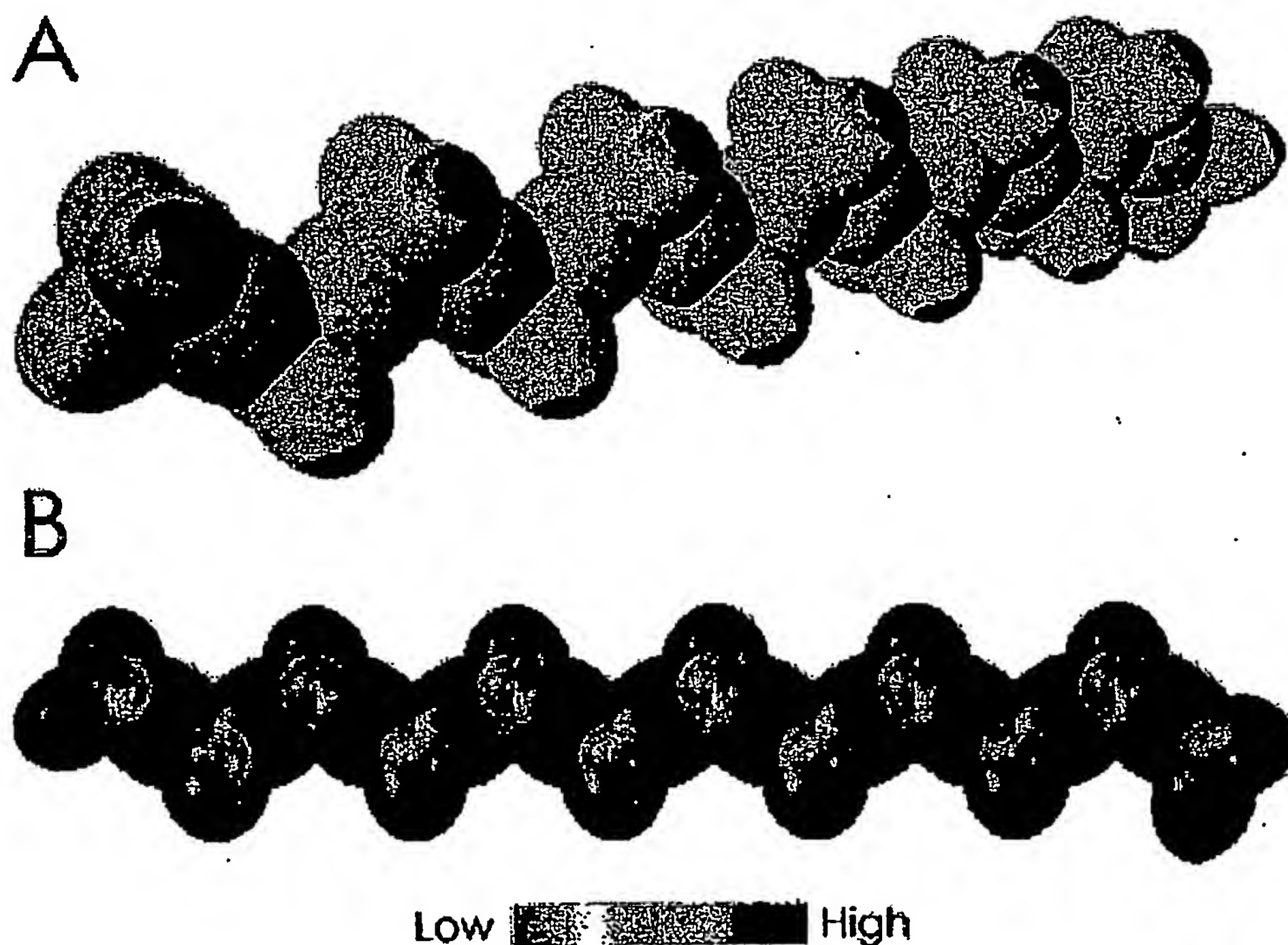
37/51

Fig.58



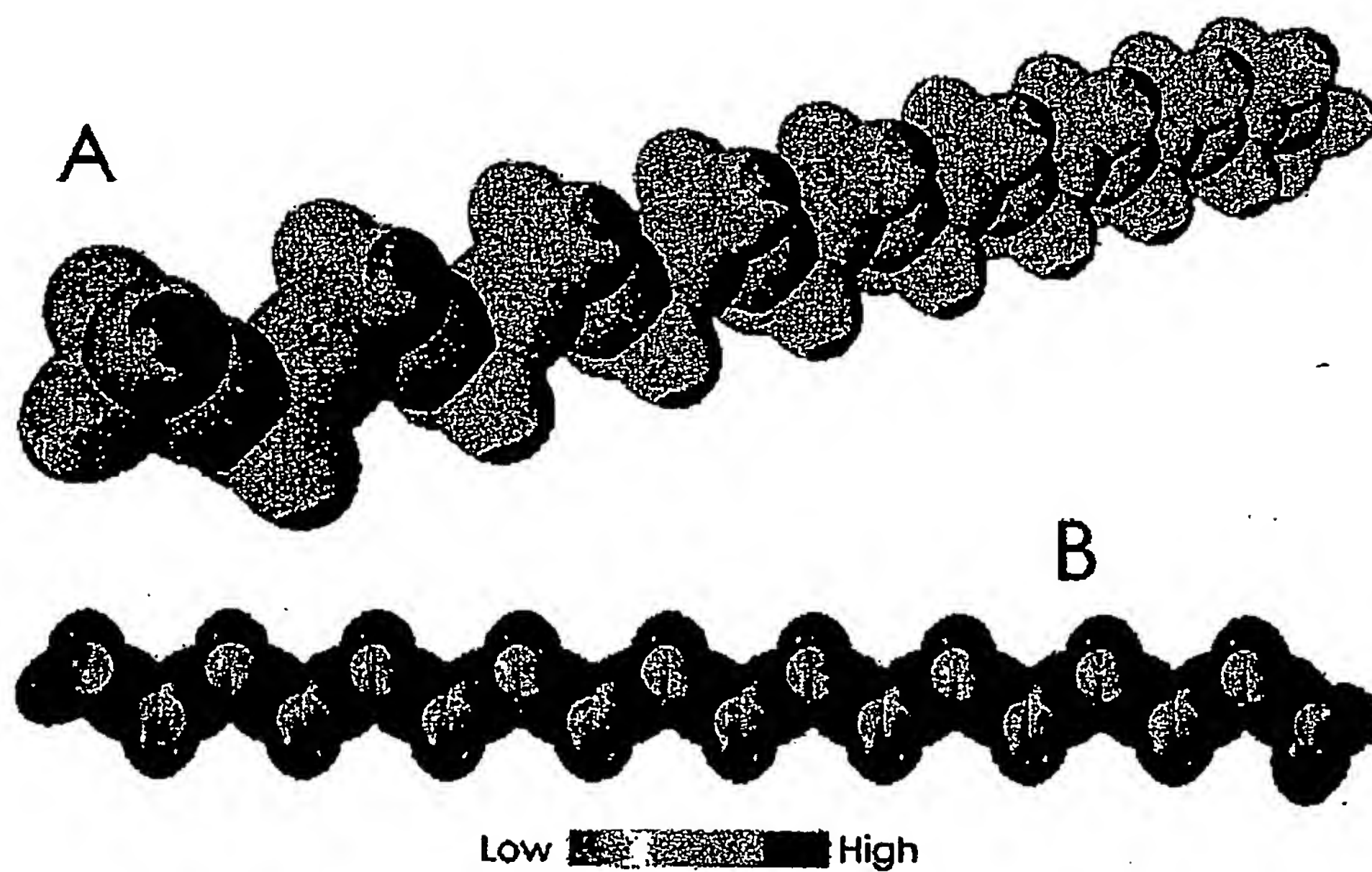
38/51

Fig. 59



39/51

Fig.60



40/51

Fig. 61.A

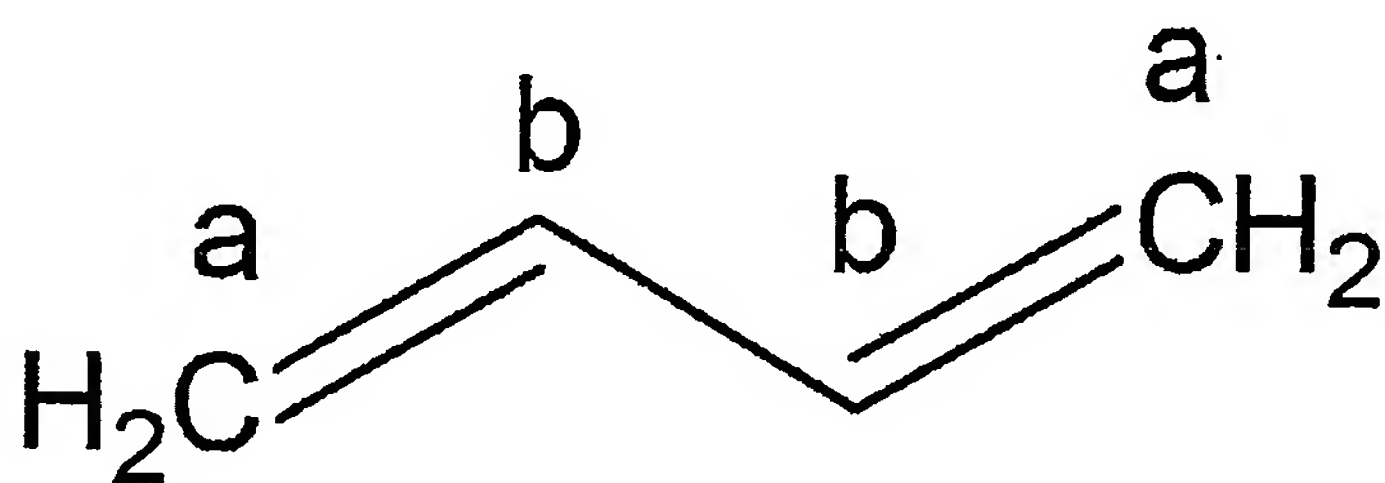


Fig. 61.B

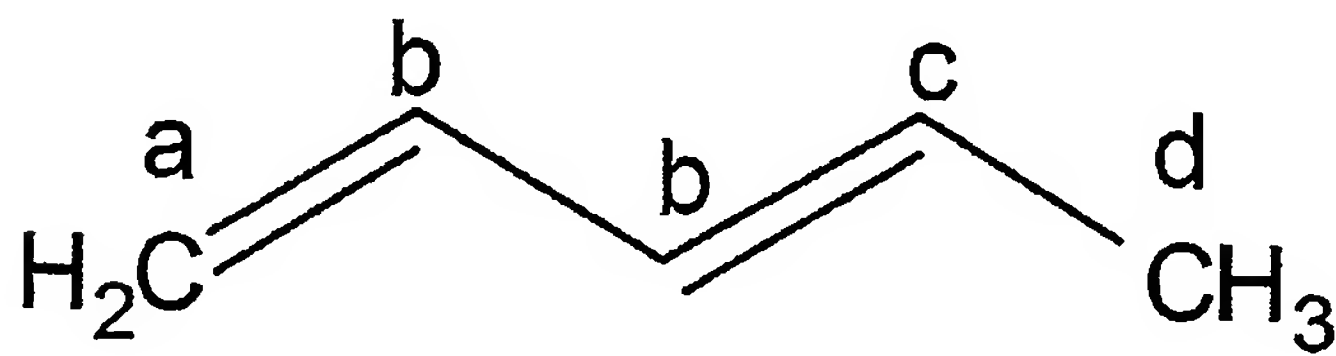
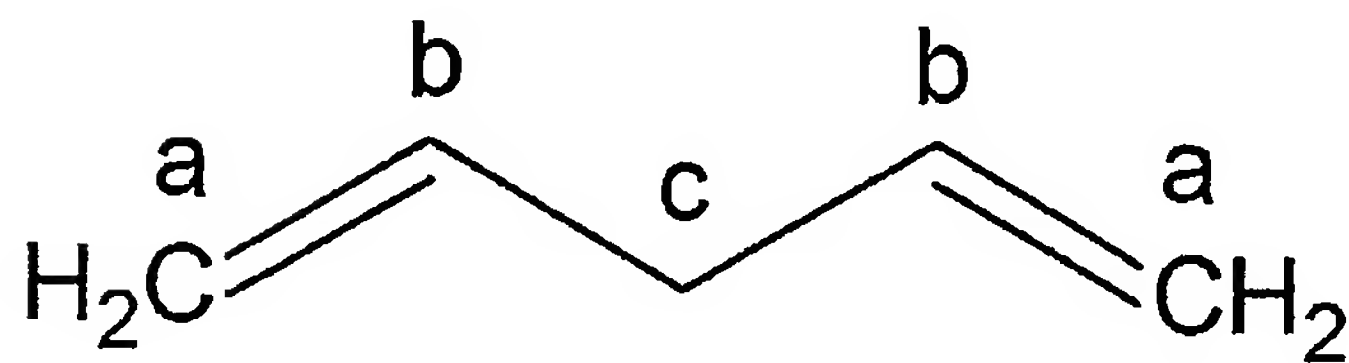


Fig. 61.C



41/51

Fig. 61.D

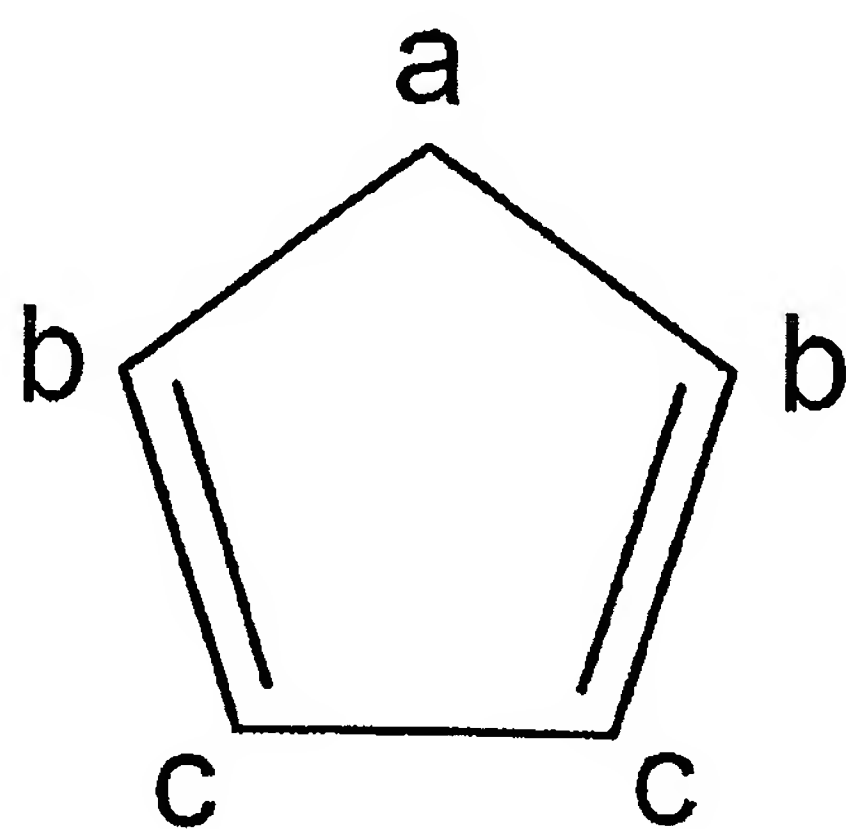
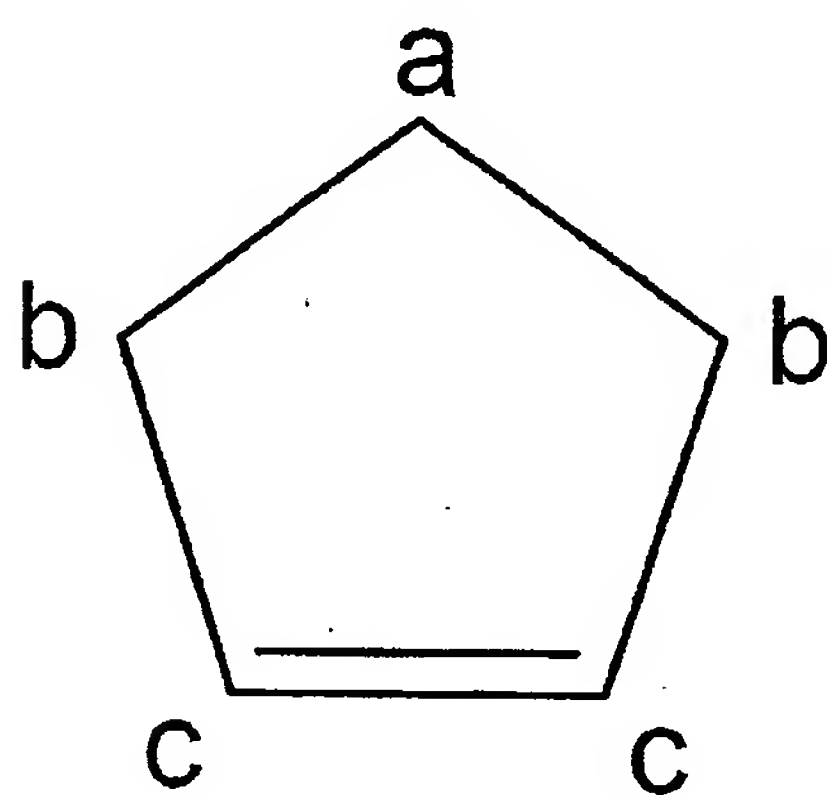


Fig. 61E



42/51

Fig. 62

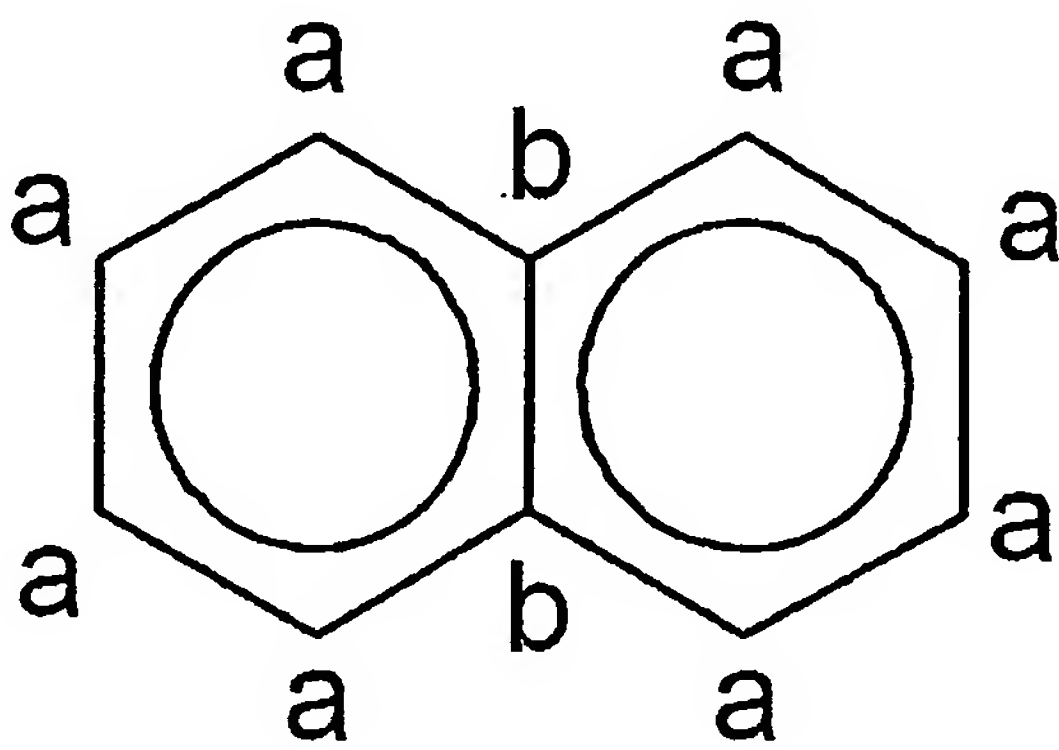


Fig. 63

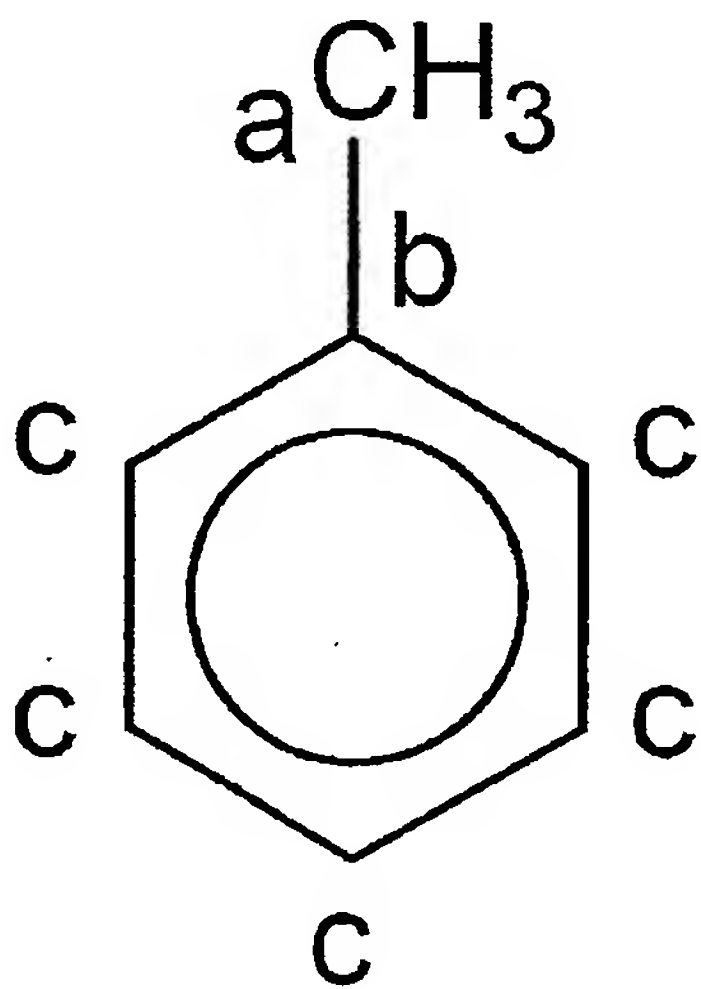
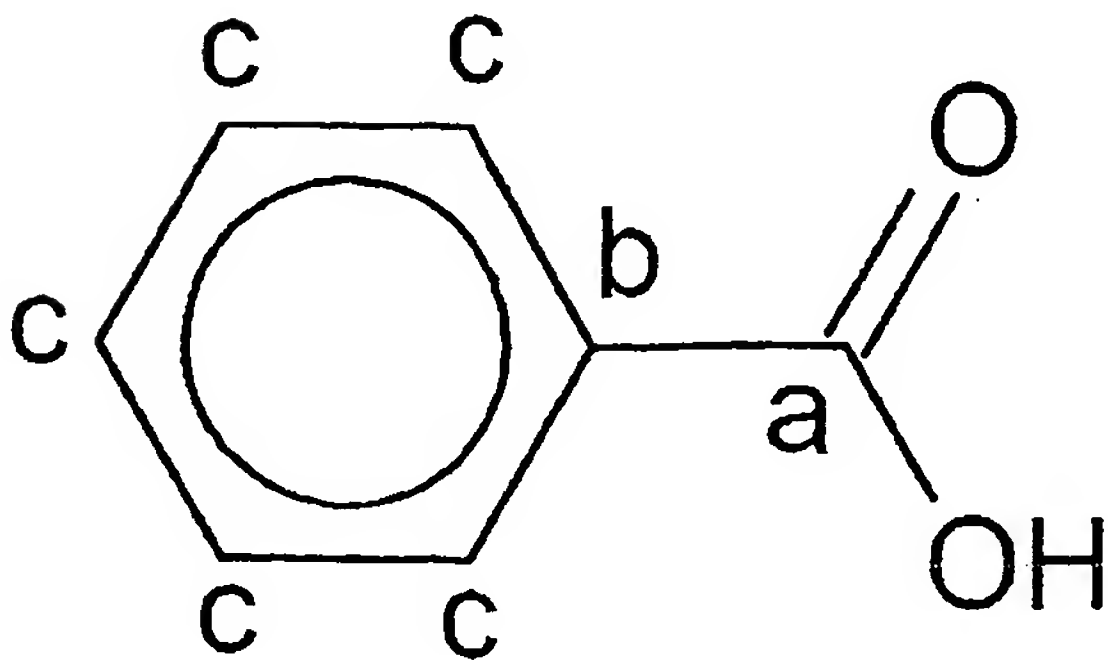


Fig. 64



43/51

Fig. 65

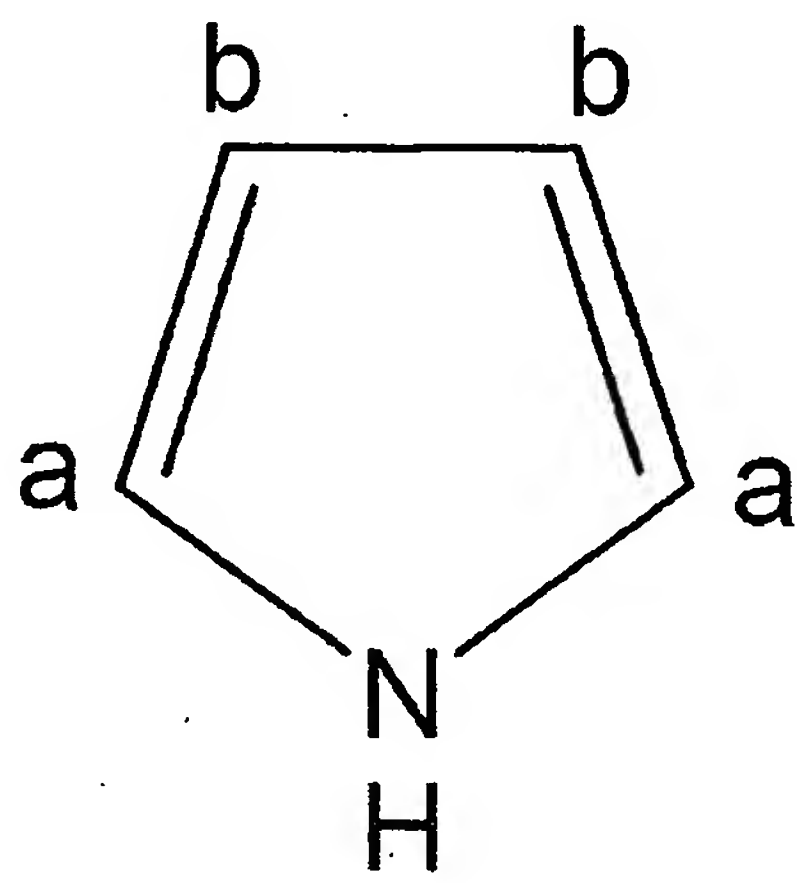
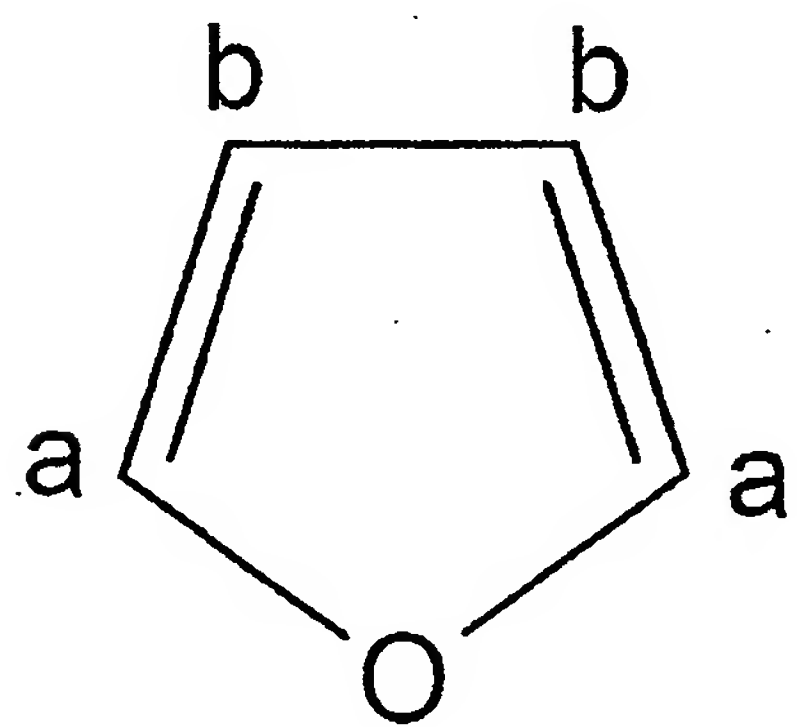


Fig. 66



44/51

Fig. 67

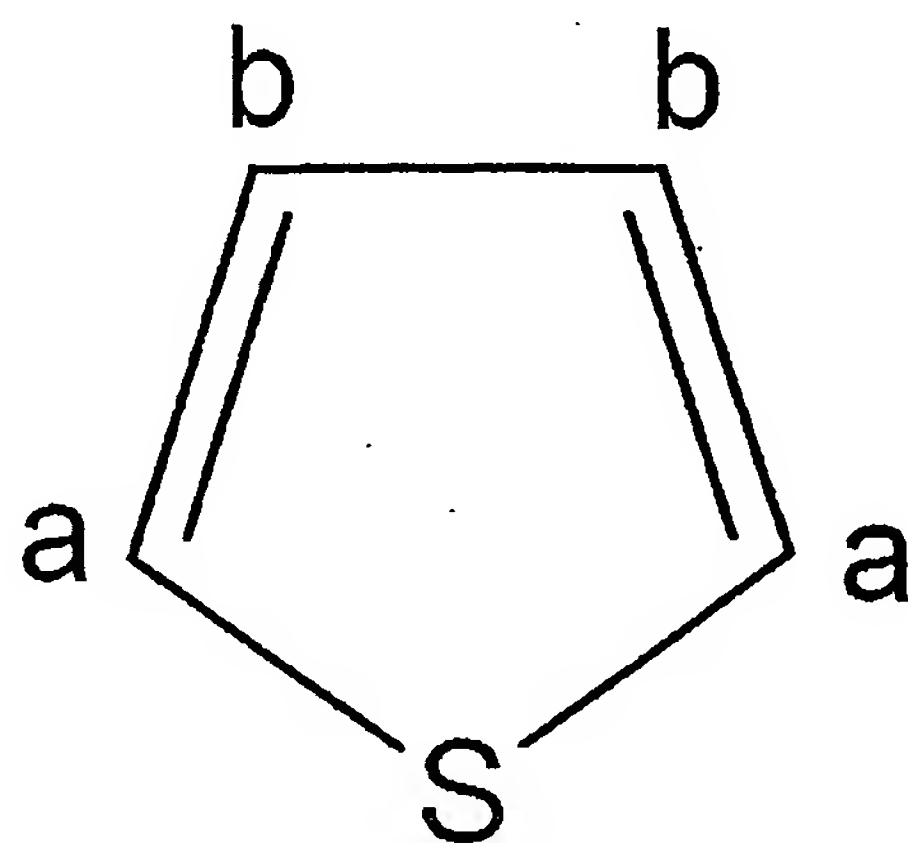
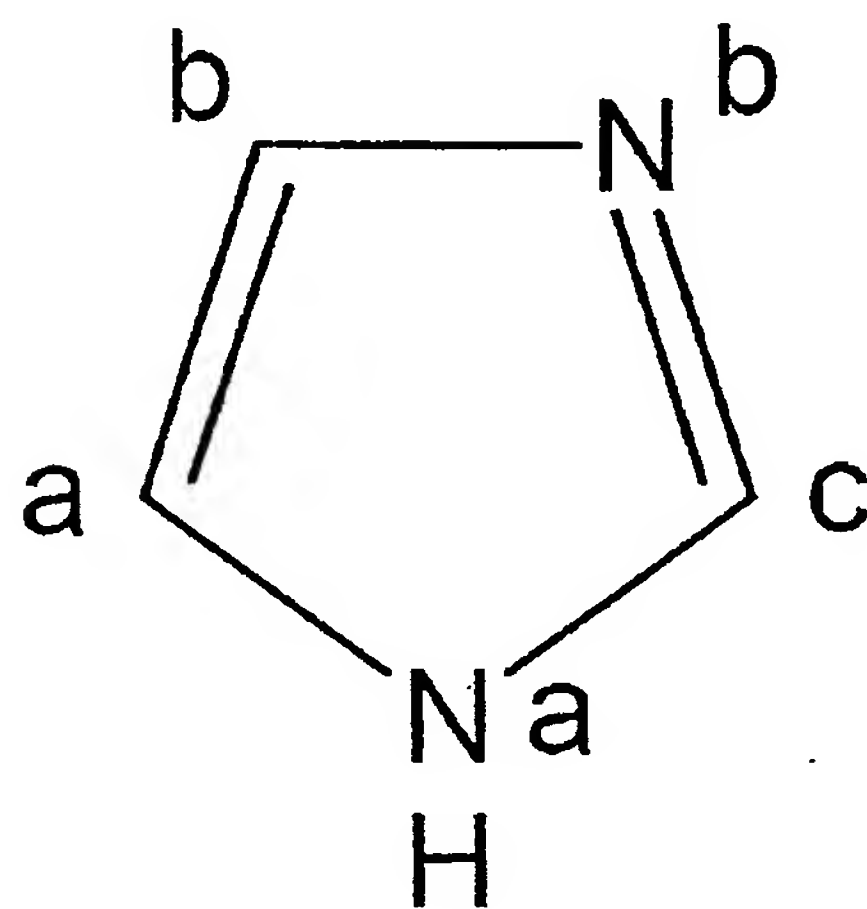


Fig. 68



45/51

Fig. 69

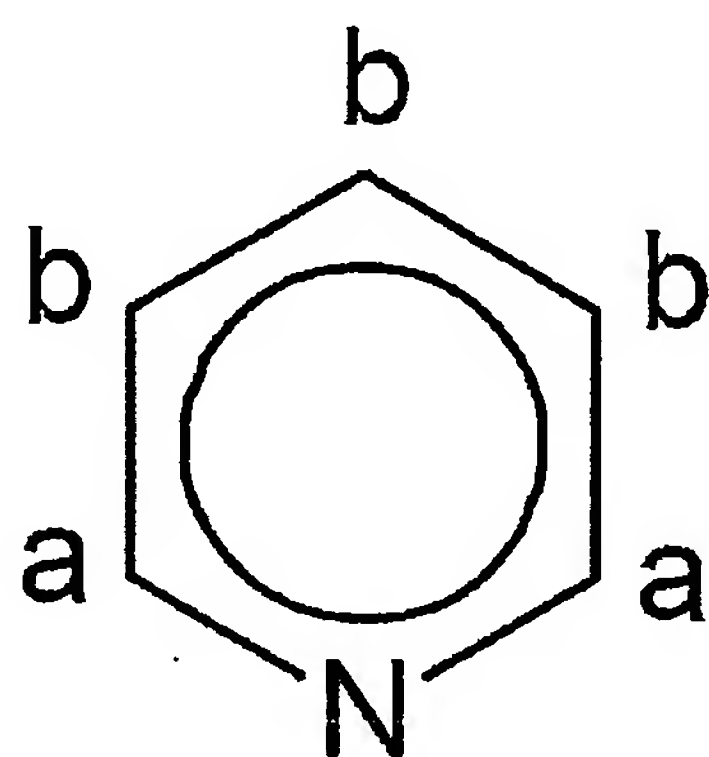
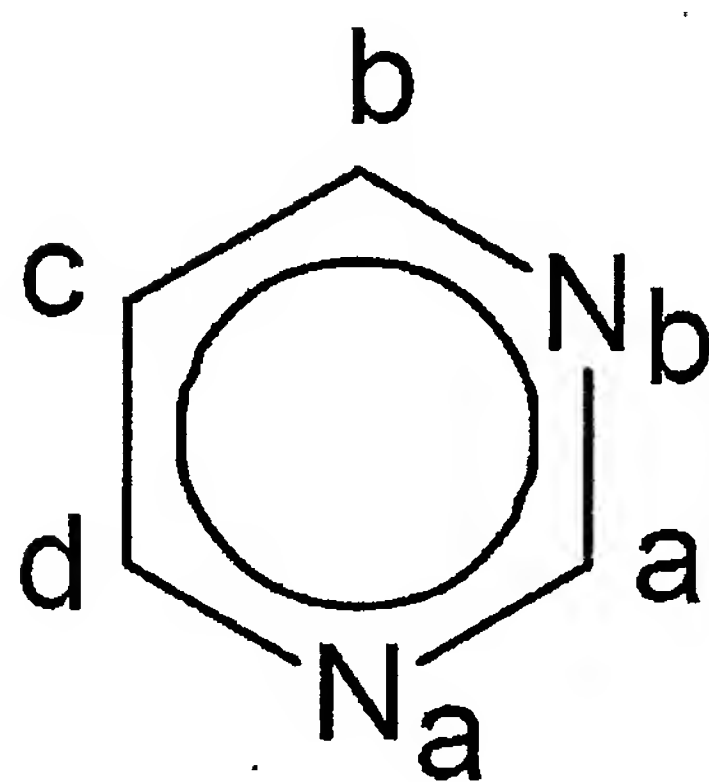


Fig. 70



46/51

Fig. 71

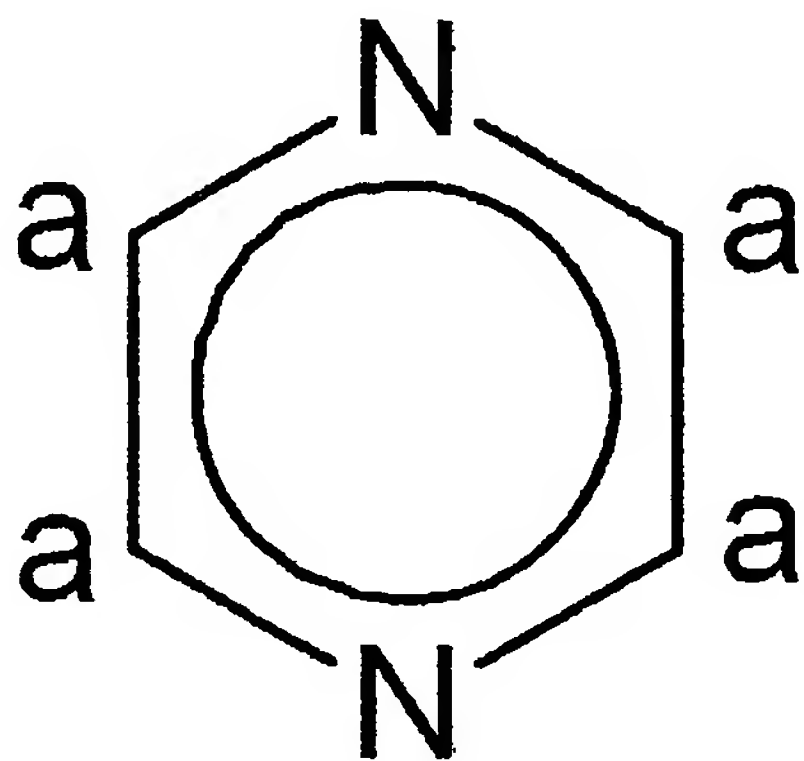
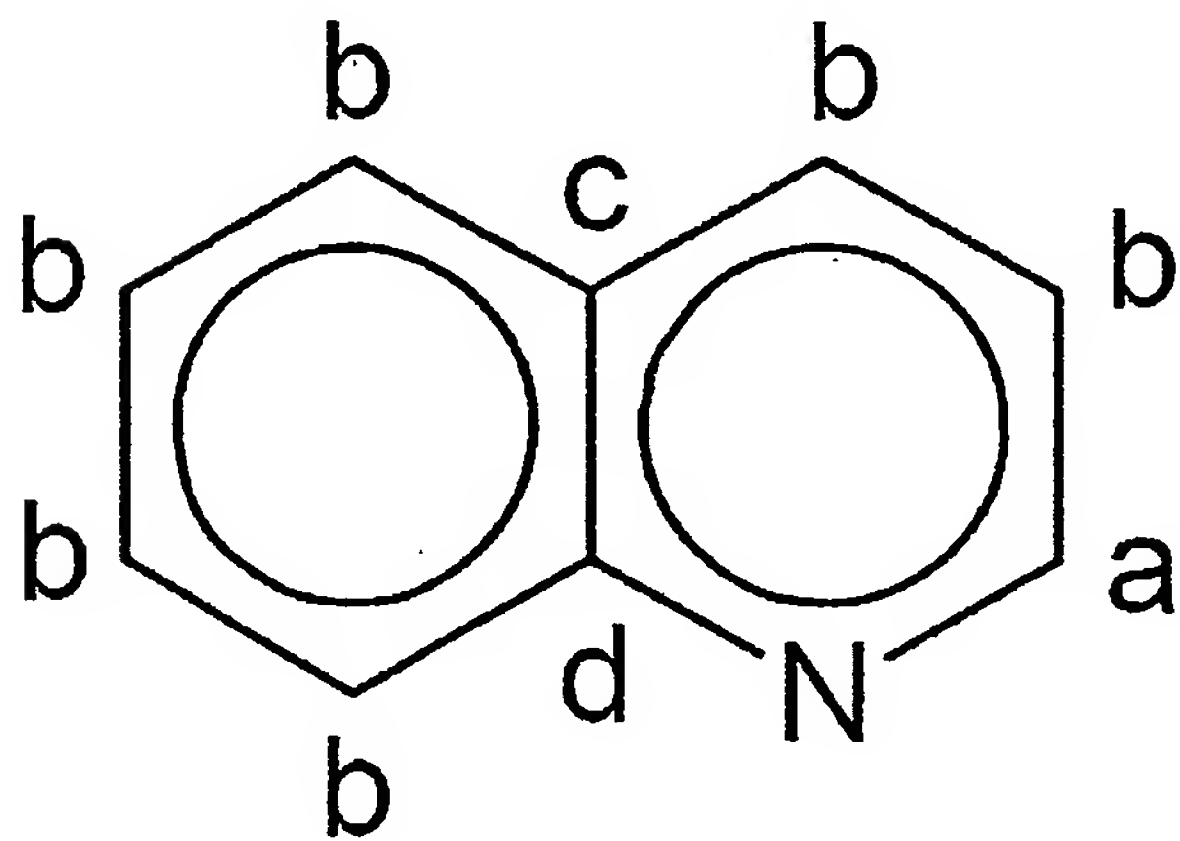


Fig.72



47/51

Fig. 73

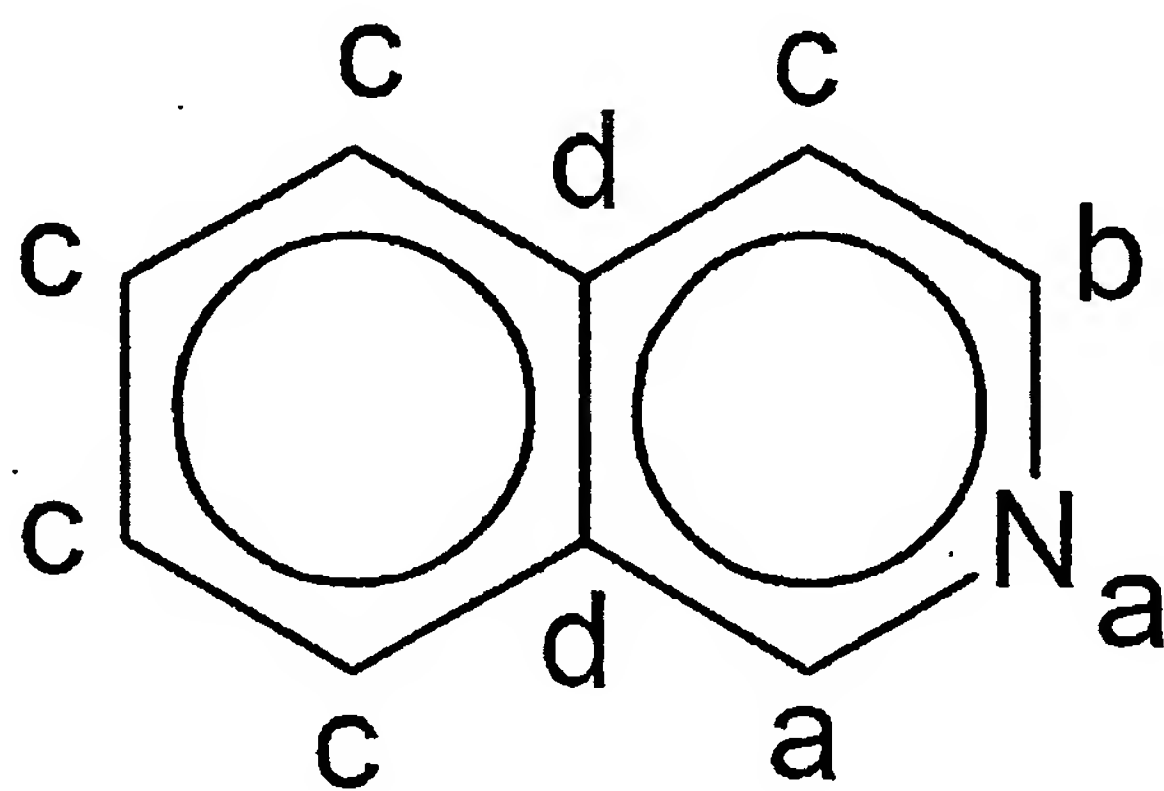
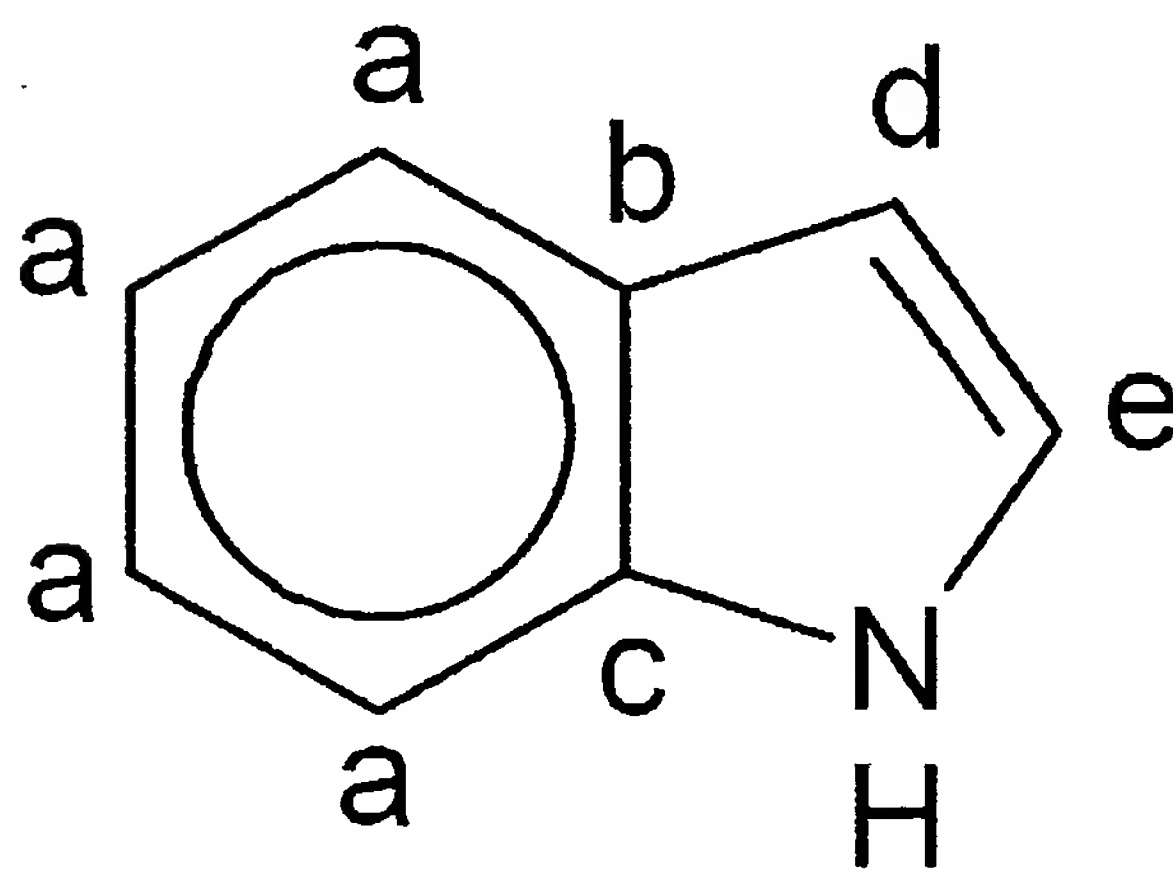
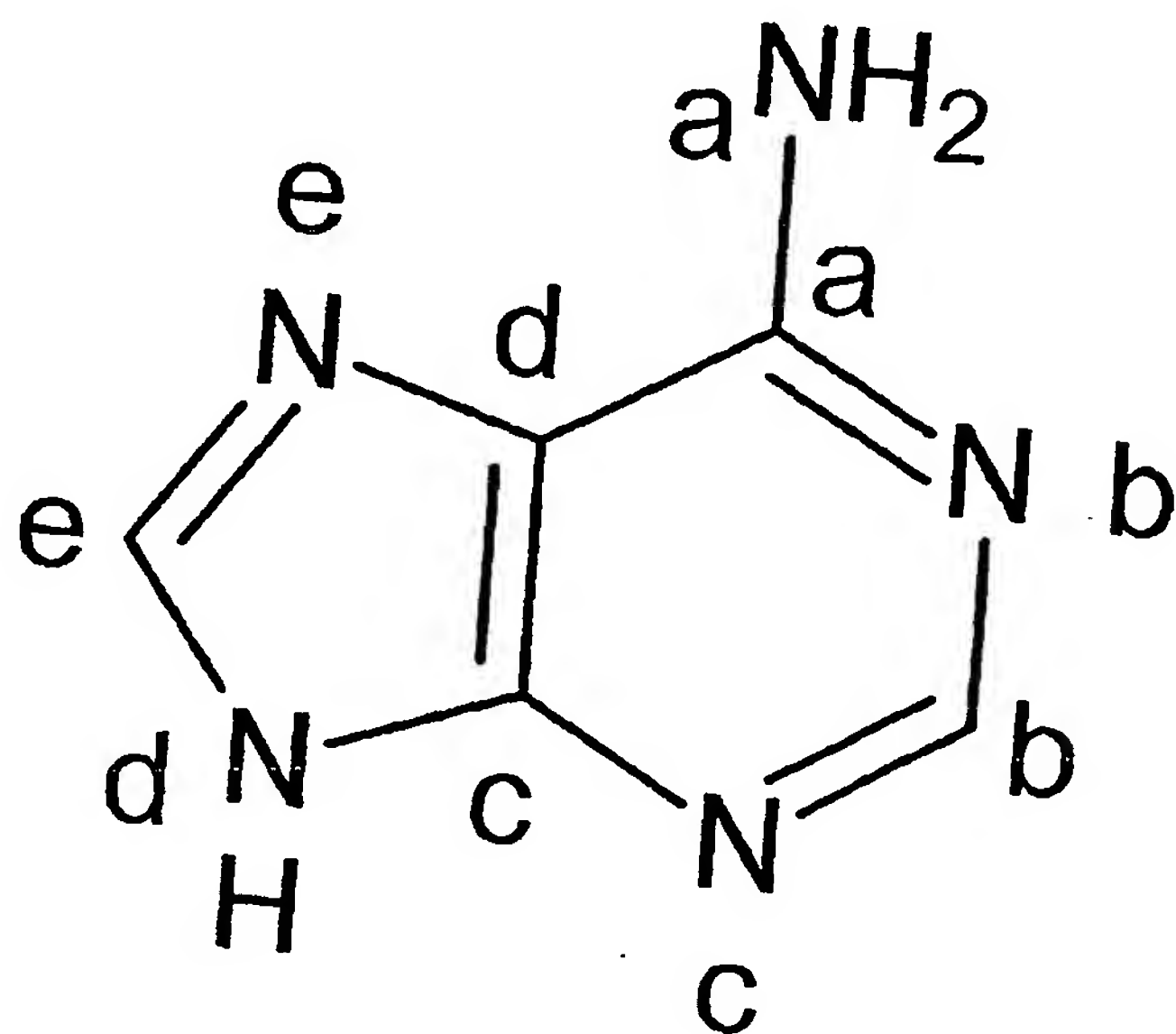


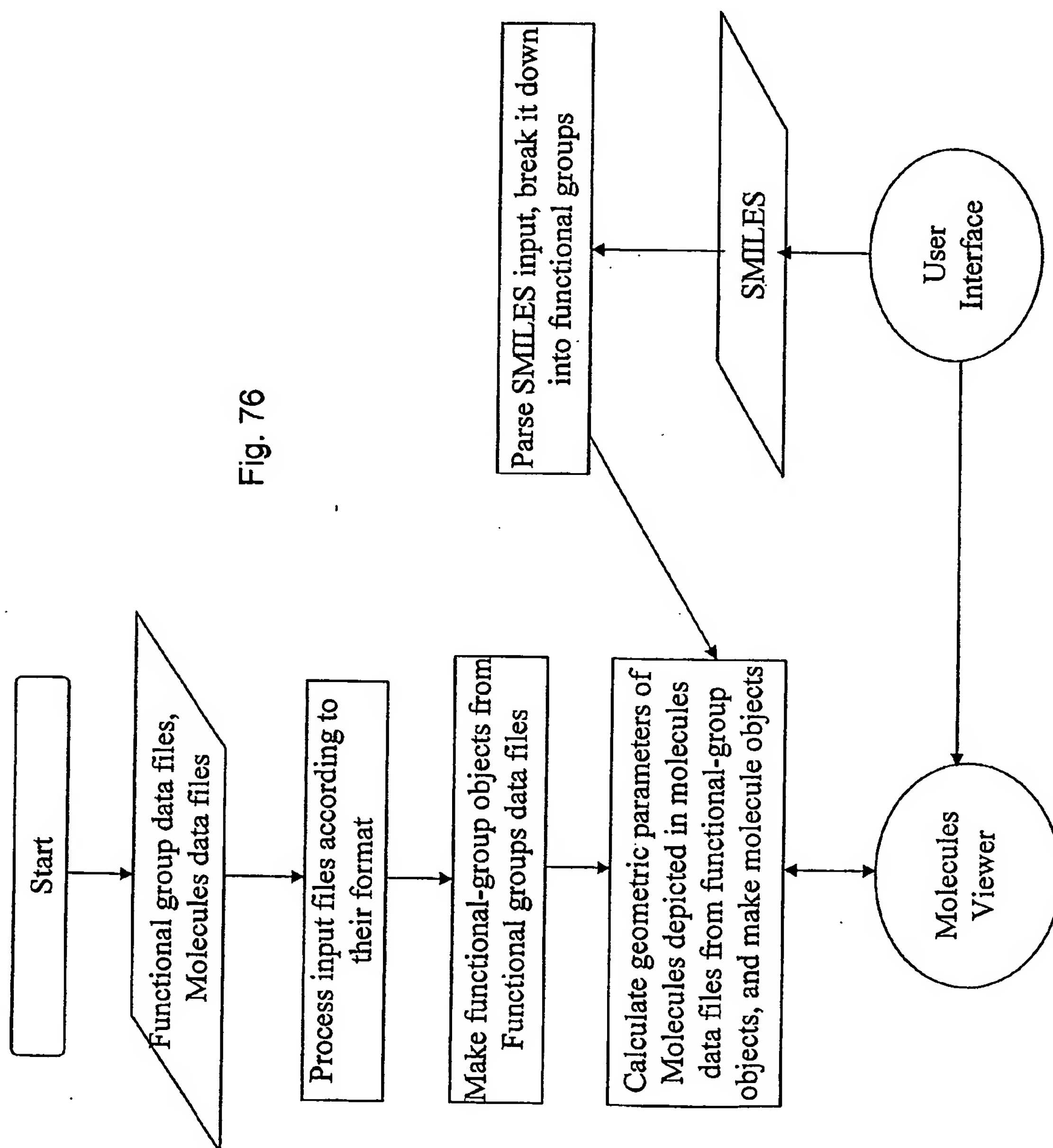
Fig. 74

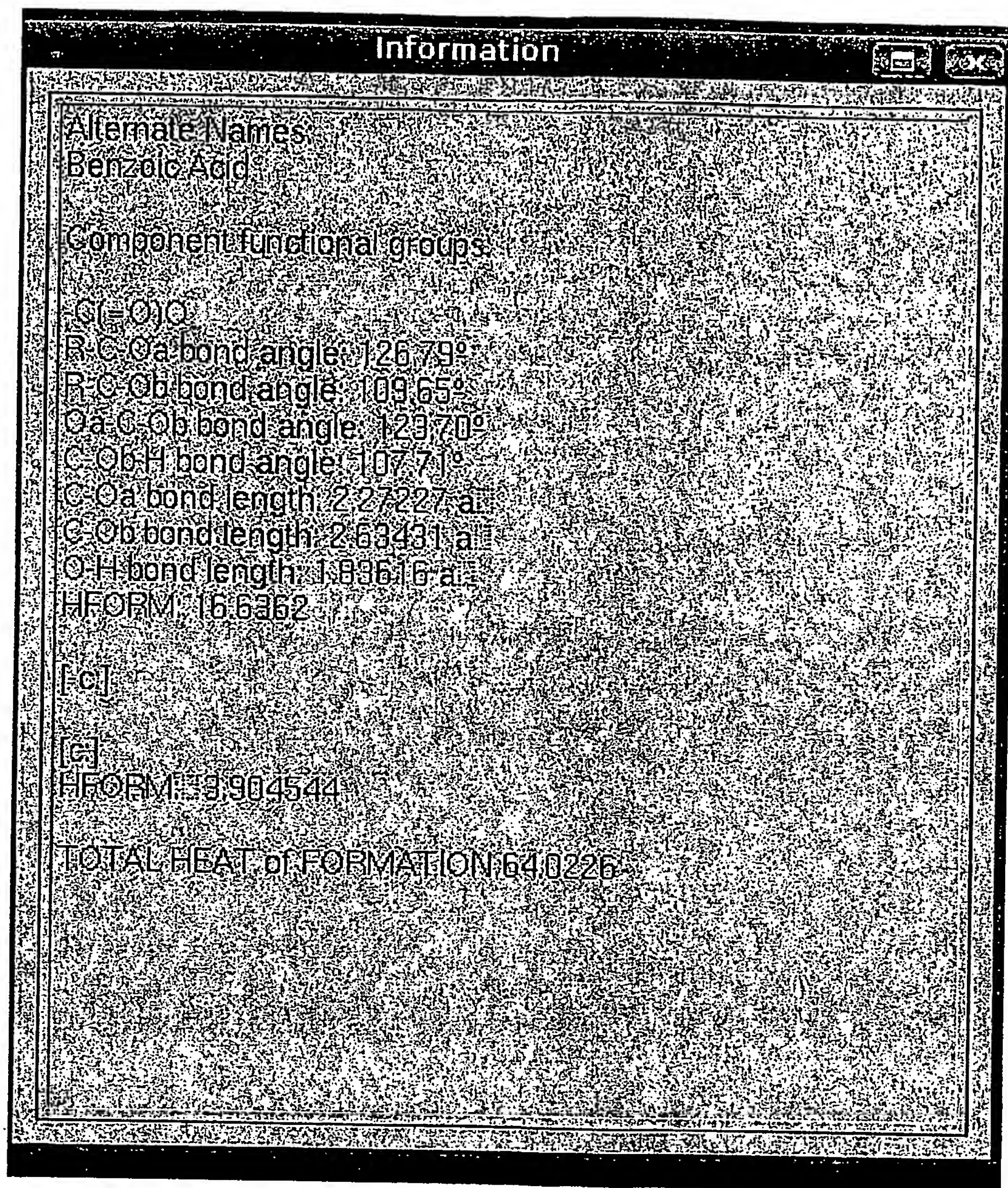


48/51

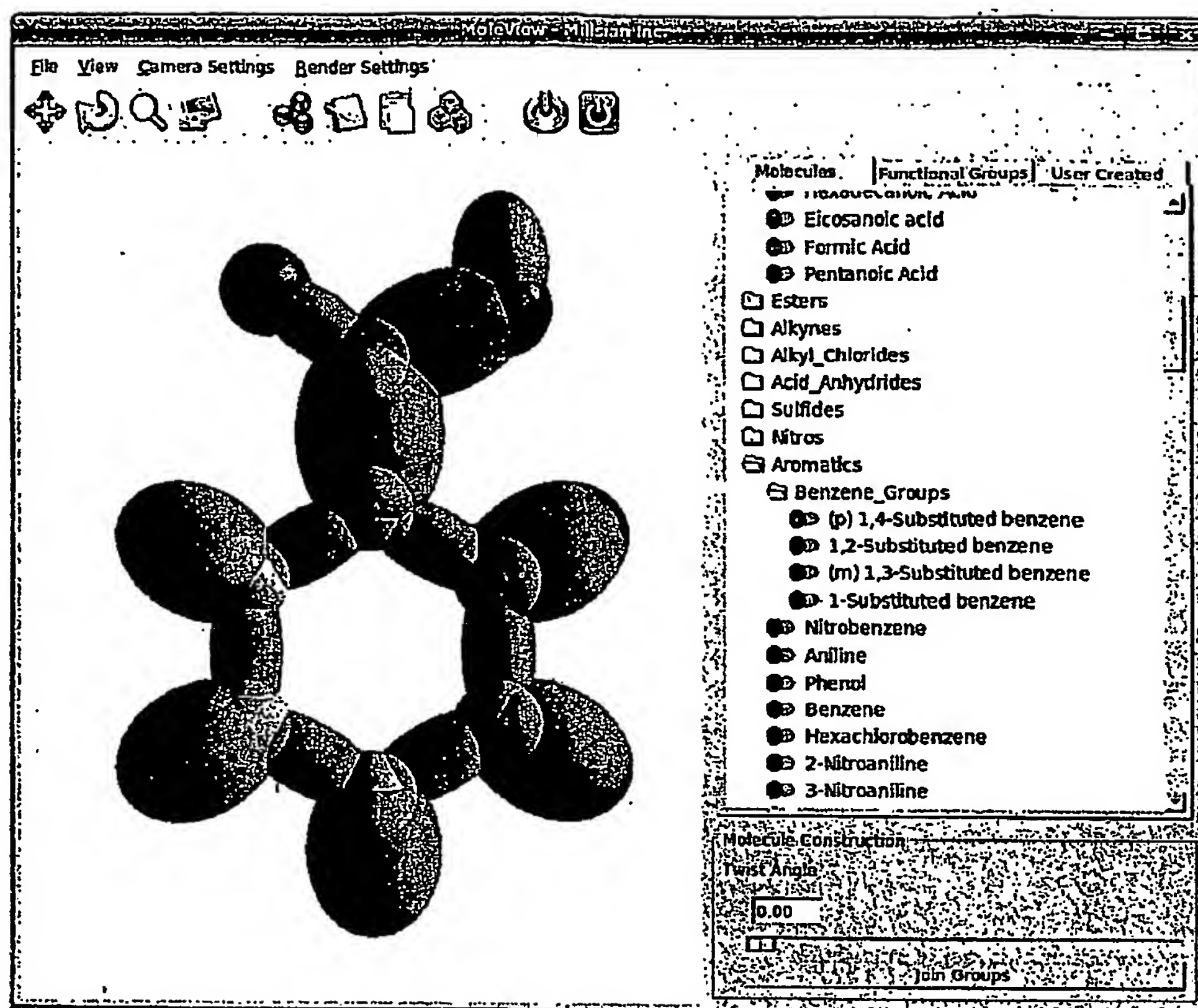
Fig. 75





50/51
Fig. 77

• **Fig.78**



Section IV

ORGANIC MOLECULAR FUNCTIONAL GROUPS AND MOLECULES

5 Organic molecules comprising an arbitrary number of atoms can be solved using the similar principles and procedures as those used to solve alkanes of arbitrary length. Alkanes can be considered to be comprised of the functional groups of CH_3 , CH_2 , and $C - C$. These groups with the corresponding geometrical parameters and energies can be added as a linear sum to give the solution of any straight chain alkane as shown in the Continuous-Chain Alkanes
10 section. Similarly, the geometrical parameters and energies of all functional groups such as alkanes, branched alkanes, alkenes, branched alkenes, alkynes, alkyl fluorides, alkyl chlorides, alkyl bromides, alkyl iodides, alkene halides, primary alcohols, secondary alcohols, tertiary alcohols, ethers, primary amines, secondary amines, tertiary amines, aldehydes, ketones, carboxylic acids, carboxylic esters, amides, N-alkyl amides, N,N-dialkyl amides,
15 urea, acid halides, acid anhydrides, nitriles, thiols, sulfides, disulfides, sulfoxides, sulfones, sulfites, sulfates, nitro alkanes, nitrites, nitrates, conjugated polyenes, aromatics, heterocyclic aromatics, substituted aromatics, and others can be solved. The functional-group solutions can be made into a linear superposition and sum, respectively, to give the solution of any organic molecule. The solutions of the functional groups can be conveniently obtained by
20 using generalized forms of the geometrical and energy equations. The equations and sections that are referenced by not contained in this text refer to those sections and equations of the book by R. L. Mills entitled, "The Grand Unified Theory of Classical Quantum Mechanics", June 2006 Edition, Cadmus Professional Communications-Science Press Division, Ephrata, PA, ISBN 0963517171, Library of Congress Control Number 2005936834; posted at
25 <http://www.blacklightpower.com/bookdownload.shtml> which is incorporated in its entirety by reference.

Consider the case wherein at least two atomic orbital hybridize as a linear combination of electrons at the same energy in order to achieve a bond at an energy minimum, and the sharing of electrons between two or more such orbitals to form a MO
30 permits the participating hybridized orbitals to decrease in energy through a decrease in the radius of one or more of the participating orbitals. The force generalized constant k' of a

H_2 -type ellipsoidal MO due to the equivalent of two point charges of at the foci is given by:

$$k' = \frac{C_1 C_2 2e^2}{4\pi\epsilon_0} \quad (15.1)$$

where C_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the molecule or molecular ion which is 0.75 (Eq. (13.59)) in the case of H bonding to a 5 central atom and 0.5 (Eq. (14.152)) otherwise, and C_2 is the factor that results in an equipotential energy match of the participating at least two molecular or atomic orbitals of the chemical bond. From Eqs. (13.58-13.63), the distance from the origin of the MO to each focus c' is given by:

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2C_1 C_2 a}} = \sqrt{\frac{aa_0}{2C_1 C_2}} \quad (15.2)$$

10 The internuclear distance is

$$2c' = 2\sqrt{\frac{aa_0}{2C_1 C_2}} \quad (15.3)$$

The length of the semiminor axis of the prolate spheroidal MO $b = c$ is given by

$$b = \sqrt{a^2 - c'^2} \quad (15.4)$$

And, the eccentricity, e , is

$$15 \quad e = \frac{c'}{a} \quad (15.5)$$

From Eqs. (11.207-11.212), the potential energy of the two electrons in the central field of the nuclei at the foci is

$$V_e = n_1 c_1 c_2 \frac{-2e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (15.6)$$

The potential energy of the two nuclei is

$$20 \quad V_p = n_1 \frac{e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \quad (15.7)$$

The kinetic energy of the electrons is

$$T = n_1 c_1 c_2 \frac{\hbar^2}{2m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (15.8)$$

And, the energy, V_m , of the magnetic force between the electrons is

$$V_m = n_1 c_1 c_2 \frac{-\hbar^2}{4m_e a \sqrt{a^2 - b^2}} \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} \quad (15.9)$$

The total energy of the H_2 -type prolate spheroidal MO, $E_T(H_2MO)$, is given by the sum of the energy terms:

$$E_T(H_2MO) = V_e + T + V_m + V_p \quad (15.10)$$

$$\begin{aligned} E_T(H_2MO) &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] \end{aligned} \quad (15.11)$$

where n_1 is the number of equivalent bonds of the MO and applies in the case of functional groups. In the case of independent MOs not in contact with the bonding atoms, the terms based on charge are multiplied by c_{BO} , the bond-order factor. It is 1 for a single bond, 4 for an independent double bond as in the case of the CO_2 and NO_2 molecules, and 9 for an independent triplet bond. Then, the kinetic energy term is multiplied by c'_{BO} which is 1 for a single bond, 2 for a double bond, and 9/2 for a triple bond. c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of an MO which is 0.75 (Eqs. (13.67-13.73)) in the case of H bonding to an unhybridized central atom and 1 otherwise, and c_2 is the factor that results in an equipotential energy match of the participating the MO and the at least two atomic orbitals of the chemical bond. Specifically, to meet the equipotential condition and energy matching conditions for the union of the H_2 -type-ellipsoidal-MO and the HOs or AOs of the bonding atoms, the factor c_2 of a H_2 -type ellipsoidal MO may given by (i) one, (ii) the ratio of the Coulombic or valence energy of the AO or HO of at least one atom of the bond and 13.605804 eV, the Coulombic energy between the electron and proton of H , (iii) the ratio of the valence energy of the AO or HO of one atom and the Coulombic energy of another, (iv) the ratio of the valence energies of the AOs or HOs of two atoms, (v) the ratio of two c_2 factors corresponding to any of cases (ii)-(iv), and (vi) the product of two different c_2 factors corresponding to any of the cases (i)-(v). Specific examples of the factor c_2 of a H_2 -type ellipsoidal MO given in previous sections are

0.936127, the ratio of the ionization energy of N 14.53414 eV and 13.605804 eV, the Coulombic energy between the electron and proton of H ;

- 0.91771, the ratio of 14.82575 eV, $-E_{Coulomb}(C, 2sp^3)$, and 13.605804 eV ;
 0.87495, the ratio of 15.55033 eV, $-E_{Coulomb}(C_{ethane}, 2sp^3)$, and 13.605804 eV ;
 0.85252, the ratio of 15.95955 eV, $-E_{Coulomb}(C_{ethylene}, 2sp^3)$, and 13.605804 eV ;
 0.85252, the ratio of 15.95955 eV, $-E_{Coulomb}(C_{benzene}, 2sp^3)$, and 13.605804 eV, and
 5 0.86359, the ratio of 15.55033 eV, $-E_{Coulomb}(C_{alkane}, 2sp^3)$, and 13.605804 eV .

In the generalization of the hybridization of at least two atomic-orbital shells to form a shell of hybrid orbitals, the hybridized shell comprises a linear combination of the electrons of the atomic-orbital shells. The radius of the hybridized shell is calculated from the total Coulombic energy equation by considering that the central field decreases by an integer for
 10 each successive electron of the shell and that the total energy of the shell is equal to the total Coulombic energy of the initial AO electrons. The total energy $E_T(atom, msp^3)$ (m is the integer of the valence shell) of the AO electrons and the hybridized shell is given by the sum of energies of successive ions of the atom over the n electrons comprising total electrons of the at least one AO shell.

$$15 \quad E_T(atom, msp^3) = -\sum_{m=1}^n IP_m \quad (15.12)$$

where IP_m is the m th ionization energy (positive) of the atom. The radius r_{msp^3} of the hybridized shell is given by:

$$r_{msp^3} = \sum_{q=Z-n}^{Z-1} \frac{-(Z-q)e^2}{8\pi\epsilon_0 E_T(atom, msp^3)} \quad (15.13)$$

Then, the Coulombic energy $E_{Coulomb}(atom, msp^3)$ of the outer electron of the $atom msp^3$
 20 shell is given by

$$E_{Coulomb}(atom, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} \quad (15.14)$$

In the case that during hybridization at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ at the initial radius r of the AO electron:

$$25 \quad E(magnetic) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r^3} = \frac{8\pi\mu_0 \mu_B^2}{r^3} \quad (15.15)$$

Then, the energy $E(\text{atom}, msp^3)$ of the outer electron of the $\text{atom } msp^3$ shell is given by the sum of $E_{\text{Coulomb}}(\text{atom}, msp^3)$ and $E(\text{magnetic})$:

$$E(\text{atom}, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r^3} \quad (15.16)$$

Consider next that the at least two atomic orbitals hybridize as a linear combination of 5 electrons at the same energy in order to achieve a bond at an energy minimum with another atomic orbital or hybridized orbital. As a further generalization of the basis of the stability of the MO, the sharing of electrons between two or more such hybridized orbitals to form a MO permits the participating hybridized orbitals to decrease in energy through a decrease in the radius of one or more of the participating orbitals. In this case, the total energy of the 10 hybridized orbitals is given by the sum of $E(\text{atom}, msp^3)$ and the next energies of successive ions of the atom over the n electrons comprising the total electrons of the at least two initial AO shells. Here, $E(\text{atom}, msp^3)$ is the sum of the first ionization energy of the atom and the hybridization energy. An example of $E(\text{atom}, msp^3)$ for $E(\text{C}, 2sp^3)$ is given in Eq. (14.503) where the sum of the negative of the first ionization energy of C, -11.27671 eV , plus the 15 hybridization energy to form the $\text{C}2sp^3$ shell given by Eq. (14.146) is $E(\text{C}, 2sp^3) = -14.63489 \text{ eV}$.

Thus, the sharing of electrons between two $\text{atom } msp^3$ HOs to form an atom-atom-bond MO permits each participating hybridized orbital to decrease in radius and energy. In order to further satisfy the potential, kinetic, and orbital energy relationships, each $\text{atom } msp^3$ 20 HO donates an excess of 25% per bond of its electron density to the atom-atom-bond MO to form an energy minimum wherein the atom-atom bond comprises one of a single, double, or triple bond. In each case, the radius of the hybridized shell is calculated from the Coulombic energy equation by considering that the central field decreases by an integer for each successive electron of the shell and the total energy of the shell is equal to the total 25 Coulombic energy of the initial AO electrons plus the hybridization energy. The total energy $E_r(\text{mol. atom}, msp^3)$ (m is the integer of the valence shell) of the HO electrons is given by the sum of energies of successive ions of the atom over the n electrons comprising total electrons of the at least one initial AO shell and the hybridization energy:

$$E_T(\text{mol.atom}, msp^3) = E(\text{atom}, msp^3) - \sum_{m=2}^n IP_m \quad (15.17)$$

where IP_m is the m th ionization energy (positive) of the atom and the sum of $-IP_1$ plus the hybridization energy is $E(\text{atom}, msp^3)$. Thus, the radius r_{msp^3} of the hybridized shell is given by:

$$r_{msp^3} = \left(\sum_{q=Z-n}^{Z-1} (Z-q) - s(0.25) \right) \frac{-e^2}{8\pi\epsilon_0 E_T(\text{mol.atom}, msp^3)} \quad (15.18)$$

where $s = 1, 2, 3$ for a single, double, and triple bond, respectively. The Coulombic energy $E_{Coulomb}(\text{mol.atom}, msp^3)$ of the outer electron of the $\text{atom } msp^3$ shell is given by

$$E_{Coulomb}(\text{mol.atom}, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} \quad (15.19)$$

In the case that during hybridization at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change for the promotion to the unpaired state is the magnetic energy $E(\text{magnetic})$ at the initial radius r of the AO electron given by Eq. (15.15). Then, the energy $E(\text{mol.atom}, msp^3)$ of the outer electron of the $\text{atom } msp^3$ shell is given by the sum of $E_{Coulomb}(\text{mol.atom}, msp^3)$ and $E(\text{magnetic})$:

$$E(\text{mol.atom}, msp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{msp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r^3} \quad (15.20)$$

$E_T(\text{atom} - \text{atom}, msp^3)$, the energy change of each $\text{atom } msp^3$ shell with the formation of the atom-atom-bond MO is given by the difference between $E(\text{mol.atom}, msp^3)$ and $E(\text{atom}, msp^3)$:

$$E_T(\text{atom} - \text{atom}, msp^3) = E(\text{mol.atom}, msp^3) - E(\text{atom}, msp^3) \quad (15.21)$$

As examples from prior sections, $E_{Coulomb}(\text{mol.atom}, msp^3)$ is one of:

$$E_{Coulomb}(\text{C}_{ethylene}, 2sp^3), \quad E_{Coulomb}(\text{C}_{ethane}, 2sp^3), \quad E_{Coulomb}(\text{C}_{acetylene}, 2sp^3), \quad \text{and} \\ E_{Coulomb}(\text{C}_{alkane}, 2sp^3);$$

$$E_{Coulomb}(\text{atom}, msp^3) \text{ is one of } E_{Coulomb}(\text{C}, 2sp^3) \text{ and } E_{Coulomb}(\text{Cl}, 3sp^3);$$

451

$E(\text{mol. atom}, msp^3)$ is one of $E(C_{\text{ethylene}}, 2sp^3)$, $E(C_{\text{ethane}}, 2sp^3)$,

$E(C_{\text{acetylene}}, 2sp^3)$, $E(C_{\text{alkane}}, 2sp^3)$;

$E(\text{atom}, msp^3)$ is one of and $E(C, 2sp^3)$ and $E(Cl, 3sp^3)$;

$E_T(\text{atom} - \text{atom}, msp^3)$ is one of $E(C - C, 2sp^3)$, $E(C = C, 2sp^3)$, and $E(C \equiv C, 2sp^3)$;

5

$\text{atom } msp^3$ is one of $C2sp^3$, $Cl3sp^3$

$E_T(\text{atom} - \text{atom}(s_1), msp^3)$ is $E_T(C - C, 2sp^3)$ and $E_T(\text{atom} - \text{atom}(s_2), msp^3)$ is

$E_T(C = C, 2sp^3)$, and

r_{msp^3} is one of r_{C2sp^3} , $r_{\text{ethane}2sp^3}$, $r_{\text{ethylene}2sp^3}$, $r_{\text{acetylene}2sp^3}$, $r_{\text{alkane}2sp^3}$, and r_{Cl3sp^3} .

In the case of the $C2sp^3$ HO, the initial parameters (Eqs. (14.142-14.146)) are

$$10 \quad r_{2sp^3} = \sum_{n=2}^5 \frac{(Z-n)e^2}{8\pi\epsilon_0 (e148.25751 \text{ eV})} = \frac{10e^2}{8\pi\epsilon_0 (e148.25751 \text{ eV})} = 0.91771a_0 \quad (15.22)$$

$$E_{\text{Coulomb}}(C, 2sp^3) = \frac{-e^2}{8\pi\epsilon_0 r_{2sp^3}} = \frac{-e^2}{8\pi\epsilon_0 0.91771a_0} = -14.82575 \text{ eV} \quad (15.23)$$

$$E(\text{magnetic}) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} = \frac{8\pi\mu_0 \mu_B^2}{(0.84317a_0)^3} = 0.19086 \text{ eV} \quad (15.24)$$

$$\begin{aligned} E(C, 2sp^3) &= \frac{-e^2}{8\pi\epsilon_0 r_{2sp^3}} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (r_3)^3} \\ &= -14.82575 \text{ eV} + 0.19086 \text{ eV} \\ &= -14.63489 \text{ eV} \end{aligned} \quad (15.25)$$

In Eq. (15.18),

$$15 \quad \sum_{q=Z-n}^{Z-1} (Z-q) = 10 \quad (15.26)$$

Eqs. (14.147) and (15.17) give

$$E_T(\text{mol. atom}, msp^3) = E_T(C_{\text{ethane}}, 2sp^3) = -151.61569 \text{ eV} \quad (15.27)$$

Using Eqs. (15.18-15.28), the final values of r_{C2sp^3} , $E_{\text{Coulomb}}(C2sp^3)$, and $E(C2sp^3)$, and the

resulting $E_T\left(\overset{BO}{C} - C, C2sp^3\right)$ of the MO due to charge donation from the HO to the MO where

$C-C$ refers to the bond order of the carbon-carbon bond for different values of the parameter s are given in Table 15.1.

Table 15.1. The final values of r_{C2sp^3} , $E_{Coulomb}(C2sp^3)$, and $E(C2sp^3)$ and the resulting $E_T(C-C, C2sp^3)$ of the MO due to charge donation from the HO to the MO where $C-C$ refers to the bond order of the carbon-carbon bond.

MO Bond Order (BO)	s 1	s 2	$r_{C2sp^3}(a_0)$ Final	$E_{Coulomb}(C2sp^3)$ (eV) Final	$E(C2sp^3)$ (eV) Final	$E_T(C-C, C2sp^3)$ (eV)
I	1	0	0.87495	-15.55033	-15.35946	-0.72457
II	2	0	0.85252	-15.95955	-15.76868	-1.13379
III	3	0	0.83008	-16.39089	-16.20002	-1.56513
IV	4	0	0.80765	-16.84619	-16.65532	-2.02043

In another generalized case of the basis of forming a minimum-energy bond with the constraint that it must meet the energy matching condition for all MOs at all HOs or AOs, the energy $E(mol.atom, msp^3)$ of the outer electron of the $atom msp^3$ shell of each bonding atom must be the average of $E(mol.atom, msp^3)$ for two different values of s :

$$E(mol.atom, msp^3) = \frac{E(mol.atom(s_1), msp^3) + E(mol.atom(s_2), msp^3)}{2} \quad (15.28)$$

In this case, $E_T(atom - atom, msp^3)$, the energy change of each $atom msp^3$ shell with the formation of each atom-atom-bond MO, is average for two different values of s :

$$E_T(atom - atom, msp^3) = \frac{E_T(atom - atom(s_1), msp^3) + E_T(atom - atom(s_2), msp^3)}{2} \quad (15.29)$$

Consider an aromatic molecule such as benzene given in the Benzene Molecule section. Each $C=C$ double bond comprises a linear combination of a factor of 0.75 of four paired electrons (three electrons) from two sets of two $C2sp^3$ HOs of the participating carbon atoms. Each $C-H$ bond of CH having two spin-paired electrons, one from an initially unpaired electron of the carbon atom and the other from the hydrogen atom, comprises the

linear combination of 75% H_2 -type ellipsoidal MO and 25% $C2sp^3$ HO as given by Eq. (13.439). However, $E_T(\text{atom} - \text{atom}, msp^3)$ of the $C - H$ -bond MO is given by $0.5E_T(C = C, 2sp^3)$ (Eq. (14.247)) corresponding to one half of a double bond that matches the condition for a single-bond order for $C - H$ that is lowered in energy due to the aromatic character of the bond.

A further general possibility is that a minimum-energy bond is achieved with satisfaction of the potential, kinetic, and orbital energy relationships by the formation of an MO comprising an allowed multiple of a linear combination of H_2 -type ellipsoidal MOs and corresponding HOs or AOs that contribute a corresponding allowed multiple (e.g. 0.5, 0.75, 1) of the bond order given in Table 15.1. For example, the alkane MO given in the Continuous-Chain Alkanes section comprises a linear combination of factors of 0.5 of a single bond and 0.5 of a double bond.

Consider a first MO and its HOs comprising a linear combination of bond orders and a second MO that shares a HO with the first. In addition to the mutual HO, the second MO comprises another AO or HO having a single bond order or a mixed bond order. Then, in order for the two MOs to be energy matched, the bond order of the second MO and its HOs or its HO and AO is a linear combination of the terms corresponding to the bond order of the mutual HO and the bond order of the independent HO or AO. Then, in general, $E_T(\text{atom} - \text{atom}, msp^3)$, the energy change of each $\text{atom } msp^3$ shell with the formation of each atom-atom-bond MO, is a weighted linear sum for different values of s that matches the energy of the bonded MOs, HOs, and AOs:

$$E_T(\text{atom} - \text{atom}, msp^3) = \sum_{n=1}^N c_{s_n} E_T(\text{atom} - \text{atom}(s_n), msp^3) \quad (15.30)$$

where c_{s_n} is the multiple of the BO of s_n . The radius r_{msp^3} of the $\text{atom } msp^3$ shell of each bonding atom is given by the Coulombic energy using the initial energy $E_{Coulomb}(\text{atom}, msp^3)$ and $E_T(\text{atom} - \text{atom}, msp^3)$, the energy change of each $\text{atom } msp^3$ shell with the formation of each atom-atom-bond MO:

$$r_{msp^3} = \frac{-e^2}{8\pi\epsilon_0 a_0 (E_{Coulomb}(\text{atom}, msp^3) + E_T(\text{atom} - \text{atom}, msp^3))} \quad (15.31)$$

where $E_{Coulomb}(C2sp^3) = -14.825751 \text{ eV}$. The Coulombic energy $E_{Coulomb}(mol.atom,msp^3)$ of the outer electron of the *atom* msp^3 shell is given by Eq. (15.19). In the case that during hybridization, at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ (Eq. (15.15)) at the initial radius r of the AO electron. Then, the energy $E(mol.atom,msp^3)$ of the outer electron of the *atom* msp^3 shell is given by the sum of $E_{Coulomb}(mol.atom,msp^3)$ and $E(magnetic)$ (Eq. (15.20)). $E_T(atom - atom,msp^3)$, the energy change of each *atom* msp^3 shell with the formation of the atom-atom-bond MO is given by the difference between $E(mol.atom,msp^3)$ and $E(atom,msp^3)$ given by Eq. (15.21). Using Eq. (15.23) for $E_{Coulomb}(C,2sp^3)$ in Eq. (15.31), the single bond order energies given by Eqs. (15.18-15.27) and shown in Table 15.1, and the linear combination energies (Eqs. (15.28-15.30)), the parameters of linear combinations of bond orders and linear combinations of mixed bond orders are given in Table 15.2.

Table 15.2. The final values of r_{C2sp^3} , $E_{Coulomb}(C2sp^3)$, and $E(C2sp^3)$ and the resulting $E_T\left(C - C, C2sp^3\right)$ of the MO comprising a linear combination of H_2 -type ellipsoidal MOs and corresponding HOs of single or mixed bond order where c_{s_n} is the multiple of the bond order parameter $E_T(atom - atom(s_n),msp^3)$ given in Table 15.1.

Table 15.1

MO Bond Order (BO)	s 1	c_{s_1}	s 2	c_{s_2}	s 3	c_{s_3}	$r_{C2sp^3}(a_0)$ Final	$E_{Coulomb}(C2sp^3)$ (eV) Final	$E(C2sp^3)$ (eV) Final	$E_T(C^{BO}-C, C2sp^3)$ (eV)
1/2I	1	0.5	0	0	0	0	0.89582	-15.18804	-14.99717	-0.36228
1/2II	2	0.5	0	0	0	0	0.88392	-15.39265	-15.20178	-0.56689
I+	1	0.5	2	0.2	0	0	0.87941	-15.47149	-15.28062	-0.64573
1/2II				5						
1/2II	2	0.2	1	0.2	2	0.2	0.87363	-15.57379	-15.38293	-0.74804
+ (I+		5		5		5				
II)										
3/4II	2	0.7	0	0	0	0	0.86793	-15.67610	-15.48523	-0.85034
		5								
I+II	1	0.5	2	0.5	0	0	0.86359	-15.75493	-15.56407	-0.92918
I+	1	0.5	3	0.5	0	0	0.85193	-15.97060	-15.77974	-1.14485
III										
I+	1	0.5	4	0.5	0	0	0.83995	-16.19826	-16.00739	-1.37250
IV										
II+	2	0.5	3	0.5	0	0	0.84115	-16.17521	-15.98435	-1.34946
III										
II+	2	0.5	4	0.5	0	0	0.82948	-16.40286	-16.21200	-1.57711
IV										
III+	3	0.5	4	0.5	0	0	0.81871	-16.61853	-16.42767	-1.79278
IV										
IV+	4	0.5	4	0.5	0	0	0.80765	-16.84619	-16.65532	-2.02043
IV										

Consider next the radius of the AO or HO due to the contribution of charge to more than one bond. The energy contribution due to the charge donation at each atom such as 5 carbon superimposes linearly. In general, the radius r_{mol2sp^3} of the $C2sp^3$ HO of a carbon atom of a given molecule is calculated using Eq. (14.514) by considering $\sum E_{T_{mol}}(MO, 2sp^3)$, the total energy donation to each bond with which it participates in bonding. The general equation for the radius is given by

$$\begin{aligned}
 r_{mol2sp^3} &= \frac{-e^2}{8\pi\epsilon_0 \left(E_{Coulomb}(C, 2sp^3) + \sum E_{T_{mol}}(MO, 2sp^3) \right)} \\
 &= \frac{e^2}{8\pi\epsilon_0 \left(e14.825751 \text{ eV} + \sum |E_{T_{mol}}(MO, 2sp^3)| \right)}
 \end{aligned}
 \tag{15.32}$$

The Coulombic energy $E_{Coulomb}(mol.atom, msp^3)$ of the outer electron of the *atom* msp^3 shell is given by Eq. (15.19). In the case that during hybridization, at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ (Eq. (15.15)) at the initial radius r of the AO electron. Then, the energy $E(mol.atom, msp^3)$ of the outer electron of the *atom* msp^3 shell is given by the sum of $E_{Coulomb}(mol.atom, msp^3)$ and $E(magnetic)$ (Eq. (15.20)).

For example, the $C2sp^3$ HO of each methyl group of an alkane contributes -0.92918 eV (Eq. (14.513)) to the corresponding single $C-C$ bond; thus, the corresponding $C2sp^3$ HO radius is given by Eq. (14.514). The $C2sp^3$ HO of each methylene group of C_nH_{2n+2} contributes -0.92918 eV to each of the two corresponding $C-C$ bond MOs. Thus, the radius (Eq. (15.32)), the Coulombic energy (Eq. (15.19)), and the energy (Eq. (15.20)) of each alkane methylene group are

$$\begin{aligned}
 r_{alkaneC_{methylene}2sp^3} &= \frac{-e^2}{8\pi\epsilon_0 \left(E_{Coulomb}(C, 2sp^3) + \sum E_{T_{alkane}}(methylene\ C-C, 2sp^3) \right)} \\
 &= \frac{e^2}{8\pi\epsilon_0 (e14.825751 \text{ eV} + e0.92918 \text{ eV} + e0.92918 \text{ eV})} \\
 &= 0.81549a_0
 \end{aligned}
 \tag{15.33}$$

$$E_{Coulomb}(C_{methylene}2sp^3) = \frac{-e^2}{8\pi\epsilon_0 (0.81549a_0)} = -16.68412 \text{ eV}
 \tag{15.34}$$

$$E(C_{methylene}2sp^3) = \frac{-e^2}{8\pi\epsilon_0 (0.81549a_0)} + \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 (0.84317a_0)^3} = -16.49325 \text{ eV}
 \tag{15.35}$$

In the determination of the parameters of functional groups, heteroatoms bonding to $C2sp^3$ HOs to form MOs are energy matched to the $C2sp^3$ HOs. Thus, the radius and the energy parameters of a bonding heteroatom are given by the same equations as those for

$C2sp^3$ HOs. Using Eqs. (15.15), (15.19-15.20), (15.24), and (15.32) in a generalized fashion, the final values of the radius of the HO or AO, $r_{Atom.HO.AO}$, $E_{Coulomb}(mol.atom,msp^3)$, and $E(C_{mol}2sp^3)$ are calculated using $\sum E_{T_{group}}(MO,2sp^3)$, the total energy donation to each bond with which an atom participates in bonding corresponding to the values of $E_T(C^{BO}-C,C2sp^3)$ of

5 the MO due to charge donation from the AO or HO to the MO given in Tables 15.1 and 15.2.

Table 15.3.A. The final values of $r_{Atom.HO.AO}$, $E_{Coulomb}(mol.atom,msp^3)$, and $E(C_{mol}2sp^3)$ calculated using the values of $E_T(C^{BO}-C,C2sp^3)$ given in Tables 15.1 and 15.2.

Atom Hybridization Designation	$E_T \left(C^{BO} - C, C2sp^3 \right)$	$E_T \left(C^{BO} - C, C2sp^3 \right)$	$E_T \left(C^{BO} - C, C2sp^3 \right)$	$E_T \left(C^{BO} - C, C2sp^3 \right)$	$E_T \left(C^{BO} - C, C2sp^3 \right)$	$r_{Atom, HO, AO}^{Final}$	$E_{Oxidant}^{Final} (mol\ atom, msp^3) (eV)$	$E(C_{ms}2sp^3) (eV) Final$
1	0	0	0	0	0	0.91771	-14.82575	-14.63489
2	-0.36229	0	0	0	0	0.89582	-15.18804	-14.99717
3	-0.46459	0	0	0	0	0.88983	-15.29034	-15.09948
4	-0.56689	0	0	0	0	0.88392	-15.39265	-15.20178
5	-0.72457	0	0	0	0	0.87495	-15.55033	-15.35946
6	-0.85034	0	0	0	0	0.86793	-15.6761	-15.48523
7	-0.92918	0	0	0	0	0.86359	-15.75493	-15.56407
8	-0.54343	-0.54343	0	0	0	0.85503	-15.91261	-15.72175
9	-1.13379	0	0	0	0	0.85252	-15.95955	-15.76868
10	-1.14485	0	0	0	0	0.85193	-15.9706	-15.77974
11	-0.46459	-0.82688	0	0	0	0.84418	-16.11722	-15.92636
12	-1.34946	0	0	0	0	0.84115	-16.17521	-15.98435
13	-1.3725	0	0	0	0	0.83995	-16.19826	-16.00739
14	-0.46459	-0.92918	0	0	0	0.83885	-16.21952	-16.02866
15	-0.72457	-0.72457	0	0	0	0.836	-16.2749	-16.08404
16	-0.5669	-0.92918	0	0	0	0.8336	-16.32183	-16.13097
17	-0.82688	-0.72457	0	0	0	0.83078	-16.37721	-16.18634
18	-1.56513	0	0	0	0	0.83008	-16.39089	-16.20002
19	-0.64574	-0.92918	0	0	0	0.82959	-16.40067	-16.20981
20	-1.57711	0	0	0	0	0.82948	-16.40286	-16.212
21	-0.72457	-0.92918	0	0	0	0.82562	-16.47951	-16.28865
22	-0.85035	-0.85035	0	0	0	0.82327	-16.52645	-16.33559
23	-1.79278	0	0	0	0	0.81871	-16.61853	-16.42767
24	-1.13379	-0.72457	0	0	0	0.81549	-16.68411	-16.49325
25	-0.92918	-0.92918	0	0	0	0.81549	-16.68412	-16.49325
26	-2.02043	0	0	0	0	0.80765	-16.84619	-16.65532

459

27	-1.13379	-0.92918	0	0	0	0.80561	-16.88872	-16.69786
28	-0.85035	-0.85035	-0.46459	0	0	0.80076	-16.99104	-16.80018
29	-0.5669	-0.72457	-0.92918	0	0	0.78916	-17.04641	-16.85554
30	-1.13379	-1.13379	0	0	0	0.79597	-17.09334	-16.90248
31	-1.34946	-0.92918	0	0	0	0.79546	-17.1044	-16.91353
32	-0.46459	-0.92918	-0.92918	0	0	0.79340	-17.14871	-16.95784
33	-0.64574	-0.85034	-0.85034	0	0	0.79232	-17.17217	-16.98131
34	-0.85035	-0.5669	-0.92918	0	0	0.79232	-17.17218	-16.98132
35	-0.72457	-0.72457	-0.92918	0	0	0.79085	-17.20408	-17.01322
36	-0.82688	-0.72457	-0.92918	0	0	0.78617	-17.30638	-17.11552
37	-0.72457	-0.92918	-0.92918	0	0	0.78155	-17.40868	-17.21782
38	-0.92918	-0.72457	-0.92918	0	0	0.78155	-17.40869	-17.21783
39	-0.54343	-0.54343	-0.5669	-0.92918	0	0.78155	-17.40869	-17.21783
40	-0.92918	-0.85034	-0.85034	0	0	0.77945	-17.45561	-17.26475
41	-0.82688	-0.92918	-0.92918	0	0	0.77699	-17.51099	-17.32013
42	-0.92918	-0.92918	-0.92918	0	0	0.77247	-17.6133	-17.42244
43	-0.85035	-0.54343	-0.5669	-0.92918	0	0.76801	-17.71561	-17.52475
44	-1.34946	-0.64574	-0.92918	0	0	0.76652	-17.75013	-17.55927
45	-0.85034	-0.54343	-0.60631	-0.92918	0	0.76631	-17.75502	-17.56415
46	-1.1338	-0.92918	-0.92918	0	0	0.7636	-17.81791	-17.62705
47	-0.46459	-0.85035	-0.85035	-0.92918	0	0.75924	-17.92022	-17.72936
48	-0.82688	-1.34946	-0.92918	0	0	0.75877	-17.93128	-17.74041
49	-1.13379	-1.13379	-1.13379	0	0	0.74646	-18.22712	-18.03626
50	-1.79278	-0.92918	-0.92918	0	0	0.73637	-18.47690	-18.28604

Table 15.3.B. The final values of $r_{Atom.HO.AO}$, $E_{Coulomb}(mol.atom,msp^3)$, and $E(C_{mol}2sp^3)$ calculated for heterocyclic groups using the values of $E_T\left(C^{BO}-C,C2sp^3\right)$ given in Tables 15.1 and 15.2.

Atom Hybridization Designation	$E_f \left(C^{BO} - C, C2sp^3 \right)$	$E_f \left(C^{BO} - C, C2sp^3 \right)$	$E_f \left(C^{BO} - C, C2sp^3 \right)$	$E_f \left(C^{BO} - C, C2sp^3 \right)$	$E_f \left(C^{BO} - C, C2sp^3 \right)$	$r_{Atom, HO, AO}$ Final	$E_{Coulomb} \left(mol, atom, msp^3 \right)$ (eV) Final	$E \left(C_{mol} 2sp^3 \right)$ (eV) Final
1	0	0	0	0	0	0.91771	-14.82575	-14.63489
2	-0.56690	0	0	0	0	0.88392	-15.39265	-15.20178
3	-0.72457	0	0	0	0	0.87495	-15.55033	-15.35946
4	-0.92918	0	0	0	0	0.86359	-15.75493	-15.56407
5	-0.54343	-0.54343	0	0	0	0.85503	-15.91261	-15.72175
6	-1.13379	0	0	0	0	0.85252	-15.95954	-15.76868
7	-0.60631	-0.60631	0	0	0	0.84833	-16.03838	-15.84752
8	-0.46459	-0.92918	0	0	0	0.83885	-16.21953	-16.02866
9	-0.72457	-0.72457	0	0	0	0.83600	-16.27490	-16.08404
10	-0.92918	-0.60631	0	0	0	0.83159	-16.36125	-16.17038
11	-0.92918	-0.72457	0	0	0	0.82562	-16.47951	-16.28864
12	-0.85035	-0.85035	0	0	0	0.82327	-16.52644	-16.33558
13	-0.92918	-0.92918	0	0	0	0.81549	-16.68411	-16.49325
14	-1.13379	-0.72457	0	0	0	0.81349	-16.68412	-16.49325
15	-1.13379	-0.92918	0	0	0	0.80561	-16.88873	-16.69786
16	-0.85035	-0.85035	-0.46459	0	0	0.80076	-16.99103	-16.80017
17	-0.85034	-0.85034	-0.56690	0	0	0.79597	-17.09334	-16.90247
18	-1.13379	-1.13380	0	0	0	0.79597	-17.09334	-16.90248
19	-0.85035	-0.54343	0.00000	-0.92918	0	0.79340	-17.14871	-16.95785
20	-0.85035	-0.56690	-0.92918	0	0	0.79232	-17.17218	-16.98132
21	-0.54343	-0.54343	-0.56690	-0.92918	0	0.78155	-17.40869	-17.21783
22	-0.85034	-0.28345	-0.54343	-0.92918	0	0.78050	-17.43216	-17.24130
23	-0.92918	-0.92918	-0.92918	0	0	0.77247	-17.61330	-17.42243
24	-0.85034	-0.54343	-0.56690	-0.92918	0	0.76801	-17.71560	-17.52474
25	-0.85034	-0.54343	-0.60631	-0.92918	0	0.76631	-17.75502	-17.56416
26	-1.13379	-0.92918	-0.92918	0	0	0.76360	-17.81791	-17.62704

27	-1.13379	-1.13380	-0.72457	0	0	0.76360	-17.81791	-17.62705
28	-0.46459	-0.85035	-0.85035	-0.92918	0	0.75924	-17.92022	-17.72935
29	-1.13380	-1.13379	-0.92918	0	0	0.75493	-18.02252	-17.83166
30	-1.13379	-1.13379	-1.13379	0	0	0.74646	-18.22713	-18.03627

The energy of the MO is matched to each of the participating outermost atomic or hybridized orbitals of the bonding atoms wherein the energy match includes the energy contribution due to the AO or HO's donation of charge to the MO. The force constant k' (Eq. (15.1)) is used to determine the ellipsoidal parameter c' (Eq. (15.2)) of the each H_2 -type-ellipsoidal-MO in terms of the central force of the foci. Then, c' is substituted into the energy equation (from Eq. (15.11)) which is set equal to n_1 times the total energy of H_2 where n_1 is the number of equivalent bonds of the MO and the energy of H_2 , -31.63536831 eV , Eq. (11.212) is the minimum energy possible for a prolate spheroidal MO. From the energy equation and the relationship between the axes, the dimensions of the MO are solved. The energy equation has the semimajor axis a as its only parameter. The solution of the semimajor axis a then allows for the solution of the other axes of each prolate spheroid and eccentricity of each MO (Eqs. (15.3-15.5)). The parameter solutions then allow for the component and total energies of the MO to be determined.

The total energy, $E_T(H_2MO)$, is given by the sum of the energy terms (Eqs. (15.6-15.11)) plus $E_T(AO/HO)$:

$$E_T(H_2MO) = V_e + T + V_m + V_p + E_T(AO/HO) \quad (15.36)$$

$$\begin{aligned} E_T(H_2MO) &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] + E_T(AO/HO) \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + E_T(AO/HO) \end{aligned} \quad (15.37)$$

where n_1 is the number of equivalent bonds of the MO, c_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the group, c_2 is the factor that results in an equipotential energy match of the participating at least two atomic orbitals of each chemical bond, and $E_T(AO/HO)$ is the total energy comprising the difference of the energy $E(AO/HO)$ of at least one atomic or hybrid orbital to which the MO is energy matched and any energy component $\Delta E_{H_2MO}(AO/HO)$ due to the AO or HO's charge donation to the MO.

$$E_T(AO/HO) = E(AO/HO) - \Delta E_{H_2MO}(AO/HO) \quad (15.38)$$

As specific examples given in previous sections, $E_T(AO/HO)$ is one from the group of

$$E_T(AO/HO) = E(O2p \text{ shell}) = -E(\text{ionization}; O) = -13.6181 \text{ eV};$$

$$E_T(AO/HO) = E(N2p \text{ shell}) = -E(\text{ionization}; N) = -14.53414 \text{ eV};$$

$$E_T(AO/HO) = E(C, 2sp^3) = -14.63489 \text{ eV};$$

$$5 \quad E_T(AO/HO) = E_{\text{Coulomb}}(Cl, 3sp^3) = -14.60295 \text{ eV};$$

$$E_T(AO/HO) = E(\text{ionization}; C) + E(\text{ionization}; C^+);$$

$$E_T(AO/HO) = E(C_{\text{ethane}}, 2sp^3) = -15.35946 \text{ eV};$$

$$E_T(AO/HO) = +E(C_{\text{ethylene}}, 2sp^3) - E(C_{\text{ethylene}}, 2sp^3);$$

$$E_T(AO/HO) = E(C, 2sp^3) - 2E_T(C=C, 2sp^3) = -14.63489 \text{ eV} - (-2.26758 \text{ eV});$$

$$10 \quad E_T(AO/HO) = E(C_{\text{acetylene}}, 2sp^3) - E(C_{\text{acetylene}}, 2sp^3) - E(C_{\text{acetylene}}, 2sp^3) = 16.20002 \text{ eV};$$

$$E_T(AO/HO) = E(C, 2sp^3) - 2E_T(C \equiv C, 2sp^3) = -14.63489 \text{ eV} - (-3.13026 \text{ eV});$$

$$E_T(AO/HO) = E(C_{\text{benzene}}, 2sp^3) - E(C_{\text{benzene}}, 2sp^3);$$

$$E_T(AO/HO) = E(C, 2sp^3) - E_T(C=C, 2sp^3) = -14.63489 \text{ eV} - (-1.13379 \text{ eV}), \text{ and}$$

$$E_T(AO/HO) = E(C_{\text{alkane}}, 2sp^3) = -15.56407 \text{ eV}.$$

$$15 \quad \text{To solve the bond parameters and energies, } c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2C_1 C_2 a}} = \sqrt{\frac{aa_0}{2C_1 C_2}} \quad (\text{Eq. (15.2)})$$

is substituted into $E_T(H_1MO)$ to give

$$\begin{aligned} E_T(H_1MO) &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{a^2 - b^2}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{a^2 - b^2}}{a - \sqrt{a^2 - b^2}} - 1 \right] + E_T(AO/HO) \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 c'} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + c'}{a - c'} - 1 \right] + E_T(AO/HO) \\ &= -\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T(AO/HO) \end{aligned} \quad (15.39)$$

The total energy is set equal to $E(\text{basis energies})$ which in the most general case is given by

the sum of a first integer n_1 times the total energy of H_2 minus a second integer n_2 times the total energy of H , minus a third integer n_3 times the valence energy of $E(AO)$ (e.g. $E(N) = -14.53414 \text{ eV}$) where the first integer can be 1, 2, 3..., and each of the second and third integers can be 0, 1, 2, 3....

$$5 \quad E(\text{basis energies}) = n_1(-31.63536831 \text{ eV}) - n_2(-13.605804 \text{ eV}) - n_3 E(AO) \quad (15.40)$$

In the case that the MO bonds two atoms other than hydrogen, $E(\text{basis energies})$ is n_1 times the total energy of H_2 where n_1 is the number of equivalent bonds of the MO and the energy of H_2 , -31.63536831 eV , Eq. (11.212) is the minimum energy possible for a prolate spheroidal MO:

$$10 \quad E(\text{basis energies}) = n_1(-31.63536831 \text{ eV}) \quad (15.41)$$

$E_T(H_2MO)$, is set equal to $E(\text{basis energies})$, and the semimajor axis a is solved.

Thus, the semimajor axis a is solved from the equation of the form:

$$-\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T(AO / HO) = E(\text{basis energies}) \quad (15.42)$$

The distance from the origin of the H_2 -type-ellipsoidal-MO to each focus c' , the internuclear distance $2c'$, and the length of the semiminor axis of the prolate spheroidal H_2 -type MO $b = c$ are solved from the semimajor axis a using Eqs. (15.2-15.4). Then, the component energies are given by Eqs. (15.6-15.9) and (15.39).

The total energy of the MO of the functional group, $E_T(MO)$, is the sum of the total energy of the components comprising the energy contribution of the MO formed between the participating atoms and $E_T(\text{atom} - \text{atom}, msp^3.AO)$, the change in the energy of the AOs or HOs upon forming the bond. From Eqs. (15.39-15.40), $E_T(MO)$ is

$$E_T(MO) = E(\text{basis energies}) + E_T(\text{atom} - \text{atom}, msp^3.AO) \quad (15.43)$$

During bond formation, the electrons undergo a reentrant oscillatory orbit with vibration of the nuclei, and the corresponding energy \bar{E}_{osc} is the sum of the Doppler, \bar{E}_D , and average vibrational kinetic energies, \bar{E}_{Kvib} :

$$\bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_{hv} \sqrt{\frac{2\bar{E}_K}{m_e c^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (15.44)$$

where n_1 is the number of equivalent bonds of the MO, k is the spring constant of the equivalent harmonic oscillator, and μ is the reduced mass. The angular frequency of the reentrant oscillation in the transition state corresponding to \bar{E}_D is determined by the force
5 between the central field and the electrons in the transition state. The force and its derivative are given by

$$f(R) = -c_{BO} \frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3} \quad (15.45)$$

and

$$f'(a) = 2c_{BO} \frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3} \quad (15.46)$$

10 such that the angular frequency of the oscillation in the transition state is given by

$$\omega = \sqrt{\frac{\left[\frac{-3}{a} f(a) - f'(a) \right]}{m_e}} = \sqrt{\frac{k}{m_e}} = \sqrt{\frac{c_{BO} \frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3}}{m_e}} \quad (15.47)$$

where R is the semimajor axis a or the semiminor axis b depending on the eccentricity of the bond that is most representative of the oscillation in the transition state, c_{BO} is the bond-order factor which is 1 for a single bond and when the MO comprises n_1 equivalent single
15 bonds as in the case of functional groups. c_{BO} is 4 for an independent double bond as in the case of the CO_2 and NO_2 molecules and 9 for an independent triplet bond. C_{1o} is the fraction of the H_2 -type ellipsoidal MO basis function of the oscillatory transition state of a chemical bond of the group, and C_{2o} is the factor that results in an equipotential energy match of the participating at least two atomic orbitals of the transition state of the chemical bond.
20 Typically, $C_{1o} = C_1$ and $C_{2o} = C_2$. The kinetic energy, E_K , corresponding to \bar{E}_D is given by Planck's equation for functional groups:

$$\bar{E}_K = \hbar\omega = \hbar \sqrt{\frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_0 R^3 m_e}} \quad (15.48)$$

The Doppler energy of the electrons of the reentrant orbit is

467

$$\bar{E}_D \cong E_{hv} \sqrt{\frac{2\bar{E}_K}{m_e c^2}} = E_{hv} \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 R^3}}}{m_e c^2}} \quad (15.49)$$

\bar{E}_{osc} given by the sum of \bar{E}_D and \bar{E}_{Kvib} is

$$\bar{E}_{osc} (group) = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_{hv} \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 R^3}}}{m_e c^2}} + E_{vib} \right) \quad (15.50)$$

E_{hv} of a group having n_1 bonds is given by $E_T (MO) / n_1$ such that

$$5 \quad \bar{E}_{osc} = n_1 (\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(E_T (MO) / n_1 \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (15.51)$$

$E_{T+osc} (Group)$ is given by the sum of $E_T (MO)$ (Eq. (15.42)) and \bar{E}_{osc} (Eq. (15.51)):

$$E_{T+osc} (Group) = E_T (MO) + \bar{E}_{osc}$$

$$= \left(\left[-\frac{n_1 e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \left[c_1 c_2 \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T (AO / HO) + E_T (atom - atom, msp^3 . AO) \right] \right. \\ \left. \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 R^3}}}{m_e c^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right] \right) \\ = \left(E(basis energies) + E_T (atom - atom, msp^3 . AO) \right) \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 R^3}}}{m_e c^2}} + n_1 \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right] \quad (15.52)$$

The total energy of the functional group $E_T(\text{group})$ is the sum of the total energy of the components comprising the energy contribution of the MO formed between the participating atoms, $E(\text{basis energies})$, the change in the energy of the AOs or HOs upon forming the bond ($E_T(\text{atom} - \text{atom}, msp^3.AO)$), the energy of oscillation in the transition state, and the change in magnetic energy with bond formation, E_{mag} . From Eq. (15.52), the total energy of the group $E_T(\text{Group})$ is

$$E_T(\text{Group}) = \left(E(\text{basis energies}) + E_T(\text{atom} - \text{atom}, msp^3.AO) \right) \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_o R^3}}}{m_e c^2}} \right] + n_1 \bar{E}_{Kvib} + E_{mag} \quad (15.53)$$

The change in magnetic energy E_{mag} which arises due to the formation of unpaired electrons in the corresponding fragments relative to the bonded group is given by

$$E_{mag} = c_3 \frac{2\pi\mu_o e^2 \hbar^2}{m_e^2 r^3} = c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} \quad (15.54)$$

where r^3 is the radius of the atom that reacts to form the bond and c_3 is the number of electron pairs.

$$E_T(\text{Group}) = \left(E(\text{basis energies}) + E_T(\text{atom} - \text{atom}, msp^3.AO) \right) \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_o R^3}}}{m_e c^2}} \right] + n_1 \bar{E}_{Kvib} + c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} \quad (15.55)$$

The total bond energy of the group $E_D(\text{Group})$ is the negative difference of the total energy of the group (Eq. (15.55)) and the total energy of the starting species given by the sum of $c_4 E_{initial}(c_4 AO / HO)$ and $c_5 E_{initial}(c_5 AO / HO)$:

$$E_D(\text{Group}) = - \left(E(\text{basis energies}) + E_T(\text{atom} - \text{atom}, \text{msp}^3, \text{AO}) \right) \left[1 + \sqrt{\frac{C_{1o} C_{2o} e^2}{4\pi\epsilon_o R^3} \frac{2\hbar}{m_e}} \right] + \left(+n_1 \bar{E}_{\text{Kvib}} + c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} - (c_4 E_{\text{initial}}(\text{AO} / \text{HO}) + c_5 E_{\text{initial}}(c_5 \text{AO} / \text{HO})) \right) \quad (15.56)$$

In the case of organic molecules, the atoms of the functional groups are energy matched to the $C2sp^3$ HO such that

$$E(\text{AO} / \text{HO}) = -14.63489 \text{ eV} \quad (15.57)$$

5 For examples of E_{mag} from previous sections:

$$E_{\text{mag}}(C2sp^3) = c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} = c_3 \frac{8\pi\mu_o \mu_B^2}{(0.91771a_0)^3} = c_3 0.14803 \text{ eV} \quad (15.58)$$

$$E_{\text{mag}}(O2p) = c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} = c_3 \frac{8\pi\mu_o \mu_B^2}{a_0^3} = c_3 0.11441 \text{ eV} \quad (15.59)$$

$$E_{\text{mag}}(N2p) = c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} = c_3 \frac{8\pi\mu_o \mu_B^2}{(0.93084a_0)^3} = c_3 0.14185 \text{ eV} \quad (15.60)$$

In the general case of the solution of an organic functional group, the geometric bond
 10 parameters are solved from the semimajor axis and the relationships between the parameters
 by first using Eq. (15.42) to arrive at a . Then, the remaining parameters are determined
 using Eqs. (15.1-15.5). Next, the energies are given by Eqs. (15.52-15.59). To meet the
 equipotential condition for the union of the H_2 -type-ellipsoidal-MO and the HO or AO of the
 atom of a functional group, the factor c_2 of a H_2 -type ellipsoidal MO in principal Eqs.
 15 (15.42) and (15.52) may given by

(i) one:

$$c_2 = 1 \quad (15.61)$$

20 (ii) the ratio that is less than one of 13.605804 eV , the magnitude of the Coulombic
 energy between the electron and proton of H given by Eq. (1.243), and the magnitude of the

Coulombic energy of the participating AO or HO of the atom, $E_{Coulomb}(MO.atom,msp^3)$ given by Eqs. (15.19) and (15.31-15.32). For $|E_{Coulomb}(MO.atom,msp^3)| > 13.605804 \text{ eV}$:

$$c_2 = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{A-B \text{ AorBsp}^3}}} = \frac{13.605804 \text{ eV}}{|E_{Coulomb}(MO.atom,msp^3)|} \quad (15.62)$$

For $|E_{Coulomb}(MO.atom,msp^3)| < 13.605804 \text{ eV}$:

$$c_2 = \frac{\frac{8\pi\epsilon_0 r_{A-B \text{ AorBsp}^3}}{e^2}}{\frac{8\pi\epsilon_0 a_0}{e^2}} = \frac{|E_{Coulomb}(MO.atom,msp^3)|}{13.605804 \text{ eV}} \quad (15.63)$$

(iii) the ratio that is less than one of 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243), and the magnitude of the valence energy, $E(valence)$, of the participating AO or HO of the atom where $E(valence)$ is the ionization energy or $E(MO.atom,msp^3)$ given by Eqs. (15.20) and (15.31-15.32). For $|E(valence)| > 13.605804 \text{ eV}$:

$$c_2 = \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{A-B \text{ AorBsp}^3}}} = \frac{13.605804 \text{ eV}}{|E(valence)|} \quad (15.64)$$

For $|E(valence)| < 13.605804 \text{ eV}$:

$$c_2 = \frac{\frac{8\pi\epsilon_0 r_{A-B \text{ AorBsp}^3}}{e^2}}{\frac{8\pi\epsilon_0 a_0}{e^2}} = \frac{|E(valence)|}{13.605804 \text{ eV}} \quad (15.65)$$

15

(iv) the ratio that is less than one of the magnitude of the Coulombic energy of the participating AO or HO of a first atom, $E_{Coulomb}(MO.atom,msp^3)$ given by Eqs. (15.19) and (15.31-15.32), and the magnitude of the valence energy, $E(valence)$, of the participating AO or HO of a second atom to which the first is energy matched where $E(valence)$ is the

ionization energy or $E(MO.atom,msp^3)$ given by Eqs. (15.20) and (15.31-15.32). For

$|E_{Coulomb}(MO.atom,msp^3)| > E(valence)$:

$$c_2 = \frac{|E(valence)|}{|E_{Coulomb}(MO.atom,msp^3)|} \quad (15.66)$$

For $|E_{Coulomb}(MO.atom,msp^3)| < E(valence)$:

$$c_2 = \frac{|E_{Coulomb}(MO.atom,msp^3)|}{|E(valence)|} \quad (15.67)$$

(v) the ratio that is less than one of the magnitude of the valence-level energies, $E_n(valence)$, of the AO or HO of the n th participating atom of two that are energy matched where $E(valence)$ is the ionization energy or $E(MO.atom,msp^3)$ given by Eqs. (15.20) and (15.31-15.32):

$$c_2 = \frac{E_1(valence)}{E_2(valence)} \quad (15.68)$$

(vi) the factor that is the ratio of the hybridization factor $c_2(1)$ of the valence AO or HO of a first atom and the hybridization factor $c_2(2)$ of the valence AO or HO of a second atom to which the first is energy matched where $c_2(n)$ is given by Eqs. (15.62-15.68); alternatively c_2 is the hybridization factor $c_2(1)$ of the valence AOs or HOs a first pair of atoms and the hybridization factor $c_2(2)$ of the valence AO or HO a third atom or second pair to which the first two are energy matched:

$$c_2 = \frac{c_2(1)}{c_2(2)} \quad (15.69)$$

(vii) the factor that is the product of the hybridization factor $c_2(1)$ of the valence AO or HO of a first atom and the hybridization factor $c_2(2)$ of the valence AO or HO of a second atom to which the first is energy matched where $c_2(n)$ is given by Eqs. (15.62-15.69);

alternatively c_2 is the hybridization factor $c_2(1)$ of the valence AOs or HOs a first pair of atoms and the hybridization factor $c_2(2)$ of the valence AO or HO a third atom or second pair to which the first two are energy matched:

$$c_2 = c_2(1)c_2(2) \quad (15.70)$$

5 The hybridization factor c_2 corresponds to the force constant k (Eqs. (11.65) and (13.58)).

In the case that the valence or Coulombic energy of the AO or HO is less than 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243), then C_2 corresponding to k' (Eq. (15.1)) is given by Eqs. (15.62-15.70).

Specific examples of the factors c_2 and C_2 of a H_2 -type ellipsoidal MO of Eq. 10 (15.51) given in following sections are

$$c_2(C2sp^3HO \text{ to } F) = \frac{E(C, 2sp^3)}{E(F)} c_2(C2sp^3HO) = \frac{-14.63489 \text{ eV}}{-17.42282 \text{ eV}} (0.91771) = 0.77087;$$

$$C_2(C2sp^3HO \text{ to } Cl) = \frac{E(Cl)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-12.96764 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.81317;$$

$$C_2(C2sp^3HO \text{ to } Br) = \frac{E(Br)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-11.81381 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.74081;$$

$$C_2(C2sp^3HO \text{ to } I) = \frac{E(I)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-10.45126 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.65537;$$

$$15 \quad c_2(C2sp^3HO \text{ to } O) = \frac{E(O)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-13.61806 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.85395;$$

$$c_2(H \text{ to } 1^\circ N) = \frac{E(N)}{E(C, 2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.35946 \text{ eV}} = 0.94627;$$

$$c_2(C2sp^3HO \text{ to } N) = \frac{E(N)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-14.53414 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.91140;$$

$$c_2(H \text{ to } 2^\circ N) = \frac{E(N)}{E(C, 2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.56407 \text{ eV}} = 0.93383;$$

$$C_2(S3p \text{ to } H) = \frac{E(S, 3p)}{E(H)} = \frac{-10.36001 \text{ eV}}{-13.60580 \text{ eV}} = 0.76144;$$

$$20 \quad C_2(C2sp^3HO \text{ to } S) = \frac{E(S)}{E(C, 2sp^3)} c_2(C2sp^3HO) = \frac{-10.36001 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.64965;$$

473

$$\begin{aligned}
 c_2(O \text{ to } S3sp^3 \text{ to } C2sp^3HO) &= \frac{E(O)}{E(S)} c_2(C2sp^3HO) \\
 &= \frac{-13.61806 \text{ eV}}{-10.36001 \text{ eV}} (0.91771); \\
 &= 1.20632
 \end{aligned}$$

$$c_2(S3sp^3) = \frac{E_{Coulomb}(S3sp^3)}{E(H)} = \frac{-11.57099 \text{ eV}}{-13.60580 \text{ eV}} = 0.85045;$$

$$C_2(C2sp^3HO \text{ to } S3sp^3) = \frac{E(S3sp^3)}{E(C,2sp^3)} c_2(S3sp^3) = \frac{-11.52126 \text{ eV}}{-14.63489 \text{ eV}} (0.85045) = 0.66951;$$

$$\begin{aligned}
 C_2(S3sp^3 \text{ to } O \text{ to } C2sp^3HO) &= \frac{E(S,3sp^3)}{E(O,2p)} c_2(C2sp^3HO) \\
 &= \frac{-11.52126 \text{ eV}}{-13.61806 \text{ eV}} (0.91771); \\
 &= 0.77641
 \end{aligned}$$

$$\begin{aligned}
 c_2(O \text{ to } N2p \text{ to } C2sp^3HO) &= \frac{E(O)}{E(N)} c_2(C2sp^3HO) \\
 &= \frac{-13.61806 \text{ eV}}{-14.53414 \text{ eV}} (0.91771); \\
 &= 0.85987
 \end{aligned}$$

5

$$c_2(N2p \text{ to } O2p) = \frac{c_2(C2sp^3HO \text{ to } N)}{c_2(C2sp^3HO \text{ to } O)} = \frac{0.91140}{0.85395} = 1.06727;$$

$$C_2(\text{benzene}C2sp^3HO) = c_2(\text{benzene}C2sp^3HO) = \frac{13.605804 \text{ eV}}{15.95955 \text{ eV}} = 0.85252;$$

$$\begin{aligned}
 c_2(\text{aryl}C2sp^3HO \text{ to } O) &= \frac{E(O)}{E(C,2sp^3)} c_2(\text{aryl}C2sp^3HO) \\
 &= \frac{-13.61806 \text{ eV}}{-14.63489 \text{ eV}} (0.85252) \quad ; \\
 &= 0.79329
 \end{aligned}$$

$$c_2(H \text{ to anline } N) = \frac{E(N)}{E(C,2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.76868 \text{ eV}} = 0.92171;$$

$$\begin{aligned}
 c_2(\text{aryl}C2sp^3HO \text{ to } N) &= \frac{E(N)}{E(C,2sp^3)} c_2(\text{aryl}C2sp^3HO) \\
 &= \frac{-14.53414 \text{ eV}}{-14.63489 \text{ eV}} (0.85252) \quad , \\
 &= 0.84665
 \end{aligned}$$

10

and

$$C_2(S3p \text{ to aryl-type } C2sp^3HO) = \frac{E(S,3p)}{E(C,2sp^3)} = \frac{-10.36001 \text{ eV}}{-15.76868 \text{ eV}} = 0.65700.$$

MO INTERCEPT ANGLES AND DISTANCES

Consider the general case of Eqs. (13.84-13.95) wherein the nucleus of a B atom and the
5 nucleus of a A atom comprise the foci of each H_2 -type ellipsoidal MO of an $A-B$ bond.

The parameters of the point of intersection of each H_2 -type ellipsoidal MO and the A -atom
AO are determined from the polar equation of the ellipse:

$$r = r_0 \frac{1+e}{1+e \cos \theta'} \quad (15.71)$$

The radius of the A shell is r_A , and the polar radial coordinate of the ellipse and the radius of
10 the A shell are equal at the point of intersection such that

$$r_A = (a - c') \frac{1 + \frac{c'}{a}}{1 + \frac{c'}{a} \cos \theta'} \quad (15.72)$$

The polar angle θ' at the intersection point is given by

$$\theta' = \cos^{-1} \left(\frac{a}{c'} \left((a - c') \frac{1 + \frac{c'}{a}}{a_0} - 1 \right) \right) \quad (15.73)$$

Then, the angle θ_{AAO} the radial vector of the A AO makes with the internuclear axis is

$$15 \quad \theta_{AAO} = 180^\circ - \theta' \quad (15.74)$$

The distance from the point of intersection of the orbitals to the internuclear axis must be the
same for both component orbitals such that the angle $\omega t = \theta_{H_2MO}$ between the internuclear
axis and the point of intersection of each H_2 -type ellipsoidal MO with the A radial vector
obeys the following relationship:

$$20 \quad r_A \sin \theta_{AAO} = b \sin \theta_{H_2MO} \quad (15.75)$$

such that

$$\theta_{H_2MO} = \sin^{-1} \frac{r_A \sin \theta_{AAO}}{b} \quad (15.76)$$

The distance d_{H_2MO} along the internuclear axis from the origin of H_2 -type ellipsoidal MO to the point of intersection of the orbitals is given by

$$d_{H_2MO} = a \cos \theta_{H_2MO} \quad (15.77)$$

The distance d_{AAO} along the internuclear axis from the origin of the A atom to the point of intersection of the orbitals is given by

$$d_{AAO} = c' - d_{H_2MO} \quad (15.78)$$

BOND ANGLES

Further consider an ACB MO comprising a linear combination of $C-A$ -bond and $C-B$ -bond MOs where C is the general central atom. A bond is also possible between the A and B atoms of the $C-A$ and $C-B$ bonds. Such $A-B$ bonding would decrease the $C-A$ and $C-B$ bond strengths since electron density would be shifted from the latter bonds to the former bond. Thus, the $\angle ACB$ bond angle is determined by the condition that the total energy of the H_2 -type ellipsoidal MO between the terminal A and B atoms is zero. The force constant k' of a H_2 -type ellipsoidal MO due to the equivalent of two point charges of at the foci is given by:

$$k' = \frac{C_1 C_2 2e^2}{4\pi\epsilon_0} \quad (15.79)$$

where C_1 is the fraction of the H_2 -type ellipsoidal MO basis function of a chemical bond of the molecule which is 0.75 (Eq. (13.59)) for a terminal $A-H$ (A is H or other atom) and 1 otherwise and C_2 is the factor that results in an equipotential energy match of the participating at least two atomic orbitals of the chemical bond and is equal to the corresponding factor of Eqs. (15.42) and (15.52). The distance from the origin of the MO to each focus c' of the $A-B$ ellipsoidal MO is given by:

$$c' = a \sqrt{\frac{\hbar^2 4\pi\epsilon_0}{m_e e^2 2C_1 C_2 a}} = \sqrt{\frac{aa_0}{2C_1 C_2}} \quad (15.80)$$

The internuclear distance is

$$2c' = 2 \sqrt{\frac{aa_0}{2C_1 C_2}} \quad (15.81)$$

The length of the semiminor axis of the prolate spheroidal $A-B$ MO $b=c$ is given by Eq. (15.4).

The component energies and the total energy, $E_T(H_2MO)$, of the $A-B$ bond are given by the energy equations (Eqs. (11.207-11.212), (11.213-11.217), and (11.239)) of H_2 except that the terms based on charge are multiplied by c_{BO} , the bond-order factor which is 1 for a single bond and when the MO comprises n_1 equivalent single bonds as in the case of functional groups. c_{BO} is 4 for an independent double bond as in the case of the CO_2 and NO_2 molecules. The kinetic energy term is multiplied by c'_{BO} which is 1 for a single bond, 2 for a double bond, and 9/2 for a triple bond. The electron energy terms are multiplied by c_1 , the fraction of the H_2 -type ellipsoidal MO basis function of a terminal chemical bond which is 0.75 (Eq. (13.233)) for a terminal $A-H$ (A is H or other atom) and 1 otherwise. The electron energy terms are further multiplied by c'_2 , the hybridization or energy-matching factor that results in an equipotential energy match of the participating at least two atomic orbitals of each terminal bond. Furthermore, when $A-B$ comprises atoms other than H , $E_T(atom-atom,msp^3.AO)$, the energy component due to the AO or HO's charge donation to the terminal MO, is added to the other energy terms to give $E_T(H_2MO)$:

$$E_T(H_2MO) = \frac{-e^2}{8\pi\epsilon_0 c'} \left[c_1 c'_2 \left(2c_{BO} - c'_{BO} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] + E_T(atom-atom,msp^3.AO) \quad (15.82)$$

The radiation reaction force in the case of the vibration of $A-B$ in the transition state corresponds to the Doppler energy, E_D , given by Eq. (11.181) that is dependent on the motion of the electrons and the nuclei. The total energy that includes the radiation reaction of the $A-B$ MO is given by the sum of $E_T(H_2MO)$ (Eq. (15.82)) and \bar{E}_{osc} given Eqs. (11.213-11.220), (11.231-11.236), and (11.239-11.240). Thus, the total energy $E_T(A-B)$ of the $A-B$ MO including the Doppler term is

477

$$E_T(A-B) = \left[\left(\frac{-e^2}{8\pi\epsilon_0 c'} \left[c_1 c'_2 \left(2c_{BO} - c'_{BO} \frac{a_0}{a} \right) \ln \frac{a+c'}{a-c'} - 1 \right] + E_T(\text{atom} - \text{atom}, msp^3.AO) \right) \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} + \frac{1}{2} \sqrt{\frac{c_{BO} \frac{c_1 c'_2 e^2}{8\pi\epsilon_0 a^3} - \frac{c_{BO} e^2}{8\pi\epsilon_0 (a+c')^3}}{\mu}} \right] \quad (15.83)$$

where C_{10} is the fraction of the H_2 -type ellipsoidal MO basis function of the oscillatory transition state of the $A-B$ bond which is 0.75 (Eq. (13.233)) in the case of H bonding to a central atom and 1 otherwise, C_{20} is the factor that results in an equipotential energy match of the participating at least two atomic orbitals of the transition state of the chemical bond, and

$\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the nuclei given by Eq. (11.154). To match the boundary

condition that the total energy of the $A-B$ ellipsoidal MO is zero, $E_T(A-B)$ given by Eq.

(15.83) is set equal to zero. Substitution of Eq. (15.81) into Eq. (15.83) gives

$$0 = \left[\left(\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \left[c_1 c'_2 \left(2c_{BO} - c'_{BO} \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T(\text{atom} - \text{atom}, msp^3.AO) \right) \right] \left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} + \frac{1}{2} \sqrt{\frac{c_{BO} \frac{c_1 c'_2 e^2}{8\pi\epsilon_0 a^3} - \frac{c_{BO} e^2}{8\pi\epsilon_0 \left(a + \sqrt{\frac{aa_0}{2C_1 C_2}} \right)^3}}{\mu}} \right] \quad (15.84)$$

The vibrational energy-term of Eq. (15.84) is determined by the forces between the central field and the electrons and those between the nuclei (Eqs. (11.141-11.145)). The electron-central-field force and its derivative are given by

$$f(a) = -c_{BO} \frac{c_1 c'_2 e^2}{4\pi\epsilon_0 a^3} \quad (15.85)$$

and

$$f'(a) = 2c_{BO} \frac{c_1 c_2' e^2}{4\pi\epsilon_0 a^3} \quad (15.86)$$

The nuclear repulsion force and its derivative are given by

$$f(a+c') = \frac{e^2}{8\pi\epsilon_0 (a+c')^2} \quad (15.87)$$

and

$$5 \quad f'(a+c') = -\frac{e^2}{4\pi\epsilon_0 (a+c')^3} \quad (15.88)$$

such that the angular frequency of the oscillation is given by

$$\omega = \sqrt{\frac{\left[\frac{-3}{a} f(a) - f'(a) \right]}{\mu}} = \sqrt{\frac{k}{m_e}} = \sqrt{\frac{c_{BO} \frac{c_1 c_2' e^2}{4\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 (a+c')^2}}{\mu}} \quad (15.89)$$

Since both terms of $\bar{E}_{osc} = \bar{E}_D + \bar{E}_{Kvib}$ are small due to the large values of a and c' , to very good approximation, a convenient form of Eq. (15.84) which is evaluated to determine the
10 bond angles of functional groups is given by

$$0 = \left[\left(\frac{-e^2}{8\pi\epsilon_0 \sqrt{\frac{aa_0}{2C_1 C_2}}} \right) c_1 c_2' \left(2 - \frac{a_0}{a} \right) \ln \frac{a + \sqrt{\frac{aa_0}{2C_1 C_2}}}{a - \sqrt{\frac{aa_0}{2C_1 C_2}}} - 1 \right] + E_T (atom - atom, msp^3 . AO) \quad (15.90)$$

$$\left[1 + \sqrt{\frac{2\hbar \sqrt{\frac{c_1 e^2}{4\pi\epsilon_0 a^3}}}{m_e c^2}} \right] + \frac{1}{2} \hbar \sqrt{\frac{\frac{c_1 e^2}{8\pi\epsilon_0 a^3} - \frac{e^2}{8\pi\epsilon_0 \left(a + \sqrt{\frac{aa_0}{2C_1 C_2}} \right)^3}}{\mu}}$$

From the energy relationship given by Eq. (15.90) and the relationship between the axes given by Eqs. (15.2-15.5), the dimensions of the $A-B$ MO can be solved. The most convenient way to solve Eq. (15.90) is by the reiterative technique using a computer.

15 A factor c_2 of a given atom in the determination of c_2' for calculating the zero of the total $A-B$ bond energy is typically given by Eqs. (15.62-15.65). In the case of a $H-H$ terminal bond of an alkyl or alkenyl group, c_2' is typically the ratio of c_2 of Eq. (15.62) for the $H-H$ bond which is one and c_2 of the carbon of the corresponding $C-H$ bond:

$$c'_2 = \frac{1}{c_2(C2sp^3)} = \frac{13.605804 \text{ eV}}{E_{Coulomb}(C-H \text{ } C2sp^3)} \quad (15.91)$$

In the case of the determination of the bond angle of the ACH MO comprising a linear combination of $C-A$ -bond and $C-H$ -bond MOs where A and C are general, C is the central atom, and c_2 for an atom is given by Eqs. (15.62-15.70), c'_2 of the $A-H$ terminal bond is typically the ratio of c_2 of the A atom for the $A-H$ terminal bond and c_2 of the C atom of the corresponding $C-H$ bond:

$$c'_2 = \frac{c_2(A(A-H)msp^3)}{c_2(C(C-H)(msp^3))} \quad (15.92)$$

In the case of the determination of the bond angle of the COH MO of an alcohol comprising a linear combination of $C-O$ -bond and $O-H$ -bond MOs where C , O , and H are carbon, oxygen, and hydrogen, respectively, c'_2 of the $C-H$ terminal bond is typically 0.91771 since the oxygen and hydrogen atoms are at the Coulomb potential of a proton and an electron (Eqs. (1.236) and (10.162), respectively) that is energy matched to the $C2sp^3$ HO.

In the determination of the hybridization factor c'_2 of Eq. (15.90) from Eqs. (15.62-15.70), the Coulombic energy, $E_{Coulomb}(MO.atom,msp^3)$, or the energy, $E(MO.atom,msp^3)$, the radius $r_{A-B \text{ } AorBsp^3}$ of the A or B AO or HO of the heteroatom of the $A-B$ terminal bond MO such as the $C2sp^3$ HO of a terminal $C-C$ bond is calculated using Eq. (15.32) by considering $\sum E_{T_{mol}}(MO,2sp^3)$, the total energy donation to each bond with which it participates in bonding as it forms the terminal bond. The Coulombic energy $E_{Coulomb}(MO.atom,msp^3)$ of the outer electron of the $atom \text{ } msp^3$ shell is given by Eq. (15.19).

In the case that during hybridization, at least one of the spin-paired AO electrons is unpaired in the hybridized orbital (HO), the energy change for the promotion to the unpaired state is the magnetic energy $E(magnetic)$ (Eq. (15.15)) at the initial radius r of the AO electron. Then, the energy $E(MO.atom,msp^3)$ of the outer electron of the $atom \text{ } msp^3$ shell is given by the sum of $E_{Coulomb}(MO.atom,msp^3)$ and $E(magnetic)$ (Eq. (15.20)).

In the specific case of the terminal bonding of two carbon atoms, the c_2 factor of each carbon given by Eq. (15.62) is determined using the Coulombic energy $E_{Coulomb}(C-C \text{ } C2sp^3)$ of the outer electron of the $C2sp^3$ shell given by Eq. (15.19) with the

radius $r_{C-C C2sp^3}$ of each $C2sp^3$ HO of the terminal $C-C$ bond calculated using Eq. (15.32)

by considering $\sum E_{T_{mol}}(MO, 2sp^3)$, the total energy donation to each bond with which it participates in bonding as it forms the terminal bond including the contribution of the methylene energy, 0.92918 eV (Eq. (14.513)), corresponding to the terminal $C-C$ bond.

5 The corresponding $E_T(\text{atom} - \text{atom}, msp^3.AO)$ in Eq. (15.90) is $E_T(C - C C2sp^3) = -1.85836 \text{ eV}$.

In the case that the terminal atoms are carbon or other heteroatoms, the terminal bond comprises a linear combination of the HOs or AOs; thus, c'_2 is the average of the hybridization factors of the participating atoms corresponding to the normalized linear sum:

$$10 \quad c'_2 = \frac{1}{2} (c'_2(\text{atom } 1) + c'_2(\text{atom } 2)) \quad (15.93)$$

In the exemplary cases of $C-C$, $O-O$, and $N-N$ where C is carbon:

$$\begin{aligned} c'_2 &= \frac{1}{2} \left(\frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{A-A_1 AO/HO}}} + \frac{\frac{e^2}{8\pi\epsilon_0 a_0}}{\frac{e^2}{8\pi\epsilon_0 r_{A-A_2 AO/HO}}} \right) \\ &= \frac{1}{2} \left(\frac{13.605804 \text{ eV}}{E_{Coulomb}(A - A_1 AO / HO)} + \frac{13.605804 \text{ eV}}{E_{Coulomb}(A - A_2 AO / HO)} \right) \end{aligned} \quad (15.94)$$

In the exemplary cases of $C-N$, $C-O$, and $C-S$,

$$c'_2 = \frac{1}{2} \left(\frac{13.605804 \text{ eV}}{E_{Coulomb}(C - B C2sp^3)} + c_2(C \text{ to } B) \right) \quad (15.95)$$

15 where C is carbon and $c_2(C \text{ to } B)$ is the hybridization factor of Eqs. (15.52) and (15.84) that matches the energy of the atom B to that of the atom C in the group. For these cases, the corresponding $E_T(\text{atom} - \text{atom}, msp^3.AO)$ term in Eq. (15.90) depends on the hybridization and bond order of the terminal atoms in the molecule, but typical values matching those used in the determination of the bond energies (Eq. (15.56)) are

$$\begin{aligned} 20 \quad E_T(C - O C2sp^3.O2p) &= -1.44915 \text{ eV}; & E_T(C - O C2sp^3.O2p) &= -1.65376 \text{ eV}; \\ E_T(C - N C2sp^3.N2p) &= -1.44915 \text{ eV}; & E_T(C - S C2sp^3.S2p) &= -0.72457 \text{ eV}; \\ E_T(O - O O2p.O2p) &= -1.44915 \text{ eV}; & E_T(O - O O2p.O2p) &= -1.65376 \text{ eV}; \end{aligned}$$

481

$$\begin{aligned}
E_T(N-N \text{ } N2p.N2p) &= -1.44915 \text{ eV}; & E_T(N-O \text{ } N2p.O2p) &= -1.44915 \text{ eV}; \\
E_T(F-F \text{ } F2p.F2p) &= -1.44915 \text{ eV}; & E_T(Cl-Cl \text{ } Cl3p.Cl3p) &= -0.92918 \text{ eV}; \\
E_T(Br-Br \text{ } Br4p.Br4p) &= -0.92918 \text{ eV}; & E_T(I-I \text{ } I5p.I5p) &= -0.36229 \text{ eV}; \\
E_T(C-F \text{ } C2sp^3.F2p) &= -1.85836 \text{ eV}; & E_T(C-Cl \text{ } C2sp^3.Cl3p) &= -0.92918 \text{ eV}; \\
5 \quad E_T(C-Br \text{ } C2sp^3.Br4p) &= -0.72457 \text{ eV}; & E_T(C-I \text{ } C2sp^3.I5p) &= -0.36228 \text{ eV}, \text{ and} \\
E_T(O-Cl \text{ } O2p.Cl3p) &= -0.92918 \text{ eV}.
\end{aligned}$$

In the case that the terminal bond is $X-X$ where X is a halogen atom, c_1 is one, and c'_2 is the average (Eq. (15.93)) of the hybridization factors of the participating halogen atoms given by Eqs. (15.62-15.63) where $E_{Coulomb}(MO.atom,msp^3)$ is determined using Eq. 10 (15.32) and $E_{Coulomb}(MO.atom,msp^3) = 13.605804 \text{ eV}$ for $X = I$. The factor C_1 of Eq. (15.90) is one for all halogen atoms. The factor C_2 of fluorine is one since it is the only halogen wherein the ionization energy is greater than that 13.605804 eV , the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). For each of the other halogens, Cl , Br , and I , C_2 is the hybridization factor of Eq. (15.52) given by 15 Eq. (15.70) with $c_2(1)$ being that of the halogen given by Eq. (15.68) that matches the valence energy of X ($E_1(valence)$) to that of the $C2sp^3$ HO ($E_2(valence) = -14.63489 \text{ eV}$, Eq. (15.25)) and to the hybridization of $C2sp^3$ HO ($c_2(2) = 0.91771$, Eq. (13.430)). $E_T(atom-atom,msp^3.AO)$ of Eq. (15.90) is the maximum for the participating atoms which is -1.44915 eV , -0.92918 eV , -0.92918 eV , and -0.33582 eV for F , Cl , Br , and I , 20 respectively.

Consider the case that the terminal bond is $C-X$ where C is a carbon atom and X is a halogen atom. The factors c_1 and C_1 of Eq. (15.90) are one for all halogen atoms. For $X = F$, c'_2 is the average (Eq. (15.95)) of the hybridization factors of the participating carbon and F atoms where c_2 for carbon is given by Eq. (15.62) and c_2 for fluorine matched to 25 carbon is given by Eq. (15.70) with $c_2(1)$ for the fluorine atom given by Eq. (15.68) that matches the valence energy of F ($E_1(valence) = -17.42282 \text{ eV}$) to that of the $C2sp^3$ HO

($E_2(\text{valence}) = -14.63489 \text{ eV}$, Eq. (15.25)) and to the hybridization of $C2sp^3$ HO ($c_2(2) = 0.91771$, Eq. (13.430)). The factor C_2 of fluorine is one since it is the only halogen wherein the ionization energy is greater than that 13.605804 eV , the magnitude of the Coulombic energy between the electron and proton of H given by Eq. (1.243). For each of
 5 the other halogens, Cl , Br , and I , c'_2 is the hybridization factor of the participating carbon atom since the halogen atom is energy matched to the carbon atom. C_2 of the terminal-atom bond matches that used to determine the energies of the corresponding $C-X$ -bond MO. Then, C_2 is the hybridization factor of Eq. (15.52) given by Eq. (15.70) with $c_2(1)$ for the halogen atom given by Eq. (15.68) that matches the valence energy of X ($E_1(\text{valence})$) to
 10 that of the $C2sp^3$ HO ($E_2(\text{valence}) = -14.63489 \text{ eV}$, Eq. (15.25)) and to the hybridization of $C2sp^3$ HO ($c_2(2) = 0.91771$, Eq. (13.430)). $E_r(\text{atom-atom,msp}^3.AO)$ of Eq. (15.90) is the maximum for the participating atoms which is -1.85836 eV , -0.92918 eV , -0.72457 eV , and -0.33582 eV for F , Cl , Br , and I , respectively.

Consider the case that the terminal bond is $H-X$ corresponding to the angle of the
 15 atoms $H CX$ where C is a carbon atom and X is a halogen atom. The factors c_1 and C_1 of Eq. (15.90) are 0.75 for all halogen atoms. For $X = F$, c'_2 is given by Eq. (15.69) with c_2 of the participating carbon and F atoms given by Eq. (15.62) and Eq. (15.65), respectively. The factor C_2 of fluorine is one since it is the only halogen wherein the ionization energy is greater than that 13.605804 eV , the magnitude of the Coulombic energy between the electron
 20 and proton of H given by Eq. (1.243). For each of the other halogens, Cl , Br , and I , c'_2 is also given by Eq. (15.69) with c_2 of the participating carbon given by Eq. (15.62) and c_2 of the participating X atom given by $c_2 = 0.91771$ (Eq. (13.430)) since the X atom is energy matched to the $C2sp^3$ HO. In these cases, C_2 is given by Eq. (15.65) for the corresponding atom X where C_2 matches the energy of the atom X to that of H .

25 Using the distance between the two atoms A and B of the general molecular group ACB when the total energy of the corresponding $A-B$ MO is zero, the corresponding bond angle can be determined from the law of cosines:

$$s_1^2 + s_2^2 - 2s_1s_2\cosine\theta = s_3^2 \quad (15.96)$$

With $s_1 = 2c'_{C-A}$, the internuclear distance of the $C-A$ bond, $s_2 = 2c'_{C-B}$, the internuclear distance of each $C-B$ bond, and $s_3 = 2c'_{A-B}$, the internuclear distance of the two terminal atoms, the bond angle $\theta_{\angle ACB}$ between the $C-A$ and $C-B$ bonds is given by

$$(2c'_{C-A})^2 + (2c'_{C-B})^2 - 2(2c'_{C-A})(2c'_{C-B})\cos\theta = (2c'_{A-B})^2 \quad (15.97)$$

$$5 \quad \theta_{\angle ACB} = \cos^{-1} \left(\frac{(2c'_{C-A})^2 + (2c'_{C-B})^2 - (2c'_{A-B})^2}{2(2c'_{C-A})(2c'_{C-B})} \right) \quad (15.98)$$

Consider the exemplary structure $C_b C_a (O_a) O_b$ wherein C_a is bound to C_b , O_a , and O_b . In the general case that the three bonds are coplanar and two of the angles are known, say θ_1 and θ_2 , then the third θ_3 can be determined geometrically:

$$\theta_3 = 360 - \theta_1 - \theta_2 \quad (15.99)$$

10 In the general case that two of the three coplanar bonds are equivalent and one of the angles is known, say θ_1 , then the second and third can be determined geometrically:

$$\theta_2 = \theta_3 = \frac{(360 - \theta_1)}{2} \quad (15.100)$$

15 ANGLES AND DISTANCES FOR AN MO THAT FORMS AN ISOSCELES TRIANGLE

In the general case where the group comprises three $A-B$ bonds having B as the central atom at the apex of a pyramidal structure formed by the three bonds with the A atoms at the base in the xy -plane. The C_{3v} axis centered on B is defined as the vertical or z -axis, and any two $A-B$ bonds form an isosceles triangle. Then, the angle of the bonds and the distances from and along the z -axis are determined from the geometrical relationships given by Eqs. (13.412-13.416):

the distance $d_{origin-B}$ from the origin to the nucleus of a terminal B atom is given by

$$d_{origin-B} = \frac{2c'_{B-B}}{2\sin 60^\circ} \quad (15.101)$$

the height along the z -axis from the origin to the A nucleus d_{height} is given by

$$25 \quad d_{height} = \sqrt{(2c'_{A-B})^2 - (d_{origin-B})^2}, \text{ and} \quad (15.102)$$

the angle θ_v of each $A-B$ bond from the z-axis is given by

$$\theta_v = \tan^{-1} \left(\frac{d_{\text{origin}-B}}{d_{\text{height}}} \right) \quad (15.103)$$

Consider the case where the central atom B is further bound to a fourth atom C and the $B-C$ bond is along the z-axis. Then, the bond $\theta_{\angle ABC}$ given by Eq. (14.206) is

$$5 \quad \theta_{\angle ABC} = 180 - \theta_v \quad (15.104)$$

DIHEDRAL ANGLE

Consider the plane defined by a general ACA MO comprising a linear combination of two $C-A$ -bond MOs where C is the central atom. The dihedral angle $\theta_{\angle BCIACA}$ between the
 10 ACA -plane and a line defined by a third bond with C , specifically that corresponding to a $C-B$ -bond MO, is calculated from the bond angle $\theta_{\angle ACA}$ and the distances between the A , B , and C atoms. The distance d_1 along the bisector of $\theta_{\angle ACA}$ from C to the internuclear-distance line between A and A , $2c'_{A-A}$, is given by

$$d_1 = 2c'_{C-A} \cos \frac{\theta_{\angle ACA}}{2} \quad (15.105)$$

15 where $2c'_{C-A}$ is the internuclear distance between A and C . The atoms A , A , and B define the base of a pyramid. Then, the pyramidal angle $\theta_{\angle ABA}$ can be solved from the internuclear distances between A and A , $2c'_{A-A}$, and between A and B , $2c'_{A-B}$, using the law of cosines (Eq. (15.98)):

$$\theta_{\angle ABA} = \cos^{-1} \left(\frac{(2c'_{A-B})^2 + (2c'_{A-B})^2 - (2c'_{A-A})^2}{2(2c'_{A-B})(2c'_{A-B})} \right) \quad (15.106)$$

20 Then, the distance d_2 along the bisector of $\theta_{\angle ABA}$ from B to the internuclear-distance line $2c'_{A-A}$, is given by

$$d_2 = 2c'_{A-B} \cos \frac{\theta_{\angle ABA}}{2} \quad (15.107)$$

The lengths d_1 , d_2 , and $2c'_{C-B}$ define a triangle wherein the angle between d_1 and the internuclear distance between B and C , $2c'_{C-B}$, is the dihedral angle $\theta_{\angle BCIACA}$ that can be
 25 solved using the law of cosines (Eq. (15.98)):

$$\theta_{\angle BC/ACA} = \cos^{-1} \left(\frac{d_1^2 + (2c'_{C-B})^2 - d_2^2}{2d_1(2c'_{B-C})} \right) \quad (15.108)$$

SOLUTION OF GEOMETRICAL AND ENERGY PARAMETERS OF MAJOR FUNCTIONAL GROUPS AND CORRESPONDING ORGANIC MOLECULES

The exemplary molecules given in the following sections were solved using the solutions of organic chemical functional groups as basis elements wherein the structures and energies were linearly added to achieve the molecular solutions. Each functional group can be treated as a building block to form any desired molecular solution from the corresponding linear combination. Each functional group element was solved using the atomic orbital and hybrid orbital spherical orbitalsphere solutions bridged by molecular orbitals comprised of the H_2 -type prolate spheroidal solution given in the Nature of the Chemical Bond of Hydrogen-Type Molecules section. The energy of each MO was matched at the HO or AO by matching the hybridization and total energy of the MO to the AOs and HOs. The energy E_{mag} (e.g. given by Eq. (15.58)) for a $C2sp^3$ HO and Eq.(15.59) for an $O2p$ AO) was subtracted for each set of unpaired electrons created by bond breakage.

The bond energy is not equal to the component energy of each bond as it exists in the molecule; although, they are close. The total energy of each group is its contribution to the total energy of the molecule as a whole. The determination of the bond energies for the creation of the separate parts must take into account the energy of the formation of any radicals and any redistribution of charge density within the pieces and the corresponding energy change with bond cleavage. Also, the vibrational energy in the transition state is dependent on the other groups that are bound to a given functional group. This will effect the functional-group energy. But, because the variations in the energy based on the balance of the molecular composition are typically of the order of a few hundreds of electron volts at most, they were neglected.

The energy of each functional-group MO bonding to a given carbon HO is independently matched to the HO by subtracting the contribution to the change in the energy of the HO from the total MO energy given by the sum of the MO contributions and $E(C,2sp^3) = -14.63489 \text{ eV}$ (Eq. (13.428)). The intercept angles are determined from Eqs.

(15.71-15.78) using the final radius of the HO of each atom. The final carbon-atom radius is determined using Eqs. (15.32) wherein the sum of the energy contributions of each atom to all the MOs in which it participates in bonding is determined. This final radius is used in Eqs. (15.19) and (15.20) to calculate the final valence energy of the HO of each atom at the
5 corresponding final radius. The radius of any bonding heteroatom that contributes to a MO is calculated in the same manner, and the energy of its outermost shell is matched to that of the MO by the hybridization factor between the carbon-HO energy and the energy of the heteroatomic shell. The donation of electron density to the AOs and HOs reduces the energy. The donation of the electron density to the MO's at each AO or HO is that which causes the
10 resulting energy to be divided equally between the participating AOs or HOs to achieve energy matching.

The molecular solutions can be used to design synthetic pathways and predict product yields based on equilibrium constants calculated from the heats of formation. New stable compositions of matter can be predicted as well as the structures of combinatorial chemistry
15 reactions. Further important pharmaceutical applications include the ability to graphically or computationally render the structures of drugs that permit the identification of the biologically active parts of the molecules to be identified from the common spatial charge-density functions of a series of active molecules. Drugs can be designed according to geometrical parameters and bonding interactions with the data of the structure of the active site of the
20 drug.

To calculate conformations, folding, and physical properties, the exact solutions of the charge distributions in any given molecule are used to calculate the fields, and from the fields, the interactions between groups of the same molecule or between groups on different molecules are calculated wherein the interactions are distance and relative orientation
25 dependent. The fields and interactions can be determined using a finite-element-analysis approach of Maxwell's equations.

AROMATIC AND HETEROCYCLIC COMPOUNDS

Aromatic and heterocyclic molecules comprise at least one of an aromatic or a cyclic conjugated alkene functional group. The latter was described in the Cyclic and Conjugated Alkenes section. The aromatic bond is uniquely stable and requires the sharing of the electrons of multiple H_2 -type MOs. The results of the derivation of the parameters of the benzene molecule given in the Benzene Molecule (C_6H_6) section can be generalized to any aromatic function group(s) of aromatic and heterocyclic compounds.

C_6H_6 can be considered a linear combination of three ethylene molecules wherein a $C-H$ bond of each CH_2 group of $H_2C=CH_2$ is replaced by a $C=C$ bond to form a six-member ring of carbon atoms. The solution of the ethylene molecule is given in the Ethylene Molecule (CH_2CH_2) section. The radius $r_{ethylene 2sp^3}$ ($0.85252a_0$) of the $C2sp^3$ shell of ethylene calculated from the Coulombic energy is given by Eq. (14.244). The Coulombic energy $E_{Coulomb}(C_{ethylene}, 2sp^3)$ ($-15.95955 eV$) of the outer electron of the $C2sp^3$ shell is given by Eq. (14.245). The energy $E(C_{ethylene}, 2sp^3)$ ($-15.76868 eV$) of the outer electron of the $C2sp^3$ shell is given by Eq. (14.246). $E_r(C=C, 2sp^3)$ ($-1.13380 eV$) (Eq. (14.247), the energy change of each $C2sp^3$ shell with the formation of the $C=C$ -bond MO is given by the difference between $E(C_{ethylene}, 2sp^3)$ and $E(C, 2sp^3)$. C_6H_6 can be solved using the same principles as those used to solve ethylene wherein the $2s$ and $2p$ shells of each C hybridize to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ hybridized orbitals (HOs) to form a molecular orbital (MO) permits each participating hybridized orbital to decrease in radius and energy. Each $2sp^3$ HO of each carbon atom initially has four unpaired electrons. Thus, the 6 H atomic orbitals (AOs) of benzene contribute six electrons and the six sp^3 -hybridized carbon atoms contribute twenty-four electrons to form six $C-H$ bonds and six $C=C$ bonds. Each $C-H$ bond has two paired electrons with one donated from the H AO and the other from the $C2sp^3$ HO. Each $C=C$ bond comprises a linear combination of a factor of 0.75 of four paired electrons (three electrons) from two sets of two $C2sp^3$ HOs of the participating carbon atoms. Each $C-H$ and each $C=C$ bond comprises a linear combination of one and two diatomic H_2 -type MOs developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section, respectively.

Consider the case where three sets of $C=C$ -bond MOs form bonds between the two carbon atoms of each molecule to form a six-member ring such that the six resulting bonds comprise eighteen paired electrons. Each bond comprises a linear combination of two MOs wherein each comprises two $C2sp^3$ HOs and 75% of a H_2 -type ellipsoidal MO divided between the $C2sp^3$ HOs:

$$\left(\begin{array}{l} 3(2 C2sp^3 + 0.75 H_2 MO) \rightarrow 3(C=C) \text{—ethylene—type—bond MO} \\ \rightarrow 6(C=C) \text{—bond MO of benzene} \end{array} \right) \quad (15.142)$$

The linear combination of each H_2 -type ellipsoidal MO with each $C2sp^3$ HO further comprises an excess 25% charge-density contribution per bond from each $C2sp^3$ HO to the $C=C$ -bond MO to achieve an energy minimum. Thus, the dimensional parameters of each bond $C=C$ -bond are determined using Eqs. (15.42) and (15.1-15.5) in a form that are the same equations as those used to determine the same parameters of the $C=C$ -bond MO of ethylene (Eqs. (14.242-14.268)) while matching the boundary conditions of the structure of benzene.

Hybridization with 25% electron donation to each $C=C$ -bond gives rise to the $C_{benzene}2sp^3$ HO-shell Coulombic energy $E_{Coulomb}(C_{benzene}, 2sp^3)$ given by Eq. (14.245). To meet the equipotential condition of the union of the six $C2sp^3$ HOs, c_2 and C_2 of Eq. (15.42) for the aromatic $C=C$ -bond MO is given by Eq. (15.62) as the ratio of 15.95955 eV, the magnitude of $E_{Coulomb}(C_{benzene}, 2sp^3)$ (Eq. (14.245)), and 13.605804 eV, the magnitude of the Coulombic energy between the electron and proton of H (Eq. (1.243)):

$$C_2(benzeneC2sp^3HO) = c_2(benzeneC2sp^3HO) = \frac{13.605804 \text{ eV}}{15.95955 \text{ eV}} = 0.85252 \quad (15.143)$$

The energies of each $C=C$ bond of benzene are also determined using the same equations as those of ethylene (Eqs. (14.251-14.253) and (14.319-14.333) with the parameters of benzene. Ethylene serves as a basis element for the $C=C$ bonding of benzene wherein each of the six $C=C$ bonds of benzene comprises $(0.75)(4) = 3$ electrons according to Eq. (15.142).

The total energy of the bonds of the eighteen electrons of the $C=C$ bonds of benzene, $E_T(C_6H_6, C=C)$, is given by $(6)(0.75)$ times $E_{T+osc}(C=C)$ (Eq. (14.492)), the total energy of the $C=C$ -bond MO of benzene including the Doppler term, minus eighteen times $E(C, 2sp^3)$ (Eq. (14.146)), the initial energy of each $C2sp^3$ HO of each C that forms the $C=C$ bonds of bond order two. Thus, the total energy of the six $C=C$ bonds of benzene with three electron per aromatic bond given by Eq. (14.493) is

$$\begin{aligned} E_T(C_6H_6, C=C) &= (6)(0.75)E_{T+osc}(C=C) - (6)(3)E(C, 2sp^3) \\ &= (6)(0.75)(-66.05796 \text{ eV}) - 18(-14.63489 \text{ eV}) \\ &= -297.26081 \text{ eV} - (-263.42798 \text{ eV}) \\ &= -33.83284 \text{ eV} \end{aligned} \quad (15.144)$$

The results of benzene can be generalized to the class of aromatic and heterocyclic compounds. E_{hv} of an aromatic bond is given by $E_T(H_2)$ (Eqs. (11.212) and (14.486)), the maximum total energy of each H_2 -type MO such that

$$\bar{E}_{osc} = n_1(\bar{E}_D + \bar{E}_{Kvib}) = n_1 \left(-31.63536831 \text{ eV} \sqrt{\frac{2\bar{E}_K}{Mc^2}} + \frac{1}{2} \hbar \sqrt{\frac{k}{\mu}} \right) \quad (15.145)$$

The factor of 0.75 corresponding to the three electrons per aromatic bond of bond order two given in the Benzene Molecule (C_6H_6) section modifies Eqs. (15.52-15.56). Multiplication of

the total energy given by Eq. (15.55) by $f_1 = 0.75$ with the substitution of Eq. (15.145) gives the total energy of the aromatic bond:

$$E_T(\text{Group}) = f_1 \left(\begin{array}{l} E(\text{basis energies}) + E_T(\text{atom-atom, msp}^3 \text{ AO}) \\ -31.63536831 \text{ eV} \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 R^3}}}{m_e}} + n_1 \bar{E}_{\text{Kvib}} + c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} \end{array} \right) \quad (15.146)$$

- 5 The total bond energy of the aromatic group $E_D(\text{Group})$ is the negative difference of the total energy of the group (Eq. (15.146)) and the total energy of the starting species given by the sum of $c_4 E_{\text{initial}}(c_4 \text{ AO / HO})$ and $c_5 E_{\text{initial}}(c_5 \text{ AO / HO})$:

$$E_D(\text{Group}) = - \left(\begin{array}{l} f_1 \left(\begin{array}{l} E(\text{basis energies}) + E_T(\text{atom-atom, msp}^3 \text{ AO}) \\ -31.63536831 \text{ eV} \sqrt{\frac{2\hbar \sqrt{\frac{C_{10} C_{20} e^2}{4\pi\epsilon_0 R^3}}}{m_e}} + n_1 \bar{E}_{\text{Kvib}} + c_3 \frac{8\pi\mu_o \mu_B^2}{r^3} \end{array} \right) \\ - (c_4 E_{\text{initial}}(\text{AO / HO}) + c_5 E_{\text{initial}}(c_5 \text{ AO / HO})) \end{array} \right) \quad (15.147)$$

Since there are three electrons per aromatic bond, c_4 is three times the number of aromatic 10 bonds.

Benzene can also be considered as comprising chemical bonds between six CH radicals wherein each radical comprises a chemical bond between carbon and hydrogen atoms. The solution of the parameters of CH is given in the Hydrogen Carbide (CH) section. Those of the benzene are given in the Benzene Molecule (C_6H_6) section. The energy components of V_e , V_p ,

- 15 T , V_m , and E_T are the same as those of the hydrogen carbide radical, except that $E_T(C = C, 2sp^3) = -1.13379 \text{ eV}$ (Eq. (14.247)) is subtracted from $E_T(CH)$ of Eq. (13.495) to match the energy of each $C-H$ -bond MO to the decrease in the energy of the corresponding $C2sp^3$ HO. In the corresponding generalization of the aromatic CH group, the geometrical parameters are determined using Eq. (15.42) and Eqs. (15.1-15.5) with
- 20 $E_T(\text{atom-atom, msp}^3 \text{ AO}) = -1.13379 \text{ eV}$.

The total energy of the benzene $C-H$ -bond MO, $E_{T_{\text{benzene}}}(C-H)$, given by Eq. (14.467) is the sum of $0.5E_T(C = C, 2sp^3)$, the energy change of each $C2sp^3$ shell per single bond due to

the decrease in radius with the formation of the corresponding $C=C$ -bond MO (Eq. (14.247)), and $E_{T_{\text{benzene}}}(CH)$, the σ MO contribution given by Eq. (14.441). In the corresponding

- 25 generalization of the aromatic CH group, the energy parameters are determined using Eqs.

(15.146-15.147) with $f_1 = 1$ and $E_T(\text{atom-atom, msp}^3 \text{ AO}) = \frac{-1.13379 \text{ eV}}{2}$. Thus, the energy

contribution to the single aromatic CH bond is one half that of the $C=C$ double bond contribution. This matches the energies of the CH and $C=C$ aromatic groups, conserves the electron number with the equivalent charge density as that of $s=1$ in Eqs. (15.18-15.21), and further gives a minimum energy for the molecule. Breakage of the aromatic $C=C$ bonds to give CH groups creates unpaired electrons in these fragments that corresponds to $c_3=1$ in Eq. (15.56) with E_{mag} given by Eq. (15.58).

Each of the $C-H$ bonds of benzene comprises two electrons according to Eq. (14.439). From the energy of each $C-H$ bond, $-E_{D_{benzene}}(^{12}CH)$ (Eq. (14.477)), the total energy of the twelve electrons of the six $C-H$ bonds of benzene, $E_T(C_6H_6, C-H)$, given by Eq. (14.494) is

$$E_T(C_6H_6, C-H) = (6)(-E_{D_{benzene}}(^{12}CH)) = 6(-3.90454 \text{ eV}) = -23.42724 \text{ eV} \quad (15.148)$$

The total bond dissociation energy of benzene, $E_D(C_6H_6)$, given by Eq. (14.495) is the negative sum of $E_T(C_6H_6, C=C)$ (Eq. (14.493)) and $E_T(C_6H_6, C-H)$ (Eq. (14.494)):

$$\begin{aligned} E_D(C_6H_6) &= -\left(E_T(C_6H_6, C=C) + E_T(C_6H_6, C-H)\right) \\ &= -((-33.83284 \text{ eV}) + (-23.42724 \text{ eV})) \\ &= 57.2601 \text{ eV} \end{aligned} \quad (15.149)$$

15 Using the parameters given in Tables 15.214 and 15.216 in the general equations (Eqs. (15.42), (15.1-15.5), and (15.146-15.147)) reproduces the results for benzene given in the Benzene Molecule (C_6H_6) section as shown in Tables 15.214 and 15.216.

The symbols of the functional groups of aromatics and hertocyclics are given in Table 15.213. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11), (15.17-15.56), and (15.146-15.147)) parameters of aromatics and hertocyclics are given in Tables 15.214, 15.215, and 15.216, respectively. The total energy of benzene given in Table 15.217 was calculated as the sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.216 corresponding to functional-group composition of the molecule. The bond angle parameters of benzene determined using Eqs. (15.79-15.108) are given in Table 25 15.218.

Table 15.213. The symbols of functional groups of aromatics and hertocyclics.

Functional Group	Group Symbol
CC (aromatic bond)	$C=C$
CH (aromatic)	CH (i)

Table 15.214. The geometrical bond parameters of aromatics and heterocyclics and experimental values [1].

Parameter	^{3e} C=C Group	CH Group
$a (a_0)$	1.47348	1.60061
$c' (a_0)$	1.31468	1.03299
Bond Length $2c' (\text{\AA})$	1.39140	1.09327
Exp. Bond Length (\AA)	1.399 (benzene)	1.101 (benzene)
$b, c (a_0)$	0.66540	1.22265
e	0.89223	0.64537

5 Table 15.216. The energy parameters (eV) of functional groups of aromatics and heterocyclics.

Parameters	^{3e} C=C Group	CH Group
f_1	0.75	1
n_1	2	1
n_2	0	0
n_3	0	0
C_1	0.5	0.75
C_2	0.85252	1
c_1	1	1
c_2	0.85252	0.91771
c_3	0	1
c_4	3	1
c_5	0	1
C_{10}	0.5	0.75
C_{20}	0.85252	1
$V_e (eV)$	-101.12679	-37.10024
$V_p (eV)$	20.69825	13.17125
$T (eV)$	34.31559	11.58941
$V_m (eV)$	-17.15779	-5.79470
$E(AO/HO) (eV)$	0	-14.63489
$\Delta E_{H_2MO}(AO/HO) (eV)$	0	-1.13379

492

$E_T (AO/HO) (eV)$	0	-13.50110
$E_T (H_2MO) (eV)$	-63.27075	-31.63539
$E_T (atom - atom, msp^3 .AO) (eV)$	-2.26759	-0.56690
$E_T (MO) (eV)$	-65.53833	-32.20226
$\omega (10^{15} rad / s)$	49.7272	26.4826
$E_K (eV)$	32.73133	17.43132
$\bar{E}_D (eV)$	-0.35806	-0.26130
$\bar{E}_{Kvib} (eV)$	0.19649 [49]	0.35532 Eq. (13.458)
$\bar{E}_{osc} (eV)$	-0.25982	-0.08364
$E_{mag} (eV)$	0.14803	0.14803
$E_T (Group) (eV)$	-49.54347	-32.28590
$E_{initial} (\epsilon_A AO/HO) (eV)$	-14.63489	-14.63489
$E_{initial} (\epsilon_S AO/HO) (eV)$	0	-13.59844
$E_D (Group) (eV)$	5.63881	3.90454

Table 15.215. The MO to HO intercept geometrical bond parameters of benzene. E_T is $E_T(atom - atom,msp^3.AO)$.

Bond	Atom	E_T (eV) Bond 1	E_T (eV) Bond 2	E_T (eV) Bond 3	E_T (eV) Bond 4	Final Total Energy $C2sp^3$ (eV)	$r_{initial}$ (a_0)	r_{final} (a_0)
$C - H (CH)$	C	-0.85035	-0.85035	-0.56690	0	-153.88327	0.91771	0.79597
$C = HC_a = C$	C_a	-0.85035	-0.85035	-0.56690	0	-153.88327	0.91771	0.79597

Bond	$E_{Conform}(C2sp^3)(eV)$ Final	$E(C2sp^3)$ (eV) Final	θ' ($^\circ$)	θ_1 ($^\circ$)	θ_2 ($^\circ$)	d_1 (a_0)	d_2 (a_0)
$C - H (CH)$	-17.09334	-16.90248	74.42	105.58	38.84	1.24678	0.21379
$C = HC_a = C$	-17.09334	-16.90248	134.24	45.76	58.98	0.75935	0.55533

Table 15.217. The total bond energies of benzene calculated using the functional group composition and the energies of Table 15.216 compared to the experimental values [2].

Formula	Name	$C^3=C$	CH	Calculated		Experimental	
				Total Bond Energy (eV)		Total Bond Energy (eV)	Relative Error
C_6H_6	Benzene	6	6	57.26008		57.26340	0.00006

Table 15.218. The bond angle parameters of benzene and experimental values [1]. E_r is $E_r(\text{atom} - \text{atom}, \text{msp}^3 \text{AO})$.

Atoms of Angle	$2c'$ Bond 1 (a_0)	$2c'$ Bond 2 (a_0)	$2c'$ Terminal Atoms (a_0)	$E_{\text{Coulombic}}$ Atom 1	Atom 1 Hybridization Designation (Table 15.3.A)	$E_{\text{Coulombic}}$ Atom 2	Atom 2 Hybridization Designation (Table 15.3.A)	c_2 Atom 1	c_2 Atom 2	C_1	C_2
$\angle CCC$ (aromatic)	2.62936	2.62936	4.5585	-17.17218	34	-17.17218	34	0.79232	0.79232	1	1
$\angle CCH$ (aromatic)											

Atoms of Angle	c_1	c'_2	E_r (eV)	θ_v ($^\circ$)	θ_1 ($^\circ$)	θ_2 ($^\circ$)	Cal. θ ($^\circ$)	Exp. θ ($^\circ$)
$\angle CCC$ (aromatic)	1	0.79232	-1.85836				120.19	120 [50-52] (benzene)
$\angle CCH$ (aromatic)					120.19		119.91	120 [50-52] (benzene)

Table 1.1. The calculated and experimental total bond energies of n-alkanes using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₃ H ₈	Propane	41.46896	41.434	-0.00085
C ₄ H ₁₀	Butane	53.62666	53.61	-0.00036
C ₅ H ₁₂	Pentane	65.78436	65.77	-0.00017
C ₆ H ₁₄	Hexane	77.94206	77.93	-0.00019
C ₇ H ₁₆	Heptane	90.09976	90.09	-0.00013
C ₈ H ₁₈	Octane	102.25746	102.25	-0.00006
C ₉ H ₂₀	Nonane	114.41516	114.40	-0.00012
C ₁₀ H ₂₂	Decane	126.57286	126.57	-0.00003
C ₁₁ H ₂₄	Undecane	138.73056	138.736	0.00004
C ₁₂ H ₂₆	Dodecane	150.88826	150.88	-0.00008
C ₁₈ H ₃₈	Octadecane	223.83446	223.85	0.00008

Table 1.2. The calculated and experimental total bond energies of branched alkanes using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₄ H ₁₀	Isobutane	53.69922	53.695	-0.00007
C ₅ H ₁₂	Isopentane	65.85692	65.843	-0.00021
C ₅ H ₁₂	Neopentane	65.86336	65.992	0.00195
C ₆ H ₁₄	2-Methylpentane	78.01462	78.007	-0.00010
C ₆ H ₁₄	3-Methylpentane	78.01462	77.979	-0.00046
C ₆ H ₁₄	2,2-Dimethylbutane	78.02106	78.124	0.00132
C ₆ H ₁₄	2,3-Dimethylbutane	77.99581	78.043	0.00061
C ₇ H ₁₆	2-Methylhexane	90.17232	90.160	-0.00014
C ₇ H ₁₆	3-Methylhexane	90.17232	90.127	-0.00051
C ₇ H ₁₆	3-Ethylpentane	90.17232	90.108	-0.00072
C ₇ H ₁₆	2,2-Dimethylpentane	90.17876	90.276	0.00107
C ₇ H ₁₆	2,2,3-Trimethylbutane	90.22301	90.262	0.00044
C ₇ H ₁₆	2,4-Dimethylpentane	90.24488	90.233	-0.00013
C ₇ H ₁₆	3,3-Dimethylpentane	90.17876	90.227	0.00054
C ₈ H ₁₈	2-Methylheptane	102.33002	102.322	-0.00008
C ₈ H ₁₈	3-Methylheptane	102.33002	102.293	-0.00036
C ₈ H ₁₈	4-Methylheptane	102.33002	102.286	-0.00043
C ₈ H ₁₈	3-Ethylhexane	102.30169	102.274	-0.00027
C ₈ H ₁₈	2,2-Dimethylhexane	102.33646	102.417	0.00079
C ₈ H ₁₈	2,3-Dimethylhexane	102.31121	102.306	-0.00005
C ₈ H ₁₈	2,4-Dimethylhexane	102.40258	102.362	-0.00040
C ₈ H ₁₈	2,5-Dimethylhexane	102.40258	102.396	-0.00006
C ₈ H ₁₈	3,3-Dimethylhexane	102.33646	102.369	0.00032
C ₈ H ₁₈	3,4-Dimethylhexane	102.31121	102.296	-0.00015
C ₈ H ₁₈	3-Ethyl-2-methylpentane	102.31121	102.277	-0.00033
C ₈ H ₁₈	3-Ethyl-3-methylpentane	102.33646	102.317	-0.00019
C ₈ H ₁₈	2,2,3-Trimethylpentane	102.38071	102.370	-0.00010
C ₈ H ₁₈	2,2,4-Trimethylpentane	102.40902	102.412	0.00003
C ₈ H ₁₈	2,3,3-Trimethylpentane	102.38071	102.332	-0.00048
C ₈ H ₁₈	2,3,4-Trimethylpentane	102.29240	102.342	0.00049
C ₈ H ₁₈	2,2,3,3-Tetramethylbutane	102.41632	102.433	0.00016
C ₉ H ₂₀	2,3,5-Trimethylhexane	114.54147	114.551	0.00008
C ₉ H ₂₀	3,3-Diethylpentane	114.49416	114.455	-0.00034
C ₉ H ₂₀	2,2,3,3-Tetramethylpentane	114.57402	114.494	-0.00070
C ₉ H ₂₀	2,2,3,4-Tetramethylpentane	114.51960	114.492	-0.00024

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₉ H ₂₀	2,2,4,4-Tetramethylpentane	114.57316	114.541	-0.00028
C ₉ H ₂₀	2,3,3,4-Tetramethylpentane	114.58266	114.484	-0.00086
C ₁₀ H ₂₂	2-Methylnonane	126.64542	126.680	0.00027
C ₁₀ H ₂₂	5-Methylnonane	126.64542	126.663	0.00014

Table 1.3. The calculated and experimental total bond energies of alkenes using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₃ H ₆	Propene	35.56033	35.63207	0.00201
C ₄ H ₈	1-Butene	47.71803	47.78477	0.00140
C ₄ H ₈	trans-2-Butene	47.93116	47.90395	-0.00057
C ₄ H ₈	Isobutene	47.90314	47.96096	0.00121
C ₅ H ₁₀	1-Pentene	59.87573	59.95094	0.00125
C ₅ H ₁₀	trans-2-Pentene	60.08886	60.06287	-0.00043
C ₅ H ₁₀	2-Methyl-1-butene	60.06084	60.09707	0.00060
C ₅ H ₁₀	2-Methyl-2-butene	60.21433	60.16444	-0.00083
C ₅ H ₁₀	3-Methyl-1-butene	59.97662	60.01727	0.00068
C ₆ H ₁₂	1-Hexene	72.03343	72.12954	0.00133
C ₆ H ₁₂	trans-2-Hexene	72.24656	72.23733	-0.00013
C ₆ H ₁₂	trans-3-Hexene	72.24656	72.24251	-0.00006
C ₆ H ₁₂	2-Methyl-1-pentene	72.21854	72.29433	0.00105
C ₆ H ₁₂	2-Methyl-2-pentene	72.37203	72.37206	0.00000
C ₆ H ₁₂	3-Methyl-1-pentene	72.13432	72.19173	0.00080
C ₆ H ₁₂	4-Methyl-1-pentene	72.10599	72.21038	0.00145
C ₆ H ₁₂	3-Methyl-trans-2-pentene	72.37203	72.33268	-0.00054
C ₆ H ₁₂	4-Methyl-trans-2-pentene	72.34745	72.31610	-0.00043
C ₆ H ₁₂	2-Ethyl-1-butene	72.21854	72.25909	0.00056
C ₆ H ₁₂	2,3-Dimethyl-1-butene	72.31943	72.32543	0.00008
C ₆ H ₁₂	3,3-Dimethyl-1-butene	72.31796	72.30366	-0.00020
C ₆ H ₁₂	2,3-Dimethyl-2-butene	72.49750	72.38450	-0.00156
C ₇ H ₁₄	1-Heptene	84.19113	84.27084	0.00095
C ₇ H ₁₄	5-Methyl-1-hexene	84.26369	84.30608	0.00050
C ₇ H ₁₄	trans-3-Methyl-3-hexene	84.52973	84.42112	-0.00129
C ₇ H ₁₄	2,4-Dimethyl-1-pentene	84.44880	84.49367	0.00053
C ₇ H ₁₄	4,4-Dimethyl-1-pentene	84.27012	84.47087	0.00238
C ₇ H ₁₄	2,4-Dimethyl-2-pentene	84.63062	84.54445	-0.00102
C ₇ H ₁₄	trans-4,4-Dimethyl-2-pentene	84.54076	84.54549	0.00006
C ₇ H ₁₄	2-Ethyl-3-methyl-1-butene	84.47713	84.44910	-0.00033
C ₇ H ₁₄	2,3,3-Trimethyl-1-butene	84.51274	84.51129	-0.00002
C ₈ H ₁₆	1-Octene	96.34883	96.41421	0.00068
C ₈ H ₁₆	trans-2,2-Dimethyl-3-hexene	96.69846	96.68782	-0.00011
C ₈ H ₁₆	3-Ethyl-2-methyl-1-pentene	96.63483	96.61113	-0.00025
C ₈ H ₁₆	2,4,4-Trimethyl-1-pentene	96.61293	96.71684	0.00107
C ₈ H ₁₆	2,4,4-Trimethyl-2-pentene	96.67590	96.65880	-0.00018
C ₁₀ H ₂₀	1-Decene	120.66423	120.74240	0.00065
C ₁₂ H ₂₄	1-Dodecene	144.97963	145.07163	0.00063
C ₁₆ H ₃₂	1-Hexadecene	193.61043	193.71766	0.00055

Table 1.4. The calculated and experimental total bond energies of alkynes using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₃ H ₄	Propyne	29.42932	29.40432	-0.00085
C ₄ H ₆	1-Butyne	41.58702	41.55495	-0.00077
C ₄ H ₆	2-Butyne	41.72765	41.75705	0.00070
C ₉ H ₁₆	1-Nonyne	102.37552	102.35367	-0.00021

Table 1.5. The calculated and experimental total bond energies of alkyl fluorides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CF ₄	Tetrafluoromethane	21.07992	21.016	-0.00303
CHF ₃	Trifluoromethane	19.28398	19.362	0.00405
CH ₂ F ₂	Difluoromethane	18.22209	18.280	0.00314
C ₃ H ₇ F	1-Fluoropropane	41.86745	41.885	0.00041
C ₃ H ₇ F	2-Fluoropropane	41.96834	41.963	-0.00012

Table 1.6. The calculated and experimental total bond energies of alkyl chlorides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CCl ₄	Tetrachloromethane	13.43181	13.448	0.00123
CHCl ₃	Trichloromethane	14.49146	14.523	0.00217
CH ₂ Cl ₂	Dichloromethane	15.37248	15.450	0.00499
CH ₃ Cl	Chloromethane	16.26302	16.312	0.00299
C ₂ H ₅ Cl	Chloroethane	28.61064	28.571	-0.00138
C ₃ H ₇ Cl	1-Chloropropane	40.76834	40.723	-0.00112
C ₃ H ₇ Cl	2-Chloropropane	40.86923	40.858	-0.00028
C ₄ H ₉ Cl	1-Chlorobutane	52.92604	52.903	-0.00044
C ₄ H ₉ Cl	2-Chlorobutane	53.02693	52.972	-0.00104
C ₄ H ₉ Cl	1-Chloro-2-methylpropane	52.99860	52.953	-0.00085
C ₄ H ₉ Cl	2-Chloro-2-methylpropane	53.21057	53.191	-0.00037
C ₅ H ₁₁ Cl	1-Chloropentane	65.08374	65.061	-0.00034
C ₅ H ₁₁ Cl	1-Chloro-3-methylbutane	65.15630	65.111	-0.00069
C ₅ H ₁₁ Cl	2-Chloro-2-methylbutane	65.36827	65.344	-0.00037
C ₅ H ₁₁ Cl	2-Chloro-3-methylbutane	65.16582	65.167	0.00002
C ₆ H ₁₃ Cl	2-Chlorohexane	77.34233	77.313	-0.00038
C ₈ H ₁₇ Cl	1-Chlorooctane	101.55684	101.564	0.00007
C ₁₂ H ₂₅ Cl	1-Chlorododecane	150.18764	150.202	0.00009
C ₁₈ H ₃₇ Cl	1-Chlorooctadecane	223.13384	223.175	0.00018

Table 1.7. The calculated and experimental total bond energies of alkyl bromides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CBr ₄	Tetrabromomethane	11.25929	11.196	-0.00566
CHBr ₃	Tribromomethane	12.87698	12.919	0.00323
CH ₃ Br	Bromomethane	15.67551	15.732	0.00360
C ₂ H ₅ Br	Bromoethane	28.03939	27.953	-0.00308
C ₃ H ₇ Br	1-Bromopropane	40.19709	40.160	-0.00093
C ₃ H ₇ Br	2-Bromopropane	40.29798	40.288	-0.00024
C ₅ H ₁₀ Br ₂	2,3-Dibromo-2-methylbutane	63.48143	63.477	-0.00007
C ₆ H ₁₃ Br	1-Bromohexane	76.67019	76.634	-0.00047
C ₇ H ₁₅ Br	1-Bromoheptane	88.82789	88.783	-0.00051
C ₈ H ₁₇ Br	1-Bromooctane	100.98559	100.952	-0.00033
C ₁₂ H ₂₅ Br	1-Bromododecane	149.61639	149.573	-0.00029
C ₁₆ H ₃₃ Br	1-Bromohexadecane	198.24719	198.192	-0.00028

Table 1.8. The calculated and experimental total bond energies of alkyl iodides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CHI ₃	Triiodomethane	10.35888	10.405	0.00444
CH ₂ I ₂	Diiodomethane	12.94614	12.921	-0.00195
CH ₃ I	Iodomethane	15.20294	15.163	-0.00263
C ₂ H ₅ I	Iodoethane	27.36064	27.343	-0.00066
C ₃ H ₇ I	1-Iodopropane	39.51834	39.516	-0.00006
C ₃ H ₇ I	2-Iodopropane	39.61923	39.623	0.00009
C ₄ H ₉ I	2-Iodo-2-methylpropane	51.96057	51.899	-0.00119

Table 1.9. The calculated and experimental total bond energies of alkene halides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₃ Cl	Chloroethene	22.46700	22.505	0.00170
C ₃ H ₅ Cl	2-Chloropropene	35.02984	35.05482	0.00071

Table 1.10. The calculated and experimental total bond energies of alcohols using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH ₄ O	Methanol	21.11038	21.131	0.00097
C ₂ H ₆ O	Ethanol	33.40563	33.428	0.00066
C ₃ H ₈ O	1-Propanol	45.56333	45.584	0.00046
C ₃ H ₈ O	2-Propanol	45.72088	45.766	0.00098
C ₄ H ₁₀ O	1-Butanol	57.72103	57.736	0.00026
C ₄ H ₁₀ O	2-Butanol	57.87858	57.922	0.00074
C ₄ H ₁₀ O	2-Methyl-1-propananol	57.79359	57.828	0.00060
C ₄ H ₁₀ O	2-Methyl-2-propananol	58.15359	58.126	-0.00048
C ₅ H ₁₂ O	1-Pentanol	69.87873	69.887	0.00011

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₅ H ₁₂ O	2-Pentanol	70.03628	70.057	0.00029
C ₅ H ₁₂ O	3-Pentanol	70.03628	70.097	0.00087
C ₅ H ₁₂ O	2-Methyl-1-butananol	69.95129	69.957	0.00008
C ₅ H ₁₂ O	3-Methyl-1-butananol	69.95129	69.950	-0.00002
C ₅ H ₁₂ O	2-Methyl-2-butananol	70.31129	70.246	-0.00092
C ₅ H ₁₂ O	3-Methyl-2-butananol	69.96081	70.083	0.00174
C ₆ H ₁₄ O	1-Hexanol	82.03643	82.054	0.00021
C ₆ H ₁₄ O	2-Hexanol	82.19398	82.236	0.00052
C ₇ H ₁₆ O	1-Heptanol	94.19413	94.214	0.00021
C ₈ H ₁₈ O	1-Octanol	106.35183	106.358	0.00006
C ₈ H ₁₈ O	2-Ethyl-1-hexanol	106.42439	106.459	0.00032
C ₉ H ₂₀ O	1-Nonanol	118.50953	118.521	0.00010
C ₁₀ H ₂₂ O	1-Decanol	130.66723	130.676	0.00007
C ₁₂ H ₂₆ O	1-Dodecanol	154.98263	154.984	0.00001
C ₁₆ H ₃₄ O	1-Hexadecanol	203.61343	203.603	-0.00005

Table 1.11. The calculated and experimental total bond energies of ethers using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₆ O	Dimethyl ether	32.84496	32.902	0.00174
C ₃ H ₈ O	Ethyl methyl ether	45.19710	45.183	-0.00030
C ₄ H ₁₀ O	Diethyl ether	57.54924	57.500	-0.00086
C ₄ H ₁₀ O	Methyl propyl ether	57.35480	57.355	0.00000
C ₄ H ₁₀ O	Isopropyl methyl ether	57.45569	57.499	0.00075
C ₆ H ₁₄ O	Dipropyl ether	81.86464	81.817	-0.00059
C ₆ H ₁₄ O	Disopropyl ether	82.06642	82.088	0.00026
C ₆ H ₁₄ O	t-Butyl ethyl ether	82.10276	82.033	-0.00085
C ₇ H ₁₆ O	t-Butyl isopropyl ether	94.36135	94.438	0.00081
C ₈ H ₁₈ O	Dibutyl ether	106.18004	106.122	-0.00055
C ₈ H ₁₈ O	Di-sec-butyl ether	106.38182	106.410	0.00027
C ₈ H ₁₈ O	Di-t-butyl ether	106.36022	106.425	0.00061
C ₈ H ₁₈ O	t-Butyl isobutyl ether	106.65628	106.497	-0.00218

Table 1.12. The calculated and experimental total bond energies of 1° amines using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH ₅ N	Methylamine	23.88297	23.857	-0.00110
C ₂ H ₇ N	Ethylamine	36.04067	36.062	0.00060
C ₃ H ₉ N	Propylamine	48.19837	48.243	0.00092
C ₄ H ₁₁ N	Butylamine	60.35607	60.415	0.00098
C ₄ H ₁₁ N	sec-Butylamine	60.45696	60.547	0.00148
C ₄ H ₁₁ N	t-Butylamine	60.78863	60.717	-0.00118
C ₄ H ₁₁ N	Isobutylamine	60.42863	60.486	0.00094

Table 1.13. The calculated and experimental total bond energies of 2° amines using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₇ N	Dimethylamine	35.76895	35.765	-0.00012
C ₄ H ₁₁ N	Diethylamine	60.22930	60.211	-0.00030
C ₆ H ₁₅ N	Dipropylamine	84.54470	84.558	0.00016
C ₆ H ₁₅ N	Diisopropylamine	84.74648	84.846	0.00117
C ₈ H ₁₉ N	Dibutylamine	108.86010	108.872	0.00011
C ₈ H ₁₉ N	Diisobutylamine	109.00522	109.106	0.00092

Table 1.14. The calculated and experimental total bond energies of 3° amines using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₃ H ₉ N	Trimethylamine	47.83338	47.761	-0.00152
C ₆ H ₁₅ N	Triethylamine	84.30648	84.316	0.00012
C ₉ H ₂₁ N	Tripropylamine	120.77958	120.864	0.00070

Table 1.15. The calculated and experimental total bond energies of aldehydes using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH ₂ O	Formaldehyde	15.64628	15.655	0.00056
C ₂ H ₄ O	Acetaldehyde	28.18711	28.198	0.00039
C ₃ H ₆ O	Propanal	40.34481	40.345	0.00000
C ₄ H ₈ O	Butanal	52.50251	52.491	-0.00022
C ₄ H ₈ O	Isobutanal	52.60340	52.604	0.00001
C ₅ H ₁₀ O	Pentanal	64.66021	64.682	0.00034
C ₇ H ₁₄ O	Heptanal	88.97561	88.942	-0.00038
C ₈ H ₁₆ O	Octanal	101.13331	101.179	0.00045
C ₈ H ₁₆ O	2-Ethylhexanal	101.20587	101.259	0.00053

Table 1.16. The calculated and experimental total bond energies of ketones using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₃ H ₆ O	Acetone	40.68472	40.672	-0.00031
C ₄ H ₈ O	2-Butanone	52.84242	52.84	-0.00005
C ₅ H ₁₀ O	2-Pentanone	65.00012	64.997	-0.00005
C ₅ H ₁₀ O	3-Pentanone	65.00012	64.997	-0.00005
C ₅ H ₁₀ O	3-Methyl-2-butanone	65.10101	65.036	-0.00099
C ₆ H ₁₂ O	2-Hexanone	77.15782	77.152	-0.00008
C ₆ H ₁₂ O	3-Hexanone	77.15782	77.138	-0.00025
C ₆ H ₁₂ O	2-Methyl-3-pentanone	77.25871	77.225	-0.00043
C ₆ H ₁₂ O	3,3-Dimethyl-2-butanone	77.29432	77.273	-0.00028
C ₇ H ₁₄ O	3-Heptanone	89.31552	89.287	-0.00032
C ₇ H ₁₄ O	4-Heptanone	89.31552	89.299	-0.00018
C ₇ H ₁₄ O	2,2-Dimethyl-3-pentanone	89.45202	89.458	0.00007

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₇ H ₁₄ O	2,4-Dimethyl-3-pentanone	89.51730	89.434	-0.00093
C ₈ H ₁₆ O	2,2,4-Trimethyl-3-pentanone	101.71061	101.660	-0.00049
C ₉ H ₁₈ O	2-Nonanone	113.63092	113.632	0.00001
C ₉ H ₁₈ O	5-Nonanone	113.63092	113.675	0.00039
C ₉ H ₁₈ O	2,6-Dimethyl-4-heptanone	113.77604	113.807	0.00027

Table 1.17. The calculated and experimental total bond energies of carboxylic acids using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH ₂ O ₂	Formic acid	21.01945	21.036	0.00079
C ₂ H ₄ O ₂	Acetic acid	33.55916	33.537	-0.00066
C ₃ H ₆ O ₂	Propanoic acid	45.71686	45.727	0.00022
C ₄ H ₈ O ₂	Butanoic acid	57.87456	57.883	0.00015
C ₅ H ₁₀ O ₂	Pentanoic acid	70.03226	69.995	-0.00053
C ₅ H ₁₀ O ₂	3-Methylbutanoic acid	70.10482	70.183	0.00111
C ₅ H ₁₀ O ₂	2,2-Dimethylpropanoic acid	70.31679	69.989	-0.00468
C ₆ H ₁₂ O ₂	Hexanoic acid	82.18996	82.149	-0.00050
C ₇ H ₁₄ O ₂	Heptanoic acid	94.34766	94.347	0.00000
C ₈ H ₁₆ O ₂	Octanoic acid	106.50536	106.481	-0.00022
C ₉ H ₁₈ O ₂	Nonanoic acid	118.66306	118.666	0.00003
C ₁₀ H ₂₀ O ₂	Decanoic acid	130.82076	130.795	-0.00020
C ₁₂ H ₂₄ O ₂	Dodecanoic acid	155.13616	155.176	0.00026
C ₁₄ H ₂₈ O ₂	Tetradecanoic acid	179.45156	179.605	0.00085
C ₁₅ H ₃₀ O ₂	Pentadecanoic acid	191.60926	191.606	-0.00002
C ₁₆ H ₃₂ O ₂	Hexadecanoic acid	203.76696	203.948	0.00089
C ₁₈ H ₃₆ O ₂	Stearic acid	228.08236	228.298	0.00094
C ₂₀ H ₄₀ O ₂	Eicosanoic acid	252.39776	252.514	0.00046

Table 1.18. The calculated and experimental total bond energies of carboxylic acid esters using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₄ O ₂	Methyl formate	32.71076	32.762	0.00156
C ₃ H ₆ O ₂	Methyl acetate	45.24849	45.288	0.00087
C ₆ H ₁₂ O ₂	Methyl pentanoate	81.72159	81.726	0.00005
C ₇ H ₁₄ O ₂	Methyl hexanoate	93.87929	93.891	0.00012
C ₈ H ₁₆ O ₂	Methyl heptanoate	106.03699	106.079	0.00040
C ₉ H ₁₈ O ₂	Methyl octanoate	118.19469	118.217	0.00018
C ₁₀ H ₂₀ O ₂	Methyl nonanoate	130.35239	130.373	0.00016
C ₁₁ H ₂₂ O ₂	Methyl decanoate	142.51009	142.523	0.00009
C ₁₂ H ₂₄ O ₂	Methyl undecanoate	154.66779	154.677	0.00006
C ₁₃ H ₂₆ O ₂	Methyl dodecanoate	166.82549	166.842	0.00010
C ₁₄ H ₂₈ O ₂	Methyl tridecanoate	178.98319	179.000	0.00009
C ₁₅ H ₃₀ O ₂	Methyl tetradecanoate	191.14089	191.170	0.00015
C ₁₆ H ₃₂ O ₂	Methyl pentadecanoate	203.29859	203.356	0.00028
C ₄ H ₈ O ₂	Propyl formate	57.76366	57.746	-0.00030
C ₄ H ₈ O ₂	Ethyl acetate	57.63888	57.548	-0.00157
C ₅ H ₁₀ O ₂	Isopropyl acetate	69.89747	69.889	-0.00013
C ₅ H ₁₀ O ₂	Ethyl propanoate	69.79658	69.700	-0.00139
C ₆ H ₁₂ O ₂	Butyl acetate	81.95428	81.873	-0.00099
C ₆ H ₁₂ O ₂	t-Butyl acetate	82.23881	82.197	-0.00051

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₆ H ₁₂ O ₂	Methyl 2,2-dimethylpropanoate	82.00612	81.935	-0.00087
C ₇ H ₁₄ O ₂	Ethyl pentanoate	94.11198	94.033	-0.00084
C ₇ H ₁₄ O ₂	Ethyl 3-methylbutanoate	94.18454	94.252	0.00072
C ₇ H ₁₄ O ₂	Ethyl 2,2-dimethylpropanoate	94.39651	94.345	-0.00054
C ₈ H ₁₆ O ₂	Isobutyl isobutanoate	106.44313	106.363	-0.00075
C ₈ H ₁₆ O ₂	Propyl pentanoate	106.26968	106.267	-0.00003
C ₈ H ₁₆ O ₂	Isopropyl pentanoate	106.37057	106.384	0.00013
C ₉ H ₁₈ O ₂	Butyl pentanoate	118.42738	118.489	0.00052
C ₉ H ₁₈ O ₂	sec-Butyl pentanoate	118.52827	118.624	0.00081
C ₉ H ₁₈ O ₂	Isobutyl pentanoate	118.49994	118.576	0.00064

Table 1.19. The calculated and experimental total bond energies of amides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH ₃ NO	Formamide	23.68712	23.697	0.00041
C ₂ H ₅ NO	Acetamide	36.15222	36.103	-0.00135
C ₃ H ₇ NO	Propanamide	48.30992	48.264	-0.00094
C ₄ H ₉ NO	Butanamide	60.46762	60.449	-0.00030
C ₄ H ₉ NO	2-Methylpropanamide	60.51509	60.455	-0.00099
C ₅ H ₁₁ NO	Pentanamide	72.62532	72.481	-0.00200
C ₅ H ₁₁ NO	2,2-Dimethylpropanamide	72.67890	72.718	0.00054
C ₆ H ₁₃ NO	Hexanamide	84.78302	84.780	-0.00004
C ₈ H ₁₇ NO	Octanamide	109.09842	109.071	-0.00025

Table 1.20. The calculated and experimental total bond energies of N-alkyl and N,N-dialkyl amides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₃ H ₇ NO	N,N-Dimethylformamide	47.53142	47.574	0.00090
C ₄ H ₉ NO	N,N-Dimethylacetamide	59.91404	59.890	-0.00041
C ₆ H ₁₃ NO	N-Butylacetamide	84.63649	84.590	-0.00055

Table 1.21. The calculated and experimental total bond energies of urea using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH ₄ N ₂ O	Urea	31.35919	31.393	0.00108

Table 1.22. The calculated and experimental total bond energies of acid halide using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₃ ClO	Acetyl chloride	28.02174	27.990	-0.00115

Table 1.23. The calculated and experimental total bond energies of acid anhydrides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₄ H ₆ O ₃	Acetic anhydride	56.94096	56.948	0.00013
C ₆ H ₁₀ O ₃	Propanoic anhydride	81.25636	81.401	0.00177

Table 1.24. The calculated and experimental total bond energies of nitriles using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₃ N	Acetonitrile	25.72060	25.77	0.00174
C ₃ H ₅ N	Propanenitrile	37.87830	37.94	0.00171
C ₄ H ₇ N	Butanenitrile	50.03600	50.08	0.00082
C ₄ H ₇ N	2-Methylpropanenitrile	50.13689	50.18	0.00092
C ₅ H ₉ N	Pentanenitrile	62.19370	62.26	0.00111
C ₅ H ₉ N	2,2-Dimethylpropanenitrile	62.47823	62.40	-0.00132
C ₇ H ₁₃ N	Heptanenitrile	86.50910	86.59	0.00089
C ₈ H ₁₅ N	Octanenitrile	98.66680	98.73	0.00069
C ₁₀ H ₁₉ N	Decanenitrile	122.98220	123.05	0.00057
C ₁₄ H ₂₇ N	Tetradecanenitrile	171.61300	171.70	0.00052

Table 1.25. The calculated and experimental total bond energies of thiols using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
HS	Hydrogen Sulfide	3.77430	3.653	-0.03320
H ₂ S	Dihydrogen Sulfide	7.56058	7.605	0.00582
CH ₄ S	Methanethiol	19.60264	19.575	-0.00141
C ₂ H ₆ S	Ethanethiol	31.76034	31.762	0.00005
C ₃ H ₈ S	1-Propanethiol	43.91804	43.933	0.00035
C ₃ H ₈ S	2-Propanethiol	44.01893	44.020	0.00003
C ₄ H ₁₀ S	1-Butanethiol	56.07574	56.089	0.00024
C ₄ H ₁₀ S	2-Butanethiol	56.17663	56.181	0.00009
C ₄ H ₁₀ S	2-Methyl-1-propanethiol	56.14830	56.186	0.00066
C ₄ H ₁₀ S	2-Methyl-2-propanethiol	56.36027	56.313	-0.00084
C ₅ H ₁₂ S	2-Methyl-1-butanethiol	68.30600	68.314	0.00012
C ₅ H ₁₂ S	1-Pentanethiol	68.23344	68.264	0.00044
C ₅ H ₁₂ S	2-Methyl-2-butanethiol	68.51797	68.441	-0.00113
C ₅ H ₁₂ S	3-Methyl-2-butanethiol	68.31552	68.381	0.00095
C ₅ H ₁₂ S	2,2-Dimethyl-1-propanethiol	68.51797	68.461	-0.00084
C ₆ H ₁₄ S	1-Hexanethiol	80.39114	80.416	0.00031
C ₆ H ₁₄ S	2-Methyl-2-pentanethiol	80.67567	80.607	-0.00085
C ₆ H ₁₄ S	2,3-Dimethyl-2-butanethiol	80.71992	80.603	-0.00145
C ₇ H ₁₆ S	1-Heptanethiol	92.54884	92.570	0.00023
C ₁₀ H ₂₂ S	1-Decanethiol	129.02194	129.048	0.00020

Table 1.26. The calculated and experimental total bond energies of sulfides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₆ S	Dimethyl sulfide	31.65668	31.672	0.00048
C ₃ H ₈ S	Ethyl methyl sulfide	43.81438	43.848	0.00078
C ₄ H ₁₀ S	Diethyl sulfide	55.97208	56.043	0.00126
C ₄ H ₁₀ S	Methyl propyl sulfide	55.97208	56.029	0.00102
C ₄ H ₁₀ S	Isopropyl methyl sulfide	56.07297	56.115	0.00075
C ₅ H ₁₂ S	Butyl methyl sulfide	68.12978	68.185	0.00081
C ₅ H ₁₂ S	t-Butyl methyl sulfide	68.28245	68.381	0.00144
C ₅ H ₁₂ S	Ethyl propyl sulfide	68.12978	68.210	0.00117
C ₅ H ₁₂ S	Ethyl isopropyl sulfide	68.23067	68.350	0.00174
C ₆ H ₁₄ S	Diisopropyl sulfide	80.48926	80.542	0.00065
C ₆ H ₁₄ S	Butyl ethyl sulfide	80.28748	80.395	0.00133
C ₆ H ₁₄ S	Methyl pentyl sulfide	80.28748	80.332	0.00056
C ₈ H ₁₈ S	Dibutyl sulfide	104.60288	104.701	0.00094
C ₈ H ₁₈ S	Di-sec-butyl sulfide	104.80466	104.701	-0.00099
C ₈ H ₁₈ S	Di-t-butyl sulfide	104.90822	104.920	0.00011
C ₈ H ₁₈ S	Diisobutyl sulfide	104.74800	104.834	0.00082
C ₁₀ H ₂₂ S	Ethyl propyl sulfide	128.91828	128.979	0.00047
C ₁₀ H ₂₂ S	Diisopentyl sulfide	129.06340	129.151	0.00068

Table 1.27. The calculated and experimental total bond energies of disulfides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₆ S ₂	Dimethyl disulfide	34.48127	34.413	-0.00199
C ₄ H ₁₀ S ₂	Diethyl disulfide	58.79667	58.873	0.00129
C ₆ H ₁₄ S ₂	Dipropyl disulfide	83.11207	83.169	0.00068
C ₈ H ₁₈ S ₂	Di-t-butyl disulfide	107.99653	107.919	-0.00072

Table 1.28. The calculated and experimental total bond energies of sulfoxides using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₆ SO	Dimethyl sulfoxide	35.52450	35.435	-0.00253
C ₄ H ₁₀ SO	Diethyl sulfoxide	59.83990	59.891	0.00085
C ₆ H ₁₄ SO	Dipropyl sulfoxide	84.15530	84.294	0.00165

Table 1.29. The calculated and experimental total bond energies of sulfones using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₆ SO ₂	Dimethyl sulfone	40.27588	40.316	0.00100

Table 1.30. The calculated and experimental total bond energies of sulfites using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
$C_2H_6SO_3$	Dimethyl sulfite	43.95058	44.042	0.00207
$C_4H_{10}SO_3$	Diethyl sulfite	68.54939	68.648	0.00143
$C_8H_{18}SO_3$	Dibutyl sulfite	117.18019	117.191	0.00009

Table 1.31. The calculated and experimental total bond energies of sulfates using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
$C_2H_6SO_4$	Dimethyl sulfate	48.70617	48.734	0.00058
$C_4H_{10}SO_4$	Diethyl sulfate	73.30077	73.346	0.00061
$C_6H_{14}SO_4$	Dipropyl sulfate	97.61617	97.609	-0.00008

Table 1.32. The calculated and experimental total bond energies of nitro alkanes using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH_3NO_2	Nitromethane	25.14934	25.107	-0.00168
$C_2H_5NO_2$	Nitroethane	37.30704	37.292	-0.00040
$C_3H_7NO_2$	1-Nitropropane	49.46474	49.451	-0.00028
$C_3H_7NO_2$	2-Nitropropane	49.56563	49.602	0.00074
$C_4H_9NO_2$	1-Nitrobutane	61.62244	61.601	-0.00036
$C_4H_9NO_2$	2-Nitroisobutane	61.90697	61.945	0.00061
$C_5H_{11}NO_2$	1-Nitropentane	73.78014	73.759	-0.00028

Table 1.33. The calculated and experimental total bond energies of nitrite using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH_3NO_2	Methyl nitrite	24.92328	24.955	0.00126

Table 1.34. The calculated and experimental total bond energies of nitrate using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH_3NO_3	Methyl nitrate	28.18536	28.117	-0.00244
$C_2H_5NO_3$	Ethyl nitrate	40.34306	40.396	0.00131
$C_3H_7NO_3$	Propyl nitrate	52.50076	52.550	0.00093
$C_3H_7NO_3$	Isopropyl nitrate	52.60165	52.725	0.00233

Table 1.35. The calculated and experimental total bond energies of conjugated alkenes using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₅ H ₈	Cyclopentene	54.83565	54.86117	0.00047
C ₄ H ₆	1,3 Butadiene	42.09159	42.12705	0.00084
C ₅ H ₈	1,3 Pentadiene	54.40776	54.42484	0.00031
C ₅ H ₈	1,4 Pentadiene	54.03745	54.11806	0.00149
C ₅ H ₆	1,3 Cyclopentadiene	49.27432	49.30294	0.00058

Table 1.36. The calculated and experimental total bond energies of aromatics and heterocyclic aromatics using closed-form equations having integers and fundamental constants only.

Formula	Name	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₆ H ₆	Benzene	57.26008	57.26340	0.00006
C ₆ H ₅ Cl	Chlorobenzene	56.55263	56.581	0.00051
C ₆ H ₄ Cl ₂	m-dichlorobenzene	55.84518	55.852	0.00012
C ₆ H ₃ Cl ₃	1,2,3-trichlorobenzene	55.13773	55.077	-0.00111
C ₆ H ₃ Cl ₃	1,3,5-trichlorobenzene	55.29542	55.255	-0.00073
C ₆ Cl ₆	Hexachlorobenzene	52.57130	52.477	-0.00179
C ₆ H ₅ NO ₂	Nitrobenzene	65.18754	65.217	0.00046
C ₇ H ₈	Toluene	69.48425	69.546	0.00088
C ₇ H ₅ O ₂	Benzoic acid	73.76938	73.762	-0.00009
C ₇ H ₅ ClO ₂	2-chlorobenzoic acid	73.06193	73.082	0.00027
C ₇ H ₅ ClO ₂	3-chlorobenzoic acid	73.26820	73.261	-0.00010
C ₇ H ₅ ClO ₂	4-chlorobenzoic acid	73.26820	73.247	-0.00028
C ₆ H ₇ N	Aniline	64.43373	64.374	-0.00093
C ₇ H ₉ N	2-methylaniline	76.62345	76.643	-0.00025
C ₇ H ₉ N	3-methylaniline	76.62345	76.661	0.00050
C ₇ H ₉ N	4-methylaniline	76.62345	76.654	0.00040
C ₆ H ₆ N ₂ O ₂	2-nitroaniline	72.47476	72.424	-0.00070
C ₆ H ₆ N ₂ O ₂	3-nitroaniline	72.47476	72.481	-0.00009
C ₆ H ₆ N ₂ O ₂	4-nitroaniline	72.47476	72.476	-0.00002
C ₇ H ₇ NO ₂	Aniline-2-carboxylic acid	80.90857	80.941	0.00041
C ₇ H ₇ NO ₂	Aniline-3-carboxylic acid	80.90857	80.813	-0.00118
C ₇ H ₇ NO ₂	Aniline-4-carboxylic acid	80.90857	80.949	0.00050
C ₆ H ₆ O	Phenol	61.75817	61.704	-0.00087
C ₆ H ₄ N ₂ O ₅	2,4-dinitrophenol	77.61308	77.642	0.00037
C ₆ H ₈ O	Anisole	73.39006	73.355	-0.00047
C ₁₀ H ₈	Naphthalene	90.74658	90.79143	0.00049
C ₄ H ₅ N	Pyrrole	44.81090	44.785	-0.00057
C ₄ H ₄ O	Furan	41.67782	41.692	0.00033
C ₄ H ₄ S	Thiophene	40.42501	40.430	0.00013
C ₃ H ₄ N ₂	Imidazole	39.76343	39.74106	-0.00056
C ₅ H ₅ N	Pyridine	51.91802	51.87927	-0.00075
C ₄ H ₄ N ₂	Pyrimidine	46.57597	46.51794	-0.00125
C ₄ H ₄ N ₂	Pyrazine	46.57597	46.51380	0.00095
C ₉ H ₇ N	Quinoline	85.40453	85.48607	0.00178
C ₉ H ₇ N	Isoquinoline	85.40453	85.44358	0.00046
C ₈ H ₇ N	Indole	78.52215	78.514	-0.00010
C ₅ H ₅ N ₅	Adenine	70.83735	70.79811	-0.00055

REFERENCES FOR SECTION IV

1. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 86th Edition, CRC Press, Taylor & Francis, Boca Raton, (2005-6), pp. 9-19 to 9-45.
- 5 2. G. Herzberg, *Molecular Spectra and Molecular Structure II. Infrared and Raman Spectra of Polyatomic Molecules*, Van Nostrand Reinhold Company, New York, New York, (1945), p. 344.
3. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 86th Edition, CRC Press, Taylor & Francis, Boca Raton, (2005-6), pp. 9-63; 9-64 to 9-76.
- 10 4. R. J. Fessenden, J. S. Fessenden, *Organic Chemistry*, Willard Grant Press. Boston, Massachusetts, (1979), p. 320.
5. cyclohexane at <http://webbook.nist.gov/>.
6. G. Herzberg, *Molecular Spectra and Molecular Structure II. Infrared and Raman Spectra of Polyatomic Molecules*, Van Nostrand Reinhold Company, New York, New York, (1945), p. 326.
- 15 7. b1 2-methyl-1-propene at <http://webbook.nist.gov/>.
8. a1 2-methyl-1-propene at <http://webbook.nist.gov/>.
9. 2-butyne at <http://webbook.nist.gov/>.
10. trifluoromethane at <http://webbook.nist.gov/>.
- 20 11. fluoroethane at <http://webbook.nist.gov/>.
12. G. Herzberg, *Molecular Spectra and Molecular Structure II. Infrared and Raman Spectra of Polyatomic Molecules*, Krieger Publishing Company, Malabar, FL, (1991), p. 195.
13. chloromethane at <http://webbook.nist.gov/>.
14. methyl bromide at <http://webbook.nist.gov/>.
- 25 15. tetrabromomethane at <http://webbook.nist.gov/>.
16. methyl iodide at <http://webbook.nist.gov/>.
17. K. P. Huber, G. Herzberg, *Molecular Spectra and Molecular Structure, IV. Constants of Diatomic Molecules*, Van Nostrand Reinhold Company, New York, (1979).
18. J. Crovisier, Molecular Database—Constants for molecules of astrophysical interest in the gas phase: photodissociation, microwave and infrared spectra, Ver. 4.2, Observatoire de Paris, Section de Meudon, Meudon, France, May 2002, pp. 34–37, available at <http://www.usr.obspm.fr/~crovisie/>.
- 30 19. methanol at <http://webbook.nist.gov/>.

20. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 46.
21. dimethyl ether at <http://webbook.nist.gov/>.
- 5 22. T. Amano, P. F. Bernath, R. W. McKellar, "Direct observation of the ν_1 and ν_3 fundamental bands of NH_2 by difference frequency laser spectroscopy", *J. Mol. Spectrosc.*, Vol. 94, (1982), pp. 100-113.
23. methylamine at <http://webbook.nist.gov/>.
24. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 79th Edition, CRC Press, Boca
10 Raton, Florida, (1998-9), pp. 9-80 to 9-85.
25. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 482.
26. acetaldehyde at <http://webbook.nist.gov/>.
- 15 27. acetone at <http://webbook.nist.gov/>.
28. 2-butanone at <http://webbook.nist.gov/>.
29. acetic acid at <http://webbook.nist.gov/>.
30. formic acid at <http://webbook.nist.gov/>.
31. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and
20 Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 138.
32. methyl formate at <http://webbook.nist.gov/>.
33. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and
25 Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 144.
34. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 147.
35. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and
30 Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 143.

36. H. J. Vledder, F. C. Mijlhoff, J. C. Leyte, C. Romers, "An electron diffraction investigation of the molecular structure of gaseous acetic anhydride", *Journal of Molecular Structure*, Vol. 7, Issues 3-4, (1971), pp. 421-429.
37. acetonitrile at <http://webbook.nist.gov/>.
- 5 38. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 86th Edition, CRC Press, Taylor & Francis, Boca Raton, (2005-6), pp. 10-202 to 10-204.
39. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 86th Edition, CRC Press, Taylor & Francis, Boca Raton, (2005-6), p. 9-79.
40. D. R. Lide, *CRC Handbook of Chemistry and Physics*, 86th Edition, CRC Press, Taylor & Francis, Boca Raton, (2005-6), pp. 9-82 to 9-86.
- 10 41. thiirane at <http://webbook.nist.gov/>.
42. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 242.
- 15 43. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 241.
44. R. M. Ibberson, M. T. F. Telling, S. Parsons, "Structural determination and phase transition behavior of dimethyl sulfate", *Acta. Cryst.*, Vol. B62, (2006), pp. 280-286.
- 20 45. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 181.
46. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 480.
- 25 47. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 187.
48. 1,3-butadiene at <http://webbook.nist.gov/>.
- 30 49. G. Herzberg, *Molecular Spectra and Molecular Structure II. Infrared and Raman Spectra of Polyatomic Molecules*, Van Nostrand Reinhold Company, New York, New York, (1945), pp. 362-369.

50. W. I. F. David, R. M. Ibberson, G. A. Jeffrey, J. R. Ruble, "The structure analysis of deuterated benzene and deuterated nitromethane by pulsed-neutron powder diffraction: a comparison with single crystal neutron analysis", *Physica B* (1992), 180 & 181, pp. 597-600..
- 5 51. G. A. Jeffrey, J. R. Ruble, R. K. McMullan, J. A. Pople, "The crystal structure of deuterated benzene," *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 414, No. 1846, (Nov. 9, 1987), pp. 47-57.
52. H. B. Burgi, S. C. Capelli, "Getting more out of crystal-structure analyses," *Helvetica Chimica Acta*, Vol. 86, (2003), pp. 1625-1640.
- 10 53. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 481.
54. R. Boese, D. Blaser, M. Nussbaumer, T. M. Krygowski, "Low temperature crystal and molecular structure of nitrobenzene", *Structural Chemistry*, Vol. 3, No. 5, (1992), pp. 15 363-368.
55. G. A. Sim, J. M. Robertson, T. H. Goodwin, "The crystal and molecular structure of benzoic acid", *Acta Cryst.*, Vol. 8, (1955), pp.157-164
56. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace 20 Jovanovich, Boston, (1991), p. 301.
57. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 303.
58. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and 25 Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), p. 207.
59. S. Martinez-Carrera, "The crystal structure of Imidazole at -150°C", *Acta. Cryst.*, Vol. 20, (1966), pp. 783-789.
60. D. Lin-Vien. N. B. Colthup, W. G. Fateley, J. G. Grasselli, *The Handbook of Infrared and 30 Raman Frequencies of Organic Molecules*, Academic Press, Inc., Harcourt Brace Jovanovich, Boston, (1991), pp. 296-297.

61. H. Sternglanz, C. E. Bugg, "Conformations of N⁶-monosubstituted adenine derivatives crystal structure of N⁶-methyladenine", *Biochimica et Biophysica Acta*, Vol. 308, (1973), pp. 1-8.

Section V1

Software Program

The present invention relates to a system of computing and rendering the nature of at least one specie selected from a group of diatomic molecules having at least one atom that is other than hydrogen, polyatomic molecules, molecular ions, polyatomic molecular ions, or molecular radicals, or any functional group therein, comprising physical, Maxwellian solutions of charge, mass, and current density functions of said specie, said system comprising: processing means for processing physical, Maxwellian equations representing charge, mass, and current density functions of said specie; and an output device in communication with the processing means for displaying said physical, Maxwellian solutions of charge, mass, and current density functions of said specie.

In one embodiment, for example, the system comprises five components: (1) the graphical user interface (GUI); (2) the routine for parsing between an input chemical structure or name and taking the input and activating a routine to call up the parts of the molecule (functional groups), which are used for determining the energies and structure to be rendered; (3) the functional-group data base that has an organization of the theoretical solutions; (4) the rendering engine, which calculates and enables manipulations of the image, such as a three-dimensional model in response to commands, as well as responds to commands for data parameters corresponding to the image such as bond energies and charge distribution and geometrical parameters; and (5) data transfer system for inputting numerical data into or out of the computational components and storage components of the main system. The system further comprises spreadsheets with solutions of the bond parameters with output in any standard spreadsheet format. The system also comprises a data-handling program to transfer data from the spreadsheets into the main program.

The output may be, for example, at least one of graphical, simulation, text, and numerical data. The output may be the calculation of at least one of: (1) a bond distance between two atoms; (2) a bond angle between three of the atoms; (3) a bond energy between two atoms; (4) orbital intercept distances and angles; and (5) charge-density functions of atomic, hybridized, and molecular orbitals, wherein the bond distance, bond angle, and bond energy are calculated from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that a bound electron(s) does not radiate under acceleration.

In other embodiments, the charge, current, energy, and geometrical parameters are output to be inputs to other programs that may be used in further applications. For example, the data of heats of formation may be input to another program to be used to predict stability (existence of compounds) equilibrium constants and to predict synthetic pathways. That is, a novel composition of matter may be discovered by calculating at least one of a bond distance between two of the atoms, a bond angle between three of the atoms, and a bond energy between two of the atoms, orbital intercept distances and angles, charge-density functions of atomic, hybridized, and molecular orbitals, the bond distance, bond angle, and bond energy being calculated from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that a bound electron(s) does not radiate under acceleration. The charge and current density functions may be used to predict the electric and magnetic fields of the species to determine other properties due to the interaction of the fields between species. These fields and the predictions of field interactions may be computed using Maxwell's equations. In one embodiment, finite-element analysis is used to predict or calculate the interaction and resulting properties, such as the freezing point, boiling point, density, viscosity, and refractive index. Furthermore, the output data can be used to give thermodynamic, spectroscopic, and other properties, aid in drug design and other applications with or without direct visualization. Furthermore, the data can be input into other programs of the system, which calculate thermodynamic and other properties, or performs a simulation, such as a chemical reaction or molecular dynamics.

The output data may be used to predict a composition of matter comprising a plurality of atoms, the improvement comprising a novel property or use discovered by calculation of at least one of a bond distance between two of the atoms, a bond angle between three of the atoms, and a bond energy between two of the atoms, orbital intercept distances and angles, charge-density functions of atomic, hybridized, and molecular orbitals, the bond distance, bond angle, and bond energy being calculated from physical solutions of the charge, mass, and current density functions of atoms and atomic ions, which solutions are derived from Maxwell's equations using a constraint that a bound electron(s) does not radiate under acceleration. The novel property, for example, may be a new pharmaceutical use, or stability at room temperature of a novel arrangement of atoms or ions.

In one embodiment, the output device of the system is a display that displays at least one of visual or graphical media. The display may be at least one of static or dynamic. At least one of vibration, rotation, and translation may be displayed. The displayed information

may be used for at least one of modeling reactivity, predicting physical properties, and aiding in drug and material design. The output device may be a monitor, video projector, printer, or one-, two- or three-dimensional rendering device. The displayed information may be used to model other molecules and provides utility to anticipate their reactivity and physical properties.

5 Additionally, data may be output and used in the same and additional applications as the rendered models and representations of the calculated physical solutions. The processing means of the system may be a general-purpose computer. The general-purpose computer may comprise a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage device, an input
10 means. The input means may comprise a serial port, USB port, microphone input, camera input, keyboard or mouse. The processing means comprises a special purpose computer or other hardware system. The system may comprise computer program products such as computer readable medium having embodied therein program code means. The computer readable media may be any available media which can be accessed by a general purpose or
15 special purpose computer. The computer readable media may comprise, for example, at least one of RAM, ROM, EPROM, CD ROM, DVD or other optical disk storage, magnetic disk storage or other magnetic storage devices, or any other medium that can embody the desired program code means and which can be accessed by a general purpose or special purpose computer. The program code means may comprise executable instructions and data, which
20 cause a general purpose computer or special purpose computer to perform a certain function of a group of functions. Commercial examples of suitable program language includes, for example, C++, C, JAVA, FORTRAN, Python and Assembly Languages, programmed with an algorithm based on the physical solutions, and the computer may be a PC, mainframe, supercomputer, or cluster of computers. Commercial examples of suitable programs include,
25 for example, APIs like OpenGL, DirectX, FOX GUI toolkit, and Qt. This program may be developed to run on at least one of operating systems like Windows XP, Windows 2000, Windows Vista, MAC OS, MAC OS X, Linux, Unix, Irix and other Unix-type operating systems.

Millsian software is designed to render 3-D models of molecules, molecular ions,
30 molecular radicals, functional groups thereof, and related structure and property information and produce useful data output and application of the parameters of these species, wherein the nature of their bound electrons and chemical bonds are solved using Dr. Randell L. Mills' Classical Quantum Mechanics theory described in the reference: R. L. Mills, "The Grand Unified Theory of Classical Quantum Mechanics", June 2006 Edition, Cadmus Professional

Communications-Science Press Division, Ephrata, PA, ISBN 0963517171, Library of Congress Control Number 2005936834; posted at

<http://www.blacklightpower.com/bookdownload.shtml>, which is incorporated by this reference in its entirety. Figure 76 provides a flow chart diagram, which is an example of a software

5 system that can be utilized for this purpose, which example is not intended to limit the scope of the disclosed inventions. The main parts of this exemplary software system illustrated in Figure 76 will now be further explained:

Start: A user can start the program by running an executable program file. That might
10 be done, for example, by double clicking the program icon on a Windows-based operating system, or typing the name of the executable file on the command line and pressing the 'Enter' key on a Linux or Unix operating system. The program initially starts by reading data files located in specific directories. The names of those directories and locations are fixed according to the type and format of the data files.

15 Data files: There are two types of data files used in the software system: functional group data files and molecule data files. Functional group data files contain information about various functional groups. Functional groups are the basic bonding elements or units that each typically comprise an atom, or at least two atoms bound together as found within a molecule (e.g. -Cl, C=C, C=O, CH₃). Functional groups typically dictate or define properties and
20 structure of the molecule. Similar functional groups in different molecules typically react in similar ways when subjected to a particular set of reaction conditions. Molecule data files contain information about molecules, molecular ions, and molecular radicals. These data files are processed according to their file formats.

File formats: Millsian software employs two kinds of file formats for storing
25 information about the structure, energies and names of molecules and functional groups: raw-data format and hierarchical format. In a raw-data file, all information is stored as is, below the header describing the type of information. As shown in the Table below, for example, the names of the molecule or functional group are listed below the #NAMES header. The names and positions of the atoms are listed below the #ATOMS header, and so on.

```

#NAMES
CH3
Alkane CH3

#SMILES
C-

#ATOMS
1      C      0      0      0
2      H      1.9775 -0.698 0
3      H      -0.9888      -0.698 1.712
4      H      -0.9888      -0.698 -1.712

#AO
1      1 1s    0.171 2
2      1 2sp3  0.864 -1

#BONDS
1      1 2      1      1.649
2      1 3      1      1.649
3      1 4      1      1.649

#BONDAXES
1      1      1      90      0      0

#DATA
RCH bond angle: 109.44°
CH bond length: 2.097 a
HFORM: 12.492

```

Most of the functional groups files and some simple molecule files are stored in raw data format. The other file format used with the Millsian software system, the hierarchical file format, represents the information in a graph style, in which nodes are connected to other nodes through links. This format is designed to construct molecules by attaching different functional groups in a desired manner. Using this file format, the user can construct complex molecules built from solved functional groups.

For example, as shown in the Table below, the pentane data file includes a #GROUPS header below which is list all of the functional groups that form a part of the pentane molecule. Under the #GROUP_LINKS header is information about how these functional groups are connected to each other to construct pentane.

5

10

#NAMES

Pentane

#SMILES

CCCCC

#GROUPS

1	C-
2	-C-
3	-C-
4	-C-
5	C-

#GROUP_LINKS

1	1 1	2 1	180
2	2 2	3 1	180
3	3 2	4 1	180
4	4 2	5 1	180

15 Processing data files: As further shown in the flow diagram of Figure 76 for the exemplary Millsian software system, the program first processes the functional group data file and constructs the functional-group objects, which are complete 3-D representations of the functional groups and their related information supplied in the corresponding file. These objects are then ready to be visually displayed through use of a molecule viewer. Next, the

20 program processes the molecule data file. If the molecule data file is in raw-data format, then the program makes a molecule object directly from it. If the file is in hierarchical format, then the program calculates geometric parameters from listed functional groups. The program stores all functional-group objects and molecule objects using internal data structures.

25 Visualization/ User Interactions: As shown in Figures 77 and 78, the molecule viewer displays the functional-group objects and molecule objects and provides basic interaction capabilities with the displayed objects, such as rotating, scaling, and moving the objects. The molecule viewer also provides other visualization options, such as viewing molecules in wire frame mode, viewing coordinate axes, and changing of the transparency and lighting. The

30 user, for example, can also select parts of a molecule for visualization, like a nucleus, atomic orbital, molecular orbital, or bond axis. The viewer also includes a drop down information window, which provides, for example, related information about molecules, such as bond angles, component functional groups, and total heat of formation. In one embodiment, a user can create new molecules by joining functional groups. For this purpose, the user can select a

first functional group. Next, the user can select an open bond from the functional group where the user desires to attach another group. Next, the user can select another or the same functional group, followed by selecting an open bond from the other group. The user can join the two selected functional groups at the selected open bonds by clicking on 'Join Groups'.

- 5 This method of joining functional groups at open bonds can be repeated to form the desired molecule.

SMILES input: The Simplified Molecular Input Line Entry Specification or SMILES is a specification for unambiguously describing the structure of chemical molecules using short ASCII strings. Through the user interface, a user can enter SMILES to construct molecules,
10 provided the new molecule comprises functional groups that are in the database. Once a SMILES is entered, a parser reads in and breaks it down into component functional groups. The software system then attaches the component functional groups to create the new molecule object, which can then be viewed using the molecule viewer.

While the claimed invention has been described in detail and with reference to specific
15 embodiments thereof, it will be apparent to one of ordinary skill in the art that various changes and modifications can be made to the claimed invention without departing from the spirit and scope thereof.

CONTINUOUS-CHAIN ALKANES (C_nH_{2n+2} , $n=3,4,5\ldots\infty$)

The continuous-chain alkanes, C_nH_{2n+2} , are the homologous series comprising terminal methyl groups at each end of the chain with $n-2$ methylene (CH_2) groups in between:



5 C_nH_{2n+2} can be solved using the same principles as those used to solve ethane and ethylene wherein the $2s$ and $2p$ shells of each C hybridize to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ hybridized orbitals (HOs) to form a molecular orbital (MO) permits each participating hybridized orbital to decrease in radius and energy. Three H AOs combine with three carbon $2sp^3$ HOs and two H AOs combine with
10 two carbon $2sp^3$ HOs to form each methyl and methylene group, respectively, where each bond comprises a H_2 -type MO developed in the Nature of the Chemical Bond of Hydrogen-Type Molecules and Molecular Ions section. The CH_3 and CH_2 groups bond by forming H_2 -type MOs between the remaining $C2sp^3$ HOs on the carbons such that each carbon forms four bonds involving its four $C2sp^3$ HOs. For the alkyl $C-C$ group, $E_r(atom-atom,msp^3.AO)$ is
15 $-1.85836 eV$ where both energy contributions are given by Eq. (14.513). It is based on the energy match between the $C2sp^3$ HOs of the chain comprising methylene groups and terminal methyl groups.

The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of straight-chain alkanes are given in Tables 15.4,
20 15.5, and 15.6, respectively. The total energy of each straight-chain alkane given in Table 15.7 was calculated as the sum over the integer multiple of each $E_D(Group)$ of Table 15.6 corresponding to functional-group composition of the molecule. The bond angle parameters of straight-chain alkanes determined using Eqs. (15.79-15.108) are given in Table 15.8. In this angle table and those given in subsequent sections when c'_2 is given as the ratio of two values of
25 c_2 designated to Atom 1 and Atom 2 and corresponding to $E_{Coulombic}$ of Atom 1 and Atom 2,

respectively, then $c'_2 = \frac{c_2(Atom\ 2)}{c_2(Atom\ 1)}$.

Table 15.4. The geometrical bond parameters of straight-chain alkanes and experimental values [1].

Parameter	C'-C' Group	C'-H (CH ₃) Group	C'-H (CH ₂) Group
<i>a</i> (a ₀)	2.12499	1.64920	1.67122
<i>c'</i> (a ₀)	1.45744	1.04856	1.05553
Bond Length 2 <i>c'</i> (Å)	1.54280	1.10974	1.11713
Exp. Bond Length (Å)	1.532 (propane) 1.531 (butane)	1.107 (C-H propane) 1.117 (C-H butane)	1.107 (C-H propane) 1.117 (C-H butane)
<i>h, c</i> (a ₀)	1.54616	1.27295	1.29569
<i>ε</i>	0.68600	0.63580	0.63159

Table 15.5. The MO to HO intercept geometrical bond parameters of straight-chain alkanes. E_r is $E_r(\text{atom} - \text{atom}, \text{mwp}, \text{AO})$.

Bond	Atom	L_r (eV) Bond 1	K_r (eV) Bond 2	E_r (eV) Bond 3	E_r (eV) Bond 4	Final Total Energy C2sp ³ (eV)	r_{bond} (a ₀)	r_{final} (a ₀)	$\epsilon_{\text{intercept}}(\text{C2sp}^3)$ (eV) Final	$E(\text{C2sp}^3)$ (eV) Final	θ' (°)	θ_i (°)	θ_2 (°)	d_1 (a ₀)	d_2 (a ₀)
C'-H (CH ₃)	C'	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	77.49	102.51	41.48	1.23564	0.18708
C'-H (CH ₂)	C'	-0.92918	-0.92918	0	0	-153.47405	0.91771	0.81549	-16.68412	-16.49325	68.47	111.53	35.84	1.35486	0.29933
H ₁ C ₂ -C ₂ H ₂ CH ₃	C ₂	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	63.82	116.18	30.08	1.81879	0.38106
H ₁ C ₂ -C ₂ H ₂ CH ₂	C ₂	-0.92918	-0.92918	0	0	-153.47405	0.91771	0.81549	-16.68412	-16.49325	56.41	123.59	26.06	1.90890	0.45117

Table 15.6. The energy parameters (eV) of functional groups of straight-chain alkanes.

Parameters	C - C Group	CH ₃ Group	CH ₂ Group
n_1	1	3	2
n_2	0	2	1
n_3	0	0	0
C_1	0.5	0.75	0.75
C_2	1	1	1
c_1	1	1	1
c_2	0.91771	0.91771	0.91771
c_3	0	0	1
c_4	2	1	1
c_5	0	3	2
C_{1o}	0.5	0.75	0.75
C_{2o}	1	1	1
V_e (eV)	-28.79214	-107.32728	-70.41425
V_p (eV)	9.33352	38.92728	25.78002
T (eV)	6.77464	32.53914	21.06675
V_m (eV)	-3.38732	-16.26957	-10.53337
$E(AOIHO)$ (eV)	-15.56407	-15.56407	-15.56407
$\Delta E_{H_2MO(AOIHO)}$ (eV)	0	0	0
$E_T(AOIHO)$ (eV)	-15.56407	-15.56407	-15.56407
$E_T(H_2MO)$ (eV)	-31.63537	-67.69451	-49.66493
$E_T(atom - atom, msp^3 AO)$ (eV)	-1.85836	0	0
$E_T(MO)$ (eV)	-33.49373	-67.69450	-49.66493
ω (10^{15} rad/s)	9.43699	24.9286	24.2751
E_K (eV)	6.21159	16.40846	15.97831
\bar{E}_D (eV)	-0.16515	-0.25352	-0.25017
\bar{E}_{Kvib} (eV)	0.12312 [2]	0.35532 (Eq. (13.458))	0.35532 (Eq. (13.458))
\bar{E}_{osc} (eV)	-0.10359	-0.22757	-0.14502
E_{mag} (eV)	0.14803	0.14803	0.14803
$E_T(Group)$ (eV)	-33.59732	-67.92207	-49.80996
$E_{initial}(c_4 AOIHO)$ (eV)	-14.63489	-14.63489	-14.63489
$E_{initial}(c_3 AOIHO)$ (eV)	0	-13.59844	-13.59844
$E_D(Group)$ (eV)	4.32754	12.49186	7.83016

Formula	Name	C-C	CH ₃	CH ₂	Calculated	Experimental	Relative Error
					Total Bond Energy	Total Bond	Error

						Energy (eV)	
Propane	2	2	2	1	41.46896	41.434	-0.00085
Butane	3	2	2	2	53.62666	53.61	-0.00036
Pentane	4	2	3	3	65.78436	65.77	-0.00017
Hexane	5	2	4	4	77.94206	77.93	-0.00019
Heptane	6	2	5	5	90.09976	90.09	-0.00013
Octane	7	2	6	6	102.25746	102.25	-0.00006
Nonane	8	2	7	7	114.41516	114.40	-0.00012
Decane	9	2	8	8	126.57286	126.57	-0.00003
Undecane	10	2	9	9	138.73056	138.736	0.00004
Dodecane	11	2	10	10	150.88826	150.88	-0.00008
Octadecane	18	2	16	16	223.83446	223.85	0.00008

Table 15.8. The bond angle parameters of straight-chain alkanes and experimental values [1]. In the calculation of θ_* , the parameters from the preceding angle were used. E_r is $E_r(\text{atom} - \text{atom}, \text{msp}^3, \text{AO})$.

Atoms of Angle	2C' Bond 1 (a_0)	2C' Bond 2 (a_0)	2C' Terminal Atoms (a_0)	E_{terminal} Atom 1	Atom 1 Hybridization Designation (Table 15.3.A)	E_{terminal} Atom 2	Atom 2 Hybridization Designation (Table 15.3.A)	C ₂ Atom 1	C ₂ Atom 2	C ₁	C ₃	C ₁	ζ_2	E_T (eV)	θ_s (°)	θ_t (°)	θ_1 (°)	Cal. θ (°)	Exp. θ (°)
Methyls $\angle HC_nH$	2.11106	2.11110 6	3.4252	15.75493 C _n	7 (Table 15.3.A)	H	(Table 15.3.A)	0.86359	1	1	1	0.75	1.15796	0				108.44	107 (propane)
$\angle C_2C_nC_n$															69.31			110.49	112 (propane) 113.8 (butane)
$\angle C_2C_nH$															69.51			110.49	111.0 (butane)
Methyl $\angle HC_nH$	2.09711	2.09711 1	3.4252	15.75493 C _n	7	H		0.86359	1	1	1	0.75	1.15796	0				109.50	
$\angle C_2C_nC_n$																70.56		109.44	
$\angle C_nC_nH$																70.56		109.44	

BRANCHED ALKANES (C_nH_{2n+2} , $n = 3, 4, 5 \dots \infty$)

The branched-chain alkanes, C_nH_{2n+2} , comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of $C-C$ bonds can be identified. The n -alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2CH$) and t -butyl ($(CH_3)_3C$) groups and the isopropyl to isopropyl, isopropyl to t -butyl, and t -butyl to t -butyl $C-C$ bonds comprise functional groups. The branched-alkane groups are solved using the same principles as those used to solve the methyl and methylene functional groups wherein the $2s$ and $2p$ AOs of each C hybridize to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ HOs to form a MO permits each participating hybridized orbital to decrease in radius and energy. $E_r(atom - atom, msp^3.AO)$ of each $C-C$ -bond MO in Eq. (15.52) due to the charge donation from the C atoms to the MO is $-1.85836 eV$ or $-1.44915 eV$ based on the energy match between the $C2sp^3$ HOs corresponding to the energy contributions equivalent to those of methylene, $-0.92918 eV$ (Eq. (14.513), or methyl, $-0.72457 eV$ (Eq. (14.151)), groups, respectively.

The symbols of the functional groups of branched-chain alkanes are given in Table 15.9. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of branched-chain alkanes are given in Tables 15.10, 15.11, and 15.12, respectively. The total energy of each branched-chain alkane given in Table 15.13 was calculated as the sum over the integer multiple of each $E_D(group)$ of Table 15.12 corresponding to functional-group composition of the molecule. The bond angle parameters of branched-chain alkanes determined using Eqs. (15.79-15.108) are given in Table 15.14.

Table 15.9. The symbols of functional groups of branched alkanes.

Functional Group	Group Symbol
CH ₃ group	$C-H (CH_3)$
CH ₂ group	$C-H (CH_2)$
CH	$C-H$
CC bond (n-C)	$C-C (a)$
CC bond (iso-C)	$C-C (b)$
CC bond (tert-C)	$C-C (c)$
CC (iso to iso-C)	$C-C (d)$
CC (t to t-C)	$C-C (e)$
CC (t to iso-C)	$C-C (f)$

Table 15.10. The geometrical bond parameters of branched alkanes and experimental values [1].

Parameter	C-C (a)	C-H (a)	C-C (b)	C-C (c)	C-C (d)	C-C (e)	C-C (f)
a (Å)	1.64920	1.67122	1.67465	2.12499	2.12499	2.10725	2.10725
c' (Å)	1.04856	1.05553	1.05661	1.45744	1.45744	1.45164	1.45164
Bond Length $2c'$ (Å)	1.10974	1.11713	1.11827	1.54280	1.54280	1.53635	1.53635
Exp. Bond Length (Å)	1.107 (C-H propane)	1.107 (C-H propane)	1.122 (isobutane)	1.532 (propane)	1.532 (propane)	1.532 (propane)	1.532 (propane)
	1.117 (C-H butane)	1.117 (C-H butane)	1.117 (isobutane)	1.531 (butane)	1.531 (butane)	1.531 (butane)	1.531 (butane)
h, c (Å)	1.27295	1.29569	1.29924	1.54616	1.54616	1.52750	1.52750
c	0.63580	0.63159	0.63095	0.68600	0.68600	0.68888	0.68888

Table 15.11. The MO to HO intercept geometrical bond parameters of branched-chain alkanes. R, R', R'' are H or alkyl groups. E_T is $E_T(\text{atom} - \text{atom}, \text{disp}, A(2))$.

Bond	Atom	E_T (eV) Bond 1	E_T (eV) Bond 2	E_T (eV) Bond 3	E_T (eV) Bond 4	Final Total Energy (eV)	r_{final} (Å)	r_{final} (Å)	$E_{\text{disp}}(\text{eV})$ Final	$E(\text{eV})$ Final	θ' (°)	θ_1 (°)	θ_2 (°)	d_1 (Å)	d_2 (Å)
C-H (CH ₃)	C	-0.92018	0	0	0	-152.54467	0.91771	0.91771	-15.75493	-15.56407	77.49	102.51	-11.48	1.23564	0.18708
C-H (CH ₂)	C	-0.92018	-0.92018	0	0	-153.47406	0.91771	0.91771	-16.68412	-16.49325	68.47	111.53	33.84	1.35486	0.29333
C-H (CH)	C	-0.92018	-0.92018	-0.92018	0	-154.40324	0.91771	0.91771	-17.61330	-17.42244	61.10	118.90	31.37	1.42988	0.37326
H ₃ C-C ₂ H ₅ -CH ₃ (C-C (a))	C	-0.92018	0	0	0	-152.54467	0.91771	0.91771	-15.75493	-15.56407	63.82	116.18	30.08	1.35879	0.38106
H ₃ C-C ₂ H ₅ -CH ₂ (C-C (b))	C	-0.92018	-0.92018	0	0	-153.47406	0.91771	0.91771	-16.68412	-16.49325	56.41	122.59	26.06	1.50820	0.45117
R-H ₃ C-C ₂ H ₅ -R' (C-C (c))	C	-0.92018	-0.92018	-0.92018	0	-154.40324	0.91771	0.91771	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
R-H ₃ C-C ₂ H ₅ -R' (C-C (d))	C	-0.92018	-0.92018	-0.92018	-0.72457	-154.71860	0.91771	0.91771	-17.92866	-17.73779	48.21	131.79	21.74	1.95794	0.50570
R-H ₃ C-C ₂ H ₅ -R' (C-C (e))	C	-0.92018	-0.92018	-0.92018	0	-154.40324	0.91771	0.91771	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
R-H ₃ C-C ₂ H ₅ -R' (C-C (f))	C	-0.72457	-0.72457	-0.72457	-0.72457	-154.51309	0.91771	0.91771	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49208
R-H ₃ C-C ₂ H ₅ -R' (C-C (g))	C	-0.72457	-0.72457	-0.72457	0	-154.19863	0.91771	0.91771	-17.40669	-17.21783	52.78	127.22	24.04	1.92443	0.47279
R-H ₃ C-C ₂ H ₅ -R' (C-C (h))	C	-0.72457	-0.72457	-0.72457	-0.72457	-154.51309	0.91771	0.91771	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49208

Table 15.12. The energy parameters (eV) of functional groups of branched-chain alkanes.

Parameters	CH ₃ Group	CH ₂ Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
n_1	3	2	1	1	1	1	1	1	1
n_2	2	1	0	0	0	0	0	0	0
n_3	0	0	0	0	0	0	0	0	0
C_1	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_2	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1
C_4	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_5	0	1	1	0	0	0	1	1	0
C_6	1	1	1	2	2	2	2	2	2
C_7	3	2	1	0	0	0	0	0	0
C_{10}	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_{2n}	1	1	1	1	1	1	1	1	1
V_e (eV)	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-29.10112	-28.79214	-29.10112	-29.10112
V_p (eV)	38.92728	25.78002	12.87680	9.33352	9.33352	9.37273	9.33352	9.37273	9.37273
T (eV)	32.53914	21.06675	10.48582	6.77464	6.77464	6.90500	6.77464	6.90500	6.90500
V_a (eV)	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.45250	-3.38732	-3.45250	-3.45250
$E_{(atom)}$ (eV)	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.35946	-15.56407	-15.35946	-15.35946
ΔE_{H_2O} (eV)	0	0	0	0	0	0	0	0	0
E_T (eV)	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.35946	-15.56407	-15.35946	-15.35946
E_T (eV)	-67.69451	-49.66493	-31.63533	-31.63537	-31.63537	-31.63535	-31.63537	-31.63535	-31.63535
E_T (atom - atom, msp, AO) (eV)	0	0	0	-1.85836	-1.85836	-1.44915	-1.85836	-1.44915	-1.44915
E_T (eV)	-67.69450	-49.66493	-31.63537	-33.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452
ω (10 ¹⁵ rad/s)	24.9286	24.2751	24.1759	9.43699	9.43699	15.4846	9.43699	9.55643	9.55643
E_K (eV)	16.40846	15.97831	15.91299	6.21159	6.21159	10.19220	6.21159	6.29021	6.29021
\bar{E}_H (eV)	-0.25352	-0.25017	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.16416	-0.16416
$\bar{E}_{K_{WH}}$ (eV)	0.35532 (Eq. (13.458))	0.35532 (Eq. (13.458))	0.35532 (Eq. (13.458))	0.12312 (Eq. (13.458))	0.17978 (Eq. (13.458))	0.09944 (Eq. (13.458))	0.12312 (Eq. (13.458))	0.12312 (Eq. (13.458))	0.12312 (Eq. (13.458))
\bar{E}_{int} (eV)	-0.22757	-0.14502	-0.07200	-0.10359	-0.07526	-0.15924	-0.10359	-0.10260	-0.10260
E_{avg} (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
E_T (eV)	-67.92207	-49.80996	-31.70737	-33.59732	-33.49373	-33.24376	-33.59732	-33.18712	-33.18712
E_{init} (eV)	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
E_{init} (eV)	-13.59844	-13.59844	-13.59844	0	0	0	0	0	0
E_D (eV)	12.49186	7.83016	3.32601	4.32754	4.29921	3.97398	4.17951	3.62128	3.91734

Table 15.13. The total bond energies of branched alkanes calculated using the functional group composition and the energies of Table 15.12 compared to the experimental values [3].

Formula	Name	CH ₄	CH ₃	CH ₂	CH	C-C (a)	C-C (b)	C-C (c)	C-C (d)	C-C (e)	C-C (f)	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₄ H ₁₀	Isobutane	3	0	1	1	0	3	0	0	0	0	53.69922	53.695	-0.00007
C ₅ H ₁₂	Isopentane	3	1	1	1	1	3	0	0	0	0	65.85692	65.843	-0.00021
C ₅ H ₁₂	Neopentane	4	0	0	0	0	0	4	0	0	0	65.86336	65.992	0.00195
C ₆ H ₁₄	2-Methylpentane	3	2	1	1	2	3	0	0	0	0	78.01462	78.007	-0.00010
C ₆ H ₁₄	3-Methylpentane	3	2	1	1	2	3	0	0	0	0	78.01462	77.979	-0.00046
C ₆ H ₁₄	2,2-Dimethylbutane	4	1	0	0	1	0	4	0	0	0	78.02106	78.124	0.00132
C ₆ H ₁₄	2,3-Dimethylbutane	4	1	0	0	1	0	4	0	0	0	77.99581	78.043	0.00061
C ₆ H ₁₆	2-Methylhexane	3	3	1	1	3	3	0	0	0	0	90.17232	90.160	-0.00014
C ₆ H ₁₆	3-Methylhexane	3	3	1	1	3	3	0	0	0	0	90.17232	90.127	-0.00051
C ₆ H ₁₆	2-Ethylpentane	3	3	1	1	3	3	0	0	0	0	90.17232	90.108	-0.00072
C ₇ H ₁₈	2,2-Dimethylpentane	4	2	0	0	2	0	4	0	0	0	90.17876	90.276	0.00107
C ₇ H ₁₈	2,2,3-Trimethylbutane	5	0	1	2	0	2	3	0	1	0	90.22301	90.262	0.00044
C ₇ H ₁₈	2,4-Dimethylpentane	4	2	0	0	2	0	4	0	0	0	90.24488	90.233	-0.00013
C ₇ H ₁₈	3,3-Dimethylpentane	4	2	0	0	2	0	4	0	0	0	90.17876	90.227	0.00054
C ₇ H ₁₈	2-Methylheptane	3	4	1	1	4	3	0	0	0	0	102.33002	102.293	-0.00008
C ₇ H ₁₈	3-Methylheptane	3	4	1	1	4	3	0	0	0	0	102.33002	102.286	-0.00043
C ₇ H ₁₈	4-Methylheptane	3	4	1	1	4	3	0	0	0	0	102.30169	102.274	-0.00027
C ₇ H ₁₈	3-Ethylhexane	3	4	1	1	4	3	0	0	0	0	102.33646	102.417	0.00079
C ₈ H ₂₀	2,2-Dimethylhexane	4	3	0	2	3	0	4	0	0	0	102.31121	102.306	-0.00005
C ₈ H ₂₀	2,3-Dimethylhexane	4	2	2	2	2	4	0	1	0	0	102.31121	102.362	-0.00040
C ₈ H ₂₀	2,4-Dimethylhexane	4	2	2	2	2	4	0	0	0	0	102.40258	102.396	-0.00006
C ₈ H ₂₀	2,5-Dimethylhexane	4	2	2	2	2	4	0	0	0	0	102.33646	102.369	0.00032
C ₈ H ₂₀	3,3-Dimethylhexane	4	3	0	3	3	0	4	0	0	0	102.31121	102.296	-0.00015
C ₈ H ₂₀	3,4-Dimethylhexane	4	2	2	2	2	4	0	1	0	0	102.31121	102.277	-0.00033
C ₈ H ₂₀	3-Ethyl-2-methylpentane	4	3	0	3	3	0	4	0	0	0	102.33646	102.317	-0.00019
C ₈ H ₂₀	3-Ethyl-3-methylpentane	4	3	0	3	3	0	4	0	0	0	102.38071	102.370	-0.00010
C ₈ H ₂₀	2,2,3-Trimethylpentane	5	1	1	1	1	2	3	0	1	0	102.40902	102.412	0.00003
C ₈ H ₂₀	2,3,3-Trimethylpentane	5	1	1	1	1	2	3	0	1	0	102.38071	102.332	-0.00048
C ₈ H ₂₀	2,3,4-Trimethylpentane	5	0	0	3	0	5	0	2	0	0	102.29240	102.342	0.00049
C ₈ H ₂₀	2,2,3,3-Tetramethylbutane	6	0	0	0	0	0	6	0	1	0	102.41632	102.433	0.00016
C ₈ H ₂₀	2,3,5-Trimethylhexane	5	1	1	3	0	7	0	1	0	0	114.54147	114.551	0.00008
C ₈ H ₂₀	3,3-Diethylpentane	4	4	0	0	4	0	4	0	0	0	114.49416	114.455	-0.00034
C ₉ H ₂₂	2,2,3,3-Tetramethylpentane	6	1	1	0	1	0	6	0	1	0	114.57402	114.494	-0.00070
C ₉ H ₂₂	2,2,3,4-Tetramethylpentane	6	0	0	2	0	3	3	1	0	1	114.51960	114.492	-0.00024
C ₉ H ₂₂	2,2,4,4-Tetramethylpentane	6	1	0	0	0	0	8	0	0	0	114.57316	114.541	-0.00028
C ₉ H ₂₂	2,3,3,4-Tetramethylpentane	6	0	0	2	0	4	2	0	0	2	114.58266	114.484	-0.00086
C ₉ H ₂₂	2-Methylheptane	3	6	1	1	6	3	0	0	0	0	126.64542	126.680	0.00027
C ₉ H ₂₂	5-Methylheptane	3	6	1	1	6	3	0	0	0	0	126.64542	126.663	0.00014

Table 15.14. The bond angle parameters of branched-chain alkanes and experimental values [1]. In the calculation of θ , the parameters from the preceding angle were used. E_f is $E_f(\text{atom} - \text{atom}, msp^3, AO)$.

[illegible]

ALKENES (C_nH_{2n} , $n = 3, 4, 5 \dots \infty$)

The straight and branched-chain alkenes, C_nH_{2n} , comprise at least one carbon-carbon double bond comprising a functional group that is solved equivalently to the double bond of ethylene. The double bond may be bound to one, two, three, or four carbon single bonds that substitute for the hydrogen atoms of ethylene. Based on the condition of energy matching of the orbital, any magnetic energy due to unpaired electrons in the constituent fragments, and differences in oscillation in the transition state, three distinct functional groups can be identified: C vinyl single bond to $-C(C)=C$, C vinyl single bond to $-C(H)=C$, and C vinyl single bond to $-C(C)=CH_2$. In addition, CH_2 of the $-C=CH_2$ moiety is an alkene functional group.

The alkyl portion of the alkene may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of C-C bonds can be identified. The n-alkane C-C bond is the same as that of straight-chain alkanes. In addition, the C-C bonds within isopropyl ($(CH_3)_2CH$) and t-butyl ($(CH_3)_3C$) groups and the isopropyl to isopropyl, isopropyl to t-butyl, and t-butyl to t-butyl C-C bonds comprise functional groups. The branched-chain-alkane groups in alkenes are equivalent to those in branched-chain alkanes. The solution of the functional groups comprises the hybridization of the 2s and 2p AOs of each C to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ HOs to form a MO permits each participating hybridized orbital to decrease in radius and energy. $E_r(atom-atom,msp^3.AO)$ of the C=C-bond MO in Eq. (15.52) due to the charge donation from the C atoms to the MO is equivalent to that of ethylene, $-2.26759 eV$, given by Eq. (14.247). $E_r(atom-atom,msp^3.AO)$ of each C-C-bond MO in Eq. (15.52) is $-1.85836 eV$ or $-1.44915 eV$ based on the energy match between the $C2sp^3$ HOs corresponding to the energy contributions equivalent to those of methylene, $-0.92918 eV$ (Eq. (14.513), or methyl, $-0.72457 eV$ (Eq. (14.151)), groups, respectively.

The symbols of the functional groups of alkenes are given in Table 15.15. The geometrical (Eqs. (15.1-15.5) and (15.41)), intercept (Eqs. (15.71-15.78)), and energy (Eqs.

(15.6-15.11) and (15.17-15.56)) parameters of alkenes are given in Tables 15.16, 15.17, and 15.18, respectively. The total energy of each alkene given in Table 15.19 was calculated as the sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.18 corresponding to functional-group composition of the molecule. For each set of unpaired electrons created by bond breakage 5, the $C2sp^3$ HO magnetic energy E_{mag} that is subtracted from the weighted sum of the $E_D(\text{Group})$ (eV) values based on composition is given by Eq. (15.58). The bond angle parameters of alkenes determined using Eqs. (15.79-15.108) are given in Table 15.20.

Table 15.15. The symbols of functional groups of alkenes.

Functional Group	Group Symbol
CC double bond	C=C
C vinyl single bond to -C(C)=C	C-C' (i)
C vinyl single bond to -C(H)=C	C-C' (ii)
C vinyl single bond to -C(C)=CH ₂	C-C' (iii)
CH ₂ alkenyl group	C-H (CH ₂) (i)
CH ₃ group	C-H (CH ₃)
CH ₃ alkyl group	C-H (CH ₃) (ii)
CH	C-H
CC bond (n-C)	C-C' (a)
CC bond (iso-C)	C-C' (b)
CC bond (tert-C)	C-C' (c)
CC (iso to iso-C)	C-C' (d)
CC (t to t-C)	C-C' (e)
CC (t to iso-C)	C-C' (f)

Table 15.16. The geometrical bond parameters of alkenes and experimental values [1].

Parameter	C=C Group	C-C' (i) Group	C-C' (ii) Group	C-C' (iii) Group	C-H (CH ₂) (i) Group	C-H (CH ₂) (ii) Group	C-H (CH ₂) (iii) Group	C-H (CH ₃) Group	C-H (CH ₃) (i) Group	C-H (CH ₃) (ii) Group	C-H (CH ₃) (iii) Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
a (a ₀)	1.4728	2.04740	2.04740	2.04740	1.64910	1.64920	1.67122	1.67465	2.12499	2.12499	2.12499	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
c' (a ₀)	1.26661	1.43087	1.43087	1.43087	1.04566	1.04856	1.05553	1.03661	1.45744	1.45744	1.45744	1.03661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length 2c' (A)	1.34052	1.51437	1.51437	1.51437	1.10668	1.10974	1.11713	1.11827	1.54280	1.54280	1.54280	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (A)	1.342 (2-methylpropene) 1.346 (2-butene) 1.349 (1,3-butadiene)				1.10 (2-methylpropene) 1.108 (avg.) 1.117 (1,3-butadiene)	1.107 (C-H propene) 1.117 (C-H butene)	1.107 (C-H propene) 1.117 (C-H butene)	1.122 (isobutane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.122 (isobutane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
h, c (a ₀)	0.75055	1.46439	1.46439	1.46439	1.26354	1.27295	1.29569	1.29924	1.54616	1.54616	1.54616	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
ε	0.86030	0.69887	0.69887	0.69887	0.63736	0.63580	0.63159	0.63095	0.68600	0.68600	0.68600	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 15.17. The MO to HO intercept geometrical bond parameters of alkenes. I_i is an alkyl group and R, R', R'' are H or alkyl groups. E_p is $E_r(\text{atom} - \text{atom}, \text{msp}^3, \text{AO})$.

Bond	Atom	E_r (eV) Bond 1	E_r (eV) Bond 2	E_r (eV) Bond 3	E_r (eV) Bond 4	Final Total Energy C_{2sp}^2 (eV)	r_{bond} (a ₀)	r_{final} (a ₀)	$E_{\text{bond}}(C_{2sp}^2)$ (eV) Final	θ' (°)	θ_1 (°)	θ_2 (°)	d_1 (a ₀)	d_2 (a ₀)
$C_1(H)C_2=C_3(H)C_4$	C_1	-1.13380	-0.92918	0	0	-153.67867	0.91771	0.80561	-16.69786	127.61	52.39	58.24	0.77492	0.49108
$C_1(H)C_2=C_3(H)C_4$	C_2	-1.13380	0	0	0	-152.74649	0.91771	0.82322	-15.76508	129.84	50.16	60.70	0.72040	0.54620
$C_1(H)C_2=C_3(H)C_4$	C_3	-1.13380	-0.72457	-0.72457	0	-154.19863	0.91771	0.78155	-17.21783	126.39	53.61	56.95	0.80289	0.46371
$R_1C_1H_2-C_2(C)=C$	C_1	-1.13380	-0.72457	-0.72457	0	-154.19863	0.91771	0.78155	-17.21783	60.88	119.12	27.79	1.81127	0.34039
$R_1C_1H_2-C_2(C)=C$	C_2	-0.72457	-0.92918	0	0	-153.26945	0.91771	0.82562	-16.47951	67.40	112.60	31.56	1.74821	0.31734
$R_1C_1H_2-C_2(C)=C$	C_3	-1.13380	-0.92918	0	0	-153.67866	0.91771	0.80561	-16.69786	64.37	115.43	29.79	1.77684	0.34596
$R_1C_1H_2-C_2(H)=C$	C_1	-0.92918	-0.92918	0	0	-153.47403	0.91771	0.81549	-16.68411	65.99	114.01	30.58	1.76270	0.33183
$R_1C_1H_2-C_2(H)=C$	C_2	-1.13380	0	0	0	-152.74649	0.91771	0.82522	-15.99555	77.15	102.85	41.13	1.23531	0.18965
$R_1C_1H_2-C_2(H)=C$	C_3	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	77.49	102.51	41.48	1.23564	0.18708
$R_1C_1H_2-C_2(H)=C$	C_4	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	68.47	111.53	35.84	1.39486	0.29933
$R_1C_1H_2-C_2(H)=C$	C_5	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	61.10	118.90	31.37	1.42988	0.37326
$H_1C_1C_2H_2C_3H_1$	C_1	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	63.82	116.18	30.08	1.33879	0.38106
$H_1C_1C_2H_2C_3H_1$	C_2	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	56.41	123.59	26.06	1.90890	0.45117
$R-H_1C_1C_2(H_1C_2-R)HC(H_2-C_3-C_4)$	C_1	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	48.30	131.70	21.90	1.97162	0.51388
$R-H_1C_1C_2(H_1C_2-R)HC(H_2-C_3-C_4)$	C_2	-0.92918	-0.72457	-0.72457	-0.72457	-154.19860	0.91771	0.78155	-17.21783	48.21	131.79	21.74	1.95734	0.50570
$R-H_1C_1C_2(H_1C_2-R)HC(H_2-C_3-C_4)$	C_3	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	48.30	131.70	21.90	1.97162	0.51388
$R-H_1C_1C_2(H_1C_2-R)HC(H_2-C_3-C_4)$	C_4	-0.72457	-0.72457	-0.72457	-0.72457	-154.19863	0.91771	0.78155	-17.21783	50.04	129.96	22.66	1.94462	0.49298
$R-H_1C_1C_2(H_1C_2-R)HC(H_2-C_3-C_4)$	C_5	-0.72457	-0.92918	-0.92918	0	-154.19863	0.91771	0.78155	-17.21783	52.78	127.22	24.04	1.92443	0.47279
$R-H_1C_1C_2(H_1C_2-R)HC(H_2-C_3-C_4)$	C_6	-0.72457	-0.72457	-0.72457	-0.72457	-154.19863	0.91771	0.78155	-17.21783	50.04	129.96	22.66	1.94462	0.49298

Table 15.18. The energy parameters (eV) of functional groups of alkenes.

Parameters	C=C Group	C-C (i) Group	C-C (ii) Group	C-C (iii) Group	CH ₃ (i) Group	CH ₃ Group	CH ₃ (ii) Group	C-H Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
n_1	2	1	1	1	2	3	2	1	1	1	1	1	1
n_2	0	0	0	0	1	2	1	0	0	0	0	0	0
n_3	0	0	0	0	0	0	0	0	0	0	0	0	0
C_1	0.5	0.5	0.5	0.5	0.75	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
C_2	0.91771	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_5	0	1	0	1	1	0	1	1	0	0	1	1	0
C_6	4	2	2	2	1	1	1	1	2	2	2	2	2
C_7	0	0	0	0	2	3	2	1	0	0	0	0	0
C_{10}	0.5	0.5	0.5	0.5	0.75	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
C_{12}	0.91771	1	1	1	1	1	1	1	1	1	1	1	1
V_p (eV)	-102.08992	-30.19634	-30.19634	-30.19634	-72.03287	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-28.79214	-29.10112	-29.10112
V_p (eV)	21.48386	9.50874	9.50874	9.50874	26.02344	38.92728	25.78002	12.87680	9.33352	9.33352	9.33352	9.33352	9.33352
T (eV)	34.67062	7.37432	7.37432	7.37432	21.99990	32.53914	21.06675	10.48582	6.77464	6.77464	6.77464	6.90500	6.90500
V_p (eV)	-17.33531	-3.68716	-3.68716	-3.68716	-10.97995	-16.26957	-10.53337	-5.4291	-3.38732	-3.38732	-3.38732	-3.45250	-3.45250
$E_{\text{initial}}(\text{eV})$	0	-14.63489	-14.63489	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
$\Delta E_{\text{HOMO}}(\text{eV})$	0	0	0	0	0	0	0	0	0	0	0	0	0
$E_T(\text{eV})$	0	-14.63489	-14.63489	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
$E_T(\text{eV})$	-63.27075	-31.63534	-31.63534	-31.63534	-49.66437	-67.69451	-49.66493	-31.63533	-31.63537	-31.63537	-31.63537	-31.63535	-31.63535
$E_T(\text{eV})$	-2.26759	-1.44915	-1.85836	-1.44915	0	0	0	0	-1.85836	-1.85836	-1.85836	-1.44915	-1.44915
$E_T(\text{eV})$	-65.53833	-33.08452	-33.49373	-33.08452	-49.66493	-67.69450	-49.66493	-31.63537	-33.49373	-33.49373	-33.49373	-33.08452	-33.08452
ω (10^{15} rad/s)	43.0680	9.97851	16.4962	9.97851	25.2077	24.9286	24.2751	24.1759	9.43699	9.43699	9.43699	9.55643	9.55643
E_K (eV)	28.34813	6.56803	10.85807	6.56803	16.59214	16.40846	15.97831	15.91299	6.21159	6.21159	6.21159	6.29021	6.29021
E_H (eV)	-0.34517	-0.16774	-0.21834	-0.16774	-0.25493	-0.25352	-0.25017	-0.24966	-0.16515	-0.16515	-0.16515	-0.16416	-0.16416
E_{HOMO} (eV)	0.17897	0.15895	0.09931	0.09931	0.35532	0.35532	0.35532	0.35532	0.12312	0.09944	0.12312	0.12312	0.12312
E_{HOMO} (eV)	-0.25568	-0.08827	-0.16869	-0.11809	-0.07727	-0.22757	-0.14502	-0.07200	-0.10359	-0.15924	-0.10359	-0.10260	-0.10260
E_{HOMO} (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
$E_T(\text{eV})$	-66.04969	-33.17279	-33.66242	-33.20260	-49.81948	-67.92207	-49.80996	-31.70737	-33.59732	-33.59732	-33.59732	-33.18712	-33.18712
E_{HOMO} (eV)	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
E_{HOMO} (eV)	0	0	0	0	-13.59844	-13.59844	-13.59844	-13.59844	0	0	0	0	0
$E_D(\text{eV})$	7.51014	3.75498	4.39264	3.78480	7.83968	12.49186	7.83016	3.32601	4.2921	3.97398	4.17951	3.62128	3.91734

Table 15.19. The total band energies of alkenes calculated using the functional group composition and the energies of Table 15.18 compared to the experimental values [2]. The magnetic energy E_{mag} that is subtracted from the weighted sum of the $E_0(\alpha\pi\pi)$ (θ°) values based on composition is given by (15.58).

values based on composition is given by (15.58).

Formula	Name	C=C	C-C	C-C	C-C	CH ₂ (i)	CH ₃	CH ₂ (ii)	CH	C-C	C-C	C-C	C-C	C-C	E _{exp}	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₃ H ₆	Propene	0	0	0	0	1	1	0	1	0	0	0	0	0	0	35.56033	35.63207	0.00201
C ₄ H ₈	1-Butene	1	0	0	0	1	1	1	1	0	0	0	0	0	0	47.71803	47.78477	0.00140
C ₄ H ₈	trans-2-Butene	1	0	0	0	2	2	0	2	0	0	0	0	0	0	47.93116	47.90395	-0.00057
C ₄ H ₈	Isobutene	1	0	0	0	1	2	0	0	0	0	0	0	0	0	47.90314	47.96096	0.00121
C ₅ H ₁₀	1-Pentene	1	0	1	0	1	1	2	1	1	0	0	0	0	0	59.87573	59.95094	0.00125
C ₅ H ₁₀	trans-2-Pentene	1	0	0	0	2	2	1	2	1	0	0	0	0	0	60.08886	60.06287	-0.00043
C ₅ H ₁₀	2-Methyl-1-butene	1	0	2	1	2	2	1	0	1	0	0	0	0	0	60.06084	60.09707	0.00060
C ₅ H ₁₀	2-Methyl-2-butene	1	1	0	0	3	3	0	1	0	0	0	0	0	0	60.21433	60.16444	-0.00083
C ₅ H ₁₀	3-Methyl-1-butene	1	0	1	0	1	3	0	2	0	0	0	0	0	0	59.97662	60.01727	0.00068
C ₆ H ₁₂	1-Hexene	1	0	1	0	1	1	3	1	3	0	0	0	0	0	72.03343	72.12954	0.00133
C ₆ H ₁₂	trans-2-Hexene	1	0	0	0	2	2	2	2	2	0	0	0	0	0	72.24656	72.23733	-0.00013
C ₆ H ₁₂	trans-3-Hexene	1	0	0	0	2	2	2	2	2	0	0	0	0	0	72.24636	72.24251	-0.00006
C ₆ H ₁₂	2-Methyl-1-pentene	1	0	2	1	2	2	2	0	2	0	0	0	0	0	72.21854	72.29433	0.00105
C ₆ H ₁₂	2-Methyl-2-pentene	1	0	0	0	3	3	1	1	1	0	0	0	0	0	72.37203	72.37206	0.00000
C ₆ H ₁₂	3-Methyl-1-pentene	1	0	1	0	1	2	1	2	1	0	0	0	0	0	72.13432	72.19173	0.00080
C ₆ H ₁₂	4-Methyl-1-pentene	1	0	1	0	2	2	1	2	0	0	0	0	0	0	72.10599	72.21038	0.00145
C ₆ H ₁₂	3-Methyl-trans-2-pentene	1	2	1	0	3	3	1	1	1	0	0	0	0	0	72.37203	72.33268	-0.00034
C ₇ H ₁₄	4-Methyl-trans-2-pentene	1	0	2	0	3	3	0	3	0	2	0	0	0	0	72.34745	72.31610	-0.00043
C ₇ H ₁₄	2-Ethyl-1-butene	1	0	0	1	2	2	2	0	4	0	0	0	0	0	72.21854	72.25909	0.00036
C ₇ H ₁₄	2,3-Dimethyl-1-butene	1	0	2	1	3	3	0	1	0	2	0	0	0	0	72.32543	72.32543	0.00008
C ₇ H ₁₄	3,3-Dimethyl-1-butene	1	0	1	1	3	0	1	0	0	3	0	0	-1	0	72.31796	72.30366	-0.00020
C ₇ H ₁₄	2,3-Dimethyl-2-butene	1	4	0	0	4	4	0	0	0	0	0	0	0	0	72.49750	72.38450	-0.00156
C ₇ H ₁₄	1-Heptene	1	0	1	0	1	1	4	1	4	0	0	0	0	0	84.19113	84.27084	0.00095
C ₇ H ₁₄	5-Methyl-1-hexene	1	0	1	0	2	2	2	2	1	3	0	0	0	0	84.26369	84.30608	0.00050
C ₇ H ₁₄	trans-3-Methyl-3-hexene	1	2	1	0	3	3	2	1	2	0	0	0	0	0	84.52973	84.42112	-0.00129
C ₇ H ₁₄	2,4-Dimethyl-1-pentene	1	0	0	2	3	3	1	1	0	3	0	0	0	0	84.44880	84.49367	0.00033
C ₇ H ₁₄	4,4-Dimethyl-1-pentene	1	0	1	1	3	1	1	1	0	0	4	0	0	0	84.27012	84.47087	0.00238
C ₇ H ₁₄	2,4-Dimethyl-2-pentene	1	2	1	0	4	4	0	2	0	2	0	0	0	0	84.63062	84.54445	-0.00102
C ₇ H ₁₄	trans-4,4-Dimethyl-2-pentene	1	0	2	0	4	4	0	2	0	0	3	0	-2	0	84.54076	84.54549	0.00006
C ₇ H ₁₄	2-Ethyl-3-methyl-1-butene	1	0	0	1	3	3	1	1	1	2	0	0	0	0	84.47715	84.44910	-0.00035
C ₇ H ₁₄	2,3,3-Trimethyl-1-butene	1	0	2	1	4	4	0	0	0	3	0	0	-2	0	84.51274	84.51129	-0.00002
C ₇ H ₁₆	1-Octene	1	0	1	1	1	1	5	1	5	0	0	0	0	0	96.34883	96.41421	0.00068
C ₇ H ₁₆	trans-2,2-Dimethyl-3-hexene	1	0	0	0	4	4	1	2	1	0	3	0	-2	0	96.69846	96.68782	-0.00011
C ₇ H ₁₆	3-Ethyl-2-methyl-1-pentene	1	0	2	1	3	3	2	1	2	2	0	0	0	0	96.63483	96.61113	-0.00025
C ₇ H ₁₆	2,4,4-Trimethyl-1-pentene	1	0	0	1	4	4	1	0	0	0	4	0	0	0	96.61293	96.71684	0.00107
C ₇ H ₁₆	2,4,4-Trimethyl-2-pentene	0	2	1	0	5	5	0	1	0	0	3	0	-3	0	96.67590	96.63880	-0.00018
C ₈ H ₁₈	1-Decene	1	0	1	1	1	1	7	1	7	0	0	0	0	0	120.66423	120.74240	0.00065
C ₈ H ₁₈	1-Dodecene	1	0	1	1	1	1	9	1	9	0	9	0	0	0	145.97963	145.07163	0.00063
C ₁₀ H ₂₂	1-Hexadecene	1	0	1	1	1	1	13	1	13	0	0	0	0	0	193.61043	193.71766	0.00055

Table 15.20. The bond angle parameters of alkenes and experimental values [1]. In the calculation of θ_i , the parameters from the preceding angle were used. E_T is $E_T(\text{atom} - \text{atom}, \text{msp}, \text{AO})$.

Atom of Angle	$2c^1$ Bond 1 (θ_0)	$2c^2$ Bond 2 (θ_0)	$2c^3$ Terminal Atom (θ_0)	E_{terminal} Atom 1 (θ_0)	Atom 1 Hybridization Designation (Table 15.3.A)	E_{terminal} Atom 2 (θ_0)	Atom 2 Hybridization Designation (Table 15.3.A)	c_2 Atom 1	c_2 Atom 2	c_1	c_2	E_T (eV)	θ_i ($^\circ$)	θ_j ($^\circ$)	Cal. θ ($^\circ$)	Exp. θ ($^\circ$)
$\angle H_1 C_1 C_2$ ($C_1 = C_2, C_3 = C_4$)	2.11323	2.86175	4.2895	-15.95954 C_1	9	-14.82375 C_2	1	0.85232	0.91771	0.75	1	0			118.36	
$\angle C_1 C_2 C_3$ ($C_1 = C_2, C_3 = C_4$)	2.86175	2.86175	4.7958	-16.08411 C_2	24	-16.08411 C_3	24	0.81549	0.81549	1	1	-1.85836			113.84	
$\angle C_1 C_2 C_3$ ($C_1 = C_2, C_3 = C_4$)	2.53321	2.86175	4.7339	-16.88873 C_1	27	-16.08411 C_2	24	0.80561	0.81549	1	1	-1.85836			123.46	124.4 (1,3,5-hexatriene C8C6C6) 121.7 (1,3,5-hexatriene C8C6C6) 124.4 (1,3-butadiene CCC) 125.3 (2-butene C6C6C6)
$\angle H_1 C_1 C_2$													118.36	123.46	118.19	
$\angle H_1 C_1 C_2$ ($H_1 C_2 = C_3 C_4$)	2.04578	2.04578	3.4756	-15.95955 C_1	9	H	H	0.85232	1	0.75	1	0			116.31	118.5 (2-methylpropene)
$\angle C_1 C_2 C_3$ ($H_1 C_2 = C_3 C_4$)															121.85	121 (2-methylpropene)
$\angle H_1 C_1 C_2$	2.11106	2.11106	3.4352	-15.75493 C_1	7	H	H	0.86359	1	0.75	1	0			108.44	107 (propene) 112 (propene) 113.8 (butane) 110.8 (isobutane) 111.0 (butane) 111.4 (isobutane)
$\angle C_1 C_2 C_3$																
$\angle H_1 C_1 C_2$	2.09711	2.09711	3.4352	-15.75493 C_1	7	H	H	0.86359	1	0.75	1	0			109.50	
$\angle C_1 C_2 C_3$															109.44	
$\angle C_1 C_2 C_3$															109.44	
$\angle C_1 C_2 C_3$ ($H_1 C_2 = C_3 C_4$)	2.91547	2.91547	4.7958	-16.68412 C_1	25	-16.68412 C_2	25	0.81549	0.81549	1	1	-1.85836			110.67	110.8 (isobutane)
$\angle C_1 C_2 C_3$ ($H_1 C_2 = C_3 C_4$)	2.91547	2.11323	4.1633	-15.55033 C_1	5	-14.82375 C_2	1	0.87495	0.91771	0.75	1	0			110.76	
$\angle C_1 C_2 C_3$ ($H_1 C_2 = C_3 C_4$)	2.91547	2.09711	4.1633	-15.55033 C_1	5	-14.82375 C_2	1	0.87495	0.91771	0.75	1	0			111.27	111.4 (isobutane)
$\angle C_1 C_2 C_3$ ($H_1 C_2 = C_3 C_4$)	2.90327	2.90327	4.7958	-15.55033 C_1	5	-14.82375 C_2	1	0.87495	0.91771	0.75	1	-1.85836			111.27	111.4 (isobutane)
$\angle C_1 C_2 C_3$													72.50		107.50	

ALKYNES (C_nH_{2n-2} , $n = 3, 4, 5 \dots \infty$)

The straight and branched-chain alkynes, C_nH_{2n-2} , have at least one carbon-carbon triple bond comprising a functional group that is solved equivalently to the triple bond of acetylene. The triple bond may be bound to one or two carbon single bonds that substitute for the hydrogen atoms of acetylene. Based on the energy matching of the mutually bound C, these C-C-bond MOs are defined as primary and secondary C-C functional groups, respectively, that are unique to alkynes. In addition, the corresponding terminal CH of a primary alkyne comprises a functional group that is solved equivalently to the methylene group of acetylene as given in the Acetylene Molecule section.

10 The alkyl portion of the alkyne may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of C-C bonds can be identified. The n-alkane C-C bond is the same as that of straight-chain alkanes. In
15 addition, the C-C bonds within isopropyl ($(CH_3)_2CH$) and t-butyl ($(CH_3)_3C$) groups and the isopropyl to isopropyl, isopropyl to t-butyl, and t-butyl to t-butyl C-C bonds comprise functional groups. The branched-chain-alkane groups in alkynes are equivalent to those in branched-chain alkanes.

The solution of the functional groups comprises the hybridization of the 2s and 2p
20 AOs of each C to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between two $C2sp^3$ HOs to form a MO permits each participating hybridized orbital to decrease in radius and energy. $E_r(atom-atom,msp^3.AO)$ of the $C \equiv C$ -bond MO in Eq. (15.52) due to the charge donation from the C atoms to the MO is equivalent to that of acetylene, $-3.13026 eV$, given by Eq. (14.342). $E_r(atom-atom,msp^3.AO)$ of each -alkyl-bond MO in
25 Eq. (15.52) is $-1.85836 eV$ or $-1.44915 eV$ based on the energy match between the $C2sp^3$ HOs corresponding to the energy contributions equivalent to those of methylene, $-0.92918 eV$ (Eq. (14.513), or methyl, $-0.72457 eV$ (Eq. (14.151)), groups, respectively. For the C-C groups each, comprising a C single bond to $C \equiv C$, $E_r(atom-atom,msp^3.AO)$ is
30 $-0.72457 eV$ based on the energy match between the $C2sp^3$ HOs for the mutually bound C of the single and triple bonds. The parameter ω of each group is matched for oscillation in the transition state based on the group being primary or secondary.

The symbols of the functional groups of alkynes are given in Table 15.21. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of alkynes are given in Tables 15.22, 15.23, and 15.24, respectively. The total energy of each alkyne given in Table 15.25 is calculated as the
5 sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.24 corresponding to functional-group composition of the molecule. The bond angle parameters of alkynes determined using Eqs. (15.79-15.108) are given in Table 15.26. Each C of the $C \equiv C$ group can further bond with only one atom, and the bond is linear as a minimum of energy as in the case of acetylene.

Table 15.21. The symbols of functional groups of alkynes.

Functional Group	Group Symbol
CC triple bond	C≡C
C single bond to C≡C (1°)	C'-C' (i)
C single bond to C≡C (2°)	C'-C' (ii)
CH (terminal)	C'-H (i)
CH ₃ group	C'-H (C'H ₃)
CH ₃ group	C'-H (C'H ₃)
CH (alkyl)	C'-H (ii)
CC bond (n-C)	C'-C' (a)
CC bond (iso-C)	C'-C' (b)
CC bond (terti-C)	C'-C' (c)
CC (iso to iso-C)	C'-C' (d)
CC (t to t-C)	C'-C' (e)
CC (t to iso-C)	C'-C' (f)

Table 15.22. The geometrical bond parameters of alkynes and experimental values [1].

Parameter	C≡C Group	C'-C' (i) Group	C'-C' (ii) Group	C'-H (i) Group	C'-H (C'H ₃) Group	C'-H (C'H ₃) Group	C'-H (ii) Group	C'-C' (a) Group	C'-C' (b) Group	C'-C' (c) Group	C'-C' (d) Group	C'-C' (e) Group	C'-C' (f) Group
σ (Å)	1.28714	1.99185	1.99185	1.48719	1.64920	1.67122	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
σ' (Å)	1.13452	1.41133	1.41133	0.99572	1.04856	1.05553	1.05661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length	1.20072	1.49369	1.49369	1.05383	1.10974	1.11713	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (Å)	1.203 (acetylene) 1.208 (2,4-hexadiynes)	1.450 (2,4-hexadiyne)	1.450 (2,4-hexadiyne)	1.060 (acetylene)	1.107 (C-H propane) 1.117 (C-H butane)	1.107 (C-H propane) 1.117 (C-H butane)	1.122 (isobutane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
h, σ (Å)	0.60793	1.40557	1.40557	1.10466	1.27295	1.29569	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
ρ	0.88143	0.70855	0.70855	0.66953	0.63580	0.63159	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 15.23. The MO to HO intercept geometrical bond parameters of alkynes. E_T is $E_T(\text{atom} - \text{atom}, \text{misp}, \text{AO})$.

Bond	Atom	E_T (eV) Bond 1	E_T (eV) Bond 2	E_T (eV) Bond 3	E_T (eV) Bond 4	Final Total Energy C2sp ³ (eV)	r_{final} (a ₀)	r_{final} (a ₀)	E_{C2sp^3} (eV) Final	$E(\text{C2sp}^3)$ (eV) Final	θ' (°)	θ_1 (°)	θ_2 (°)	d_1 (a ₀)	d_2 (a ₀)
$RC_1C_2C_3-H$	C ₁	-1.50513	0	0	0	-153.18082	0.91771	0.83008	-16.39088	-16.20002	90.99	89.01	48.71	0.98144	0.91428
$C_1C_2C_3H$	C ₁	-1.50513	-0.36220	0	0	-153.54311	0.91771	0.81213	-16.75317	-16.50221	137.17	42.83	65.25	0.53890	0.59562
$C_1C_2C_3H$	C ₂	-1.50513	0	0	0	-153.18082	0.91771	0.83008	-16.39088	-16.20002	137.91	42.09	66.24	0.51853	0.61599
$C_1-C_2=C_3H$	C ₃	-0.30220	-0.92018	0	0	-153.00716	0.91771	0.84418	-16.11722	-15.92636	75.71	104.29	55.59	1.61974	0.20841
$C-H(CH_2)$	C ₁	-0.92018	0	0	0	-153.54487	0.91771	0.86359	-15.75403	-15.56407	77.40	102.51	41.48	1.23564	0.18708
$C-H(CH_2)$	C ₂	-0.92018	-0.92018	0	0	-153.47406	0.91771	0.81349	-16.68412	-16.49325	68.47	111.53	35.84	1.34486	0.20933
$C-H(CH)$	C ₃	-0.92018	-0.92018	-0.92018	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	61.10	118.90	31.37	1.42088	0.37326
$H_3C_1C_2H_2CH_3-$ (C-C (a))	C ₁	-0.92018	0	0	0	-152.54487	0.91771	0.86359	-15.75403	-15.56407	63.82	116.18	30.08	1.83879	0.38106
$H_3C_1C_2H_2CH_3-$ (C-C (a))	C ₂	-0.92018	-0.92018	0	0	-153.47406	0.91771	0.81349	-16.68412	-16.49325	56.41	123.59	26.06	1.90800	0.45117
$R-H_2C_1C_2(H_1C_1-R)HCH_2-$ (C-C (b))	C ₁	-0.92018	-0.92018	-0.92018	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C_1C_2(R'-H_1C_1)C_3(R''-H_1C_1)CH_2-$ (C-C (c))	C ₂	-0.92018	-0.72457	-0.72457	-0.72457	-154.71860	0.91771	0.75189	-17.92866	-17.73779	48.21	131.79	21.74	1.95754	0.50370
$isoC_1C_2(H_1C_1-R')HCH_2-$ (C-C (d))	C ₁	-0.92018	-0.92018	-0.92018	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$isoC_1(R'-H_1C_1)C_2(R''-H_1C_1)CH_2-$ (C-C (e))	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49298
$isoC_1C_2(H_1C_1-R')HCH_2-$ (C-C (f))	C ₂	-0.72457	-0.92018	-0.92018	0	-154.19163	0.91771	0.78155	-17.40869	-17.21783	52.78	127.22	24.04	1.92443	0.47279
$isoC_1(R'-H_1C_1)C_2(R''-H_1C_1)CH_2-$ (C-C (g))	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49298

Table 1.5.24. The energy parameters (eV) of functional groups of alkynes.

Parameters	C=C Group	C-C (i) Group	C-C (ii) Group	C-H (i) Group	CH ₃ Group	CH ₃ Group	CH ₃ Group	C-H (ii) Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
n_1	3	1	1	1	3	3	2	1	1	1	1	1	1	1
n_2	0	0	0	0	2	2	1	0	0	0	0	0	0	0
n_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_1	0.5	0.5	0.5	0.75	0.75	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_4	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_5	2	1	1	0	0	0	1	1	0	0	0	1	1	0
C_6	6	2	2	1	1	1	1	1	2	2	2	2	2	2
C_7	0	0	0	1	3	3	2	1	0	0	0	0	0	0
C_{10}	0.5	0.5	0.5	0.75	0.75	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_{20}	1	1	1	1	1	1	1	1	1	1	1	1	1	1
V_c (eV)	-182.53826	-31.29307	-31.29307	-40.62396	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-28.79214	-28.79214	-28.79214	-28.79214	-28.79214
V_r (eV)	35.97770	9.64042	9.64042	13.66428	38.92728	25.78002	12.87680	9.33352	9.33352	9.33352	9.33352	9.33352	9.33352	9.33352
T (eV)	70.90876	7.85528	7.85528	13.65796	32.53914	21.06675	10.48582	6.77464	6.77464	6.77464	6.77464	6.77464	6.77464	6.77464
V_a (eV)	-35.45438	-3.92764	-3.92764	-6.82898	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.38732	-3.38732	-3.38732	-3.38732	-3.38732
$E(\nu_{\text{C-H}})$ (eV)	-16.20002	-14.63489	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
$\Delta E_{\text{H-L}}(\nu_{\text{H-H}})$ (eV)	0	-0.72457	-0.72457	-3.130269	0	0	0	0	0	0	0	0	0	0
$E_T(\nu_{\text{C-H}})$ (eV)	-16.20002	-13.91032	-13.91032	-11.50462	-15.56407	-15.56407	-14.63489	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
$E_T(\nu_{\text{C-H}})$ (eV)	-94.90616	-31.63533	-31.63533	-31.63532	-67.69451	-49.66493	-31.63533	-31.63533	-31.63533	-31.63533	-31.63533	-31.63533	-31.63533	-31.63533
$E_T(\text{atom} - \text{atom.msp}^3 \text{AO})$ (eV)	-3.13026	-0.72457	-0.72457	0	0	0	0	0	-1.85836	-1.85836	-1.44915	-1.85836	-1.44915	-1.44915
$E_T(\nu_{\text{C-H}})$ (eV)	-98.03637	-32.35994	-32.35994	-31.63537	-67.69450	-49.66493	-31.63537	-31.63537	-33.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452
ω (10^{15} rad/s)	20.0186	10.3988	17.5426	30.8370	24.9286	24.2751	24.1759	24.1759	9.43699	9.43699	15.4846	9.43699	9.55643	9.55643
E_K (eV)	13.17659	6.84470	11.54882	20.29747	16.40846	15.97831	15.91299	15.91299	6.21159	6.21159	10.19220	6.21159	6.29021	6.29021
E_D (eV)	-0.23468	-0.16749	-0.21754	-0.28197	-0.25352	-0.25017	-0.24966	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.16416	-0.16416
E_{K-H} (eV)	0.27773	0.08989	0.08989	0.35332	0.35332	0.35332	0.35332	0.35332	0.12312	0.12312	0.09944	0.12312	0.12312	0.12312
$E_{\text{H-H}}$ (eV)	-0.09581	-0.12255	-0.17260	-0.10430	-0.22757	-0.14502	-0.07200	-0.07200	-0.10359	-0.10359	-0.15924	-0.10359	-0.10260	-0.10260
$E_{\text{H-H}}$ (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
$E_T(\nu_{\text{H-H}})$ (eV)	-98.02775	-32.48249	-32.53254	-31.73967	-67.92207	-49.80996	-31.70737	-33.59732	-33.59732	-33.49373	-33.24376	-33.59732	-33.18712	-33.18712
$E_{\text{H-H}}(\nu_{\text{H-H}})$ (eV)	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
$E_{\text{H-H}}(\nu_{\text{H-H}})$ (eV)	0	0	0	-13.59844	-13.59844	-13.59844	-13.59844	-13.59844	0	0	0	0	0	0
$E_D(\nu_{\text{H-H}})$ (eV)	10.21841	3.21271	3.26276	3.50634	12.49186	7.83016	3.32601	4.32754	4.32754	4.29921	3.97398	4.17951	3.62128	3.91734

Table 15.25. The total bond energies of alkynes calculated using the functional group composition and the energies of Table 15.24 compared to the experimental values [2].

Formula	Name	$C \equiv C$	$C-H$ (i)	$C-H$ (ii)	$C-H$ (a)	$C-H$ (b)	$C-H$ (c)	$C-H$ (d)	$C-H$ (e)	$C-H$ (f)	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C_3H_4	Propyne	1	1	1	0	0	0	0	0	0	29.42332	29.40432	-0.00085
C_4H_6	1-Butyne	1	1	1	0	1	0	0	0	0	41.58702	41.55495	-0.00077
C_4H_6	2-Butyne	1	0	2	0	0	0	0	0	0	41.72765	41.75705	0.00070
C_5H_{10}	1-Pentyne	1	1	1	0	0	0	0	0	0	102.37552	102.55367	-0.00021

Table 15.26. The bond angle parameters of alkynes and experimental values [1]. In the calculation of θ_p , the parameters from the preceding angle were used. E_T is $E_T(Atom - atom, nsp^3 AO)$.

Atom of Angle	$2\theta'$ Bond 1 ($^\circ$)	$2\theta'$ Bond 2 ($^\circ$)	$2\theta'$ Terminal Atom ($^\circ$)	E_{T, nsp^3} Atom 1	Atom 1 Hybridization Designation (Table 15.3.A)	E_{T, nsp^3} Atom 2	Atom 2 Hybridization Designation (Table 15.3.A)	C_2 Atom 1	C_2 Atom 2	C_1	C_2	C_1	C_2	C_1	C_2	E_T (eV)	θ_p ($^\circ$)	θ_1 ($^\circ$)	θ_2 ($^\circ$)	Cal. θ ($^\circ$)	Exp. θ ($^\circ$)
$\angle C_1 C_2 C_3$ ($C_1 C_2 \equiv C_3 H$)																					
$\angle HC_2 H$	2.11106	2.11106	3.4252	-15.75493	7	H	H	0.86359	1	1	1	1	1	1	1	0				180	
$\angle C_1 C_2 C_3$																				108.44	107 (propane)
$\angle C_1 C_2 H$																				110.49	112 (propane) 113.8 (butane) 110.8 (isobutane)
$\angle HC_2 H$	2.09711	2.09711	3.4252	-15.75493	7	H	H	0.86359	1	1	1	1	1	1	1	0	69.51			110.49	111.0 (butane) 111.4 (isobutane)
$\angle C_1 C_2 C_3$																				109.50	
$\angle C_1 C_2 H$	2.91547	2.91547	4.7958	-16.68412	25	C_2	C_2	0.81549	0.81549	1	1	1	1	1	1	-1.85836	70.56			109.44	
$\angle C_1 C_2 H$	2.91547	2.91547	4.1633	-15.55033	5	C_1	C_1	0.87495	0.87495	0.75	1	0.75	1	0.75	1	0				110.67	110.8 (isobutane)
$\angle C_1 C_2 H$	2.91547	2.91547	4.1633	-15.55033	5	C_1	C_1	0.87495	0.87495	0.75	1	0.75	1	0.75	1	0				110.76	
$\angle C_1 C_2 C_3$	2.90327	2.90327	4.7958	-15.55033	5	C_2	C_2	0.87495	0.87495	0.75	1	0.75	1	0.75	1	-1.85836	72.50			111.27	111.4 (isobutane)
$\angle C_1 C_2 C_3$																				111.27	111.4 (isobutane)

ALKYL FLUORIDES ($C_nH_{2n+2-m}F_m$, $n=1,2,3,4,5\ldots\infty$ $m=1,2,3\ldots\infty$)

The branched-chain alkyl fluorides, $C_nH_{2n+2-m}F_m$, may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds wherein at least one H is replaced by a fluorine. The $C-F$ bond comprises a functional group for each case of F replacing a H of methane in the series $H_{4-m}C-F_m$, $m=1,2,3,4$, and F replacing a H of an alkane. The methyl, methylene, methylene functional groups are equivalent to those of branched-chain alkanes. Six types of $C-C$ bonds can be identified. The n -alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2CH$) and t -butyl ($(CH_3)_3C$) groups and the isopropyl to isopropyl, isopropyl to t -butyl, and t -butyl to t -butyl $C-C$ bonds comprise functional groups that are equivalent to those of branched-chain alkanes.

The solution of the $C-F$ functional groups comprises the hybridization of the $2s$ and $2p$ AOs of each C to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO and the F AO to form a molecular orbital (MO) permits each participating orbital to decrease in radius and energy. In alkyl fluorides, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a corresponding energy of $E(C,2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)), and the F AO has an energy of $E(F) = -17.42282 \text{ eV}$. To meet the equipotential condition of the union of the $C-F$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor c_2 of Eq. (15.52) for the $C-F$ -bond MO given by Eqs. (15.68) and (15.70) is

$$c_2(C2sp^3HO \text{ to } F) = \frac{E(C,2sp^3)}{E(F)} c_2(C2sp^3HO) = \frac{-14.63489 \text{ eV}}{-17.42282 \text{ eV}} (0.91771) = 0.77087 \quad (15.110)$$

$E_r(\text{atom-atom}, msp^3.AO)$ of the $C-F$ -bond MO in Eq. (15.52) based on the charge donation from F to the MO is determined by the linear combination that results in a energy that is a minimum which does not exceed the energy of the AO of the F atom to which it is energy matched.

The symbols of the functional groups of branched-chain alkyl fluorides are given in Table 15.27. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of branched-chain alkyl fluorides are

given in Tables 15.28, 15.29, and 15.30, respectively. The total energy of each branched-chain alkyl fluoride given in Table 15.31 was calculated as the sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.30 corresponding to functional-group composition of the molecule. For each set of unpaired electrons created by bond breakage, the $C2sp^3$ HO magnetic energy E_{mag} that is subtracted from the weighted sum of the $E_D(\text{Group})$ (eV) values based on composition is given by Eq. (15.58). In the case of trifluoromethane, E_{mag} is positive since the term due to the fluorine atoms cancels that of the CH group. The $C-C$ bonds to the CHF group (one H bond to C) were each treated as an iso $C-C$ bond. The $C-C$ bonds to the CF group (no H bonds to C) were each treated as a tert-butyl $C-C$. E_{mag} was subtracted for each t-butyl group. The bond angle parameters of branched-chain alkyl fluorides determined using Eqs. (15.61-15.70), (15.79-15.108) and (15.110) are given in Table 15.32.

Table 15.27. The symbols of functional groups of branched-chain alkyl fluorides.

Functional Group	Group Symbol
CF of $CF_m H_{4-m}$	$C-F$ (i)
CF of $C_n H_{2n+2-m} F_m$	$C-F$ (ii)
CH_3 group	$C-H$ (CH_3)
CH_2 group	$C-H$ (CH_2)
CH	$C-H$
CC bond (n-C)	$C-C$ (a)
CC bond (iso-C)	$C-C$ (b)
CC bond (tert-C)	$C-C$ (c)
CC (iso to iso-C)	$C-C$ (d)
CC (t to t-C)	$C-C$ (e)
CC (t to iso-C)	$C-C$ (f)

Table 15.28. The geometrical bond parameters of branched-chain alkyl fluorides and experimental values [1].

Parameter	C-F (i) Group	C-F (ii) Group	C-H (i) Group	C-H (ii) Group	C-H (iii) Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
a (a_0)	1.72139	1.72139	1.64920	1.67122	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
c' (a_0)	1.31202	1.31202	1.04836	1.05553	1.05661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length $2c'$ (\AA)	1.38858	1.38858	1.10974	1.11713	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (\AA)	1.382 (methyl fluoride)	1.382 (methyl fluoride)	1.107 (C-H propane) 1.117 (C-H butane)	1.107 (C-H propane) 1.117 (C-H butane)	1.122 (isobutane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
h_{c-c} (a_0)	1.11435	1.11435	1.27295	1.29569	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
c	0.76219	0.76219	0.63580	0.63159	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 15.29. The MO to HO intercept geometrical bond parameters of branched-chain alkyl fluorides. R, R', R'' are H or alkyl groups. E_r is $E_r(\text{atom}, \text{exp}, \text{MO})$.

Bond	Atom	E_r (eV) Bond 1	E_r (eV) Bond 2	E_r (eV) Bond 3	E_r (eV) Bond 4	Final Total Energy (eV)	r_{final} (a_0)	r_{final} (a_0)	E_{final} (eV)	$E(\text{exp})$ (eV)	θ' ($^\circ$)	θ_1 ($^\circ$)	θ_2 ($^\circ$)	d_1 (a_0)	d_2 (a_0)
H_2C-C-F (C-F (i))	C	-1.34946	0	0	0	-152.96515	0.91771	0.84115	-16.17521	-15.98435	100.77	79.23	47.86	1.15488	0.15714
H_2C-C-F (C-F (ii))	F	-1.34946	0	0	0	-152.96515	0.91771	0.84115	-16.17521	-15.98435	100.77	79.23	47.86	1.15488	0.15714
H_2C-C-F (C-F (iii))	C	-1.34946	-0.92918	0	0	-153.89433	0.91771	0.79346	-17.10440	-16.91353	97.02	82.98	45.11	1.21483	0.09718
H_2C-C-F (C-F (iv))	F	-1.34946	0	0	0	-152.96515	0.91771	0.84115	-16.17521	-15.98435	100.77	79.23	47.86	1.15488	0.15714
$C-C-H$ (CH ₃)	C	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	77.49	102.51	41.48	1.23564	0.18708
$C-C-H$ (CH ₂)	C	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	-16.49325	68.47	111.53	35.84	1.35486	0.29933
$C-C-H$ (CH)	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	61.10	118.90	31.37	1.42988	0.37326
H_2C-C-H_2 (C-C (ii))	C	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	63.82	116.18	30.08	1.33879	0.38106
H_2C-C-H_2 (C-C (iii))	C	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	-16.49325	56.41	123.59	26.06	1.50890	0.45117
$R-H_2C-C(H_2-C-R')HCH_3$ (C-C (iv))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C(H_2-C-R')HCH_3$ (C-C (v))	C	-0.92918	-0.92918	-0.92918	-0.92918	-154.71860	0.91771	0.75889	-17.93866	-17.73779	48.21	131.79	21.74	1.95734	0.50370
$HO-C-C(H_2-C-R')HCH_3$ (C-C (vi))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$HO-C-C(H_2-C-R')HCH_3$ (C-C (vii))	C	-0.92918	-0.92918	-0.92918	-0.92918	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49298
$HO-C-C(H_2-C-R')HCH_3$ (C-C (viii))	C	-0.92918	-0.92918	-0.92918	0	-154.19863	0.91771	0.78155	-17.40869	-17.21783	52.78	127.22	24.04	1.92443	0.47279
$HO-C-C(H_2-C-R')HCH_3$ (C-C (ix))	C	-0.92918	-0.92918	-0.92918	-0.92918	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49298

Table 15.30. The energy parameters (eV) of functional groups of branched-chain alkyl fluorides.

Parameters	C-F (i) Group	C-F (ii) Group	C-H Group	C-H ₂ Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
n_1	1	1	3	2	1	1	1	1	1	1	1
n_2	0	0	2	1	0	0	0	0	0	0	0
n_3	0	0	0	0	0	0	0	0	0	0	0
C_1	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_2	1	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1
S_1	0.77087	0.77087	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
S_2	0	0	0	1	1	0	0	0	1	1	0
S_3	2	2	1	1	1	2	2	2	2	2	2
S_4	0	0	3	2	1	0	0	0	0	0	0
C_1'	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_2'	1	1	1	1	1	1	1	1	1	1	1
V_1 (eV)	-32.02108	-32.02108	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-29.10112	-28.79214	-29.10112	-29.10112
V_2 (eV)	10.37015	10.37015	38.92728	25.78002	12.87680	9.3352	9.3352	9.37273	9.3352	9.37273	9.37273
V_3 (eV)	9.10097	9.10097	32.53914	21.06675	10.48582	6.77464	6.77464	6.90500	6.77464	6.90500	6.90500
V_4 (eV)	-4.65048	-4.65048	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.45250	-3.38732	-3.45250	-3.45250
E_1 (eV)	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.35946	-15.56407	-15.35946	-15.35946
$\Delta E_{11,100}$ (eV)	0	0	0	0	0	0	0	0	0	0	0
E_2 (eV)	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.35946	-15.56407	-15.35946	-15.35946
E_3 (eV)	-31.63534	-31.63534	-67.69451	-49.66493	-31.63533	-31.63537	-31.63537	-31.63535	-31.63537	-31.63535	-31.63535
E_4 (eV)	-2.69892	-2.69892	0	0	0	-1.85836	-1.85836	-1.44915	-1.85836	-1.44915	-1.44915
E_5 (eV)	-34.33429	-34.33429	-67.69450	-49.66493	-31.63537	-33.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452
ω (10 ⁴ rad/s)	24.8506	12.9435	24.9286	24.2751	24.1759	9.43699	9.43699	15.4846	9.43699	9.55643	9.55643
E_6 (eV)	16.33707	8.51966	16.40846	15.97831	15.91299	6.21159	6.21159	10.19220	6.21159	6.29021	6.29021
E_7 (eV)	-0.27472	-0.19826	-0.25352	-0.25017	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.16416	-0.16416
E_8 (eV)	0.13849	0.10911	0.35532	0.35532	0.35532	0.12312	0.17978	0.09944	0.12312	0.12312	0.12312
E_9 (eV)	-0.20547	-0.14371	-0.22757	-0.14502	-0.07200	-0.10359	-0.07526	-0.15924	-0.10359	-0.10260	-0.10260
E_{10} (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
E_{11} (eV)	-34.53976	-34.47800	-67.92207	-49.80996	-31.70737	-33.59732	-33.49373	-33.24376	-33.59732	-33.18712	-33.18712
E_{12} (eV)	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
E_{13} (eV)	0	0	-13.59844	-13.59844	-13.59844	0	0	0	0	0	0
E_{14} (eV)	5.26998	5.20822	12.49186	7.83016	3.32601	4.32754	4.29921	3.97398	4.17951	3.62128	3.91734

Table 15.31. The total bond energies of branched-chain alkyl fluorides calculated using the functional group composition and the energies of Table 15.30 compared to the experimental values [2]. The magnetic energy E_{mag} that is subtracted from the weighted sum of the $E_{11}(i \rightarrow j)$ (eV) values based on composition is given by (15.58).

Formula	C-F (i)	C-F (ii)	C-H	C-H ₂	C-H	C-C (a)	C-C (b)	C-C (c)	C-C (d)	C-C (e)	C-C (f)	E_{mag}	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CF ₄	4	0	0	0	0	0	0	0	0	0	0	0	21.07992	21.016	-0.00003
CHF ₃	3	0	0	0	1	0	0	0	0	0	0	0	19.24398	19.363	0.00405
CH ₂ F ₂	2	0	0	1	0	0	0	0	0	0	0	0	18.37012	18.280	-0.00496
C ₂ H ₅ F	0	1	1	2	0	0	0	0	0	0	0	-1	41.86745	41.885	0.00041
C ₂ H ₄ F ₂	0	1	2	0	1	0	2	0	0	0	0	-1	41.96334	41.963	-0.00012

Atoms of Angle	$2C'$ Bond 1 (σ_e)	$2C'$ Bond 2 (σ_e)	$2C'$ Terminal Atoms (σ_e)	$E_{\text{electrode}}$ Atom 1	Atom 1 Hybridization Designation (Table 13.3.A)	$E_{\text{electrode}}$ Atom 2	Atom 2 Hybridization Designation (Table 13.3.A)	ζ_2 Atom 1	ζ_3 Atom 2	C_1	C_2	ζ_1	ζ_2	E_γ (eV)	θ_v ($^\circ$)	θ_1 ($^\circ$)	θ_2 ($^\circ$)	Cal. θ ($^\circ$)	Exp. θ ($^\circ$)
$\angle FC_2F$ ($C'' - F$ (i))	2.62403	2.62403	4.3128	-16.17521 F	12	-16.17521 F	12	0.84115	0.84115	1	1	1	0.84115	-1.44915				110.53	108.8 (fluoroform)
$\angle HC_2F$ ($C'' - F$ (i))	2.11106	2.62403	3.8987	-17.10440 C''	31	-17.42283 F	F	0.79546	0.78092 (Eq. (15.64))	0.75	1	0.75	0.98172	0				110.38	
$\angle C_2C_2F$ ($C'' - F$ (ii))	2.91547	2.62403	4.5826	-16.68412 C''	25	-17.42283 F	F	0.81549	0.77087 (Eq. (15.110))	1	1	1	0.79318	-1.55856				111.53	110.3 (1,2-difluoroethane)
$\angle C_2C_2H$ ($C'' - F$ (ii))	2.91547	2.11106	4.1633	-15.55033 C''	5	-14.82575 C''	1	0.87495	0.91771	0.75	1	0.75	1.04887	0				110.83	111.0 (1,1-difluoroethane)
Methyl $\angle HC_2H$	2.11106	2.11106	3.4252	-15.75493	7	H		0.86359	1	1	1	0.75	1.15796	0				108.44	107 (propane)
$\angle C_2C_2C_1$															69.51			110.49	112 (propane) 113.8 (butane) 110.8 (isobutane)
$\angle C_2C_2H$															69.51			110.49	111.0 (butane) 111.4 (isobutane)
Methyl $\angle HC_2H$	2.09711	2.09711	3.4252	-15.75493	7	H		0.86359	1	1	1	0.75	1.15796	0				109.50	
$\angle C_2C_2C_2$																70.56		109.44	
$\angle C_2C_2H$																70.56		109.44	
$\angle C_2C_2C_2$ non C''	2.91547	2.91547	4.7958	-16.68412 C''	25	-16.68412 C''	25	0.81549	0.81549	1	1	1	0.81549	-1.83836				110.67	110.8 (isobutane)
$\angle C_2C_2H$ non C''	2.91547	2.11323	4.1633	-15.55033 C''	5	-14.82575 C''	1	0.87495	0.91771	0.75	1	0.75	1.04887	0				110.76	
$\angle C_2C_2H$ non C''	2.91547	2.09711	4.1633	-15.55033 C''	5	-14.82575 C''	1	0.87495	0.91771	0.75	1	0.75	1.04887	0				111.27	111.4 (isobutane)
$\angle C_2C_2C_2$ non C''	2.90327	2.90327	4.7958	-15.55033 C''	5	-14.82575 C''	1	0.87495	0.91771	0.75	1	0.75	1.04887	-1.83836				111.27	111.4 (isobutane)
$\angle C_2C_2C_2$															72.50			107.50	

ALKYL CHLORIDES ($C_n H_{2n+2-m} Cl_m$, $n = 1, 2, 3, 4, 5 \dots \infty$ $m = 1, 2, 3 \dots \infty$)

The branched-chain alkyl chlorides, $C_n H_{2n+2-m} Cl_m$, may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds wherein at least one H is replaced by a chlorine. The $C-Cl$ bond comprises a functional group for each case of Cl replacing a H of methane for the series $H_{4-m} C-Cl_m$, $m = 1, 2, 3$, with the $C-Cl$ bond of CCl_4 comprising another functional group due to the limitation of the minimum energy of Cl matched to that of the $C2sp^3$ HO. In addition, the $C-Cl$ bond due to Cl replacing a H of an alkane is a function group. The methyl, methylene, methylene functional groups are equivalent to those of branched-chain alkanes. Six types of $C-C$ bonds can be identified. The n-alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2 CH$) and t-butyl ($(CH_3)_3 C$) groups and the isopropyl to isopropyl, isopropyl to t-butyl, and t-butyl to t-butyl $C-C$ bonds comprise functional groups that are equivalent to those of branched-chain alkanes.

The solution of the $C-Cl$ functional groups comprises the hybridization of the $2s$ and $2p$ AOs of each C to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO and the Cl AO to form a MO permits each participating orbital to decrease in radius and energy. In alkyl chlorides, the energy of chlorine is less than the Coulombic energy between the electron and proton of H given by Eq. (1.243). Thus, c_2 in Eq. (15.52) is one, and the energy matching condition is determined by the C_2 parameter. Then, $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a corresponding energy of $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)), and the Cl AO has an energy of $E(Cl) = -12.96764 \text{ eV}$. To meet the equipotential condition of the union of the $C-Cl$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor C_2 of Eq. (15.52) for the $C-Cl$ -bond MO given by Eqs. (15.68) and (15.70) is

$$C_2(C2sp^3 HO \text{ to } Cl) = \frac{E(Cl)}{E(C, 2sp^3)} c_2(C2sp^3 HO) = \frac{-12.96764 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.81317 \quad (15.110)$$

The valence energy of the carbon $2p$ is -11.2603 eV and that of the Cl AO is -12.96764 eV .

The energy difference is more than that of $2E_r(C-C, 2sp^3)$ given by Eq. (14.151) for a single

bond. Thus, $E_r(\text{atom} - \text{atom}, msp^3, AO)$ of the $C - Cl$ -bond MO in Eq. (15.52) due to the charge donation from the C and Cl atoms to the MO is -1.44915 eV based on the energy match between the $C2sp^3$ HO and the Cl AO corresponding to the energy contributions equivalent to those of methyl groups, -0.72457 eV (Eq. (14.151)).

5 The symbols of the functional groups of branched-chain alkyl chlorides are given in Table 15.33. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of branched-chain alkyl chlorides are given in Tables 15.34, 15.35, and 15.36, respectively. The total energy of each branched-chain alkyl chloride given in Table 15.37 was calculated as the sum over the integer multiple of each

10 $E_D(\text{Group})$ of Table 15.36 corresponding to functional-group composition of the molecule. For each set of unpaired electrons created by bond breakage, the $C2sp^3$ HO magnetic energy E_{mag} that was subtracted from the weighted sum of the $E_D(\text{Group})$ (eV) values based on composition is given by Eq. (15.58). The $C - C$ bonds to the $CHCl$ group (one H bond to C) were each treated as an iso $C - C$ bond. The $C - C$ bonds to the CCl group (no H bonds to C) were

15 each treated as a tert-butyl $C - C$. E_{mag} was subtracted for each t-butyl group. The bond angle parameters of branched-chain alkyl chlorides determined using Eqs. (15.61-15.70), (15.79-15.108) and (15.111) are given in Table 15.38.

Table 15.33. The symbols of functional groups of branched-chain alkyl chlorides.

Functional Group	Group Symbol
CCl of $CCl_m H_{4-m}$	$C - Cl$ (i)
CCl of CCl_4	$C - Cl$ (ii)
CCl of $C_n H_{2n+2-m} Cl_m$	$C - Cl$ (iii)
CH_3 group	$C - H$ (CH_3)
CH_2 group	$C - H$ (CH_2)
CH	$C - H$
CC bond (n-C)	$C - C$ (a)
CC bond (iso-C)	$C - C$ (b)
CC bond (tert-C)	$C - C$ (c)
CC (iso to iso-C)	$C - C$ (d)
CC (t to t-C)	$C - C$ (e)
CC (t to iso-C)	$C - C$ (f)

Table 15.55. The MO to HO intercept geometrical bond parameters of branched-chain alkyl chlorides. R, R', R'' are H or alkyl groups. E_T is $E_T(\text{atom} - \text{atom}, \text{msp}^3 A(\cdot))$.

Bond	Atom	K_T (eV) Bond 1	H_T (eV) Bond 2	H_T (eV) Bond 3	K_T (eV) Bond 4	Final Total Energy '2sp' (eV)	r_{final} (a_0)	r_{final} (a_u)	E_{residual} (eV) Final	$E(C2sp^2)$ (eV) Final	θ ($^\circ$)	θ_1 ($^\circ$)	θ_2 ($^\circ$)	d_i (a_u)	u_a (a_u)
$H_{\text{bond}}C_m-C(l)_m$, $m=1,2,3$ ($C_m-C(l)$ (i))	C_m	-0.72457	0	0	0	-152.34026	0.91771	0.87495	-15.55033	-15.55045	69.63	110.38	30.00	1.99599	0.30463
$H_{\text{bond}}C_m-C(l)_m$, $m=1,2,3$ ($C_m-C(l)$ (ii))	$C(l)$	-0.72457	0	0	0		1.05158	0.87495	-15.55033		69.63	110.38	30.00	1.99599	0.30463
$C_m^2H_3$ ($C_m-C(l)$ (iii))	C_m	-0.46459	0	0	0	-152.08028	0.91771	0.88593	-15.29034	-15.09948	66.08	113.02	29.87	2.05530	0.34401
$C_m^2H_3$ ($C_m-C(l)$ (iv))	$C(l)$	-0.46459	0	0	0		1.05158	0.88593	-15.29034		66.08	113.02	29.87	2.05530	0.34401
$-H_1C_1C_2-C(l)$ ($C_2-C(l)$ (v))	C_m	-0.72457	-0.92918	0	0	-153.26045	0.91771	0.82562	-16.47951	-16.28864	63.18	116.82	27.48	2.06384	0.37248
$-H_1C_1C_2-C(l)$ ($C_2-C(l)$ (vi))	$C(l)$	-0.72457	0	0	0		1.05158	0.87495	-15.55033		69.62	110.38	30.00	1.99599	0.30463
$C_m-H(C_2H_3)$	C_m	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	77.49	102.51	41.48	1.23564	0.18708
$C_m-H(C_2H_3)$	$C(l)$	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	-16.49325	64.47	111.53	35.84	1.35486	0.29233
$H_1C_1C_2H_3H_3$ ($C_m-C(l)$ (vii))	C_m	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	61.10	118.90	31.37	1.42918	0.37226
$H_1C_1C_2H_3H_3$ ($C_m-C(l)$ (viii))	$C(l)$	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	63.82	116.18	30.08	1.83879	0.38106
$R-H_1C_1C_2(H_1C_2-R)HC(H_3)$ ($C_m-C(l)$ (ix))	C_m	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	-16.49325	56.41	123.59	26.06	1.90800	0.45117
$R-H_1C_1C_2(H_1C_2-R)HC(H_3)$ ($C_m-C(l)$ (x))	$C(l)$	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.00	1.97162	0.51388
$R-H_1C_1C_2(H_1C_2-R)HC(H_3)$ ($C_m-C(l)$ (xi))	C_m	-0.92918	-0.72457	-0.72457	-0.72457	-154.71860	0.91771	0.75489	-17.92166	-17.73779	48.21	131.79	21.74	1.93734	0.50570
$R-H_1C_1C_2(H_1C_2-R)HC(H_3)$ ($C_m-C(l)$ (xii))	$C(l)$	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.00	1.97162	0.51388
$R-H_1C_1C_2(H_1C_2-R)HC(H_3)$ ($C_m-C(l)$ (xiii))	C_m	-0.72457	-0.72457	-0.72457	-0.72457	-154.51390	0.91771	0.76765	-17.92166	-17.73779	50.04	129.96	22.66	1.94462	0.49298
$R-H_1C_1C_2(H_1C_2-R)HC(H_3)$ ($C_m-C(l)$ (xiv))	$C(l)$	-0.72457	-0.92918	-0.92918	0	-154.19863	0.91771	0.78155	-17.40869	-17.21783	52.78	127.22	24.04	1.92443	0.47279
$R-H_1C_1C_2(H_1C_2-R)HC(H_3)$ ($C_m-C(l)$ (xv))	C_m	-0.72457	-0.72457	-0.72457	-0.72457	-154.51390	0.91771	0.76765	-17.92166	-17.73779	50.04	129.96	22.66	1.94462	0.49298

Table 15.36. The energy parameters (eV) of functional groups of branched-chain alkyl chlorides.

Parameters	C-Cl Group	C-Cl (ii) Group	C-Cl (iii) Group	C-H ₂ Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
n_1	1	1	1	2	1	1	1	1	1	1	1
n_2	0	0	0	1	0	0	0	0	0	0	0
n_3	0	0	0	0	0	0	0	0	0	0	0
C_1	0.5	0.5	0.5	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_2	0.81317	0.81317	0.81317	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1
C_4	1	1	1	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_5	1	1	1	1	1	0	0	0	1	1	0
C_6	2	2	2	1	1	2	2	2	2	2	2
C_7	0	0	0	2	1	0	0	0	0	0	0
C_8	0.5	0.5	0.5	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_9	0.81317	0.81317	0.81317	1	1	1	1	1	1	1	1
V_1 (eV)	-29.68411	-28.95265	-29.68411	-70.41425	-35.12015	-28.79214	-28.79214	-29.10112	-28.79214	-29.10112	-29.10112
V_2 (eV)	8.04432	7.96922	8.04432	25.78002	12.87680	9.33352	9.33352	9.37273	9.33352	9.37273	9.37273
T (eV)	6.38036	6.10748	6.38036	21.06675	10.48882	6.77464	6.77464	6.90500	6.77464	6.90500	6.90500
V_3 (eV)	-3.19018	-3.05374	-3.19018	-10.33337	-5.24291	-3.38732	-3.38732	-3.45250	-3.38732	-3.45250	-3.45250
I_1 (eV)	-14.63489	-14.63489	-14.63489	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
$\Delta E_{H_{2p}} (eV)$	-1.44915	-0.92918	-1.44915	0	0	0	0	0	0	0	0
I_2 (eV)	-13.18374	-13.70571	-13.18374	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
I_3 (eV)	-31.63536	-31.63540	-31.63536	-49.66493	-31.63533	-31.63537	-31.63537	-31.63535	-31.63537	-31.63535	-31.63535
I_4 (eV)	-1.44915	-0.92918	-1.44915	0	0	-1.85836	-1.85836	-1.44915	-1.85836	-1.44915	-1.44915
I_5 (eV)	-33.08452	-32.56455	-33.08452	-49.66493	-31.63537	-33.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452
ω (10 ¹⁵ rad/s)	7.42995	7.22380	7.42995	24.2751	24.1759	9.43699	9.43699	10.19220	9.43699	10.19220	9.55643
E_F (eV)	4.89052	4.75483	4.89052	15.97831	15.91299	6.21159	6.21159	6.21159	6.21159	6.21159	6.29021
E_n (eV)	-0.14475	-0.14048	-0.14475	-0.23552	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.20896	-0.16416
$E_{F_{H_{2p}}}$ (eV)	0.08039	0.08039	0.09113	0.35532	0.35532	0.12312	0.12312	0.09944	0.12312	0.12312	0.12312
$E_{F_{H_{2p}}}$ (eV)	-0.10445	-0.10019	-0.10445	-0.14502	-0.07200	-0.10359	-0.10359	-0.15924	-0.10359	-0.15924	-0.10260
$E_{H_{2p}}$ (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
$E_{F_{H_{2p}}}$ (eV)	-33.18897	-32.66473	-33.18897	-49.80996	-31.70737	-33.59732	-33.59732	-33.24376	-33.59732	-33.24376	-33.18712
$E_{F_{H_{2p}}}$ (eV)	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
$E_{F_{H_{2p}}}$ (eV)	0	0	0	-13.59844	-13.59844	0	0	0	0	0	0
$E_{F_{H_{2p}}}$ (eV)	3.77116	3.39496	3.96108	7.85016	3.32601	4.32734	4.29921	3.97398	4.17951	3.62128	3.91734

Table 15.37. The total bond energies of branched-chain alkyl chlorides calculated using the functional group composition and the energies of Table 15.36 compared to the experimental values [2]. The magnetic energy E_{mag} that is subtracted from the weighted sum of the $E_{\text{g}}(\text{nm})$ (eV) values based on composition is given by (15.58).

Formula	Name	C-C (i)	C-C (ii)	C-C (iii)	C-H ₂	C-H (i)	C-H (ii)	C-C (a)	C-C (b)	C-C (c)	C-C (d)	C-C (e)	C-C (f)	E_{mag}	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CCl_4	Tetrachloromethane	0	0	4	0	0	0	0	0	0	0	0	0	-1	13.43181	13.448	0.00123
CHCl_3	Trichloromethane	3	0	0	0	0	0	0	0	0	0	0	0	-1	14.49146	14.523	0.00217
CH_2Cl_2	Dichloromethane	2	0	0	0	0	0	0	0	0	0	0	0	0	15.37248	15.450	0.00499
CH_3Cl	Chloromethane	1	0	0	0	0	0	0	0	0	0	0	0	0	16.26302	16.312	0.00299
$\text{C}_2\text{H}_5\text{Cl}$	Chloroethane	0	0	1	1	0	0	0	0	0	0	0	0	0	28.61064	28.571	-0.00138
$\text{C}_3\text{H}_7\text{Cl}$	1-Chloropropane	0	0	1	2	0	0	0	0	0	0	0	0	0	40.76834	40.723	-0.00112
$\text{C}_4\text{H}_9\text{Cl}$	2-Chloropropane	0	0	1	3	0	0	0	0	0	0	0	0	0	40.86923	40.858	-0.00028
$\text{C}_4\text{H}_9\text{Cl}$	1-Chlorobutane	0	0	1	3	0	0	0	0	0	0	0	0	0	52.92604	52.903	-0.00044
$\text{C}_4\text{H}_9\text{Cl}$	2-Chlorobutane	0	0	1	3	0	0	0	0	0	0	0	0	0	52.92693	52.972	-0.00104
$\text{C}_5\text{H}_{11}\text{Cl}$	1-Chloro-2-methylpropane	0	0	1	4	0	0	0	0	0	0	0	0	-1	52.99860	52.933	-0.00085
$\text{C}_5\text{H}_{11}\text{Cl}$	1-Chloropentane	0	0	1	4	0	0	0	0	0	0	0	0	0	53.21057	53.191	-0.00037
$\text{C}_5\text{H}_{11}\text{Cl}$	2-Chloro-2-methylbutane	0	0	1	4	0	0	0	0	0	0	0	0	0	65.08379	65.061	0.00034
$\text{C}_5\text{H}_{11}\text{Cl}$	1-Chloro-3-methylbutane	0	0	1	4	0	0	0	0	0	0	0	0	0	65.15630	65.111	-0.00069
$\text{C}_5\text{H}_{11}\text{Cl}$	2-Chloro-3-methylbutane	0	0	1	4	0	0	0	0	0	0	0	0	-1	65.56827	65.344	-0.00037
$\text{C}_6\text{H}_{13}\text{Cl}$	2-Chlorohexane	0	0	1	5	0	0	0	0	0	0	0	0	0	77.34233	77.313	-0.00058
$\text{C}_6\text{H}_{13}\text{Cl}$	1-Chlorooctane	0	0	1	7	0	0	0	0	0	0	0	0	0	101.55684	101.564	0.00007
$\text{C}_8\text{H}_{17}\text{Cl}$	1-Chlorododecane	0	0	1	11	0	0	0	0	0	0	0	0	0	150.87640	150.202	0.00009
$\text{C}_{10}\text{H}_{21}\text{Cl}$	1-Chlorogundecane	0	0	1	11	0	0	0	0	0	0	0	0	0	223.13384	223.175	0.00018

[illegible]

ALKYL BROMIDES ($C_n H_{2n+2-m} Br_m$, $n = 1, 2, 3, 4, 5 \dots \infty$ $m = 1, 2, 3 \dots \infty$).

The branched-chain alkyl bromides, $C_n H_{2n+2-m} Br_m$, may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds wherein at least one H is replaced by a bromine. The $C-Br$ bond comprises a functional group for each case of Br replacing a H of methane for the series $H_{4-m} C-Br_m$, $m = 1, 2, 3$, with the $C-Br$ bond of CBr_4 comprising another functional group due to the limitation of the minimum energy of Br matched to that of the $C2sp^3$ HO. In addition, the $C-Br$ bond due to Br replacing a H of an alkane is a function group. The methyl, methylene, methylene functional groups are equivalent to those of branched-chain alkanes. Six types of $C-C$ bonds can be identified. The n-alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2 CH$) and t-butyl ($(CH_3)_3 C$) groups and the isopropyl to isopropyl, isopropyl to t-butyl, and t-butyl to t-butyl $C-C$ bonds comprise functional groups that are equivalent to those of branched-chain alkanes.

The solution of the $C-Br$ functional groups comprises the hybridization of the $2s$ and $2p$ shells of each C to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ hybridized orbital (HO) and the Br AO to form a molecular orbital (MO) permits each participating orbital to decrease in radius and energy. In alkyl bromides, the energy of bromine is less than the Coulombic energy between the electron and proton of H given by Eq. (1.243). Thus, c_2 in Eq. (15.52) is one, and the energy matching condition is determined by the C_2 parameter. Then, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a corresponding energy of $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)), and the Br AO has an energy of $E(Br) = -11.81381 \text{ eV}$. To meet the equipotential condition of the union of the $C-Br$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor C_2 of Eq. (15.52) for the $C-Br$ -bond MO given by Eqs. (15.68) and (15.70) is

$$C_2(C2sp^3 HO \text{ to } Br) = \frac{E(Br)}{E(C, 2sp^3)} c_2(C2sp^3 HO) = \frac{-11.81381 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.74081 \quad (15.112)$$

The valence energy of the carbon $2p$ is -11.2603 eV and that of the Br AO is -11.81381 eV . The energy difference is less than that of $E_T(C-C, 2sp^3)$ given by Eq. (14.151) for a single bond. Thus, $E_T(atom-atom, msp^3.AO)$ of the alkyl $C-Br$ -bond MO in Eq. (15.52) due to the charge donation from the C and Br atoms to the MO is -0.92918 eV (Eq. (14.513) based on the maximum single-bond-energy contribution of the $C2sp^3$ HO. $E_T(atom-atom, msp^3.AO)$ of the series $CBr_m H_{4-m}$ $m=1,2,3$ is equivalent to those of methyl groups, -0.72457 eV (Eq. (14.151)). For CBr_4 , $E_T(atom-atom, msp^3.AO)$ of the $C-Br$ -bond MO in Eq. (15.52) due to the charge donation from the C and Br atoms to the MO is -0.36229 eV (Eqs. (15.18-15.20 and Eq. (15.29) with a linear combination of $s=1$, $E_T(atom-atom, msp^3.AO) = -0.72457 \text{ eV}$ and $E_T(atom-atom, msp^3.AO) = 0$) based on the maximum charge density on the $C2sp^3$ HO.

The symbols of the functional groups of branched-chain alkyl bromides are given in Table 15.39. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of branched-chain alkyl bromides are given in Tables 15.40, 15.41, and 15.42, respectively. The total energy of each branched-chain alkyl bromide given in Table 15.43 was calculated as the sum over the integer multiple of each $E_D(Group)$ of Table 15.42 corresponding to functional-group composition of the molecule. For each set of unpaired electrons created by bond breakage, the $C2sp^3$ HO magnetic energy E_{mag} that was subtracted from the weighted sum of the $E_D(Group)$ (eV) values based on composition is given by Eq. (15.58). The $C-C$ bonds to the $CHBr$ group (one H bond to C) were each treated as an iso $C-C$ bond. The $C-C$ bonds to the CBr group (no H bonds to C) were each treated as a tert-butyl $C-C$. E_{mag} is subtracted for each t-butyl group. In the case of 2,3-dibromo-2-methylbutane, E_{mag} is positive since the terms due to the two bromine atoms cancel that of the t-butyl and CH groups. The bond angle parameters of branched-chain alkyl bromides determined using Eqs. (15.61-15.70), (15.79-15.108) and (15.112) are given in Table 15.44.

Table 15.39. The symbols of functional groups of branched-chain alkyl bromides.

Functional Group	Group Symbol
CBr of $CBr_m H_{4-m}$	$C - Br$ (i)
CBr of CBr_4	$C - Br$ (ii)
CBr of $C_n H_{2n+2-m} Br_m$	$\dot{C} - Br$ (iii)
CH_3 group	$C - H$ (CH_3)
CH_2 group	$C - H$ (CH_2)
CH	$C - H$
CC bond (n-C)	$C - C$ (a)
CC bond (iso-C)	$C - C$ (b)
CC bond (tert-C)	$C - C$ (c)
CC (iso to iso-C)	$C - C$ (d)
CC (t to t-C)	$C - C$ (e)
CC (t to iso-C)	$C - C$ (f)

Table 13.40. The geometrical bond parameters of branched-chain alkyl bromides and experimental values [1].

Parameter	C-Br (I) Group	C-Br (II) Group	C-Br (III) Group	C-H (CH) Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
a (Å)	2.49163	2.52509	2.47329	1.67122	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
c (Å)	1.83395	1.84622	1.82719	1.04856	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length $2c$ (Å)	1.94097	1.93396	1.93381	1.10974	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (Å)	1.953 (methyl bromide)	1.955 (carbon tetrabromide)	1.93 (1,1,1-trichloroethane) 1.950 (1,2-dibromochloroethane)	1.107 (C-H propane) 1.117 (C-H butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
b/c (Å)	1.68667	1.72265	1.66689	1.27295	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
c	0.75604	0.73115	0.73877	0.63380	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 13.41. The MO to HO intercept geometrical bond parameters of branched-chain alkyl bromides. R, R', R'' are H or alkyl groups. E_r is $E_r(\text{atom} - \text{atom}, \text{msp}^*, \text{AO})$.

Bond	Atom	R_r (eV) Bond 1	R_r (eV) Bond 2	R_r (eV) Bond 3	R_r (eV) Bond 4	Final Total Energy (2sp) (eV)	r_{msp} (Å)	r_{msp} (Å)	E_{msp} (eV) Final	$E(2sp)$ (eV) Final	θ' (°)	θ_1 (°)	θ_2 (°)	d_1 (Å)	d_2 (Å)
$H_1 - C_1 - Br_1$ (I) (C ₁ - Br (I))	C ₁	-0.36229	0	0	0	-151.97798	0.91771	0.89582	-15.18804	-14.99717	68.10	111.90	29.52	2.16808	0.33415
$H_1 - C_1 - Br_1$ (II) (C ₁ - Br (II))	H ₁	-0.36229	0	0	0	-151.97798	1.15169	0.89582	-15.18804	-14.99717	68.10	111.90	29.52	2.16808	0.33415
$H_1 - C_1 - Br_1$ (III) (C ₁ - Br (III))	C ₁	-0.18114	0	0	0	-151.97798	0.91771	0.90664	-15.00689	-14.81603	66.10	113.90	28.76	2.21357	0.36734
$H_1 - C_1 - Br_1$ (IV) (C ₁ - Br (IV))	Br ₁	-0.18114	0	0	0	-151.97798	1.15169	0.90664	-15.00689	-14.81603	66.10	113.90	28.76	2.21357	0.36734
$H_1 - C_1 - Br_1$ (V) (C ₁ - Br (V))	C ₁	-0.46459	-0.92918	0	0	-153.00946	0.91771	0.88983	-16.21052	-16.02866	62.67	117.53	26.55	2.21237	0.36318
$H_1 - C_1 - Br_1$ (VI) (C ₁ - Br (VI))	H ₁	-0.46459	0	0	0	-153.00946	1.15169	0.88983	-16.21052	-16.02866	62.67	117.53	26.55	2.21237	0.36318
$H_1 - C_1 - Br_1$ (VII) (C ₁ - Br (VII))	C ₁	-0.92918	-0.92918	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	77.49	102.51	41.48	1.23564	0.18708
$H_1 - C_1 - Br_1$ (VIII) (C ₁ - Br (VIII))	C ₁	-0.92918	-0.92918	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	77.49	102.51	41.48	1.23564	0.18708
$H_1 - C_1 - Br_1$ (IX) (C ₁ - Br (IX))	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	61.10	118.90	31.37	1.42098	0.37226
$H_1 - C_1 - Br_1$ (X) (C ₁ - Br (X))	C ₁	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	63.82	116.18	30.08	1.45879	0.38106
$H_1 - C_1 - Br_1$ (XI) (C ₁ - Br (XI))	C ₁	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.08412	-16.49325	56.41	123.59	26.06	1.90890	0.45117
$H_1 - C_1 - Br_1$ (XII) (C ₁ - Br (XII))	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$H_1 - C_1 - Br_1$ (XIII) (C ₁ - Br (XIII))	C ₁	-0.92918	-0.92918	-0.92918	-0.72457	-154.71860	0.91771	0.75889	-17.92866	-17.73779	48.21	131.79	21.74	1.95734	0.50370
$H_1 - C_1 - Br_1$ (XIV) (C ₁ - Br (XIV))	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$H_1 - C_1 - Br_1$ (XV) (C ₁ - Br (XV))	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	129.96	22.66	1.94463	0.49208
$H_1 - C_1 - Br_1$ (XVI) (C ₁ - Br (XVI))	C ₁	-0.72457	-0.72457	-0.72457	0	-154.19863	0.91771	0.78155	-17.40869	-17.21763	52.78	127.22	24.04	1.95443	0.47279
$H_1 - C_1 - Br_1$ (XVII) (C ₁ - Br (XVII))	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	129.96	22.66	1.94463	0.49208

Parameters	C-Br (i) Group	C-Br (ii) Group	C-Br (iii) Group	χH_1 Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
n_1	1	1	1	3	2	1	1	1	1	1	1
n_2	0	0	0	2	1	0	0	0	0	0	0
n_3	0	0	0	0	0	0	0	0	0	0	0
C_1	0.5	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
C_2	0.74081	0.74081	0.74081	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1	1
C_4	1	1	1	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_5	0	0	0	0	1	0	0	0	1	1	0
C_6	2	2	2	1	1	2	2	2	2	2	2
C_7	0	0	0	3	2	0	0	0	0	0	0
C_8	0.5	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
C_9	0.74081	0.74081	0.74081	1	1	1	1	1	1	1	1
C_{10}	1	1	1	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_{11}	0	0	0	0	1	0	0	0	1	1	0
C_{12}	2	2	2	1	1	2	2	2	2	2	2
C_{13}	0	0	0	3	2	0	0	0	0	0	0
C_{14}	0.5	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
C_{15}	0.74081	0.74081	0.74081	1	1	1	1	1	1	1	1
C_{16}	1	1	1	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_{17}	0	0	0	0	1	0	0	0	1	1	0
C_{18}	2	2	2	1	1	2	2	2	2	2	2
C_{19}	0	0	0	3	2	0	0	0	0	0	0
C_{20}	0.5	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
C_{21}	0.74081	0.74081	0.74081	1	1	1	1	1	1	1	1
C_{22}	1	1	1	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_{23}	0	0	0	0	1	0	0	0	1	1	0
C_{24}	2	2	2	1	1	2	2	2	2	2	2
C_{25}	0	0	0	3	2	0	0	0	0	0	0
C_{26}	0.5	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
C_{27}	0.74081	0.74081	0.74081	1	1	1	1	1	1	1	1
C_{28}	1	1	1	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_{29}	0	0	0	0	1	0	0	0	1	1	0
C_{30}	2	2	2	1	1	2	2	2	2	2	2
C_{31}	0	0	0	3	2	0	0	0	0	0	0
C_{32}	0.5	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5
C_{33}	0.74081	0.74081	0.74081	1	1	1	1	1	1	1	1
C_{34}	1	1	1	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_{35}	0	0	0	0	1	0	0	0	1	1	0
C_{36}	2	2	2	1	1	2	2	2	2	2	2
C_{37}	0	0	0	3	2	0	0	0	0	0	0
C_{38}	0.5	0.5	0.5	0.7							

Table 15.43. The total bond energies of branched-chain alkyl bromides calculated using the functional group composition and the energies of Table 15.42 compared to the experimental values [2]. The magnetic energy E_{mag} that is subtracted from the weighted sum of the $E_{g, (sum)}$ (eV) values based on composition is given by (15.58).

Formula	Name	C-Br (I)	C-Br (II)	C-Br (III)	C-H ₁	C-H ₂	C-H (I)	C-H (II)	C-C (a)	C-C (b)	C-C (c)	C-C (d)	C-C (e)	C-C (f)	E_{mag}	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CBr ₄	Tetrabromomethane	0	4	0	0	0	0	0	0	0	0	0	0	0	-1	11.25929	11.196	-0.00366
CHBr ₃	Tribromomethane	3	0	0	0	0	0	0	0	0	0	0	0	0	0	12.87698	12.919	0.00323
CH ₂ Br ₂	Dibromomethane	1	0	0	0	0	0	0	0	0	0	0	0	0	0	15.67551	15.732	0.00360
CH ₃ Br	Bromomethane	0	0	0	0	0	0	0	0	0	0	0	0	0	0	28.03939	27.953	-0.00308
C ₂ H ₅ Br	1-Bromopropane	0	0	0	1	2	0	0	0	0	0	0	0	0	0	40.19709	40.160	-0.00093
C ₃ H ₇ Br	2-Bromopropane	0	0	0	1	0	0	0	0	0	0	0	0	0	0	40.29798	40.288	-0.00024
C ₄ H ₉ Br	2,3-Dibromo-2-methylbutane	1	0	0	0	0	0	0	0	0	0	0	0	0	0	63.48143	63.477	-0.00007
C ₄ H ₉ Br	1-Bromobutane	0	0	0	1	5	0	0	0	0	0	0	0	0	0	76.67019	76.634	-0.00047
C ₅ H ₁₁ Br	1-Bromopentane	0	0	0	1	6	0	0	0	0	0	0	0	0	0	88.82789	88.783	-0.00051
C ₆ H ₁₃ Br	1-Bromohexane	0	0	0	1	7	0	0	0	0	0	0	0	0	0	100.98559	100.952	-0.00033
C ₇ H ₁₅ Br	1-Bromododecane	0	0	0	1	11	0	0	0	0	0	0	0	0	0	149.61639	149.573	-0.00029
C ₈ H ₁₇ Br	1-Bromohexadecane	0	0	0	1	15	0	0	0	0	0	0	0	0	0	198.24719	198.193	-0.00028

Table 15.44. The bond angle parameters of branched-chain alkyl bromides and experimental values [1]. In the calculation of θ_{ij} , the parameters from the preceding angle were used. E_i is $E_{g, (atom, map, AO)}$.

Atom of Angle	2C ¹ Bond 1 (n ₁)	2C ² Bond 2 (n ₂)	2C ³ Bond 3 (n ₃)	E ₁ ^{Hybridization} Atom 1 (Table 15.3.A)	E ₂ ^{Hybridization} Atom 2 (Table 15.3.A)	C ₁ Atom 1	C ₂ Atom 2	C ₃ Atom 3	q ₁	q ₂	E ₃ (eV)	θ ₁ (°)	θ ₂ (°)	θ ₃ (°)	Cal. θ (°)	Exp. θ (°)
∠BrC ¹ Br (C ² - Br (II))	3 (6790)	3 (6790)	6 (1816)	-15.19803 Br	-15.19804 Br	2	0.89582	0.89582	1	0.74081 (Br (15.112))	-1.92018				112.03	111.2 (methyl bromide) 113.2 (dibromomethane) 111.7 (bromobenzene)
∠HC ¹ Br (C ² - Br (III))	2 (1106)	3 (6790)	4 (812)	-15.53033 C ³	-11.81381 Br	3	0.82495	0.91771	0.75	0.86529 (Br (15.65))	0				110.53	109 (dibromomethane)
∠HC ¹ H (C ² - Br (III))	2 (9971)	2 (9971)	3 (4252)	-15.75493 C ³	II	7	0.86539	I	I	I	0				109.50	
∠C ¹ C ² H ₃ (C ³ - Br (III))																
∠C ¹ C ² Br (C ³ - Br (III))	2 (1547)	3 (65437)	3 (2347)	-15.75493 C ³	-11.81381 Br	7	0.86539	0.86539	I	0.74081 (Br (15.112))	-0.72157				109.44	110 (1,2-dibromomethane) 109.0 (1,1,1-tribromomethane)
∠HC ¹ H (Methylene)	2 (1106)	2 (1106)	3 (4252)	-15.75493 C ³	II	7	0.86539	I	I	I	0				108.44	107 (propane)
∠C ¹ C ² C ³																
∠C ¹ C ² H (Methyl)	2 (9971)	2 (9971)	3 (4252)	-15.75493 C ³	II	7	0.86539	I	I	I	0					
∠C ¹ C ² C ³																
∠C ¹ C ² C ³ (Methyl)	2 (91547)	2 (91547)	4 (7958)	-16.68412 C ³	-16.68412 C ³	25	0.81549	0.81549	I	I	-1.85016				109.44	113.8 (propane) 113.8 (butane) 110.8 (isobutane)
∠C ¹ C ² H (Methyl)	2 (91547)	2 (11325)	4 (1633)	-15.53033 C ³	-14.82575 C ³	5	0.87495	0.91771	0.75	I	0				110.49	111.0 (butane) 111.4 (isobutane)
∠C ¹ C ² H (Methyl)	2 (91547)	2 (9971)	4 (1633)	-15.53033 C ³	-14.82575 C ³	5	0.87495	0.91771	0.75	I	0				109.50	
∠C ¹ C ² C ³ (Methyl)	2 (90327)	2 (90327)	4 (7958)	-15.53033 C ³	-14.82575 C ³	5	0.87495	0.91771	0.75	I	-1.85016				109.44	110.8 (isobutane)
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹ C ² C ³ (Methyl)																
∠C ¹																

ALKYL IODIDES ($C_n H_{2n+2-m} I_m$, $n=1,2,3,4,5\ldots\infty$ $m=1,2,3\ldots\infty$)

The branched-chain alkyl iodides, $C_n H_{2n+2-m} I_m$, may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds wherein at least one H is replaced by an iodine atom. The $C-I$ bond comprises a functional group for I replacing a H of methane (CH_3I) or for I replacing a H of an alkane corresponding to the series $C_n H_{2n+2-m} I_m$. The $C-I$ bond of each of CH_2I_2 and CHI_3 comprise separate functional groups due to the limitation of the minimum energy of I matched to that of the $C2sp^3$ HO. The methyl, methylene, methylene functional groups are equivalent to those of branched-chain alkanes. Six types of $C-C$ bonds can be identified. The n-alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2CH$) and t-butyl ($(CH_3)_3C$) groups and the isopropyl to isopropyl, isopropyl to t-butyl, and t-butyl to t-butyl $C-C$ bonds comprise functional groups that are equivalent to those of branched-chain alkanes.

The solution of the $C-I$ functional groups comprises the hybridization of the $2s$ and $2p$ AOs of each C to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO and the I AO to form a MO permits each participating orbital to decrease in radius and energy. In alkyl iodides, the energy of iodine is less than the Coulombic energy between the electron and proton of H given by Eq. (1.243). Thus, c_2 in Eq. (15.52) is one, and the energy matching condition is determined by the C_2 parameter. Then, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a corresponding energy of $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)). The I AO has an energy of $E(I) = -10.45126 \text{ eV}$. To meet the equipotential condition of the union of the $C-I$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor C_2 of Eq. (15.51) for the $C-I$ -bond MO given by Eqs. (15.68) and (15.70) is

$$C_2(C2sp^3 HO \text{ to } I) = \frac{E(I)}{E(C, 2sp^3)} c_2(C2sp^3 HO) = \frac{-10.45126 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.65537 \quad (15.113)$$

The valence energy of the carbon $2p$ is -11.2603 eV and that of the I AO is -10.45126 eV . The energy difference is positive. Thus, based on the maximum charge density on the $C2sp^3$

HO $E_r(\text{atom} - \text{atom}, \text{msp}^3 \text{AO})$ of the $C-I$ -bond MO in Eq. (15.52) due to the charge donation from the C and I atoms to the MO is -0.36229 eV (Eqs. (15.18-15.20 and Eq. (15.29) with a linear combination of $s=1$, $E_r(\text{atom} - \text{atom}, \text{msp}^3 \text{AO}) = -0.72457 \text{ eV}$ and $E_r(\text{atom} - \text{atom}, \text{msp}^3 \text{AO}) = 0$) for methyl and alkyl iodides, -0.18114 eV for diiodomethane, 5 and 0 for CHI_3 .

The symbols of the functional groups of branched-chain alkyl iodides are given in Table 15.45. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of branched-chain alkyl iodides are given in Tables 15.46, 15.47, and 15.48, respectively. The total energy of each branched-chain 10 alkyl iodide given in Table 15.49 was calculated as the sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.48 corresponding to functional-group composition of the molecule. For each set of unpaired electrons created by bond breakage, the $C2sp^3$ HO magnetic energy E_{mag} that was subtracted from the weighted sum of the $E_D(\text{Group})$ (eV) values based on composition is given by Eq. (15.58). The $C-C$ bonds to the CHI group (one H bond to C) were each 15 treated as an iso $C-C$ bond. The $C-C$ bonds to the CI group (no H bonds to C) were each treated as a tert-butyl $C-C$. E_{mag} is subtracted for each t-butyl group. The bond angle parameters of branched-chain alkyl iodides determined using Eqs. (15.61-15.70), (15.79-15.108) and (15.113) are given in Table 15.50.

20 Table 15.45. The symbols of functional groups of branched-chain alkyl iodides.

Functional Group	Group Symbol
CI of CH_3I and $\text{C}_n\text{H}_{2n+2-m}\text{I}_m$	$C-I$ (i)
CI of CH_2I_2	$C-I$ (ii)
CI of CHI_3	$C-I$ (iii)
CH_3 group	$C-H$ (CH_3)
CH_2 group	$C-H$ (CH_2)
CH	$C-H$
CC bond (n-C)	$C-C$ (a)
CC bond (iso-C)	$C-C$ (b)
CC bond (tert-C)	$C-C$ (c)
CC (iso to iso-C)	$C-C$ (d)
CC (t to t-C)	$C-C$ (e)
CC (t to iso-C)	$C-C$ (f)

Table 15.46. The geometrical bond parameters of branched-chain alkyl iodides and experimental values [1].

Parameter	C-I (I) Group	C-I (II) Group	C-I (III) Group	C-H (CH ₃) Group	C-H (CH ₃) Group	C-H (CH ₃) Group	C-H (CH ₃) Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
σ (Å)	2.67103	2.68865	2.70662	1.64920	1.67122	1.67465	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
c^* (Å)	2.01881	2.02546	2.03222	1.04856	1.05553	1.05661	1.05661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length $2c^*$ (Å)	2.15662	2.14365	2.15081	1.10974	1.11713	1.11827	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (Å)	2.132 (methyl iodide)	2.132 (methyl iodide)	2.15 (carbon tetrachloride)	1.107 (C-H propane) 1.117 (C-H butane)	1.107 (C-H propane) 1.117 (C-H butane)	1.122 (isobutane)	1.122 (isobutane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
b, c (Å)	1.74894	1.76815	1.78770	1.27295	1.29569	1.29924	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
σ	0.75582	0.75534	0.75083	0.63580	0.63139	0.63095	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 15.47. The MO to HO intercept geometrical bond parameters of branched-chain alkyl iodides. E_r is $E_r(\text{atom} - \text{atom}, \text{map}^2, \text{AO})$.

Bond	Atom	E_r (eV) Bond 1	E_r (eV) Bond 2	E_r (eV) Bond 3	E_r (eV) Bond 4	Final Total Energy C2sp ² (eV)	r_{final} (a ₀)	$E_{\text{calculated}}$ (eV) Final	$E(\text{C2sp}^2)$ (eV) Final	θ' (°)	θ_1 (°)	θ_2 (°)	d_1 (a ₀)	d_2 (a ₀)
$\text{CH}_3/\text{C}_\alpha - \text{I}$ (i)	C _α	-0.18114	0	0	0	-151.79683	0.91771	-15.00689	-14.81003	69.63	110.37	29.08	2.33442	0.31560
$\text{CH}_3/\text{C}_\beta - \text{I}$ (ii)	I	-0.18114	0	0	0		1.30183	-15.00689		69.63	110.37	29.08	2.33442	0.31560
$-\text{H}_2\text{C}-\text{C}_\alpha/\text{C}_\beta - \text{I}$ (i)	C _α	-0.18114	-0.92918	0	0	-152.72602	0.91771	-15.03608	-15.74521	63.16	116.84	25.82	2.40436	0.38554
$-\text{H}_2\text{C}-\text{C}_\alpha/\text{C}_\beta - \text{I}$ (ii)	I	-0.18114	0	0	0		1.30183	-15.00689		69.63	110.37	29.08	2.33442	0.31560
$\text{C}_\alpha\text{H}_2/\text{C}_\beta - \text{I}$ (i)	C _α	-0.09057	0	0	0	-151.70606	0.91771	-14.91032	-14.72546	68.61	111.39	28.71	2.33818	0.33272
$\text{C}_\alpha\text{H}_2/\text{C}_\beta - \text{I}$ (ii)	I	-0.09057	0	0	0		1.30183	-14.91032		68.61	111.39	28.71	2.33818	0.33272
$\text{C}_\alpha\text{H}_2/\text{C}_\beta - \text{I}$ (iii)	C _α	0	0	0	0	-151.61509	0.91771	-14.82375	-14.63489	67.56	112.44	28.32	2.38256	0.35035
$\text{C}_\alpha\text{H}_2/\text{C}_\beta - \text{I}$ (iv)	I	0	0	0	0		1.30183	-14.82375		67.56	112.44	28.32	2.38256	0.35035
$\text{C}' - \text{H} (\text{CH}_3)$	C	-0.92918	0	0	0	-152.54487	0.91771	-15.73493	-15.56407	71.49	102.51	41.48	1.23564	0.18708
$\text{C}' - \text{H} (\text{CH}_3)$	C	-0.92918	-0.92918	0	0	-153.47406	0.91771	-16.68412	-16.49325	68.47	111.53	35.84	1.35486	0.29053
$\text{C}' - \text{H} (\text{CH}_3)$	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	-17.61330	-17.42244	61.10	118.90	31.37	1.42918	0.37326
$\text{H}_2\text{C}-\text{C}_\alpha/\text{C}_\beta - \text{H}_2\text{C} -$ $\text{H}_2\text{C}-\text{C}_\alpha/\text{C}_\beta - \text{H}_2\text{C} -$ $(\text{C} - \text{C} (\text{e}))$	C _α	-0.92918	0	0	0	-152.54487	0.91771	-15.73493	-15.56407	63.82	116.18	30.08	1.83879	0.38106
$\text{H}_2\text{C}-\text{C}_\alpha/\text{C}_\beta - \text{H}_2\text{C} -$ $\text{H}_2\text{C}-\text{C}_\alpha/\text{C}_\beta - \text{H}_2\text{C} -$ $(\text{C} - \text{C} (\text{e}))$	C _β	-0.92918	-0.92918	0	0	-153.47406	0.91771	-16.68412	-16.49325	56.41	123.59	26.06	1.90190	0.45117
$\text{R} - \text{H}_2\text{C}-\text{C}_\alpha (\text{H}_2\text{C} - \text{R}) \text{HCH}_2 -$ $(\text{C} - \text{C} (\text{b}))$	C _α	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$\text{R} - \text{H}_2\text{C}-\text{C}_\alpha (\text{H}_2\text{C} - \text{R}) \text{HCH}_2 -$ $(\text{C} - \text{C} (\text{b}))$	C _β	-0.92918	-0.72457	-0.72457	-0.72457	-154.71860	0.91771	-17.92866	-17.73779	48.21	131.79	21.74	1.95734	0.50570
$\text{R} - \text{H}_2\text{C}-\text{C}_\alpha (\text{H}_2\text{C} - \text{R}) \text{HCH}_2 -$ $(\text{C} - \text{C} (\text{e}))$	C _α	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$\text{R} - \text{H}_2\text{C}-\text{C}_\alpha (\text{H}_2\text{C} - \text{R}) \text{HCH}_2 -$ $(\text{C} - \text{C} (\text{d}))$	C _β	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49298
$\text{R} - \text{H}_2\text{C}-\text{C}_\alpha (\text{H}_2\text{C} - \text{R}) \text{HCH}_2 -$ $(\text{C} - \text{C} (\text{f}))$	C _α	-0.72457	-0.92918	-0.92918	0	-154.19863	0.91771	-17.40869	-17.21783	52.78	127.22	24.04	1.92443	0.47279
$\text{R} - \text{H}_2\text{C}-\text{C}_\alpha (\text{H}_2\text{C} - \text{R}) \text{HCH}_2 -$ $(\text{C} - \text{C} (\text{f}))$	C _β	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49298

Table 15.48. The energy parameters (eV) of functional groups of branched-chain alkyl iodides.											
Parameters	C-I (i) Group	C-I (ii) Group	C-I (iii) Group	CH ₃ Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
η_1	1	1	3	2	1	1	1	1	1	1	1
η_2	0	0	2	1	0	0	0	0	0	0	0
η_3	0	0	0	0	0	0	0	0	0	0	0
ζ_1	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
ζ_2	0.65537	0.65537	1	1	1	1	1	1	1	1	1
ζ_3	1	1	1	1	1	1	1	1	1	1	1
ζ_4	1	1	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
ζ_5	1	0	0	1	1	0	0	0	1	1	0
ζ_6	2	2	1	1	1	2	2	2	2	2	2
ζ_7	0	0	3	2	1	0	0	0	0	0	0
ζ_{10}	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
ζ_{11}	0.65537	0.65537	1	1	1	1	1	1	1	1	1
ν_1 (eV)	-26.59109	-26.34902	-107.32728	-70.41625	-35.12015	-28.79214	-28.79214	-29.10112	-28.79214	-29.10112	-29.10112
ν_2 (eV)	6.73951	6.71739	38.92728	25.78002	12.87680	9.33352	9.33352	9.37273	9.33352	9.37273	9.37273
T (eV)	4.97768	4.90005	32.53914	21.06675	10.48582	6.77464	6.77464	6.90500	6.77464	6.90500	6.90500
ν_m (eV)	-2.48884	-2.45002	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.45250	-3.38732	-3.45250	-3.45250
$E(\omega_{\text{rot}})$ (eV)	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.35946	-15.56407	-15.35946	-15.35946
$\Delta E_{\text{rot}}(\omega_{\text{rot}})$ (eV)	-0.36229	-0.18114	0	0	0	0	0	0	0	0	0
$f_T(\omega_{\text{rot}})$ (eV)	-14.29907	-14.45375	-14.63489	-15.56407	-14.63489	-15.56407	-15.56407	-15.35946	-15.56407	-15.35946	-15.35946
$E_T(\omega_{\text{rot}})$ (eV)	-31.63334	-31.63335	-31.63340	-49.66493	-31.63333	-31.63337	-31.63337	-31.63335	-31.63337	-31.63335	-31.63335
$E_T(\omega_{\text{rot}} - \omega_{\text{vib}})$ (eV)	-0.36229	-0.18114	0	0	0	-1.85836	-1.85836	-1.44915	-1.85836	-1.44915	-1.44915
$E_T(\omega_{\text{rot}})$ (eV)	-31.99766	-31.81651	-31.63337	-49.66493	-31.63337	-31.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452
ω (10^{13} rad/s)	10.2318	5.36799	9.90080	24.9286	24.3751	9.43699	9.43699	15.4846	9.43699	9.55643	9.55643
E_x (eV)	6.73472	3.53331	6.51688	16.40846	15.97831	6.21159	6.21159	10.19220	6.21159	6.29021	6.29021
E_n (eV)	-0.16428	-0.11832	-0.15977	-0.25352	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.16416	-0.16416
E_{Kvib} (eV)	0.06608	0.06608	0.35532	0.35532	0.35532	0.12312	0.12312	0.09944	0.12312	0.12312	0.12312
E_{Kvib} (eV)	-0.13124	-0.08527	-0.12673	-0.22757	-0.14502	-0.10359	-0.07526	-0.15924	-0.10359	-0.10260	-0.10260
E_{vib} (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
E_{rot} (eV)	-32.12889	-31.90179	-31.76210	-49.80996	-31.70737	-33.59732	-33.49373</				

[illegible]

Atoms of Angle	$2c_1$ (rad)	$2c_2$ (rad)	$2c_3$ (rad)	F_{residual} Atom 1	Atom 1 Hybridization Designation (Table 15.3.A)	c_1 Atom 1	c_2 Atom 2	C_1	C_2	c_1	c_2	ζ_1	E_T (eV)	θ_+ (°)	θ_1 (°)	θ_2 (°)	Cal. θ (°)	Exp. θ (°)
$\angle C_1 - I - (ii)$	4.03092	4.03092	6.7205	-10.45126 /	I	I	I	I	0.65337 (Eq. (15.113))	I	I	I	-0.56228				112.10	
$\angle HC_1 - I$	2.11106	4.03763	5.2203	-10.45126 /	I		0.91771	0.75	0.76115 (Eq. (15.63))	0.75		1.00000	0				112.39	
$\angle HC_1 - I - (ii)$	2.09711	2.09711	3.4232	-15.73493	H		I	I	I	0.75		1.15796	0				109.50	111.2 (methyl iodide)
$\angle HC_1 - H$	2.91547	4.03763	5.7019	-15.53033 /	I		0.87495	I	0.65337 (Eq. (15.113))	I		0.87495	-0.56228				111.86	
$\angle C_1 - I - (ii)$	2.11106	2.11106	3.4232	-15.73493	H		I	I	I	0.75		1.15796	0				108.44	107 (propane)
$\angle C_1 - C_2 - C_3$														69.51			110.49	112 (propane) 113.8 (butane) 110.8 (isobutane) 111.0 (isobutane) 111.4 (isobutane)
$\angle C_1 - C_2 - H$	2.09711	2.09711	3.4232	-15.73493	H		0.86359	I	I	0.75		1.15796	0				109.50	
$\angle HC_1 - H$	2.91547	2.91547	4.7958	-16.68412 /	I		0.81549	I	I	I		0.81549	-1.85836				110.67	110.8 (isobutane)
$\angle C_1 - C_2 - C_3$	2.11106	2.11106	3.4232	-15.53033 /	I		0.91771	0.75	I	0.75		1.04887	0				110.76	
$\angle C_1 - C_2 - H$	2.91547	2.91547	4.7958	-15.53033 /	I		0.87495	0.75	I	0.75		1.04887	0				111.27	111.4 (isobutane)
$\angle C_1 - C_2 - C_3$	2.11106	2.11106	3.4232	-15.53033 /	I		0.91771	0.75	I	0.75		1.04887	-1.85836				111.27	111.4 (isobutane)
$\angle C_1 - C_2 - H$	2.91547	2.91547	4.7958	-15.53033 /	I		0.87495	0.75	I	0.75		1.04887	-1.85836				107.50	

ALKENYL HALIDES ($C_n H_{2n-m} X_m$, $n = 3, 4, 5 \dots \infty$ $m = 1, 2, 3 \dots \infty$)

The branched-chain alkenyl halides, $C_n H_{2n+2-m} X_m$ with $X = F, Cl, Br, I$, may comprise alkyl and alkenyl functional groups wherein at least one H is replaced by a halogen atom. In the case that a halogen atom replaces an alkyl H , the $C-X$ bond comprises the alkyl-halogen functional groups given in their respective sections. The alkenyl halogen $C-X$ bond comprises a separate functional group for each case of X bonding to the $C=C$ -bond functional group given in the Alkenes section. In addition the CH group of the moiety $XCH=C$ comprises a functional group unique to alkenyl halides. The straight and branched-chain alkenes, $C_n H_{2n}$, comprise at least one carbon-carbon double bond comprising a functional group that is solved equivalently to the double bond of ethylene. The double bond may be bound to one, two, three, or four carbon single bonds that substitute for the hydrogen atoms of ethylene. The three distinct functional groups given in the Alkenes section are C vinyl single bond to $-C(C)=C$, C vinyl single bond to $-C(H)=C$, and C vinyl single bond to $-C(C)=CH_2$. In addition, CH_2 of the $-C=CH_2$ moiety is also an alkene functional group solved in the Alkenes section.

Consider the case where $X = Cl$ substitutes for a carbon single bond or a hydrogen atom. Based on the condition of energy matching of the orbital, any magnetic energy due to unpaired electrons in the constituent fragments, and differences in oscillation in the transition state, two distinct $C-Cl$ functional groups can be identified: Cl vinyl single bond to $-C(C)=C$ and Cl vinyl single bond to $-C(H)=C$. The alkenyl-halide CH group is equivalent to that solved in the Hydrogen Carbide (CH) section except that $\Delta E_{H,MO}(AO/HO) = -1.13379 \text{ eV}$ in order to energy match to the $C-Cl$ and $C=C$ bonds.

The alkyl portion of the alkenyl halide may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of $C-C$ bonds can be identified. The n-alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2CH$) and t-butyl ($(CH_3)_3C$) groups and the isopropyl to isopropyl, isopropyl to t-butyl, and t-butyl to t-butyl $C-C$ bonds comprise functional groups. The branched-chain-alkane groups in alkene halides are equivalent to those in branched-chain alkanes.

$E_T(\text{atom} - \text{atom}, msp^3.AO)$ of the $C = C$ -bond MO in Eq. (15.52) due to the charge donation from the C atoms to the MO is equivalent to that of ethylene, -2.26759 eV , given by Eq. (14.247). $E_T(\text{atom} - \text{atom}, msp^3.AO)$ of each $C - C$ -bond MO in Eq. (15.52) is -1.85836 eV or -1.44915 eV based on the energy match between the $C2sp^3$ HOs corresponding to the energy contributions equivalent to those of methylene, -0.92918 eV (Eq. (14.513), or methyl, -0.72457 eV (Eq. (14.151)), groups, respectively.

The solution of each $C - X$ functional group comprises the hybridization of the $2s$ and $2p$ AOs of the C atom to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO and the X AO to form a MO permits each participating orbital to decrease in radius and energy. The alkenyl $C - X$ -bond functional groups comprise single bonds and are equivalent to those of the corresponding alkyl halides except that the halogen AO and the $C - X$ -bond MO are each energy matched to the alkene $C2sp^3$ HO. In alkenyl halides with $X = Cl, Br, \text{ or } I$, the energy of the halogen atom is less than the Coulombic energy between the electron and proton of H given by Eq. (1.243). Thus, c_2 in Eq. (15.52) is one, and the energy matching condition is determined by the C_2 parameter. For example, the hybridization factor C_2 of Eq. (15.52) for the alkenyl $C - Cl$ -bond MO given by Eq. (15.111) is $C_2(C2sp^3HO \text{ to } Cl) = 0.81317$.

$E_T(\text{atom} - \text{atom}, msp^3.AO)$ of the alkenyl $C - Cl$ -bond MO in Eq. (15.52) due to the charge donation from the C and Cl atoms to the MO is -0.72457 eV for the Cl vinyl single bond to $-C(H)=C$ $C - Cl$ group and -0.92918 eV for the Cl vinyl single bond to $-C(C)=C$ $C - Cl$ group. It is based on the energy match between the Cl atom and the $C2sp^3$ HO of an unsubstituted vinyl group and a substituted vinyl group given by Eqs. (14.151) and (14.513), respectively.

The symbols of the functional groups of branched-chain alkenyl chlorides are given in Table 15.51. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of branched-chain alkenyl chlorides are given in Tables 15.52, 15.53, and 15.54, respectively. The total energy of each branched-chain alkenyl chloride given in Table 15.55 was calculated as the sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.54 corresponding to functional-group composition of the molecule.

The bond angle parameters of branched-chain alkenyl chlorides determined using Eqs. (15.61-15.70), (15.79-15.108) and (15.111) are given in Table 15.56.

Table 15.51. The symbols of functional groups of branched-chain alkenyl chlorides.

Functional Group	Group Symbol
Cl vinyl single bond to $-C(H)=C$	$C-Cl$ (i)
Cl vinyl single bond to $-C(C)=C$	$C-Cl$ (ii)
CC double bond	$C=C$
C vinyl single bond to $-C(C)=C$	$C-C$ (i)
C vinyl single bond to $-C(H)=C$	$C-C$ (ii)
C vinyl single bond to $-C(C)=CH_2$	$C-C$ (iii)
CH (alkenyl halide)	$C-H$ (i)
CH_2 alkenyl group	$C-H$ (CH_2) (i)
CH_3 group	$C-H$ (CH_3)
CH_2 alkyl group	$C-H$ (CH_2) (ii)
CH (alkyl)	$C-H$ (ii)
CC bond (n-C)	$C-C$ (a)
CC bond (iso-C)	$C-C$ (b)
CC bond (tert-C)	$C-C$ (c)
CC (iso to iso-C)	$C-C$ (d)
CC (t to t-C)	$C-C$ (e)
CC (t to iso-C)	$C-C$ (f)

Table 15.52. The geometrical bond parameters of branched-chain alkanyl chlorides and experimental values (1)

Parameter	C-C (i) Group	C-C (ii) Group	C=C Group	C-C (i) Group	C-C (ii) Group	C-C (iii) Group	C-H (i) Group	C-H (ii) Group	C-H (CH ₃) Group	C-H (CH ₃) Group	C-H (i) Group	C-H (ii) Group	C-H (CH ₃) Group	C-H (CH ₃) Group	C-H (i) Group	C-H (ii) Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
σ (Å)	2.15818	2.19158	1.47228	2.04740	2.04740	2.04740	1.60061	1.64920	1.67122	1.67465	2.12499	2.12499	2.12499	2.12499	2.12499	2.12499	2.12499	2.12499	2.10725	2.10725	2.10725	2.10725
c' (Å)	1.62912	1.64243	1.26661	1.43087	1.43087	1.43087	1.03299	1.04856	1.05553	1.05661	1.45744	1.45744	1.45744	1.45744	1.45744	1.45744	1.45744	1.45744	1.45164	1.45164	1.45164	1.45164
Bond Length $2c'$ (Å)	1.72419	1.73827	1.34032	1.51437	1.51437	1.51437	1.09327	1.10974	1.11713	1.11827	1.54280	1.54280	1.54280	1.54280	1.54280	1.54280	1.54280	1.54280	1.53635	1.53635	1.53635	1.53635
Exp. Bond Length (Å)	1.730 (vinyl chloride) 1.73 (1,1- dichloroethylene)	1.730 (vinyl chloride) 1.73 (1,1- dichloroethylene)	1.342 (2- methylpropane) 1.346 (2-butane) 1.349 (1,3-butadiene)	1.46439	1.46439	1.46439	1.22265	1.27295	1.29569	1.29924	1.54616	1.54616	1.54616	1.54616	1.54616	1.54616	1.54616	1.54616	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
b, e (Å)	1.41552	1.45403	0.75085	1.46439	1.46439	1.46439	1.22265	1.27295	1.29569	1.29924	1.54616	1.54616	1.54616	1.54616	1.54616	1.54616	1.54616	1.54616	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
σ	0.75486	0.74874	0.86030	0.69887	0.69887	0.69887	0.64337	0.63580	0.63159	0.61095	0.68600	0.68600	0.68600	0.68600	0.68600	0.68600	0.68600	0.68600	0.68888	0.68888	0.68888	0.68888

Table 15.53. The MO to HO intercept geometrical bond parameters of branched-chain alkyl chlorides. R_1, R_2, R_3 are H or alkyl groups. E_p is $E_p(\text{atom-atom, } \text{exp}, \text{AO})$.

Bond	Atom	E_p (eV) Bond 1	E_p (eV) Bond 2	E_p (eV) Bond 3	E_p (eV) Bond 4	Final Total Energy (2ap) (eV)	r_{exp} (Å)	r_{calc} (Å)	$E(\text{2ap})$ (eV) Final	θ (°)	θ_1 (°)	θ_2 (°)	d_1 (Å)	d_2 (Å)
$C_1 = C_2(CD) - H(CD)$ (i)	C_1	-1.11946	-0.72457	0	0	-153.47406	0.91771	0.91549	-16.68412	76.99	103.01	40.53	1.21653	0.18354
$-C_1 = C_2(HX)$ (i)	C_1	-1.11370	-0.72457	0	0	-153.47406	0.91771	0.91549	-16.68412	79.43	100.57	34.49	1.77872	0.14960
$-C_1 = C_2(CD)$ (i)	C_1	-0.72457	0	0	0		1.05158	0.87405	-15.35033	85.36	94.64	38.03	1.69995	0.07083
$-C_1 = C_2(CD)$ (ii)	C_1	1.13379	-0.46459	-0.92918	0	-154.14326	0.91771	0.78405	-17.35332	72.17	107.83	30.88	1.80353	0.34010
$-C_1 = C_2(CD)$ (iii)	C_1	-0.46459	0	0	0		1.05158	0.88043	-15.39034	83.62	96.38	37.46	1.74125	0.09882
$C_1(H)C_2 = C_3H_2C_4$	C_1	-1.13180	-0.92918	0	0	-153.67867	0.91771	0.80561	-16.84873	127.61	52.39	58.24	0.77492	0.40168
$C_1(H)C_2 = C_3H_2$	C_1	-1.13180	0	0	0	-153.74049	0.91771	0.83252	-15.95955	129.84	50.16	60.70	0.71040	0.54000
$C_1(C_2)C_3 = C_4H_2C_5$	C_1	-1.13180	-0.72457	-0.72457	0	-154.19863	0.91771	0.78155	-17.40860	126.39	53.61	56.95	0.80289	0.46371
$R_1C_1H_2 - C_2(C) = C_3$	C_1	-1.13180	-0.72457	-0.72457	0	-154.19863	0.91771	0.78155	-17.40860	60.88	110.12	27.79	1.81127	0.38039
$R_1C_1H_2 - C_2(C) = C_3$	C_1	-0.72457	-0.92918	0	0	-153.28045	0.91771	0.82562	-16.47951	67.40	112.60	31.26	1.74821	0.31734
$R_1C_1H_2 - C_2(H) = C_3$	C_1	-1.13180	-0.92918	0	0	-153.67865	0.91771	0.80561	-16.84873	64.37	115.43	29.79	1.77684	0.34596
$R_1C_1H_2 - C_2(H) = C_3$	C_1	-0.92918	-0.92918	0	0	-153.47405	0.91771	0.91549	-16.68411	65.99	114.01	30.58	1.76270	0.33183
$C^* - H(CD)$ (i)	C^*	-1.13180	0	0	0	-152.70040	0.91771	0.83252	-15.95955	77.15	102.85	41.13	1.23531	0.18965
$C^* - H(CD)$ (ii)	C^*	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.56407	77.49	102.51	41.48	1.23564	0.18708
$C^* - H(CD)$ (iii)	C^*	-0.92918	-0.92918	0	0	-153.47405	0.91771	0.91549	-16.68412	68.47	111.53	35.84	1.35486	0.39933
$H_1C_1C_2H_2C_3 -$	C_1	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	61.10	118.90	31.37	1.42968	0.37326
$H_1C_1C_2H_2C_3 -$	C_1	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.56407	63.82	116.18	30.08	1.83879	0.38106
$H_1C_1C_2H_2C_3 -$	C_1	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.91549	-16.68412	56.41	123.59	26.06	1.80890	0.45117
$R - H_1C_1C_2(H_2C_2 - R)H(CD)$	C_1	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	48.30	151.70	21.90	1.97162	0.51388
$R - H_1C_1C_2(H_2C_2 - R)H(CD)$	C_1	-0.92918	-0.72457	-0.72457	-0.72457	-154.71860	0.91771	0.75889	-17.73779	48.21	151.79	21.74	1.95734	0.50370
$tertC_1C_2(H_2C_2 - R)H(CD)$	C_1	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	48.30	151.70	21.90	1.97162	0.51388
$tertC_1C_2(H_2C_2 - R)H(CD)$	C_1	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	50.04	129.96	22.66	1.94462	0.49298
$tertC_1C_2(H_2C_2 - R)H(CD)$	C_1	-0.72457	-0.92918	-0.92918	0	-154.19863	0.91771	0.78155	-17.40860	52.78	137.22	24.04	1.92443	0.47279
$tertC_1C_2(H_2C_2 - R)H(CD)$	C_1	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	50.04	129.96	22.66	1.94462	0.49298

[illegible]

the energies of Table 5.34 compared to the experimental values [2].

Formula	$C-C$ (i)	$C-C$ (ii)	$C-C$ (iii)	$C-H$ (i)	CH_2 (ii)	CH (iii)	$C-C$ (a)	$C-C$ (b)	$C-C$ (c)	$C-C$ (d)	$C-C$ (e)	$C-C$ (f)	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
1,3,5-C ₆ H ₆	1	0	0	1	0	0	0	0	0	0	0	0	22.503	22.503	0.00170
2-Chloronaphthalene	0	0	0	0	0	0	0	0	0	0	0	0	21.46700	21.55432	0.00071

Table 15.56. The bond angle parameters of branched-chain alkanyl chlorides and experimental values (θ_i). In the calculation of θ_i , the parameters from the preceding angle were used. E_r is $E_r(\text{atom} - \text{atom}, \text{dist}, \text{AO})$.

Atom of Angle	$2C_1$ Dist 1 (σ_1)	$2C_2$ Dist 2 (σ_2)	$2C_3$ Dist 3 (σ_3)	R_1 Atom 1 Hybridization Exptl. (Table 15.3 A)	E_r Atom 1 Hybridization Exptl. (Table 15.3 A)	C_1 Atom 1 Hybridization Exptl. (Table 15.3 A)	C_2 Atom 2 Hybridization Exptl. (Table 15.3 A)	C_3 Atom 3 Hybridization Exptl. (Table 15.3 A)	ζ_1	ζ_2	ζ_3	E_r (eV)	θ_1 ($^\circ$)	θ_2 ($^\circ$)	θ_3 ($^\circ$)	Cal. θ ($^\circ$)	Exp. θ ($^\circ$)
$\angle H(C_1)C_1$ ($C_1 - C_1$)	2.05898	2.25835	4.5049	15	-12.90764 (C_1)	0.91771	0.91600	0.91771	0.75	1.0775	0	0				116.94	124 (ethyl chloride)
$\angle C_2(C_1)C_1$ ($C_1 - C_1$)													116.94			121.53	124 (ethyl chloride)
$\angle C_3(C_1)C_1$ ($C_1 - C_1$)													116.94			121.53	124 (ethyl chloride)
$\angle C_4(C_1)C_1$ ($C_1 - C_1$)																123.19	124 (ethyl chloride)
$\angle C_5(C_1)C_1$ ($C_1 - C_1$)																118.36	124 (ethyl chloride)
$\angle C_6(C_1)C_1$ ($C_1 - C_1$)																113.84	124 (ethyl chloride)
$\angle C_7(C_1)C_1$ ($C_1 - C_1$)																	124.4 (1,3-dichloropropane) 121.7 (1,3-dichloropropane) 124.4 (1,3-dichloropropane) 123.3 (1,3-dichloropropane) 124.4 (1,3-dichloropropane) 123.3 (1,3-dichloropropane)
$\angle H(C_2)C_2$ ($C_2 - C_2$)	2.04578	2.04578	3.4756	9	-15.95935 (C_2)	0.85252	0.85252	0.85252	0.75	1.1790	0	0				116.31	118.5 (2-methylpropane)
$\angle C_1(C_2)C_2$ ($C_2 - C_2$)																121.83	121 (2-methylpropane)
$\angle C_3(C_2)C_2$ ($C_2 - C_2$)																109.44	107 (propane)
$\angle C_4(C_2)C_2$ ($C_2 - C_2$)																110.49	112 (propane) 113.8 (butane) 110.8 (butane) 111.0 (butane)
$\angle C_5(C_2)C_2$ ($C_2 - C_2$)																110.49	111.4 (butane)
$\angle C_6(C_2)C_2$ ($C_2 - C_2$)																109.50	110.8 (butane)
$\angle C_7(C_2)C_2$ ($C_2 - C_2$)																109.44	110.8 (butane)
$\angle C_8(C_2)C_2$ ($C_2 - C_2$)																110.67	110.8 (butane)
$\angle C_9(C_2)C_2$ ($C_2 - C_2$)																110.76	110.8 (butane)
$\angle C_{10}(C_2)C_2$ ($C_2 - C_2$)																111.37	111.4 (butane)
$\angle C_{11}(C_2)C_2$ ($C_2 - C_2$)																111.37	111.4 (butane)
$\angle C_{12}(C_2)C_2$ ($C_2 - C_2$)																107.50	107.50 (butane)

ALCOHOLS ($C_n H_{2n+2} O_m$, $n=1,2,3,4,5,\dots,\infty$)

The alkyl alcohols, $C_n H_{2n+2} O_m$, comprise an OH functional group and two types of $C-O$ functional groups, one for methyl alcohol and the other for general alkyl alcohols. The alkyl portion of the alkyl alcohol may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of $C-C$ bonds can be identified. The n -alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2 CH$) and t -butyl ($(CH_3)_3 C$) groups and the isopropyl to isopropyl, isopropyl to t -butyl, and t -butyl to t -butyl $C-C$ bonds comprise functional groups. The branched-chain-alkane groups in alcohols are equivalent to those in branched-chain alkanes.

The OH functional group was solved in the Hydroxyl Radical (OH) section. Each $C-O$ group is solved by hybridizing the $2s$ and $2p$ AOs of the C atom to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO and the O AO to form a MO permits each participating orbital to decrease in radius and energy. In alkyl alcohols, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a corresponding energy of $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)) and the O AO has an energy of $E(O) = -13.61806 \text{ eV}$. To meet the equipotential condition of the union of the $C-O$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor c_2 of Eq. (15.52) for the $C-O$ -bond MO given by Eqs. (15.68) and (15.70) is

$$c_2(C2sp^3 HO \text{ to } O) = \frac{E(O)}{E(C, 2sp^3)} c_2(C2sp^3 HO) = \frac{-13.61806 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.85395 \quad (15.114)$$

$E_T(\text{atom-atom}, msp^3 AO)$ of the $C-O$ -bond MO in Eq. (15.52) due to the charge donation from the C and O atoms to the MO is -1.65376 eV for the CH_3-OH $C-O$ group. It is based on the energy match between the OH group and the $C2sp^3$ HO of a methyl group and is given by the linear combination of -0.92918 eV (Eq. (14.513)) and -0.72457 eV (Eq. (14.151)), respectively. For the alkyl $C-O$ group, $E_T(\text{atom-atom}, msp^3 AO)$ is -1.85836 eV . It is based on the energy match between the O AO and the $C2sp^3$ HO of a methylene group where both energy contributions are given by Eq. (14.513).

The symbols of the functional groups of branched-chain alkyl alcohols are given in Table 15.57. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of alkyl alcohols are given in Tables 15.58, 15.59, and 15.60, respectively. The total energy of each alkyl alcohol given in Table 5 15.61 was calculated as the sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.60 corresponding to functional-group composition of the molecule. The bond angle parameters of alkyl alcohols determined using Eqs. (15.79-15.108) are given in Table 15.62.

Table 15.57. The symbols of functional groups of alkyl alcohols.

Functional Group	Group Symbol
OH group	OH
CH ₃ OH C-O	C-O (i)
Alkyl C-O	C-O (ii)
CH ₃ group	C-H (CH ₃)
CH ₂ group	C-H (CH ₂)
CH	C-H
CC bond (n-C)	C-C (a)
CC bond (iso-C)	C-C (b)
CC bond (tert-C)	C-C (c)
CC (iso to iso-C)	C-C (d)
CC (t to t-C)	C-C (e)
CC (t to iso-C)	C-C (f)

Table 15.58. The geometrical bond parameters of alkyl alcohols and experimental values [1].

Parameter	OH Group	C-O (i) Group	C-O (ii) Group	C-H (CH ₃) Group	C-H (CH ₂) Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
ν (cm ⁻¹)	1.26430	1.79473	1.78255	1.67122	1.67122	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
c' (Å)	0.91808	1.33968	1.33512	1.04856	1.05553	1.05661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length 2c' (Å)	0.971651	1.41785	1.41303	1.10974	1.11713	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp Bond Length (Å)	0.971 (ethanol) 0.9451 (methanol)	1.4246 (methanol)	1.431 (ethanol)	1.107 (C-H propane) 1.117 (C-H butane)	1.107 (C-H propane) 1.117 (C-H butane)	1.122 (isobutane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
h, c (Å)	0.80925	1.19429	1.18107	1.27295	1.29569	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
σ	0.72615	0.74645	0.74900	0.63380	0.63159	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 15.59. The MO to HO intercept geometrical bond parameters of alkyl alcohols. E_r is $E_r(\text{atom} - \text{atom}, \text{misp}^3, \text{AO})$.

Bond	Atom	E_r (eV) Bond 1	E_r (eV) Bond 2	E_r (eV) Bond 3	E_r (eV) Bond 4	Final Total Energy C2sp ³ (eV)	r_{bond} (a ₀)	r_{bond} (a ₀)	E_{C2sp^3} (eV) Final	θ' (°)	θ_1 (°)	θ_2 (°)	d_1 (a ₀)	d_2 (a ₀)
H_1C-O-H	O	-0.82688	0	0	0		1.00000	0.85923	-15.65263	115.49	64.51	64.51	0.34405	0.37403
$-H_1C-O-H$	O	-0.92918	0	0	0		1.00000	0.86359	-15.75493	115.09	64.91	64.12	0.35182	0.36625
H_1C-OH	C	-0.82688	0	0	0	-152.44257	0.91771	0.86923	-15.65263	96.59	83.41	46.30	1.23986	0.09981
$(C-O(i))$	O	-0.82688	0	0	0		1.00000	0.86923	-15.65263	96.59	83.41	46.30	1.23986	0.09981
H_1C-OH	C	-0.92918	-0.92918	0	0	-153.47405	0.91771	0.81549	-16.68411	93.09	82.80	46.50	1.23564	0.18708
$(C-O(ii))$	O	-0.92918	0	0	0		1.00000	0.86359	-15.75493	97.20	102.51	41.48	1.23564	0.18708
$-H_1C-OH$	C	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.56407	77.49	111.53	35.84	1.35486	0.29933
$(C-H(CH_3))$	C	-0.92918	-0.92918	0	0	-153.47405	0.91771	0.81549	-16.68412	68.47	118.90	31.37	1.42988	0.57326
$(C-H(CH_2))$	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	61.10	116.18	30.08	1.83179	0.38106
$(C-H(CH))$	C	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.56407	63.82	123.59	26.06	1.90890	0.45117
$H_1C-C_1H_2CH_3$	C ₁	-0.92918	-0.92918	0	0	-153.47405	0.91771	0.81549	-16.68412	56.41	131.70	21.90	1.97162	0.51388
$(C-C(a))$	C ₁	-0.92918	-0.92918	0	0	-154.40324	0.91771	0.77247	-17.61330	48.30	131.70	21.90	1.97162	0.51388
$R-H_1C_1(R'-H_2C_2)(R''-H_3C_3)CH_2-$	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.75889	-17.92866	48.21	131.79	21.74	1.95734	0.50570
$(C-C(b))$	C ₁	-0.92918	-0.92918	-0.92918	-0.72457	-154.71860	0.91771	0.75889	-17.92866	48.21	131.79	21.74	1.95734	0.50570
$R-H_1C_1(R'-H_2C_2)(R''-H_3C_3)CH_2-$	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	48.30	131.70	21.90	1.97162	0.51388
$(C-C(c))$	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	48.30	131.70	21.90	1.97162	0.51388
$isoC_1C_2(H_1C_1-R')HCH_2-$	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	48.30	131.70	21.90	1.97162	0.51388
$(C-C(d))$	C ₁	-0.92918	-0.92918	-0.92918	-0.72457	-154.71860	0.91771	0.75889	-17.92866	50.04	129.96	22.66	1.94462	0.49298
$tertC_1(R'-H_2C_2)C_2(R''-H_3C_3)CH_2-$	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.31399	0.91771	0.76765	-17.92866	50.04	129.96	22.66	1.94462	0.49298
$(C-C(e))$	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.31399	0.91771	0.76765	-17.92866	50.04	129.96	22.66	1.94462	0.49298
$isoC_1C_2(H_1C_1-R')HCH_2-$	C ₁	-0.72457	-0.72457	-0.72457	0	-154.19863	0.91771	0.78155	-17.40869	52.78	127.22	24.04	1.92443	0.47279
$(C-C(f))$	C ₁	-0.72457	-0.72457	-0.72457	0	-154.19863	0.91771	0.78155	-17.40869	52.78	127.22	24.04	1.92443	0.47279
$tertC_1(R'-H_2C_2)C_2(R''-H_3C_3)CH_2-$	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.31399	0.91771	0.76765	-17.92866	50.04	129.96	22.66	1.94462	0.49298
$(C-C(g))$	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.31399	0.91771	0.76765	-17.92866	50.04	129.96	22.66	1.94462	0.49298

Table 15.60. The energy parameters (eV) of functional groups of alkyl alcohols.

Parameters	OH Group	C-O (i) Group	C-O (ii) Group	C'H ₃ Group	CH ₃ Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
η_1	1	1	1	3	2	1	1	1	1	1	1	1
η_2	0	0	0	2	1	0	0	0	0	0	0	0
η_3	0	0	0	0	0	0	0	0	0	0	0	0
C_1	0.75	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_2	1	1	1	1	1	1	1	1	1	1	1	1
C_3	0.75	1	1	1	1	1	1	1	1	1	1	1
C_4	1	0.85395	0.85395	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_5	1	0	0	0	1	1	0	0	0	1	1	0
C_6	1	2	2	1	1	1	2	2	2	2	2	2
C_7	1	0	0	3	2	1	0	0	0	0	0	0
C_8	0.75	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_9	1	1	1	1	1	1	1	1	1	1	1	1
V_1 (eV)	-40.92709	-33.47304	-33.78820	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-29.10112	-28.79214	-29.10112	-29.10112
V_2 (eV)	14.81988	10.15605	10.19068	38.92728	25.78002	12.87680	9.33552	9.33552	9.33552	9.33552	9.33552	9.33552
T (eV)	16.18567	9.32537	9.47749	32.53914	21.06675	10.48582	6.77464	6.77464	6.90500	6.77464	6.90500	6.90500
V_3 (eV)	-8.09284	-4.66268	-4.73874	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.45250	-3.38732	-3.45250	-3.45250
E_1 (eV)	-13.6181	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
$\Delta E_{1,2}$ (eV)	0	-1.63376	-1.85836	0	0	0	0	0	0	0	0	0
E_2 (eV)	-13.6181	-12.98113	-12.77653	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
E_3 (eV)	-31.63247	-31.63344	-31.63529	-67.69451	-49.66493	-31.63533	-31.63537	-31.63537	-31.63535	-31.63537	-31.63535	-31.63535
E_4 (eV)	0	-1.63376	-1.85836	0	0	0	-1.85836	-1.85836	-1.44915	-1.85836	-1.44915	-1.44915
E_5 (eV)	-31.63537	-33.28912	-33.49373	-67.69450	-49.66493	-31.63537	-33.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452
ω (10 ¹⁵ rad/s)	44.1776	22.3978	12.2831	24.9286	24.2751	24.1759	9.43699	9.43699	15.4846	9.43699	15.4846	9.43699
E_6 (eV)	29.07844	14.74264	8.08494	16.40846	15.97831	15.91299	6.21159	6.21159	10.19220	6.21159	10.19220	6.29021
E_7 (eV)	-0.33749	-0.25287	-0.18841	-0.25352	-0.25017	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.16416	-0.16416
E_8 (eV)	0.46311	0.12808	0.13328	0.35532	0.35532	0.35532	0.12312	0.12312	0.09944	0.12312	0.12312	0.12312
E_9 (eV)	[17-18]	[19]	[20]	(Eq. (13.458))	(Eq. (13.458))	(Eq. (13.458))	[21]	[4]	[5]	[2]	[2]	[2]
E_{10} (eV)	-0.10594	-0.18883	-0.12177	-0.22757	-0.14502	-0.07200	-0.10359	-0.07526	-0.15924	-0.10359	-0.10260	-0.10260
E_{11} (eV)	0.11441	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
E_{12} (eV)	-31.74130	-33.47795	-33.61550	-67.92207	-49.80996	-31.70737	-33.59732	-33.59732	-33.24376	-33.59732	-33.24376	-33.59732
E_{13} (eV)	-13.6181	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
E_{14} (eV)	-13.59844	0	0	-13.59844	-13.59844	-13.59844	0	0	0	0	0	0
E_{15} (eV)	4.41035	4.20817	4.34572	12.49186	7.83016	3.32601	4.32754	4.29921	3.97398	4.17951	3.62128	3.91734

Table 15.61. The total bond energies of alkyl alcohols calculated using the functional group composition and the energies of Table 15.60 compared to the experimental values [3].

Formula	Name	OH Group	C-O (i) Group	C-O (ii) Group	C ₂ H ₅	C ₂ H ₃	C ₂ H ₂	C ₂ H	C-C (a)	C-C (b)	C-C (c)	C-C (d)	C-C (e)	C-C (f)	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH ₃ O	Methanol	1	0	1	1	0	0	0	0	0	0	0	0	0	21.1038	21.131	0.00097
C ₂ H ₅ O	Ethanol	1	0	1	1	1	0	1	0	0	0	0	0	0	33.40563	33.428	0.00066
C ₃ H ₇ O	1-Propanol	1	0	1	1	2	0	2	0	0	0	0	0	0	45.56333	45.584	0.00046
C ₃ H ₇ O	2-Propanol	1	0	1	2	0	1	2	0	0	0	0	0	0	45.72088	45.766	0.00098
C ₄ H ₉ O	1-Butanol	1	0	1	1	3	0	3	0	0	0	0	0	0	57.72103	57.736	0.00026
C ₄ H ₉ O	2-Butanol	1	0	1	2	1	1	3	0	0	0	0	0	0	57.87858	57.922	0.00074
C ₅ H ₁₁ O	2-Methyl-1-propananol	1	0	1	2	1	1	0	0	0	0	0	0	0	57.79359	57.828	0.00060
C ₅ H ₁₁ O	2-Methyl-2-propananol	1	0	1	3	0	0	0	3	0	0	0	0	0	58.15359	58.126	-0.00048
C ₅ H ₁₁ O	1-Pentanol	1	0	1	1	4	0	4	0	0	0	0	0	0	69.87873	69.887	0.00011
C ₅ H ₁₁ O	2-Pentanol	1	0	1	2	2	1	4	0	0	0	0	0	0	70.03628	70.057	0.00029
C ₅ H ₁₁ O	3-Pentanol	1	0	1	2	2	1	4	0	0	0	0	0	0	70.03628	70.097	0.00087
C ₆ H ₁₃ O	2-Methyl-1-butanol	1	0	1	2	2	1	1	2	0	0	0	0	0	69.95129	69.957	0.00008
C ₆ H ₁₃ O	2-Methyl-1-butanol	1	0	1	2	2	1	1	2	0	0	0	0	0	69.95129	69.950	-0.00002
C ₆ H ₁₃ O	2-Methyl-2-butanol	1	0	1	3	1	0	1	3	0	0	0	0	0	70.31129	70.246	-0.00092
C ₆ H ₁₃ O	3-Methyl-2-butanol	1	0	1	3	0	2	0	0	3	0	0	0	0	69.96081	70.083	0.00174
C ₆ H ₁₃ O	1-Hexanol	1	0	1	1	5	0	5	0	0	0	0	0	0	82.03643	82.034	0.00021
C ₆ H ₁₃ O	2-Hexanol	1	0	1	2	3	1	5	0	0	0	0	0	0	82.19398	82.236	0.00052
C ₆ H ₁₃ O	1-Heptanol	1	0	1	1	6	0	6	0	0	0	0	0	0	94.19413	94.214	0.00021
C ₆ H ₁₃ O	1-Octanol	1	0	1	1	7	0	7	0	0	0	0	0	0	106.35183	106.358	0.00006
C ₇ H ₁₅ O	2-Ethyl-1-hexanol	1	0	1	2	13	1	4	3	0	0	0	0	0	106.42439	106.459	0.00032
C ₈ H ₁₇ O	1-Nonanol	1	0	1	1	8	0	8	0	0	0	0	0	0	118.50953	118.521	0.00010
C ₉ H ₁₉ O	1-Decanol	1	0	1	1	9	0	9	0	0	0	0	0	0	130.66723	130.676	0.00007
C ₁₀ H ₂₁ O	1-Dodecanol	1	0	1	1	11	0	11	0	0	0	0	0	0	154.98263	154.984	0.00001
C ₁₆ H ₃₃ O	1-Hexadecanol	1	0	1	1	15	0	15	0	0	0	0	0	0	203.61343	203.603	-0.00005

Atom or Angle	Zc ¹ Bond 1 (a ₀)	2c ² Bond 2 (a ₀)	2c ³ Terminal Atoms (a ₀)	R ₁ ¹ Atom 1	Atom 1 Hybridization Designation (Table 15.3.A)	E ₁ ¹ Atom 1	Atom 2 Hybridization Designation (Table 15.3.A)	E ₂ ¹ Atom 2	c ₁ Atom 1	c ₂ Atom 2	C ₁	C ₂	c ₃	E ₃ (eV)	θ ₁ (°)	θ ₂ (°)	Cal. θ (°)	Exp. θ (°)	
∠C ¹ OH (C ¹ -O (i))	2.67935	1.83616	3.6697	-14.82575	I	-14.82575	I	-14.82575	I	0.91771	0.75	I	0.75	0.91771	0			107.24	108.53 (methanol)
∠C ¹ OH (C ¹ -O (ii))	2.67024	1.83616	3.6515	-14.82575	I	-14.82575	I	-14.82575	I	0.91771	0.75	I	0.75	0.91771	0			106.78	105 (ethanol)
∠C ¹ C ² O (C ¹ -O (ii))	2.91547	2.67024	4.5826	-16.68412	2S	-13.61806	O	-13.61806	0.81549	0.83395 (Eq. (15.114))	I	I	I	0.83472	-1.65376			110.17	107.8 (ethanol)
Methane ∠HC ¹ H	2.11106	2.11106	3.4252	-15.75493	7	H	H	H	0.86359	I	I	I	0.75	1.15796	0			108.44	107 (propane)
∠C ¹ C ² C ³																		112	112 (propane)
∠C ¹ C ² H														69.51				110.49	113.8 (butane)
Methyl ∠HC ¹ H	2.09711	2.09711	3.4252	-15.75493	7	H	H	H	0.86359	I	I	I	0.75	1.15796	0			109.50	111.4 (isobutane)
∠C ¹ C ² C ³														70.56				109.44	
∠C ¹ C ² H														70.56				109.64	
∠C ¹ C ² C ³ iso C ³	2.91347	2.91547	4.7958	-16.68412	2S	-16.68412	2S	-16.68412	0.81549	0.81549	I	I	I	0.81549	-1.85836			110.67	110.8 (isobutane)
∠C ¹ C ² H iso C ³	2.91547	2.11323	4.1633	-15.55033	S	-14.82575	I	-14.82575	0.87495	0.91771	0.75	I	0.75	1.04887	0			110.76	
∠C ¹ C ² H iso C ³	2.91547	2.09711	4.1633	-15.55033	S	-14.82575	I	-14.82575	0.87495	0.91771	0.75	I	0.75	1.04887	0			111.27	111.4 (isobutane)
∠C ¹ C ² C ³ iso C ³	2.90337	2.90327	4.7958	-15.55033	S	-14.82575	I	-14.82575	0.87495	0.91771	0.75	I	0.75	1.04887	-1.85836			111.27	111.4 (isobutane)
∠C ¹ C ² C ³														72.50				107.50	

ETHERS ($C_nH_{2n+2}O_m$, $n = 2, 3, 4, 5 \dots \infty$)

The alkyl ethers, $C_nH_{2n+2}O_m$, comprise two types of $C-O$ functional groups, one for methyl or t-butyl groups corresponding to the C and the other for general alkyl groups. The alkyl portion of the alkyl ether may comprise at least two terminal methyl groups (CH_3) at each
 5 end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of $C-C$ bonds can be identified. The n-alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2CH$) and t-butyl ($(CH_3)_3C$) groups and the
 10 isopropyl to isopropyl, isopropyl to t-butyl, and t-butyl to t-butyl $C-C$ bonds comprise functional groups. The branched-chain-alkane groups in ethers are equivalent to those in branched-chain alkanes.

Each $C-O$ group is solved by hybridizing the $2s$ and $2p$ AOs of the C atom to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO
 15 and the O AO to form a MO permits each participating orbital to decrease in radius and energy. In alkyl ethers, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) and an energy of $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)) and the O AO has an energy of $E(O) = -13.61806 \text{ eV}$. To meet the equipotential condition of the union of the $C-O$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor c_2 of Eq. (15.52) for the
 20 $C-O$ -bond MO given by Eq. (15.113) is $c_2(C2sp^3HO \text{ to } O) = 0.85395$. $E_r(atom - atom, msp^3.AO)$ of the $C-O$ -bond MO in Eq. (15.52) due to the charge donation from the C and O atoms to the MO is -1.44915 eV for the CH_3-O- and $(CH_3)_3C-O-$ $C-O$ groups. It is based on the energy match between the O AO, initially at the Coulomb potential of a proton and an electron (Eqs. (1.236) and (10.162), respectively), and the $C2sp^3$
 25 HO of a methyl group as given by Eq. (14.151). For the alkyl $C-O$ group, $E_r(atom - atom, msp^3.AO)$ is -1.65376 eV . It is based on the energy match between the O AO and the $C2sp^3$ HO of a methylene group and is given by the linear combination of -0.72457 eV (Eq. (14.151)) and -0.92918 eV (Eq. (14.513)), respectively.

The symbols of the functional groups of branched-chain alkyl ethers are given in Table 15.63. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of alkyl ethers are given in Tables 15.64, 15.65, and 15.66, respectively. The total energy of each alkyl ether given in Table 15.67
5 was calculated as the sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.66 corresponding to functional-group composition of the molecule. The bond angle parameters of alkyl ethers determined using Eqs. (15.79-15.108) are given in Table 15.68.

Table 15.63. The symbols of functional groups of alkyl ethers.

Functional Group	Group Symbol
C-O (CH_3-O- and $(CH_3)_3C-O-$)	C-O (i)
C-O (alkyl)	C-O (ii)
CH ₃ group	C-H (CH_3)
CH ₂ group	C-H (CH_2)
CH	C-H
CC bond (n-C)	C-C (a)
CC bond (iso-C)	C-C (b)
CC bond (tert-C)	C-C (c)
CC (iso to iso-C)	C-C (d)
CC (t to t-C)	C-C (e)
CC (t to iso-C)	C-C (f)

Table 15.64. The geometrical bond parameters of alkyl ethers and experimental values [1].

Parameter	C-O (i) Group	C-O (ii) Group	C-H (CH_3) Group	C-H (CH_2) Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
a (a_0)	1.80717	1.79473	1.64920	1.67122	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
c' (a_0)	1.34431	1.33968	1.04856	1.05553	1.05661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length $2c'$ (\AA)	1.42276	1.41785	1.10974	1.11713	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (\AA)	1.416 (dimethyl ether)	1.418 (ethyl methyl ether (avg.))	1.107 (C-H propane) 1.117 (C-H butane)	1.107 (C-H propane) 1.117 (C-H butane)	1.122 (isobutane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
b, c (a_0)	1.20776	1.19429	1.27295	1.29569	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
e	0.74388	0.74645	0.63580	0.63159	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 15.65. The MO to HO intercept geometrical bond parameters of alkyl ethers. E_r is $E_r(\text{atom} - \text{atom}, \text{m.p.}, \text{A})$.

Bond	Atom	E_r (eV) Bond 1	E_r (eV) Bond 2	E_r (eV) Bond 3	E_r (eV) Bond 4	Final Total Energy C2ap (eV)	r_{final} (a ₀)	E_{Final} (eV) Final	$E(\text{C2ap})$ (eV) Final	θ_i (°)	θ_z (°)	d_i (a ₀)	d_z (a ₀)
C-H (CH ₃)	C ₁	-0.72457	0	0	0	-152.34026	0.91771	-15.55033	-15.55046	101.15	42.40	1.21777	0.16921
H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₁	-0.72457	0	0	0	-152.34026	0.91771	-15.55033	-15.55046	95.98	46.10	1.23319	0.09112
H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	O	-0.72457	-0.72457	0	0		1.00000	-16.27490		87.34	43.74	1.30555	0.03876
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₂	-0.82688	-0.92918	0	0	-153.37175	0.91771	-16.58181	-16.58095	87.59	43.35	1.30312	0.03456
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	O	-0.72457	-0.82688	0	0		1.00000	-16.37720		86.67	43.98	1.29138	0.04829
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₁	-0.82688	-0.82688	0	0	-152.34487	0.91771	-15.75493	-15.56407	87.13	43.66	1.29829	0.04138
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₂	-0.92918	-0.92918	0	0	-153.47406	0.91771	-16.68412	-16.49335	102.51	41.48	1.23564	0.18703
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₁	-0.92918	-0.92918	0	0	-154.40324	0.91771	-17.61330	-17.42244	111.53	33.84	1.34466	0.29333
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₂	-0.92918	-0.92918	0	0	-152.34487	0.91771	-15.75493	-15.56407	116.18	30.08	1.83879	0.38106
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₁	-0.92918	-0.92918	0	0	-153.47406	0.91771	-16.68412	-16.49335	123.59	26.06	1.90190	0.45117
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₂	-0.92918	-0.92918	0	0	-154.40324	0.91771	-17.61330	-17.42244	131.70	21.90	1.97163	0.51388
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₁	-0.92918	-0.92918	-0.72457	-0.72457	-154.71860	0.91771	-17.92866	-17.73779	131.70	21.74	1.95734	0.50570
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₂	-0.92918	-0.92918	-0.72457	-0.72457	-154.40324	0.91771	-17.61330	-17.42244	131.70	21.90	1.97162	0.51388
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	-17.92866	-17.73779	129.96	22.66	1.94463	0.49298
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₂	-0.72457	-0.92918	-0.72457	-0.72457	-154.19863	0.91771	-17.40869	-17.21793	127.22	24.04	1.92843	0.47279
-H ₃ C ₁ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ H ₅ (CH ₃) ₁ C ₂ -O-C ₂ (CH ₃) ₃ (C-O (i))	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	-17.92866	-17.73779	129.96	22.66	1.94462	0.49298

Table 15.66. The energy parameters (eV) of functional groups of alkyl ethers.

Parameters	C-O (i) Group	C-O (ii) Group	H ₁ Group	H ₂ Group	C-H Group	C'-C' (a) Group	C'-C' (b) Group	C'-C' (c) Group	C'-C' (d) Group	C'-C' (e) Group	C'-C' (f) Group
η_1	1	1	3	2	1	1	1	1	1	1	1
η_2	0	0	2	1	0	0	0	0	0	0	0
η_3	0	0	0	0	0	0	0	0	0	0	0
ζ_1	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
ζ_2	1	1	1	1	1	1	1	1	1	1	1
ζ_3	1	1	1	1	1	1	1	1	1	1	1
ζ_4	0.83395	0.83395	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
ζ_5	0	0	0	1	1	0	0	0	1	1	0
ζ_6	2	2	1	1	1	2	2	2	2	2	2
ζ_7	0	0	3	2	1	0	0	0	0	0	0
ζ_8	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
ζ_9	1	1	1	1	1	1	1	1	1	1	1
ζ_{10}	-33.15757	-33.47304	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-29.10112	-28.79214	-29.10112	-29.10112
ζ_{11}	10.12103	10.15605	38.92728	25.78002	12.87680	9.33532	9.33532	9.37273	9.33532	9.37273	9.37273
ζ_{12}	9.17589	9.32537	32.53914	21.06675	10.48582	6.77464	6.77464	6.90500	6.77464	6.90500	6.90500
ζ_{13}	-4.58695	-4.66268	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.45250	-3.38732	-3.45250	-3.45250
ζ_{14}	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.55946	-15.56407	-15.55946	-15.55946
ζ_{15}	-1.44915	-1.65376	0	0	0	0	0	0	0	0	0
ζ_{16}	-13.18574	-12.98113	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.55946	-15.56407	-15.55946	-15.55946
ζ_{17}	-31.65533	-31.65344	-67.69451	-49.66493	-31.65333	-31.65337	-31.65337	-31.65335	-31.65337	-31.65335	-31.65335
ζ_{18}	-1.44915	-1.65376	0	0	0	-1.83836	-1.83836	-1.44915	-1.83836	-1.44915	-1.44915
ζ_{19}	-33.08452	-33.28912	-67.69450	-49.66493	-31.65337	-33.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452
ω (10^4 rad/s)	12.0329	12.1583	24.9286	24.2751	24.1759	9.43699	9.43699	15.4846	9.43699	9.55643	9.55643
ζ_{20}	7.92028	8.00277	16.40846	15.97831	15.91299	6.21159	6.21159	10.19220	6.21159	6.29021	6.29021
ζ_{21}	-0.18420	-0.18631	-0.23532	-0.23017	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.16416	-0.16416
ζ_{22}	0.13663	0.16118	0.35532	0.35532	0.35532	0.12312	0.12312	0.09944	0.12312	0.12312	0.12312
ζ_{23}	-0.11589	-0.10572	-0.22757	-0.14502	-0.07200	-0.10359	-0.07526	-0.15924	-0.10359	-0.10260	-0.10260
ζ_{24}	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
ζ_{25}	-33.20040	-33.39484	-67.92207	-49.80996	-31.70737	-33.59732	-33.49373	-33.24376	-33.59732	-33.18712	-33.18712
ζ_{26}	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
ζ_{27}	0	0	-13.59844	-13.59844	-13.59844	0	0	0	0	0	0
ζ_{28}	3.93062	4.12506	12.49186	7.83016	3.37601	4.37754	4.29921	3.97398	4.17951	3.62128	3.91734

Table 15.67. The total bond energies of alkyl ethers calculated using the functional group composition and the energies of Table 15.66 compared to the experimental values [3].

Formula	Name	Group	$\text{C}-\text{O}$ (i)	Group	$\text{C}-\text{O}$ (ii)	$\text{C}-\text{H}$	$\text{C}-\text{H}_2$	$\text{C}-\text{H}$	$\text{C}-\text{C}$ (a)	$\text{C}-\text{C}$ (b)	$\text{C}-\text{C}$ (c)	$\text{C}-\text{C}$ (d)	$\text{C}-\text{C}$ (e)	$\text{C}-\text{C}$ (f)	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
$\text{C}_2\text{H}_6\text{O}$	Dimethyl ether	1	0	2	2	0	0	0	0	0	0	0	0	0	32.8496	32.902	0.00174
$\text{C}_3\text{H}_8\text{O}$	Ethyl methyl ether	1	1	2	2	1	0	0	0	0	0	0	0	0	45.19710	45.183	-0.00030
$\text{C}_4\text{H}_{10}\text{O}$	Diethyl ether	0	2	2	2	2	0	0	0	0	0	0	0	0	57.54924	57.500	-0.00086
$\text{C}_5\text{H}_{12}\text{O}$	Methyl propyl ether	1	1	2	2	2	0	0	0	0	0	0	0	0	57.35480	57.355	0.00000
$\text{C}_6\text{H}_{14}\text{O}$	Isopropyl methyl ether	1	1	3	3	0	0	0	0	0	0	0	0	0	57.45569	57.499	0.00073
$\text{C}_7\text{H}_{16}\text{O}$	Dipropyl ether	0	2	2	2	4	0	0	0	0	0	0	0	0	81.86464	81.817	-0.00059
$\text{C}_8\text{H}_{18}\text{O}$	Diisopropyl ether	0	2	4	4	0	0	0	0	0	0	0	0	0	82.06642	82.088	0.00026
$\text{C}_9\text{H}_{20}\text{O}$	t-Butyl ethyl ether	1	1	4	4	1	0	0	0	0	0	0	0	0	94.36135	94.438	0.00085
$\text{C}_{10}\text{H}_{22}\text{O}$	n-Butyl isopropyl ether	1	1	5	5	0	0	0	0	0	0	0	0	0	106.18004	106.122	-0.00055
$\text{C}_{11}\text{H}_{24}\text{O}$	Dibutyl ether	0	2	2	2	6	0	0	0	0	0	0	0	0	106.38182	106.410	0.00027
$\text{C}_{12}\text{H}_{26}\text{O}$	Di-sec-butyl ether	0	2	4	4	2	0	0	0	0	0	0	0	0	106.65628	106.425	-0.00218
$\text{C}_{13}\text{H}_{28}\text{O}$	Di-n-butyl ether	2	0	6	6	0	0	0	0	0	0	0	0	0	106.49072	106.497	0.00006
$\text{C}_{14}\text{H}_{30}\text{O}$	t-Butyl isobutyl ether	1	1	5	5	1	0	0	0	0	0	0	0	0	106.49072	106.497	0.00006

Table 15.68. The bond angle parameters of alkyl ethers and experimental values [1]. In the calculation of θ , the parameters from the preceding angle were used. E_T is $E_T(\text{atom} - \text{atom}, \text{exp}) - A(\theta)$.

Atom 1/Atom 2	ZC ¹ Bond 1 (a ₁)	ZC ² Bond 2 (a ₂)	ZC ³ Terminal Atom (a ₃)	E ₁ Atom 1	Atom 1 Hybridization Designation (Table 15.3.A)	E ₂ Atom 2	Atom 2 Hybridization Designation (Table 15.3.A)	C ₁	C ₂	C ₃	q ₁	ε ₁	E _v (eV)	θ _v (°)	θ ₁ (°)	θ ₂ (°)	Cal. θ (°)	Exp. θ (°)
ZC ₂ (H) ₂ (C ₂ - C ₁) (II)	2.68862	2.68862	4.4721	-17.40869	3N	0.78155	3B	1	1	1	1	0.78155	-1.85836				112.54	112 (dimethyl ether)
ZC ₂ (H) ₂ (C ₂ - C ₁) (II)	2.68862	2.67935	4.4385	-17.51099	4I	0.77699		1	1	1	1	0.77699	-1.85836				111.55	111.9 (ethyl methyl ether)
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)	2.91547	2.67935	4.5607	-16.04412	2S	0.81549	O	1	1	1	1	0.83472	-1.65376				109.13	109.4 (ethyl methyl ether)
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)	2.11106	2.11106	3.4232	-15.75493	7	0.86359	H	1	1	1	0.75	1.15796	0				108.44	107 (propane)
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)														69.51			110.49	112 (propane) 113.8 (butane) 110.8 (isobutane)
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)														69.51			110.49	111.0 (butane) 111.4 (isobutane)
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)	2.09711	2.09711	3.4232	-15.75493	7	0.86359	H	1	1	1	0.75	1.15796	0				109.50	
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)														70.56			109.44	
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)														70.56			109.44	
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)	2.91547	2.91547	4.7918	-16.04412	2S	0.81549	2S	1	1	1	1	0.81549	-1.85836				110.67	110.8 (isobutane)
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)	2.91547	2.11325	4.1653	-15.53033	5	0.87495	1	0.75	1	1	0.75	1.04897	0				110.76	
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)	2.91547	2.10971	4.1653	-15.53033	5	0.87495	1	0.75	1	1	0.75	1.04897	0				111.27	111.4 (isobutane)
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)	2.90327	2.90327	4.7958	-15.53033	5	0.87495	1	0.75	1	1	0.75	1.04897	-1.85836				111.27	111.4 (isobutane)
ZC ₂ (H) ₂ (C ₂ - C ₁) (III)														72.50			107.50	

PRIMARY AMINES ($C_n H_{2n+2+m} N_m$, $n=1,2,3,4,5\ldots\infty$)

The primary amines, $C_n H_{2n+2+m} N_m$, comprise an NH_2 functional group and a $C-N$ functional group. The alkyl portion of the primary amine may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and
 5 methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of $C-C$ bonds can be identified. The n -alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2 CH$) and t -butyl ($(CH_3)_3 C$) groups and the isopropyl to isopropyl, isopropyl to t -butyl, and t -butyl to t -butyl
 10 $C-C$ bonds comprise functional groups. The branched-chain-alkane groups in primary amines are equivalent to those in branched-chain alkanes.

The primary amino (NH_2) functional group was solved using the procedure given in the Dihydrogen Nitride (NH_2) section. Using the results of Eqs. (13.245-13.368), the primary amino parameters in Eq. (15.52) are $n_1 = 2$, $C_1 = 0.75$, $C_2 = 0.93613$ (Eqs. (13.248-13.249)),
 15 $C_{1n} = 1.5$, and $c_1 = 0.75$. In primary amines, the $C2sp^3$ HO of the $C-NH_2$ -bond MO has an energy of $E(C,2sp^3) = -15.35946 \text{ eV}$ (Eq. (15.18) with $s=1$ and Eqs. (15.19-15.20)) and the N AO has an energy of $E(N) = -14.53414 \text{ eV}$. To meet the equipotential condition of the union of the $N-H$ H_2 -type-ellipsoidal-MO with the $C2sp^3$ HO, the hybridization factor c_2 of Eq. (15.52) for the $N-H$ -bond MO given by Eq. (15.68) is

$$20 \quad c_2(H \text{ to } 1^\circ N) = \frac{E(N)}{E(C,2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.35946 \text{ eV}} = 0.94627 \quad (15.115)$$

The $C-N$ group is solved by hybridizing the $2s$ and $2p$ AOs of the C atom to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO and the N AO to form a MO permits each participating orbital to decrease in radius and energy. In primary amines, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a
 25 corresponding energy of $E(C,2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)), and the N AO has an energy of $E(N) = -14.53414 \text{ eV}$. To meet the equipotential condition of the union of the

$C-N$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor c_2 of Eq. (15.52) for the $C-N$ -bond MO given by Eqs. (15.68) and (15.70) is

$$c_2(C2sp^3HO \text{ to } N) = \frac{E(N)}{E(C,2sp^3)} c_2(C2sp^3HO) = \frac{-14.53414 \text{ eV}}{-14.63489 \text{ eV}} (0.91771) = 0.91140 \quad (15.116)$$

$E_T(\text{atom-atom}, msp^3.AO)$ of the $C-N$ -bond MO in Eq. (15.52) due to the charge donation from the C and N atoms to the MO is -1.44915 eV . It is based on the energy match between the N of the NH_2 group and the $C2sp^3$ HO corresponding to the energy contributions to the single bond that are equivalent to those of methyl groups, -0.72457 eV (Eq. (14.151)), where the $N-H$ bonds are also energy matched to the $C-N$ bond.

The symbols of the functional groups of branched-chain primary amines are given in Table 15.69. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of primary amines are given in Tables 15.70, 15.71, and 15.72, respectively. The total energy of each primary amine given in Table 15.73 was calculated as the sum over the integer multiple of each $E_D(\text{Group})$ of Table 15.72 corresponding to functional-group composition of the molecule. The bond angle parameters of primary amines determined using Eqs. (15.79-15.108) are given in Table 15.74.

Table 15.69. The symbols of functional groups of primary amines.

Functional Group	Group Symbol
NH_2 group	NH_2
C-N	$C-N$
CH_3 group	$C-H (CH_3)$
CH_2 group	$C-H (CH_2)$
CH	$C-H$
CC bond (n-C)	$C-C$ (a)
CC bond (iso-C)	$C-C$ (b)
CC bond (tert-C)	$C-C$ (c)
CC (iso to iso-C)	$C-C$ (d)
CC (t to t-C)	$C-C$ (e)
CC (t to iso-C)	$C-C$ (f)

Table 1.5.70. The geometrical bond parameters of primary amines and experimental values [1].

Parameter	NH ₂ Group	C-N Group	C-H (CH ₃) Group	C-H (CH ₂) Group	C-H (CH) Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
<i>a</i> (Å)	1.28083	1.92682	1.64920	1.67122	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
<i>c'</i> (Å)	0.95506	1.38810	1.04856	1.05553	1.05661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length 2 <i>c'</i> (Å)	1.0108	1.46910	1.10974	1.11713	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (Å)	1.010 (methylamine)	1.471 (methylamine)	1.107 (C-H propane) 1.117 (C-H butane)	1.107 (C-H propane) 1.117 (C-H butane)	1.122 (isobutane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)	1.532 (propane) 1.531 (butane)
<i>h_c</i> (Å)	0.85345	1.33634	1.27295	1.29569	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
<i>r_e</i>	0.74556	0.72041	0.63580	0.63159	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 1.5.71. The MO to HO intercept geometrical bond parameters of primary amines. *R*, *R'*, *R''* are H or alkyl groups. *E_p* is *E_p* (atom - atom, m.e.v.² ÅO).

Bond	Atom	<i>E_p</i> (eV) Bond 1	<i>E_p</i> (eV) Bond 2	<i>E_p</i> (eV) Bond 3	<i>E_p</i> (eV) Bond 4	Final Total Energy (eV)	<i>r_{MO}</i> (Å)	<i>r_{MO}</i> (Å)	<i>E_{MO}</i> (eV) Final	<i>E_{MO}</i> (eV) Final	<i>θ'</i> (°)	<i>θ₁</i> (°)	<i>θ₂</i> (°)	<i>d₁</i> (Å)	<i>d₂</i> (Å)
H ₂ CN(H)-H	N	-0.72457	0	0	0	-152.34026	0.91084	0.87495	-15.55033	-15.55033	118.00	62.00	64.85	0.54432	0.41075
-H ₂ CN(H)-H	N	-0.72457	0	0	0	-152.34026	0.91084	0.87495	-15.55033	-15.55033	118.00	62.00	64.85	0.54432	0.41075
H ₂ C-NH ₂	C	-0.72457	0	0	0	-152.34026	0.91771	0.87495	-15.55033	-15.55033	85.28	94.72	40.73	1.46010	0.77200
H ₂ C-NH ₂	N	-0.72457	0	0	0	-152.34026	0.91084	0.87495	-15.55033	-15.55033	85.28	94.72	40.73	1.46010	0.77200
-H ₂ C-NH ₂	C	-0.72457	-0.92918	0	0	-153.26945	0.91771	0.82362	-16.47951	-16.28864	80.20	99.80	37.50	1.52858	0.14048
-H ₂ C-NH ₂	N	-0.72457	0	0	0	-152.34026	0.91084	0.87495	-15.55033	-15.55033	85.28	94.72	40.73	1.46010	0.77200
C-H (CH ₃)	C	-0.92918	0	0	0	-152.34487	0.91771	0.86359	-15.75493	-15.56407	77.49	102.51	41.48	1.23564	0.18708
C-H (CH ₂)	C	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81349	-16.68412	-16.49325	68.47	111.53	35.84	1.35486	0.29933
C-H (CH)	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	61.10	118.90	31.37	1.42988	0.77226
H ₂ C-C ₂ H ₅ -CH ₃ - (C-C (a))	C ₂	-0.92918	0	0	0	-152.34487	0.91771	0.86359	-15.75493	-15.56407	63.82	116.18	30.08	1.83879	0.38106
H ₂ C-C ₂ H ₅ -CH ₃ - (C-C (b))	C ₂	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81349	-16.68412	-16.49325	56.41	123.59	26.06	1.90890	0.45117
R-H ₂ C-C ₂ H ₅ -CH ₃ - (C-C (c))	C ₂	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
R-H ₂ C-C ₂ H ₅ -CH ₃ - (C-C (d))	C ₂	-0.92918	-0.72457	-0.72457	-0.72457	-154.71860	0.91771	0.75889	-17.92866	-17.73779	48.21	131.79	21.74	1.97374	0.50370
isoC ₂ H ₅ -C ₂ H ₅ -CH ₃ - (C-C (e))	C ₂	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
tertC ₂ H ₅ -C ₂ H ₅ -CH ₃ - (C-C (f))	C ₂	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	139.96	22.66	1.94462	0.49298
tertC ₂ H ₅ -C ₂ H ₅ -CH ₃ - (C-C (g))	C ₂	-0.72457	-0.92918	-0.92918	0	-154.19863	0.91771	0.78155	-17.40869	-17.21783	52.78	127.22	24.04	1.92443	0.47279
isoC ₂ H ₅ -C ₂ H ₅ -CH ₃ - (C-C (h))	C ₂	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	139.96	22.66	1.94462	0.49298

Table 15.72. The energy parameters (eV) of functional groups of primary amines.

Parameters	NH_2 Group	$C-N$ Group	CH_2 Group	CH_3 Group	$C-H$ Group	$C-C(a)$ Group	$C-C(b)$ Group	$C-C(c)$ Group	$C-C(d)$ Group	$C-C(e)$ Group	$C-C(f)$ Group	$C-C(g)$ Group
n_1	2	1	3	2	1	1	1	1	1	1	1	1
n_2	0	0	2	1	0	0	0	0	0	0	0	0
n_3	1	0	0	0	0	0	0	0	0	0	0	0
C_1	0.75	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5	0.5
C_2	0.93613	1	1	1	1	1	1	1	1	1	1	1
C_3	0.75	1	1	1	1	1	1	1	1	1	1	1
C_4	0.94627	0.91140	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_5	0	0	0	1	1	0	0	0	1	1	1	0
C_6	1	2	1	1	1	2	2	2	2	2	2	2
C_7	2	0	3	2	1	0	0	0	0	0	0	0
C_8	1.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5	0.5
C_9	1	1	1	1	1	1	1	1	1	1	1	1
V_1 (eV)	-77.89897	-32.46339	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-29.10112	-28.79214	-29.10112	-29.10112	-29.10112
V_2 (eV)	28.49191	9.80175	38.92728	25.78002	12.87680	9.3352	9.3352	9.3352	9.3352	9.3352	9.3352	9.3352
V_3 (eV)	30.40957	8.42409	32.53914	21.06675	10.48582	6.77464	6.77464	6.90500	6.77464	6.90500	6.90500	6.90500
V_4 (eV)	-15.20478	-4.21204	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.45250	-3.38732	-3.45250	-3.45250	-3.45250
E_1 (eV)	-14.53414	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
$\Delta E_{1,10}$ (eV)	0	-1.44915	0	0	0	0	0	0	0	0	0	0
E_2 (eV)	-14.53414	-13.18574	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
E_3 (eV)	-14.53414	0	0	0	0	0	0	0	0	0	0	0
E_4 (eV)	-48.73642	-31.63534	-67.69451	-49.66493	-31.63533	-31.63537	-31.63537	-31.63535	-31.63537	-31.63535	-31.63535	-31.63535
E_5 (eV)	0	-1.44915	0	0	0	-1.85836	-1.85836	-1.44915	-1.85836	-1.44915	-1.44915	-1.44915
E_6 (eV)	-48.73660	-33.08452	-67.69450	-49.66493	-31.63537	-33.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452	-33.08452
ω (10 ³ rad/s)	64.2189	18.9231	24.9286	24.2751	24.1759	9.43699	9.43699	15.4846	9.43699	15.4846	9.55643	9.55643
E_7 (eV)	42.27003	12.45552	16.40846	15.97831	15.91299	6.21159	6.21159	10.19220	6.21159	10.19220	6.29021	6.29021
E_8 (eV)	-0.40690	-0.23100	-0.23552	-0.25017	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.16416	-0.16416	-0.16416
E_9 (eV)	0.40929	0.12944	0.35532	0.35532	0.35532	0.12312	0.12312	0.09944	0.12312	0.12312	0.12312	0.12312
E_{10} (eV)	-0.20226	-0.16628	-0.22757	-0.14502	-0.07200	-0.10359	-0.10359	-0.15924	-0.10359	-0.10260	-0.10260	-0.10260
E_{11} (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
E_{12} (eV)	-49.14112	-33.25079	-67.92307	-49.80996	-31.70737	-33.59732	-33.59732	-33.24376	-33.59732	-33.18712	-33.18712	-33.18712
E_{13} (eV)	-14.53414	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
E_{14} (eV)	-13.59844	0	-13.59844	-13.59844	-13.59844	0	0	0	0	0	0	0
E_{15} (eV)	7.41010	3.98101	12.49186	7.83016	3.32601	4.32754	4.29921	3.97398	4.17951	3.62128	3.91734	3.91734

Table 15.73. The total bond energies of primary amines calculated using the functional group composition and the energies of Table 15.72 compared to the experimental values [3].

Formula	Name	NH_2 Group	$C-N$ Group	CH_2 Group	CH_3 Group	CH Group	$C-C(a)$ Group	$C-C(b)$ Group	$C-C(c)$ Group	$C-C(d)$ Group	$C-C(e)$ Group	$C-C(f)$ Group	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
CH_5N	Methylamine	1	1	1	0	0	0	0	0	0	0	0	23.857	23.857	-0.00110
C_2H_7N	Ethylamine	1	1	1	1	0	0	0	0	0	0	0	36.052	36.052	0.00060
C_3H_9N	Propylamine	1	1	1	2	0	0	0	0	0	0	0	48.243	48.243	0.00092
$C_4H_{11}N$	Butylamine	1	1	1	3	0	0	0	0	0	0	0	60.415	60.415	0.00098
$C_5H_{13}N$	sec-Butylamine	1	1	2	1	1	0	0	0	0	0	0	60.45696	60.457	0.00148
$C_6H_{15}N$	t-Butylamine	1	1	3	0	0	0	0	0	0	0	0	60.717	60.717	-0.00118
$C_7H_{17}N$	Isobutylamine	1	1	2	1	1	0	0	0	0	0	0	60.486	60.486	0.00094

SECONDARY AMINES ($C_n H_{2n+2+m} N_m$, $n = 2, 3, 4, 5 \dots \infty$)

The secondary amines, $C_n H_{2n+2+m} N_m$, comprise an NH functional group and two types of $C-N$ functional groups, one for the methyl group corresponding to the C of $C-N$ and the other for general alkyl secondary amines. The alkyl portion of the secondary amine may
 5 comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of $C-C$ bonds can be identified. The n -alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl
 10 ($(CH_3)_2 CH$) and t -butyl ($(CH_3)_3 C$) groups and the isopropyl to isopropyl, isopropyl to t -butyl, and t -butyl to t -butyl $C-C$ bonds comprise functional groups. The branched-chain-alkane groups in secondary amines are equivalent to those in branched-chain alkanes.

The secondary amino (NH) functional group was solved using the procedure given in the Hydrogen Nitride (NH) section. Using the results of Eqs. (13.245-13.316), the secondary
 15 amino parameters in Eq. (15.52) are $n_1 = 1$, $C_1 = 0.75$, $C_2 = 0.93613$ (Eqs. (13.248-13.249)), $C_{1o} = 0.75$, and $c_1 = 0.75$. In secondary amines, the $C2sp^3$ HO of the $C-NH$ -bond MO has an energy of $E(C, 2sp^3) = -15.56407 \text{ eV}$ (Eqs. (14.514-14.516)); Eq. (15.29) with $s=1$ and $s=2$, Eq. (15.31), and Eqs. (15.19-15.20)) and the N AO has an energy of $E(N) = -14.53414 \text{ eV}$. To meet the equipotential condition of the union of the $N-H$ H_2 -
 20 type-ellipsoidal-MO with the $C2sp^3$ HO, the hybridization factor c_2 of Eq. (15.52) for the $N-H$ -bond MO given by Eq. (15.68) is

$$c_2(H \text{ to } 2^\circ N) = \frac{E(N)}{E(C, 2sp^3)} = \frac{-14.53414 \text{ eV}}{-15.56407 \text{ eV}} = 0.93383 \quad (15.117)$$

The $C-N$ group is solved by hybridizing the $2s$ and $2p$ AOs of the C atom to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO and the
 25 N AO to form a MO permits each participating orbital to decrease in radius and energy. In secondary amines, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a corresponding energy of $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)), and the N AO has an energy of $E(N) = -14.53414 \text{ eV}$. To meet the equipotential condition of the union of the

$C-N$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor c_2 of Eq. (15.52) for the $C-N$ -bond MO given by Eq. (15.116) is $c_2(C2sp^3HO \text{ to } N) = 0.91140$.

As given in the Continuous-Chain Alkanes (C_nH_{2n+2} , $n=3,4,5,\dots,\infty$) section, each methylene group forms two single bonds, and the energy of each $C2sp^3$ HO of each CH_2 group alone is given by that in ethylene, -1.13379 eV (Eq. (14.511)). In secondary amines, the N of the NH group also binds to two $C2sp^3$ HOs and the corresponding $E_T(atom - atom, msp^3.AO)$ of each $C-N$ -bond MO in Eq. (15.52) due to the charge donation from the C and N atoms to the MO is -1.13379 eV . It is based on the energy match between the N of the NH group to the two $C2sp^3$ HOs corresponding to the energy contributions to each of the two single bonds that are equivalent to those of independent methylene groups, -1.13379 eV (Eq. (14.511)), where the $N-H$ bond is also energy matched to the $C-N$ bonds. $E_T(atom - atom, msp^3.AO)$ of the $C-N$ -bond MO in Eq. (15.52) due to the charge donation from the C and N atoms to the MO is -1.13379 eV . It is based on the energy match between the N of the NH group to two $C2sp^3$ HOs corresponding to the energy contributions to the single bond that are equivalent to those of methyl groups, -0.72457 eV (Eq. (14.151)), where the $N-H$ bonds are also energy matched to the $C-N$ bond.

The symbols of the functional groups of branched-chain secondary amines are given in Table 15.75. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of secondary amines are given in Tables 15.76, 15.77, and 15.78, respectively. As in the case of NH_2 (Eq. (13.339)), $C_{1o} = 2C_1$ rather than $C_{1o} = C_1$ in Eq. (15.52) for the $C-N$ bond. The total energy of each secondary amine given in Table 15.79 was calculated as the sum over the integer multiple of each $E_D(Grp)$ of Table 15.78 corresponding to functional-group composition of the molecule. The bond angle parameters of secondary amines determined using Eqs. (15.79-15.108) are given in Table 15.80.

Table 15.75. The symbols of functional groups of secondary amines.

Functional Group	Group Symbol
NH group	NH
C-N (methyl)	$C-N$ (i)
C-N (alkyl)	$C-N$ (ii)
CH_3 group	$C-H$ (CH_3)
CH_2 group	$C-H$ (CH_2)
CH	$C-H$
CC bond (n-C)	$C-C$ (a)
CC bond (iso-C)	$C-C$ (b)
CC bond (tert-C)	$C-C$ (c)
CC (iso to iso-C)	$C-C$ (d)
CC (t to t-C)	$C-C$ (e)
CC (t to iso-C)	$C-C$ (f)

Table 15.76. The geometrical bond parameters of secondary amines and experimental values [1].

Parameter	NH Group	C-N (i) Group	C-N (ii) Group	C-H (CH ₃) Group	C-H (CH ₂) Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
σ (Å)	1.26224	1.94862	1.94862	1.64920	1.67122	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
σ' (Å)	0.94811	1.39593	1.39593	1.04856	1.05553	1.05661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length 2 σ' (Å)	1.00343	1.47739	1.47739	1.10974	1.11713	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (Å)	1.00 (dimethylamine)	1.455 (dimethylamine)		1.107 (C-H propane)	1.107 (C-H propane)	1.122 (isobutane)	1.532 (propane)	1.532 (propane)	1.532 (propane)	1.532 (propane)	1.532 (propane)	1.532 (propane)
h_c (Å)	0.83327	1.35960	1.35960	1.27295	1.29569	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
σ	0.75113	0.71637	0.71637	0.63380	0.63159	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 15.77. The MO to HO intercept geometrical bond parameters of secondary amines. R, R', R'' are H or alkyl groups. E_f is $E_f(\text{atom} - \text{atom}, \text{exp}, \text{AO})$.

Bond	Atom	E_T (eV) Bond 1	E_T (eV) Bond 2	E_T (eV) Bond 3	E_T (eV) Bond 4	Final Total C2- σ^* (eV)	r_{exp} (a_0)	r_{calc} (a_0)	E_{calc} (eV) Final	$E(C2-\sigma^*)$ (eV) Final	θ^* ($^\circ$)	θ_1 ($^\circ$)	θ_2 ($^\circ$)	d_1 (a_0)	d_2 (a_0)
$H_2C-N(C_2H_5)-H$	N	-0.36690	-0.36690	0	0	-152.18259	0.93084	0.83232	-15.93954	-15.93954	118.18	61.82	64.40	0.54546	0.40264
$-H_2C-N(R_{\text{exp}})-H$	N	-0.36690	-0.36690	0	0	-152.18259	0.93084	0.83232	-15.93954	-15.93954	118.18	61.82	64.40	0.54546	0.40264
$H_2C-NH-C_2H_5$	C	-0.36690	0	0	0	-152.18259	0.91771	0.83392	-15.93265	-15.20178	84.14	93.86	40.30	1.48625	0.09032
$H_2C-NH-C_2H_5$	N	-0.36690	-0.36690	0	0	-152.18259	0.93084	0.83232	-15.93954	-15.93954	80.95	99.05	38.26	1.53008	0.13415
$-H_2C-NH-C_2H_5$	C	-0.36690	-0.92918	0	0	-153.11177	0.91771	0.83360	-16.32183	-16.13097	78.89	101.11	36.99	1.55650	0.16057
$-H_2C-NH-C_2H_5$	N	-0.36690	-0.36690	0	0	-152.54487	0.93084	0.83232	-15.93954	-15.93954	80.95	99.05	38.26	1.53008	0.13415
$C-H (CH_3)$	C	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	77.49	102.51	41.48	1.23564	0.18708
$C-H (CH_2)$	C	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	-16.49325	68.47	111.53	35.84	1.35486	0.29933
$C-H (CH)$	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	61.10	118.90	31.37	1.43988	0.37326
$H_2C-C_2H_5-C_2H_5$ (C-C (a))	C	-0.92918	0	0	0	-152.54487	0.91771	0.86359	-15.75493	-15.56407	63.82	116.18	30.08	1.83879	0.38106
$H_2C-C_2H_5-C_2H_5$ (C-C (b))	C	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	-16.49325	56.41	123.59	26.06	1.90890	0.45117
$R-H_2C-C_2H_5-C_2H_5$ (C-C (c))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (d))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (e))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (f))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (g))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (h))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (i))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (j))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (k))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (l))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (m))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (n))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (o))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (p))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (q))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (r))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (s))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (t))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (u))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (v))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (w))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (x))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (y))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_2C-C_2H_5-C_2H_5$ (C-C (z))	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388

Table 15.78. The energy parameters (eV) of functional groups of secondary amines.

Parameters	NH Group	C-N (i) Group	C-N (ii) Group	CH ₃ Group	CH ₂ Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
n_1	1	1	1	3	2	1	1	1	1	1	1	1
n_2	0	0	0	2	1	0	0	0	0	0	0	0
n_3	0	0	0	0	0	0	0	0	0	0	0	0
c_1	0.75	0.5	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
c_2	0.93613	1	1	1	1	1	1	1	1	1	1	1
c_3	0.75	1	1	1	1	1	1	1	1	1	1	1
c_4	0.93383	0.91140	0.91140	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
c_5	1	0	0	0	1	1	0	0	0	1	1	0
c_6	1	2	2	1	1	1	2	2	2	2	2	2
c_7	1	0	0	3	2	1	0	0	0	0	0	0
c_{10}	0.75	1	1	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
c_{11}	1	1	1	1	1	1	1	1	1	1	1	1
V_e (eV)	-39.21967	-31.98456	-31.98456	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-28.79214	-28.79214	-28.79214	-28.79214
V_p (eV)	14.35050	9.74677	9.74677	38.92728	25.78002	12.87680	9.3352	9.3352	9.3352	9.3352	9.3352	9.3352
T (eV)	15.53581	8.20698	8.20698	32.53914	21.06675	10.48582	6.77464	6.77464	6.77464	6.77464	6.77464	6.77464
V_s (eV)	-7.76790	-4.10349	-4.10349	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.38732	-3.38732	-3.38732	-3.38732
E_{100} (eV)	-14.53414	-14.63489	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
ΔE_{100} (eV)	0	-1.13379	-1.13379	0	0	0	0	0	0	0	0	0
E_p (eV)	-14.53414	-13.50110	-13.50110	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407	-15.56407
E_p (eV)	-31.63541	-31.63540	-31.63540	-67.69451	-49.66493	-31.63533	-31.63537	-31.63537	-31.63537	-31.63537	-31.63537	-31.63537
E_p (eV)	0	-1.13379	-1.13379	0	0	0	-1.85836	-1.85836	-1.44915	-1.44915	-1.44915	-1.44915
E_p (eV)	-31.63537	-32.76916	-32.76916	-67.69450	-49.66493	-31.63537	-33.49373	-33.49373	-33.49373	-33.49373	-33.49373	-33.49373
ω (10 ⁴ kcal/mol)	47.0696	15.1983	15.1983	24.9286	24.2751	24.1759	9.43699	9.43699	9.43699	9.43699	9.43699	9.43699
E_p (eV)	30.98202	10.00377	10.00377	16.40846	15.97831	15.91299	6.21159	6.21159	6.21159	6.21159	6.21159	6.21159
E_p (eV)	-0.34836	-0.20505	-0.20505	-0.25352	-0.25017	-0.24966	-0.16515	-0.16515	-0.16515	-0.16515	-0.16515	-0.16515
E_p (eV)	0.40696	0.12944	0.12944	0.35532	0.35532	0.35532	0.12312	0.12312	0.12312	0.12312	0.12312	0.12312
E_p (eV)	-0.14488	-0.14033	-0.14033	-0.22757	-0.14502	-0.07200	-0.10359	-0.07200	-0.10359	-0.10359	-0.10359	-0.10359
E_p (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
E_p (eV)	-31.78025	-32.90949	-32.90949	-67.92207	-49.80996	-31.70737	-33.59732	-33.59732	-33.59732	-33.59732	-33.59732	-33.59732
E_p (eV)	-14.53414	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
E_p (eV)	-13.59844	0	0	-13.59844	-13.59844	-13.59844	0	0	0	0	0	0
E_p (eV)	3.50582	3.63971	3.63971	12.49186	7.80016	3.32601	4.32754	4.29921	4.17951	3.62128	3.91734	3.91734

Table 15.79. The total bond energies of secondary amines calculated using the functional group composition and the energies of Table 15.78 compared to the experimental values [3].

Formula	Name	NH Group	C-N (i) Group	C-N (ii) Group	CH ₃ Group	CH ₂ Group	CH Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₂ H ₅ N	Dimethylamine	1	1	0	2	0	0	0	0	0	0	0	0	35.76895	35.765	-0.00012
C ₃ H ₇ N	Diethylamine	1	0	2	2	2	0	0	0	0	0	0	0	60.211	60.211	-0.00010
C ₄ H ₉ N	Dipropylamine	1	0	2	2	4	0	0	0	0	0	0	0	84.5470	84.558	0.00016
C ₅ H ₁₁ N	Diisopropylamine	1	0	2	4	0	2	0	0	0	0	0	0	108.872	108.872	0.00011
C ₆ H ₁₃ N	Diisobutylamine	1	0	2	4	0	2	0	0	0	0	0	0	129.106	129.106	0.00092

[illegible]

TERTIARY AMINES ($C_nH_{2n+3}N$, $n = 3, 4, 5 \dots \infty$)

The tertiary amines, $C_nH_{2n+3}N$, have three $C-N$ bonds to methyl or alkyl groups wherein $C-N$ comprises a functional group. The alkyl portion of the tertiary amine may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise 5 methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of $C-C$ bonds can be identified. The n-alkane $C-C$ bond is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2CH$) and t-butyl ($(CH_3)_3C$) groups and the isopropyl to isopropyl, isopropyl to t-butyl, 10 and t-butyl to t-butyl $C-C$ bonds comprise functional groups. The branched-chain-alkane groups in tertiary amines are equivalent to those in branched-chain alkanes.

The $C-N$ group is solved by hybridizing the $2s$ and $2p$ AOs of the C atom to form a single $2sp^3$ shell as an energy minimum, and the sharing of electrons between the $C2sp^3$ HO and the N AO to form a MO permits each participating orbital to decrease in radius and energy. 15 In tertiary amines, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a corresponding energy of $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)), and the N AO has an energy of $E(N) = -14.53414 \text{ eV}$. To meet the equipotential condition of the union of the $C-N$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor c_2 of Eq. (15.52) for the $C-N$ -bond MO given by Eq. (15.116) is $c_2(C2sp^3HO \text{ to } N) = 0.91140$.

20 As given in the Continuous-Chain Alkanes (C_nH_{2n+2} , $n = 3, 4, 5 \dots \infty$) section, the energy of each $C2sp^3$ HO must be a linear combination of that of the CH_3 and CH_2 groups that serve as basis elements. Each CH_3 forms one $C-C$ bond, and each CH_2 group forms two. Thus, the energy of each $C2sp^3$ HO of each CH_3 and CH_2 group alone is given by that in ethane, -0.72457 eV (Eq. (14.151)), and ethylene, -1.13379 eV (Eq. (14.511)), respectively. In order 25 to match the energy of the component HOs and MOs for the entire molecule, the energy $E_{r_{alkane}}(C-C, 2sp^3)$ given as a linear combination of these basis elements is -0.92918 eV (Eq. (14.513)). In tertiary amines, the N binds to three $C2sp^3$ HOs and the corresponding $E_r(atom - atom, msp^3.AO)$ of each $C-N$ -bond MO in Eq. (15.52) due to the charge donation

from the *C* and *N* atoms to the MO is -0.92918 eV . It comprises a linear combination of the energy for a primary amine, -0.72457 eV and a secondary amine, -1.13379 eV .

The symbols of the functional groups of branched-chain tertiary amines are given in Table 15.81. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and
5 energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of tertiary amines are given in Tables 15.82, 15.83, and 15.84, respectively. The total energy of each tertiary amine given in Table 15.85 was calculated as the sum over the integer multiple of each $E_p(\text{Group})$ of Table 15.84 corresponding to functional-group composition of the molecule. The bond angle parameters of tertiary amines determined using Eqs. (15.79-15.108) are given in Table 15.86.

Table 15.81. The symbols of functional groups of tertiary amines.

Functional Group	Group Symbol
C-N	C-N
CH ₃ group	C-H (CH ₃)
CH ₂ group	C-H (CH ₂)
CH	C-H
CC bond (n-C)	C-C (a)
CC bond (iso-C)	C-C (b)
CC bond (tert-C)	C-C (c)
CC (iso to iso-C)	C-C (d)
CC (t to t-C)	C-C (e)
CC (t to iso-C)	C-C (f)

Table 15.82. The geometrical bond parameters of tertiary amines and experimental values [1].

Parameter	C-N Group	C-H (CH ₃) Group	C-H (CH ₂) Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
σ (Å)	1.96313	1.64920	1.67122	1.67465	2.12499	2.12499	2.10725	2.12499	2.10725	2.10725
σ' (Å)	1.40112	1.04856	1.05553	1.05661	1.45744	1.45744	1.45164	1.45744	1.45164	1.45164
Bond Length 2 σ' (Å)	1.48288	1.10974	1.11713	1.11827	1.54280	1.54280	1.53635	1.54280	1.53635	1.53635
Exp. Bond Length (Å)	1.458 (trimethylamine)	1.107 (C-H propane)	1.107 (C-H propane)	1.122 (isobutane)	1.532 (propane)	1.532 (propane)	1.532 (propane)	1.532 (propane)	1.532 (propane)	1.532 (propane)
		1.117 (C-H butane)	1.117 (C-H butane)		1.531 (butane)	1.531 (butane)	1.531 (butane)	1.531 (butane)	1.531 (butane)	1.531 (butane)
$h.c$ (Å)	1.37505	1.27295	1.29569	1.29924	1.54616	1.54616	1.52750	1.54616	1.52750	1.52750
ϵ	0.71372	0.63580	0.63159	0.63095	0.68600	0.68600	0.68888	0.68600	0.68888	0.68888

Table 15.83. The MO to HO intercept geometrical bond parameters of tertiary amines. R, R', R'' are H or alkyl groups. E_f is $E_f(\text{atom} - \text{atom}, \text{misp}^2, \text{AO})$.

Bond	Atom	E_f (eV) Bond 1	E_f (eV) Bond 2	E_f (eV) Bond 3	E_f (eV) Bond 4	Final Total Energy C2, sp ² (eV)	r_{bond} (a ₀)	r_{bond} (a ₀)	E_{Coulomb} (eV) Final	$E(\text{C2sp}^2)$ (eV) Final	θ' (°)	θ_1 (°)	θ_2 (°)	d_1 (a ₀)	d_2 (a ₀)
$N-(C, H_2)_1$	C ₁	-0.46459		0	0	-152.08028	0.91771	0.85983	-15.29034	-15.09948	83.37	96.63	40.00	1.50383	0.10271
$N-(C, H_2)_1$	N	-0.46459	-0.46459	-0.46459	0		0.93084	0.83885	-16.21933		78.02	101.98	36.64	1.57525	0.17413
$N-(C, H_2)_2$	C ₁	-0.46459	-0.92918	0	0	-153.00946	0.91771	0.83885	-16.21933	-16.02866	78.02	101.98	36.64	1.57525	0.17413
$N-(C, H_2)_2$	N	-0.46459	-0.46459	-0.46459	0		0.93084	0.83885	-16.21933		78.02	101.98	36.64	1.57525	0.17413
$C-H (CH_3)$	C	-0.92918	0	0	0	-152.54487	0.91771	0.86339	-15.75493	-15.56407	77.49	102.51	41.48	1.23564	0.18708
$C-H (CH_2)$	C	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	-16.49325	68.47	111.53	35.84	1.35486	0.29033
$C-H (CH)$	C	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	61.10	118.90	31.37	1.42988	0.37326
$H_1C-C_1-H_2CH_1-$ (C-C (a))	C ₁	-0.92918	0	0	0	-152.54487	0.91771	0.86339	-15.75493	-15.56407	63.82	116.18	30.08	1.83879	0.38106
$H_1C-C_1-H_2CH_1-$ (C-C (b))	C ₁	-0.92918	-0.92918	0	0	-153.47406	0.91771	0.81549	-16.68412	-16.49325	56.41	123.59	26.06	1.50890	0.45117
$R-H_1C-C_1(H_2C-R')HC(H_2-$ (C-C (c))	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$R-H_1C-C_1(R'-H_2C)C_1(R''-H_2C)CH_2-$ (C-C (d))	C ₁	-0.92918	-0.72457	-0.72457	-0.72457	-154.71860	0.91771	0.75889	-17.92866	-17.73779	48.21	131.79	21.74	1.95734	0.50570
$isoC_1C_1(H_2C-R')HC(H_2-$ (C-C (e))	C ₁	-0.92918	-0.92918	-0.92918	0	-154.40324	0.91771	0.77247	-17.61330	-17.42244	48.30	131.70	21.90	1.97162	0.51388
$tertC_1(R'-H_2C)C_1(R''-H_2C)CH_2-$ (C-C (f))	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49298
$tertC_1(R'-H_2C-R')HC(H_2-$ (C-C (g))	C ₁	-0.72457	-0.92918	-0.92918	0	-154.19863	0.91771	0.78155	-17.40869	-17.21783	52.78	127.22	24.04	1.92443	0.47279
$tertC_1(R'-H_2C)C_1(R''-H_2C)CH_2-$ (C-C (h))	C ₁	-0.72457	-0.72457	-0.72457	-0.72457	-154.51399	0.91771	0.76765	-17.92866	-17.73779	50.04	129.96	22.66	1.94462	0.49298

Table 15.84. The energy parameters (eV) of functional groups of tertiary amines.

Parameters	C-N Group	C-H ₃ Group	C-H ₂ Group	C-H Group	C-C (a) Group	C-C (b) Group	C-C (c) Group	C-C (d) Group	C-C (e) Group	C-C (f) Group
n_1	1	3	2	1	1	1	1	1	1	1
n_2	0	2	1	0	0	0	0	0	0	0
n_3	0	0	0	0	0	0	0	0	0	0
C_1	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_2	1	1	1	1	1	1	1	1	1	1
C_3	1	1	1	1	1	1	1	1	1	1
C_4	0.91140	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771	0.91771
C_5	0	0	1	1	0	0	0	1	1	0
C_6	2	1	1	1	2	2	2	2	2	2
C_7	0	3	2	1	0	0	0	0	0	0
C_{12}	0.5	0.75	0.75	0.75	0.5	0.5	0.5	0.5	0.5	0.5
C_{13}	1	1	1	1	1	1	1	1	1	1
V_c (eV)	-31.67393	-107.32728	-70.41425	-35.12015	-28.79214	-28.79214	-29.10112	-28.79214	-29.10112	-29.10112
V_p (eV)	9.71067	38.92728	25.78002	12.87680	9.33352	9.33352	9.37273	9.33352	9.37273	9.37273
T (eV)	8.06719	32.53914	21.06675	10.48582	6.77464	6.77464	6.90500	6.77464	6.90500	6.90500
V_m (eV)	-4.03359	-16.26957	-10.53337	-5.24291	-3.38732	-3.38732	-3.45250	-3.38732	-3.45250	-3.45250
$E_{(1,10\text{ m})}$ (eV)	-14.63489	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.35946	-15.56407	-15.35946	-15.35946
$\Delta E_{(1,10\text{ m})}$ (eV)	-0.92918	0	0	0	0	0	0	0	0	0
$E_{(1,10\text{ m})}$ (eV)	-13.70571	-15.56407	-15.56407	-14.63489	-15.56407	-15.56407	-15.35946	-15.56407	-15.35946	-15.35946
$E_{(1,10\text{ m})}$ (eV)	-31.63537	-67.69451	-49.66493	-31.63533	-31.63537	-31.63537	-31.63535	-31.63537	-31.63535	-31.63535
$E_{(1,10\text{ m})}$ (eV)	-0.92918	0	0	0	-1.85836	-1.85836	-1.44915	-1.85836	-1.44915	-1.44915
$E_{(1,10\text{ m})}$ (eV)	-32.56455	-67.69450	-49.66493	-31.63537	-33.49373	-33.49373	-33.08452	-33.49373	-33.08452	-33.08452
ω (10 ¹¹ rad/s)	18.1298	24.9286	24.2751	24.1759	9.43699	9.43699	15.4846	9.43699	15.4846	9.55643
E_c (eV)	11.93333	16.40846	15.97831	15.91299	6.21159	6.21159	10.19220	6.21159	6.29021	6.29021
E_p (eV)	-0.22255	-0.25352	-0.25017	-0.24966	-0.16515	-0.16515	-0.20896	-0.16515	-0.16416	-0.16416
E_{res} (eV)	0.12944	0.35532	0.35532	0.35532	0.12312	0.17978	0.09944	0.12312	0.12312	0.12312
E_{res} (eV)	-0.15783	-0.22757	-0.14502	-0.07200	-0.10359	-0.07526	-0.15924	-0.10359	-0.10260	-0.10260
E_{res} (eV)	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803	0.14803
$E_{(1,10\text{ m})}$ (eV)	-32.72238	-67.97207	-49.80996	-31.70737	-33.59732	-33.59732	-33.24376	-33.59732	-33.18712	-33.18712
$E_{(1,10\text{ m})}$ (eV)	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489	-14.63489
$E_{(1,10\text{ m})}$ (eV)	0	-13.59844	-13.59844	-13.59844	0	0	0	0	0	0
$E_{(1,10\text{ m})}$ (eV)	3.45260	12.49186	7.83016	3.32601	4.32754	4.29921	3.97398	4.17951	3.62128	3.91734

Table 15.85. The total bond energies of tertiary amines calculated using the functional group composition and the energies of Table 15.84 compared to the experimental values [3].

Formula	Name	C-N Group	C-H ₃	C-H ₂	C-H	C-C (a)	C-C (b)	C-C (c)	C-C (d)	C-C (e)	C-C (f)	Calculated Total Bond Energy (eV)	Experimental Total Bond Energy (eV)	Relative Error
C ₃ H ₉ N	Trimethylamine	3	3	0	0	0	0	0	0	0	0	47.83338	47.761	-0.00152
C ₆ H ₁₅ N	Triethylamine	3	3	3	0	3	0	0	0	0	0	84.30648	84.316	0.00012
C ₉ H ₂₁ N	Tripropylamine	3	3	6	0	6	0	0	0	0	0	120.77958	120.864	0.00070

Names of Angle	Z_{C_1} Bond 1 (σ_1)	Z_{C_2} Bond 2 (σ_2)	Z_{C_3} Torsion axis (σ_3)	$E_{\text{calculated}}$ or E_{exp} Atom 1	Atom 1 Hybridization Designation (Table 15.3.A)	$E_{\text{calculated}}$ Atom 2	Atom 2 Hybridization Designation (Table 15.3.A)	C_1 Atom 1	C_2 Atom 2	C_3	ζ_1	ζ_2	E_T (eV)	θ_r ($^\circ$)	θ_l ($^\circ$)	θ_2 ($^\circ$)	Cal. θ ($^\circ$)	Exp. θ ($^\circ$)
$\angle C_1NC$	2.80224	2.80124	4.6043	-17.14871	I	-17.14871	6	0.79340	0.79340	I		0.79140	-1.85836				110.48	110.9 (trimethylamine)
Methylene $\angle HC_2H$	2.11106	2.11106	3.4252	-15.75493	7	H	H	0.86359	I	I	0.75	1.15796	0				103.44	107 (dimethylamine)
$\angle C_2C_1C_3$																		107 (propane)
$\angle C_2C_3H$														69.51			110.49	113.8 (butane)
Methyl $\angle HC_2H$	2.09711	2.09711	3.4252	-15.75493	7	H	H	0.86359	I	I	0.75	1.15796	0				110.49	111.4 (isobutane)
$\angle C_3C_1C_2$														70.56			109.44	
$\angle C_3C_2H$														70.56			109.44	
$\angle C_1C_2C_3$ in C_{2v}	2.91547	2.91547	4.7958	-16.68412 C_1	25	-16.68412 C_2	25	0.81549	0.81549	I	I	0.81549	-1.85836				110.67	110.8 (isobutane)
$\angle C_1C_3H$ in C_{2v}	2.91547	2.11323	4.1633	-15.55033 C_2	5	-14.82575 C_1	I	0.87495	0.91771	0.75	0.75	1.04887	0				110.76	
$\angle C_2C_3H$ in C_{2v}	2.91547	2.09711	4.1633	-15.55033 C_1	5	-14.82575 C_2	I	0.87495	0.91771	0.75	0.75	1.04887	0				111.27	111.4 (isobutane)
$\angle C_1C_2C_3$ test C_{2v}	2.90327	2.90327	4.7958	-15.55033 C_1	5	-14.82575 C_2	I	0.87495	0.91771	0.75	0.75	1.04887	-1.85836				111.27	111.4 (isobutane)
$\angle C_1C_2C_3$														72.50			107.50	

ALDEHYDES ($C_nH_{2n}O$, $n = 1, 2, 3, 4, 5 \dots \infty$)

The alkyl aldehydes, $C_nH_{2n}O$, each have a $HC=O$ moiety that comprises a $C=O$ functional group and a CH functional group. The single bond of carbon to the carbonyl carbon atom, $C-C(O)H$, is a functional group. In addition to the $C=O$ functional group, 5 formaldehyde comprises a CH_2 functional group. The alkyl portion of the alkyl aldehyde may comprise at least two terminal methyl groups (CH_3) at each end of the chain, and may comprise methylene (CH_2), and methylene (CH) functional groups as well as C bound by carbon-carbon single bonds. The methyl and methylene functional groups are equivalent to those of straight-chain alkanes. Six types of $C-C$ bonds can be identified. The n -alkane $C-C$ bond 10 is the same as that of straight-chain alkanes. In addition, the $C-C$ bonds within isopropyl ($(CH_3)_2CH$) and t -butyl ($(CH_3)_3C$) groups and the isopropyl to isopropyl, isopropyl to t -butyl, and t -butyl to t -butyl $C-C$ bonds comprise functional groups. The branched-chain-alkane groups in aldehydes are equivalent to those in branched-chain alkanes.

The CH functional group was solved in the Hydrogen Carbide (CH) section except that 15 E_{mag} is not subtracted since unpaired electrons are not created with fragmentation of the CH functional group of aldehydes. The CH_2 functional group of formaldehyde is solved in the Dihydrogen Carbide (CH_2) section except that the energy of each $C-H$ MO is matched to the initial energy of the $C2sp^3$ HO (Eq. (15.25)). The $C=O$ and $C-C(O)H$ groups are solved by hybridizing the $2s$ and $2p$ AOs of each C atom to form a single $2sp^3$ shell as an energy 20 minimum, and the sharing of electrons between the $C2sp^3$ HO and the O AO or between two $C2sp^3$ HOs, respectively, to form a MO permits each participating orbital to decrease in radius and energy. In alkyl aldehydes, the $C2sp^3$ HO has a hybridization factor of 0.91771 (Eq. (13.430)) with a corresponding energy of $E(C, 2sp^3) = -14.63489 \text{ eV}$ (Eq. (15.25)) and the O AO has an energy of $E(O) = -13.61806 \text{ eV}$. To meet the equipotential condition of the union 25 of the $C=O$ H_2 -type-ellipsoidal-MO with these orbitals, the hybridization factor c_2 of Eq. (15.52) for the $C=O$ -bond MO given by Eq. (15.114) is $c_2(C2sp^3HO \text{ to } O) = 0.85395$. The unpaired electrons created by bond breakage of the double $C=O$ bond requires that two times

the $O2p$ AO magnetic energy E_{mag} (Eq. (15.60)) be subtracted from the total energy to give $E_D(Grp)$ (eV) for $C=O$.

$E_T(atom-atom,msp^3.AO)$ of the $C=O$ -bond MO in Eq. (15.52) due to the charge donation from the C and O atoms to the MO is -2.69893 eV which is an energy minimum for the double bond between the pair of $C2sp^3$ HO electrons of the C atom and the pair of AO electrons of the O atom. It is given as a linear combination of the energy contributions corresponding to a double bond, -1.13379 eV (Eq. (14.247)), and a triple bond, -1.56513 eV (Eq. (14.342)). The triple bond contribution includes the $C2sp^3$ HO electron of the $C-H$ bond in addition to the pair involved directly in the double bond with O .

10 $E_T(atom-atom,msp^3.AO)$ of the $C-C(O)H$ group is equivalent to that of an alkane, -1.85836 eV, where both energy contributions are given by Eq. (14.513). It is based on the energy match between the $C2sp^3$ HOs of the aldehyde. In order to match energy between the groups bonded to the $C=O$, electron-density is shared. Due to the interaction in the transition state between the groups based on the sharing, $C_{1u} = 2C_1$ rather than $C_{1u} = C_1$ in Eq. (15.52) for

15 the $C-C(O)H$ bond.

The symbols of the functional groups of alkyl aldehydes are given in Table 15.87. The geometrical (Eqs. (15.1-15.5) and (15.42)), intercept (Eqs. (15.71-15.78)), and energy (Eqs. (15.6-15.11) and (15.17-15.56)) parameters of alkyl aldehydes are given in Tables 15.88, 15.89, and 15.90, respectively. The total energy of each alkyl aldehyde given in Table 15.91 was

20 calculated as the sum over the integer multiple of each $E_D(Grp)$ of Table 15.90 corresponding to functional-group composition of the molecule. The bond angle parameters of alkyl aldehydes determined using Eqs. (15.79-15.108) are given in Table 15.92.